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Origin of the gravitational constant and particle masses in a scale-invariant scalar-tensor theory

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A viable scalar-tensor theory of gravitation is formulated by imposing global scale invariance to the matter part. Nonvanishing masses m of elementary particles as well as the gravitational constant G emerge through the cosmological background value of the scalar field. The scalar field maintains a dynamical degree of freedom in exchange for conformal invariance enjoyed otherwise by the gravity part. The temporal developments of G, m, and the scale factor of the Universe are determined simultaneously by solving coupled differential equations. In the simplest single-scalar model the result is not a variable-G theory in the usual sense. Departures from the standard theory occur through the time-dependent cosmological term. Of particular interest among the solutions are the asymptotically standard solutions.

I. INTRODUCTION

Scalar-tensor theory was first conceived in an attempt to make a theory of gravitation in comformity with Mach's principle.[1,2] It offers a natural way of extending the standard theory of Einstein.[3–8] Particularly noteworthy is the fact that the Lagrangian contains no dimensional constants in the unit system with $c=\hbar=1$, which we assume to be true constants throughout this paper. The usual gravitational constant G having a dimension of length squared emerges through a cosmological background value (BGV) of the scalar field. It is one of the natural consequences of the theory that G may vary with the cosmic time t.

The absence of a dimensional constant implies an invariance under global scale transformation[9] and eliminates one of the major disparities between the standard gravity theory and the theory of elementary particles; it is increasingly evident that the coupling constants of all the fundamental interactions among elementary particles are dimensionless.

In many of the scalar-tensor theories[2,3,6] the global scale invariance has been limited to the gravity part; the invariance is broken explicitly in the matter part by introducing fixed-mass terms. As is well known, however, in the local field theory of elementary particles, the invariance can be maintained if the masses are generated spontaneously by means of nonvanishing vacuum expectation values (VEV's) of certain scalar fields.[10] Obviously the simplest scheme one can conceive is that there is only one such fundamental scalar field which is identified with the scalar gravitational field.

It is also encouraging to observe[4] that the troublesome scalar long-range force then no longer occurs in the weak-field limit, thus keeping the theory viable without jeopardizing the successful experimental tests of the standard theory. This approach allows one to understand all the dimensional "constants" in nature, i.e., G and particle masses, in terms of a common origin, the cosmological BGV of the scalar field. Because of the expected variability of the BGV, G and particle masses may also change as the Universe expands.

Now in our simplest theory with only one scalar field (called the "single-scalar model" in Ref. 4), G and a particle mass m change precisely in such a way that the dimensionless product Gm^2 remains unchanged. As a consequence, in the "microscopic unit frame" in which m stays constant, G also stays unchanged, unlike what is expected in Dirac's large numbers hypothesis (LNH).[11] This inherent discrepancy with LNH may be avoided by assuming a more complex theory by introducing two scalar gravitational fields, for example (corresponding to the "two-scalar model" in Ref. 4), as suggested by Bekenstein.[12] We still believe that our simplest sensible alternative to the standard theory deserves detailed studies, partly because the observational situations on the variable G are still controversial.[13]

Theories which embody LNH have also been formulated in terms of the scalar-tensor theory with a single scalar field, but by choosing a special value of the coupling parameter of the scalar field such that the theory is conformally invariant.[14–17] The scalar field is then deprived of dynamical degree of freedom, resulting in no physical long-range force which might affect the tests of general relativity. One is now free to choose the scalar field and hence $G(t)$ in accordance with LNH, if one wishes.

We insist, however, that any physical quantity which develops with time must be of dynamical nature.[18] From this point of view we avoid the conformal coupling. As we have pointed out, the theory with a dynamical scalar field can be made viable if the matter part is globally scale invariant. (The matter part is allowed to be conformally invariant as well, like in the model given in Sec. II, whereas the gravity part is not.)

As was also emphasized above, not only m but also G are time independent in the microscopic unit frame. It may appear that nothing differs from the standard theory. We do expect some differences, however, because the scalar field is still present, acting, roughly speaking, like a time-dependent cosmological term. It is unlikely that deviations from the standard theory can be detected by analyzing planetary motions or similar phenomena at the present epoch. It is still highly probable that modifications should be significant in phenomena which take place on a cosmological time scale.

In this paper we confine our considerations to the basic structure of the theory, leaving more details to future publications, including practical applications. In Sec. II we present the formulations. We begin with writing the Lagrangian in the "primary unit frame" in which the sector of the scalar field takes the simplest form. The relation to the microscopic unit frame is also exhibited. In Sec. III we derive a dynamical equation which determines the temporal behavior of the BGV of the scalar field together with the scale factor of the Universe. Some of the solutions of this nonlinear equation are given in Sec. IV. Of particular interest are the "asymptotically standard solutions" which tend to the standard solutions as $t\to\infty$. This type of solution suggests an interesting scenario that the standard theory provides an accurate description of the Universe in relatively recent epochs, while deviations might have been significant in earlier epochs. Section V contains concluding remarks.

II. FORMULATION

We consider a combined system of a tensor gravitational field, a scalar gravitational field ϕ, and the matter, which we choose, for the sake of illustration, to be an interacting system of a massless electron and a photon. It is straightforward to extend the model by including other types of gauge theories with some scalar Higgs fields.[19] The basic Lagrangian is given by

$$\mathscr{L}=(-g)^{1/2}(\tfrac{1}{2}f^{-2}\phi^2R+L)\,, \tag{2.1a}$$

where

$$L=-\tfrac{1}{2}\epsilon g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+L_M+L_I\,. \tag{2.1b}$$

The dimensionless constant f^2 is reasonably assumed to be of order unity,[20] while $\epsilon=\pm1$ according to whether ϕ is a normal field of positive metric or a ghost field of negative metric. The matter Lagrangian L_M is given by

$$L_M=-\tfrac{1}{2}\bar{\psi}(\mathscr{D}-\overleftarrow{\mathscr{D}})\psi-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \tag{2.2a}$$

where

$$\mathscr{D}_\mu\psi=(\partial_\mu+\tfrac{1}{4}A^{\alpha\beta}{}_\mu\sigma_{\alpha\beta}+ieA_\mu)\psi\,, \tag{2.2b}$$

$$\bar{\psi}\overleftarrow{\mathscr{D}}_\mu=\bar{\psi}(\overleftarrow{\partial}_\mu-\tfrac{1}{4}A^{\alpha\beta}{}_\mu\sigma_{\alpha\beta}-ieA_\mu)\,, \tag{2.2c}$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\,, \tag{2.2d}$$

with $A^{\alpha\beta}{}_\mu$ being the Lorentz connection expressed in terms of the Ricci rotation coefficients (α,β are for local Lorentz transformations). The interaction Lagrangian [apart from the gauge couplings included in (2.2b) and (2.2c)] is

$$L_I=L_1+L_2\,, \tag{2.3a}$$

where[21]

$$L_1=-c_0\phi^4\,, \tag{2.3b}$$

$$L_2=-g\bar{\psi}\psi\phi\,. \tag{2.3c}$$

The Lagrangian (2.1) is characterized by the absence of dimensional constants; all the constants f, e, c_0, and g are dimensionless. Consequently the theory possesses global scale invariance. It should be understood that these constants as well as c and $\hbar$ are true constants.

If ϕ has a BGV $v(t)$ which varies sufficiently slowly as a function of the cosmic time t, the factor $\frac{1}{2}f^{-2}v^2(t)$ in the first term of (2.1a) gives a time-dependent gravitational constant,

$$G(t)=(f^2/8\pi)v^{-2}(t)\,. \tag{2.4}$$

Likewise substituting $v(t)$ for ϕ in (2.3c) yields the time-dependent electron mass $m(t)$,

$$m(t)=gv(t)\,. \tag{2.5}$$

It immediately follows that they satisfy the relation[22]

$$G(t)m^2(t)=f^2g^2/8\pi=\text{const}\,. \tag{2.6}$$

The constancy of this product exemplifies a general feature of the present theory that dimensionless numbers are true constants.

The field equations are derived as follows:

$$f^{-2}\phi^2 G_{\mu\nu}=T_{\mu\nu}-f^{-2}(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)\phi^2 \ , \tag{2.7a}$$

$$f^{-2}\phi R+\epsilon\Box\phi+\partial L_I/\partial\phi=0 \ , \tag{2.7b}$$

$$-\mathscr{D}\psi+\partial L_I/\partial\bar{\psi}=0 \ , \tag{2.7c}$$

$$\nabla_\nu F^{\nu\mu}-ie\bar{\psi}\gamma^\mu\psi=0 \ . \tag{2.7d}$$

Here $T_{\mu\nu}$ is the symmetric energy-momentum tensor including ϕ and the matter, as given by

$$T_{\mu\nu}=\epsilon\partial_\mu\phi\partial_\nu\phi +\tfrac{1}{4}\bar{\psi}(\gamma_\mu\mathscr{D}_\nu+\gamma_\nu\mathscr{D}_\mu-\overleftarrow{\mathscr{D}}^\mu\gamma_\nu-\overleftarrow{\mathscr{D}}^\nu\gamma_\mu)\psi +g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma}+g_{\mu\nu}L \ . \tag{2.8}$$

By taking a trace of (2.7a) and using (2.7c) we transform (2.7b) into

$$Z^{-1}\Box\phi^2=2\left[4-\phi\frac{\partial}{\partial\phi}-\tfrac{3}{2}\psi\frac{\partial}{\partial\psi}-\tfrac{3}{2}\bar{\psi}\frac{\partial}{\partial\bar{\psi}}\right]L_I \ , \tag{2.9a}$$

where

$$Z^{-1}=\epsilon+6f^{-2} \ . \tag{2.9b}$$

Since L_I contains no dimensional constants, the right-hand side of (2.9a) vanishes. We then obtain

$$Z^{-1}\Box\phi^2=0 \ . \tag{2.10}$$

For the conformal coupling $\epsilon f^2=-6$, i.e., $Z^{-1}=0$ from (2.9b), Eq. (2.10) ceases to be a dynamical equation of ϕ^2. We avoid this choice because we want a theory by which ϕ is determined dynamically. But this requires us to accept an important (though sometimes taken for granted) assumption that there is a preferred unit frame among the other conformally transformed unit frames. Only in this special frame, hereafter called the primary unit frame, the ϕ sector of the Lagrangian takes a simple form as in (2.1).

Once we choose $Z^{-1}\neq 0$, Eq. (2.10) implies that ϕ^2 has no direct coupling to the matter. This decoupling frees f^2 from being constrained by the well-established experimental tests of general relativity, thus leaving the theory viable. If we broke scale invariance explicitly by introducing a fixed-mass term $-m_0\bar{\psi}\psi$ with a constant m_0, the right-hand side of (2.9a) would be $6m_0\bar{\psi}\psi$, yielding a direct matter source.

Suppose we apply a conformal transformation

$$g_{\mu\nu}\to g_{*\mu\nu}=\Omega^{-2}g_{\mu\nu} \ , \tag{2.11a}$$

$$\phi\to\phi_*=\Omega\phi \ , \tag{2.11b}$$

$$\psi\to\psi_*=\Omega^{3/2}\psi, \ \ A_\mu\to A_{*\mu}=A_\mu \ . \tag{2.11c}$$

Choose a special Ω given by

$$\Omega(x)=v_*\phi^{-1}(x) \ , \tag{2.12a}$$

so that

$$\phi_*=v_* \tag{2.12b}$$

is a constant. The interaction term L_2 becomes

$$(-g)^{1/2}L_2=-(-g_*)^{1/2}m_*\bar{\psi}_*\psi_* \ , \tag{2.13a}$$

where the electron mass is given by

$$m_*=gv_* \ , \tag{2.13b}$$

which is a true constant. For this reason the new unit frame may be called the microscopic unit frame.

We obtain

$$\mathscr{L}=(-g_*)^{1/2}\left[\frac{1}{16\pi G_*}R_*+L'\right] \ , \tag{2.14a}$$

where

$$G_*=(f^2/8\pi)v_*^{-2} \tag{2.14b}$$

is a truly constant gravitational constant and

$$L'=-\tfrac{1}{2}\epsilon_* g_*^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi+L_{*M}+L_{*1} -m_*\bar{\psi}_*\psi_* \ , \tag{2.14c}$$

with

$$\Phi=|Z|^{-1/2}v_*\ln\phi, \ \ \epsilon_*=\mathrm{sign}(Z^{-1}) \ , \tag{2.14d}$$

and L_{*M} being (2.2a) with starred fields, and $L_{*1}=-c_0v_*^4$. Notice that $e_*=e$ due to the well-known conformal invariance of the massless electrodynamics.[23]

The field Φ couples only to the tensor gravitational field and represents a deviation from the standard theory in spite of the constancy of G_* and m_* in this unit frame.

III. COSMOLOGICAL EQUATIONS

We assume that ϕ^2 in the primary unit frame is separated into BGV $u(t)$ and a fluctuating part

$\sigma(x)$:

$$\phi^2(x)=u(t)+\sigma(x)\ , \tag{3.1}$$

where $u=v^2$, with $v(t)$ the BGV of ϕ as introduced before. In accordance with (2.10) with $Z^{-1}\neq 0$, we may reasonably assume

$$\Box u(t)=0 \tag{3.2a}$$

and

$$\Box\sigma(x)=0\ . \tag{3.2b}$$

Equation (3.2b) shows that the locally fluctuating field $\sigma(x)$ is a free field playing no significant role in the limit of flat spacetime, a crucial consequence of the scale invariance assumed for the matter part.

It is convenient to rewrite (2.7a) as

$$G_{\mu\nu}=f^2\mathcal{T}_{\mu\nu}\ , \tag{3.3a}$$

where

$$\mathcal{T}_{\mu\nu}=\phi^{-2}[T_{\mu\nu}-f^{-2}(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)\phi^2] \tag{3.3b}$$

is the source of the tensor gravitational field. We separate this also into the BGV $\mathcal{T}^{(0)}_{\mu\nu}$ and the rest $\mathcal{T}^{(1)}_{\mu\nu}$, where

$$\mathcal{T}^{(0)}_{\mu\nu}=u^{-1}[\epsilon\partial_\mu v\partial_\nu v +g_{\mu\nu}(-\tfrac{1}{2}\epsilon g^{\rho\sigma}\partial_\rho v\partial_\sigma v-c_0v^4) -f^{-2}(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)v^2]\ . \tag{3.4}$$

It seems reasonable to interpret the spacetime-dependent part $\mathcal{T}^{(1)}_{\mu\nu}$ as the physical energy-momentum tensor of the combined system of the matter and the field $\sigma(x)$.

We now assume that the Universe is described by a spatially flat Robertson-Walker metric,

$$g_{00}=-1,\ \ g_{ij}=a^2(t)\delta_{ij}\ \ (i,j\neq 0)\ . \tag{3.5}$$

Equation (3.2a) is then put into the form

$$\frac{d}{dt}(a^3\dot{u})=0\ . \tag{3.6}$$

For $\mathcal{T}^{(1)}_{\mu\nu}$ we assume the ideal fluid as usual,

$$\mathcal{T}^{(1)}_{\mu\nu}=u^{-1}[(\rho+p)u_\mu u_\nu+pg_{\mu\nu}]\ , \tag{3.7}$$

with the equation of state

$$p=z\rho\ .$$

The 00 component of (3.3a) reads as

$$3H^2=f^2u^{-1}(\tfrac{1}{2}\epsilon\dot{v}^2+c_0u^2-3f^{-2}H\dot{u}+\rho)\ , \tag{3.8a}$$

where

$$H=\dot{a}/a\ . \tag{3.8b}$$

Other components give no new results.

We must also impose the condition

$$\nabla_\mu\mathcal{T}^{\mu\nu}=0\ . \tag{3.9}$$

The only independent component is for $\nu=0$. We obtain

$$\dot{\mathcal{T}}_{00}+3Hu^{-1}[\epsilon\dot{v}^2+f^{-2}\ddot{u} -f^{-2}H\dot{u}+(1+z)\rho]=0\ , \tag{3.10}$$

where $\mathcal{T}_{00}$ is taken from the right-hand side of (3.8a) divided by f^2. By differentiating (3.8a) with respect to t and combining the result with (3.10) we obtain [by using (3.8a) again to eliminate ρ]

$$\ddot{u}+\tfrac{1}{2}(1-z)f^2\epsilon\dot{v}^2+3(1+z)uH^2+2\dot{H}u +(2+3z)H\dot{u}=(1+z)f^2c_0u^2\ . \tag{3.11}$$

Equation (3.6) is satisfied trivially if we choose

$$\dot{u}=0\ .$$

Equation (3.11) then simplifies to

$$3(1+z)H^2+2\dot{H}=(1+z)f^2c_0u^2\ , \tag{3.12}$$

which is a standard equation for $H(t)$. For $c_0=0$ we recover the usual Friedmann solution

$$a(t)=a_0t^{\gamma_0}\ , \tag{3.13a}$$

with

$$\gamma_0=\tfrac{2}{3}(1+z)^{-1}\ . \tag{3.13b}$$

We next assume $\dot{u}\neq 0$ and solve (3.6) to obtain

$$a(t)=K\dot{u}^{-1/3}(t)\ , \tag{3.14a}$$

which together with (3.8b) is substituted into (3.11):

$$\dddot{u}u\dot{u}^{-1}+B_1\ddot{u}^2u\dot{u}^{-2}+B_2\ddot{u}+B_3\dot{u}^2u^{-1}+B_4u^2=0\ , \tag{3.14b}$$

with

$$B_1=-\tfrac{1}{2}(3+z),\ \ B_2=-\tfrac{1}{2}(1-3z)\ , \quad B_3=-\tfrac{3}{16}\epsilon f^2(1-z),\ \ B_4=\tfrac{3}{2}c_0f^2(1+z)\ . \tag{3.14c}$$

This is our fundamental equation that determines $u(t)$, and hence $a(t)$ through (3.14a). The energy density $\rho(t)$ is then given by (3.8a).

Before solving (3.14b) we briefly discuss the conformal transformation to the microscopic unit frame. By substituting $v(t)$ into ϕ in (2.12a) we obtain

$$\Omega(t)=v_0/v(t)=m_0/m(t)\ , \tag{3.15}$$

where we have chosen $\Omega(t_0)=1$ for the present epoch t_0 by adjusting $v_0=v(t_0)$. In the second equality of (3.15) we have used (2.5) with $m_0=m(t_0)$. From (2.11a) we find

$$ds_*=\Omega^{-1}ds=[m(t)/m_0]ds\ . \tag{3.16}$$

This implies that the time and length in the microscopic unit frame are measured in units of $m^{-1}(t)$, in agreement with the physical situation that the time scale of atomic clocks, for example, is provided by the atomic levels which are determined by the Rydberg constant $(me^4)^{-1}$.

The metric $g_{00}=-1$ in (3.5) is transformed to $g_{*00}=-\Omega^{-2}$. This can be brought back again to the form of (3.5) by introducing the time coordinate t_* in the microscopic unit frame according to $dt_*=\Omega^{-1}(t)dt$. The new scale factor a_* is also defined by $a_*(t_*)=\Omega^{-1}(t)a(t)$. Now the first term in (2.14c) multiplied by G_* [with ϕ in (2.14d) replaced by $v(t)$] may be interpreted as a time-dependent cosmological term

$$\Lambda(t)=-4\pi\,|\,Z\,|^{-1}G_*v_*{}^2(\dot{v}/v)^2\ , \tag{3.17}$$

giving a departure from the standard theory. By using (2.14b) and choosing $\dot{v}/v\sim t^{-1}$, we obtain an approximate expression

$$\Lambda(t)\sim t^{-2}\ , \tag{3.18}$$

which for $t\sim 10^{10}$ yr turns out to be close to the observational upper limit 10^{-57} cm^{-2}. The value might be even smaller if $v(t)$ is nearly constant as suggested by the asymptotically standard solutions to be discussed in the next section.

One can derive an equation for $\Phi(t_*)$ corresponding to (3.14b) ($c_0=z=0$, $\dot{\Phi}=d\Phi/dt_*$):

$$\dddot{\Phi}\dot{\Phi}^{-1}-\tfrac{3}{2}\ddot{\Phi}^2\dot{\Phi}^{-2}-6\pi\epsilon_*G_*\dot{\Phi}^2=0\ , \tag{3.19}$$

which is in fact verified to be equivalent to (3.14b).

IV. SIMPLE SOLUTIONS

We first assume $c_0=0$ and solve (3.14b):

$$\dddot{u}\,u\dot{u}^{-1}+B_1\ddot{u}^2u\dot{u}^{-2}+B_2\ddot{u}+B_3\dot{u}^2u^{-1}=0\ . \tag{4.1}$$

One immediately finds a power-type solution

$$u(t)=At^{\beta}\ , \tag{4.2a}$$

where A is an arbitrary constant, while β is subject to the condition

$$(\beta-1)(\beta-2)+B_1(\beta-1)^2+B_2\beta(\beta-1)+B_3\beta^2=0\ . \tag{4.2b}$$

Substituting (4.2a) into (3.14b) we also find

$$a(t)=a_0t^{\gamma} \tag{4.3a}$$

with

$$\gamma=\tfrac{1}{3}(1-\beta)\ . \tag{4.3b}$$

The obvious restriction $\gamma>0$ gives

$$\beta<1\ . \tag{4.4}$$

It should be noticed that in the limit of a constant u, i.e., $\beta\to 0$, Eq. (4.3b) gives $\gamma=\frac{1}{3}$ which differs from (3.13b). This is not surprising because we have started from (3.6) which would not exist if $\dot{u}=0$ was chosen at the outset.

We next consider the case $c_0\neq 0$. For the power-type solution t^{β}, the first four terms of (3.14b) give $t^{\beta-2}$, while the last term yields $t^{2\beta}$. These behaviors agree with each other only for $\beta=-2$. Substituting $u\sim t^{-2}$ into (3.14b) gives

$$12+9B_1+6B_2+4B_3+B_4=0\ . \tag{4.5}$$

In other words, the power-type solution $\sim t^{-2}$ with $c_0\neq 0$ is allowed only if the parameters ϵf^2 and c_0 satisfy (4.5) exactly. Since such a precise tuning of the parameters is rather unnatural, we do not study the choice $c_0\neq 0$ any further.

The solution (4.2a) has only two adjustable parameters, the overall scale and the origin of t, indicating the presence of more general solutions. We seek them by making an ansatz

$$u(t)=t^{\beta}F(t)\ , \tag{4.6a}$$

with the condition

$$F(0)=1\ . \tag{4.6b}$$

Substituting (4.6a) into (4.1) yields

$$(\beta-1)(\beta-2)F'''FF'^{-1}+B_1(\beta-1)^2F''^2FF'^{-2}+B_2\beta(\beta-1)F''+B_3\beta^2F'^2F^{-1}=0\ , \tag{4.7a}$$

where

$$F'=F+\frac{t}{\beta}\dot{F}\ , \tag{4.7b}$$

$$F''=F+2\frac{t}{\beta-1}\dot{F}+\frac{t^2}{\beta(\beta-1)}\ddot{F}\ , \tag{4.7c}$$

$$F'''=F+3\frac{t}{\beta-2}\dot{F}+3\frac{t^2}{(\beta-1)(\beta-2)}\ddot{F}+\frac{t^3}{\beta(\beta-1)(\beta-2)}\dddot{F} \ . \qquad (4.7d)$$

We first assume the form

$$F(t)=1+\lambda t^{\alpha} \qquad (4.8a)$$

with the condition

$$\alpha>0 \ , \qquad (4.8b)$$

and $|\lambda t^{\alpha}| \ll 1$. We substitute (4.8a) into (4.7) and expand the result into a power series of λt^{α}. The sum of the zeroth-order terms vanishes for β that satisfies (4.2b). The first-order terms give

$$\lambda t^{\alpha}(\alpha/\beta)[\alpha^2+(E+1)\alpha+E] \qquad (4.9)$$

with $E=(2B_1+B_2+3)\beta-2B_1-4$. Equation (4.7a) is satisfied for any small λt^{α} if the quantity in square brackets in (4.9) vanishes. We obtain two solutions for α; the one $\alpha=-1$ is ruled out in view of (4.8b), while the other is

$$\alpha=-E=(1-z)(\tfrac{1}{2}\beta+1) \ . \qquad (4.10)$$

The condition (4.8b) is met if

$$\beta>-2 \ . \qquad (4.11)$$

The third parameter λ can be absorbed into the scale of t apart from the sign. For this reason we write the approximate solutions as

$$F_{\pm}(t)=1\pm t^{\alpha} \qquad (4.12)$$

with $t^{\alpha} \ll 1$. For a given β subject to (4.2b) we now choose t_0 such that $t_0{}^{\alpha} \ll 1$ and solve (4.7) numerically for the "initial values"

$$F_{\pm 0}=1\pm t_0{}^{\alpha} \ , \qquad (4.13a)$$

$$F'_{\pm 0}=1\pm\left[1+\frac{\alpha}{\beta}\right]t_0{}^{\alpha} \ , \qquad (4.13b)$$

$$F''_{\pm 0}=1\pm\left[1+\frac{2\alpha}{\beta-1}+\frac{\alpha(\alpha-1)}{\beta(\beta-1)}\right]t_0{}^{\alpha} \ . \qquad (4.13c)$$

The solutions may be called F_+ and F_- solutions, respectively, according to the signs in (4.13).

The solutions obtained so far for $z=0$ (dust) may be classified into two major categories.

(I) F_- solutions with $-2<\beta<0$ and F_+ solutions with $0<\beta<1$*:* The function $F(t)$ starting from $F(0)=1$ either vanishes (F_-) or diverges (F_+) at $t=t_1$, which is finite in F_+ solutions and F_- solutions with $-1<\beta<0$, or infinite in F_- solutions with $-2<\beta\leq 1$, as illustrated in Fig. 1(a). The corresponding time t_{*1} in the microscopic unit frame is always finite. We also find that the scale factor $a_*(t_*)$ in the microscopic unit frame collapses at this time with an infinite derivative, like the standard Friedmann solution with $k=+1$.[24] An example is shown in Fig. 1(b).[25] It is easy to show by using the power-type solution that $a_*(t_*)$ behaves like $t_*{}^{1/3}$ as $t_*\to 0$ for any β. This implies that the deceleration parameter q_* defined with

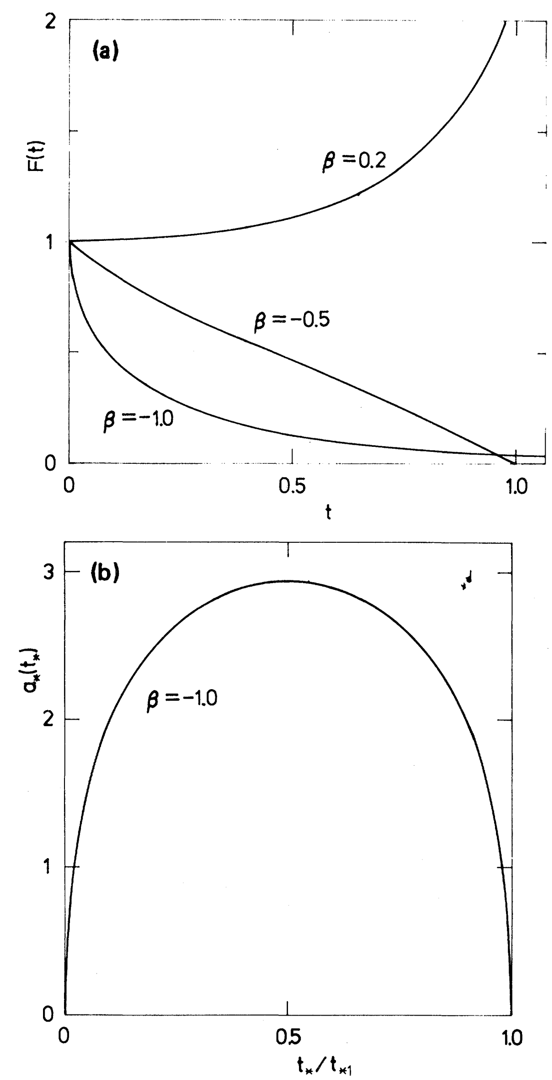

FIG. 1. Typical solutions of the category (I). (a) $F(t)$ for F_+ with $\beta=0.2$, F_- with $\beta=-0.5$ and $\beta=-1.0$. The scale of t is arbitrary, but chosen such that $t_1=1.0$ for $\beta=0.2$ and $\beta=-0.5$. (b) $a_*(t_*)$ in arbitrary scale for the solution F_- with $\beta=-1.0$. Curves for other β's look nearly the same.

respect to t_* approaches the value 2 (from above) as $t_* \to 0$. From the typical curve in Fig. 1(b) one finds that q_* should be very large in a large portion of the interval $0 < t_* < t_{*1}$. The observation seems to indicate that $q_* \lesssim 2$. This would be compatible with the theory only if we are now at the epoch t_{*0} which is much smaller than t_{*1}.

(II) F_- solutions with $0<\beta<1$ and F_+ solutions with $-2<\beta<0$: In all the solutions in this category the function $u(t)$ approaches a constant for $t \to \infty$. See Fig. 2(a) for some examples. We also find that $a(t) \sim \dot{u}^{-1/3}(t)$ tends to the standard solution $a(t) \sim t^{2/3}$ as given by (3.13). These features can be seen most clearly in Figs. 2(b) and 2(c), where

$$\delta(t) = \frac{1}{H}\frac{\dot{u}}{u} = -3\frac{\dot{u}^2}{\ddot{u}u} = -\frac{3\beta}{\beta-1}\frac{F'^2}{F''F} \tag{4.14a}$$

and

$$q(t) = -\frac{\ddot{a}a}{\dot{a}^2}$$

$$= -3B_1 - 4 + B_2\delta - \tfrac{1}{3}B_3\delta^2 \tag{4.14b}$$

are plotted. Notice that $q \to \frac{1}{2}$ as $t \to \infty$, the standard value for the Einstein—de Sitter solution ($z=0$).

The solutions of the second category may be called "asymptotically standard solutions." We find that the asymptotic standardization occurs whenever $u(t)$ behaves as a power function of t for $t \to 0$ and develops smoothly toward $t \to \infty$, and also toward $t_* \to \infty$. This is by no means trivial since, as we showed before, the standard solution (3.13), which is derived by choosing $\dot{u}=0$ at the outset, can be entirely different from the solutions of (3.14) (even with the limit $\beta \to 0$ taken afterwards).

It should be remembered that we have left the scale of t completely arbitrary, by choosing $\lambda=1$ in (4.8a). Only if we are able to compare our theoretical calculations with phenomena that take place on a cosmological time scale can we determine where we are on the plots of Figs. 1 and 2, and with what value of β, and hence what values of ϵf^2 through (4.2b) and (3.14c). We must also decide which time, t or t_*, should be used in analyzing observational results. It might also be necessary to study how the solutions differ if we assume the spatially closed or open models of the Universe. We should be content here with rather qualitative discussions on the solutions we have obtained by a numerical method. The results of our approach based on a more analytic method including a rigorous proof of the asymptotic standardization will be reported elsewhere, though some of the results have been already used above in discussing general properties of the numerical solutions.

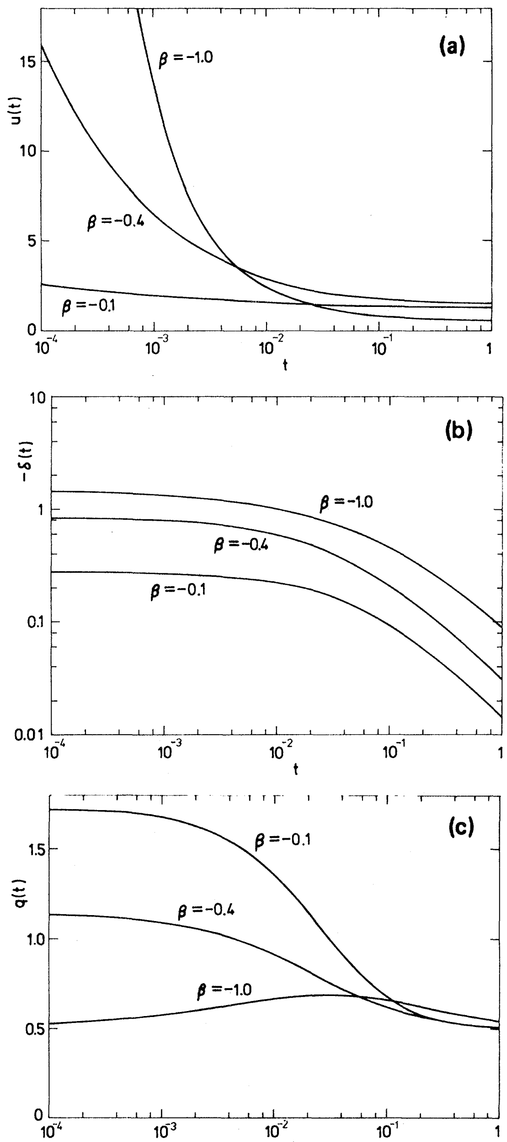

FIG. 2. Typical solutions of the category (II). (a) $u(t)$, (b) $-\delta(t)$, and (c) $q(t)$, for F_+ solutions with $\beta=-0.1$, -0.4, and -1.0, respectively. The scale of t indicated is for $\beta=-0.1$. Those for $\beta=-0.4$ and $\beta=-1.0$ should be multiplied by 10 and 100, respectively.

V. CONCLUDING REMARKS

Our scalar-tensor theory incorporating the microscopic matter system in the form of local field theory has been chosen to be invariant under global scale transformation but not under conformal transformation. This is almost a unique choice if one tries to maintain the dynamical degree of freedom of the scalar field, and also to survive the current experimental tests of gravity within a reasonable range of the value of the coupling parameter. The scalar field equation together with the Einstein equation results in the coupled differential equations that determine the temporal development of the BGV of the scalar field [and hence $G(t)$ and $m(t)$] in conjunction with the scale factor of the Universe, essentially in the same spirit as in Ref. 2. One then arrives at a unified view that the BGV of a single scalar field provides the ultimate origin of all the dimensional quantities in nature.

Also stated tacitly is that dimensional parameters which are not put in the Lagrangian have been brought in through the initial (or boundary) conditions in solving the differential equations, a characteristic shared by the mechanism of spontaneous symmetry breaking. In fact, each solution of our third-order differential equation contains three integration constants, the origin of t, the overall scale of $u(t)$ and the scale of t. Although the first one is trivial, the last two do have dimensions. In this respect we have something in common with Terazawa's view that the mass scales in nature have their origin in the nonvanishing Hubble's parameter and the cosmological average of the energy-momentum tensor.[17]

The theory, which certainly differs from the standard theory, turns out not to be a variable-G or variable-m theory in the sense that G stays constant in the unit frame in which m is constant. In other words, the Einstein gauge and the atomic gauge[15] coincide with each other. If unambiguous evidence is accumulated to support a changing G (in the atomic gauge), one must go beyond the simplest model.[26,27]

We have not attempted to exhaust all the possible solutions of our nonlinear equations (3.14b) or (4.1). Among the solutions discovered so far, however, the asymptotically standard solutions are of particular interest. A conjecture might be proposed that the standard theory applies to the present (or relatively recent) Universe just because we are already in the asymptotic region; in much earlier epochs the Universe could have been different considerably from what one infers on the basis of the standard theory. According to our theory, deviations would have occurred as a larger cosmological term as one goes back to the earlier epochs. Notice that the time-dependent cosmological term (3.17) is given essentially by $H^2\delta^2$, as one finds from (4.14a). Deriving realistic and quantitative results and applying them to cosmological and other phenomena will be left for future investigations.

Other possible solutions which might help our understanding of cosmology are those giving the behavior $a(t)\sim t^{\gamma}$ with $\gamma \geq 1$ (hence $q \leq 0$) for $t\rightarrow 0$, yet reproducing $q>0$ at the present epoch. We then have $\int_0^t a^{-1}(t')dt'=\infty$, thus offering a resolution of the horizon problem.[28] If we insist on a solution of the form $u(t)=t^{\beta}F(t)$ with $F(0)=1$, we must accept $\beta<-2$ as required by (4.3b). But this is in contradiction with (4.11). The horizon problem might be outside our present approach. Nevertheless, the outstanding importance of the problem seems to justify searching for different kinds of solutions. Solutions with $c_0\neq 0$ may also be worth examining in more detail.

We abandoned the highly attractive principle of conformal invariance. As a consequence the primary unit frame assumes a preferred status among all the other unit frames obtained by applying conformal transformations. Only in this preferred unit frame the ϕ sector of the Lagrangian takes a simple form, as in (2.1). It is certainly one of the fundamental questions if we are paying too high a price for insisting on having dynamical equations for $G(t)$ and $m(t)$.

Basically the same theoretical development can also be applied to the two-scalar model in which one of the scalar gravitational fields is expected to give an additional finite-range term to the Newtonian potential.[4,29] This second simplest model might deserve further studies in connection with the horizon problem as well as allowing variability of dimensionless numbers, as suggested by LNH.

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[8]G. V. Bicknell and A. H. Klotz, J. Phys. A 9, 1637 (1976); 9, 1647 (1976).
[9]A (global) scale transformation, $x^\mu \to \lambda x^\mu$, sometimes called dilatation, is a coordinate transformation with ds^2 unchanged, whereas a conformal transformation, also referred to as a Weyl rescaling, is defined by $ds^2 \to \Omega^{-2}(x) ds^2$, being not a coordinate transformation. The invariance under dilatation always follows if no dimensional constants are present in the Lagrangian (the fields must be transformed appropriately). A (global) special conformal transformation, also part of coordinate transformations, is a generalization of dilatation; the invariance holds only for limited classes of Lagrangians. It is known that any theory invariant under a Weyl rescaling is also invariant under special conformal and scale transformations. [See B. Zumino, in *Lectures on Elementary Particles and Quantum Field Theory,* 1970 Brandeis University Summer Institute in Theoretical Physics, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, Mass., 1970), Vol. 2.] Obviously conformal invariance is a stronger requirement than global scale invariance. No theory can be conformally invariant if dilatation symmetry is violated.
[10]In the mass-field approach of Hoyle and Narlikar (Ref. 14), no dimensional constants are introduced corresponding to the masses of particles. A massive field theory can then be scale invariant making it possible to render the theory conformally invariant. If the mass field is decomposed into a constant and a spacetime-dependent part, the former gives a mass in the usual sense. This is part of the mechanism known as a spontaneous symmetry breaking in the local field theory of elementary particles. In this paper we reinterpret the constant part, or the vacuum expectation value in Ref. 4, as a slowly varying BGV, hence bringing the two concepts, the mass field and the spontaneous symmetry breaking, closer to each other.
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[20]Our f^2 is related to the Brans-Dicke ω by $\omega = \epsilon f^2/4$. We emphasize, however, that f^2 is not constrained by the experiments resulting in the bound $|\omega| \gg 30$, because our scalar field is effectively decoupled from the local matter distribution [see Eqs. (2.10) and (3.2b)].
[21]The coupling of ϕ to the matter as in L_2 was ruled out in Ref. 2. It would give an additional term $(\partial L_2/\partial\phi) g^{\mu\nu}\partial_\nu\phi$ to $\nabla_\nu T^{\nu\mu}$ (matter) $= 0$ which usually results in the geodesic line equation. We find, however, that the spatial component of this term vanishes, while the component with $\mu = 0$ occurs precisely in accordance with time variability of m. See also Ref. 7.
[22]The same relation is also true in Refs. 5, 7, 14, and 17.
[23]Conformal invariance is enjoyed also by any non-Abelian gauge theories with massless fermions and scalar fields conformally coupled to gravity.
[24]Similar behavior has been obtained also by T. Kaneko and H. Sugawara, Report No. KEK-TH 43, 1982 (unpublished).
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