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Contribution of a neutrino magnetic coupling to the muon magnetic moment

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We consider a massive Dirac neutrino and compute the contribution of its magnetic coupling to the anomalous magnetic moment of the muon using an unsubtracted dispersion relation and assuming a small neutrino structure. We find a result negligibly small.

The interest in the possibility that the neutrino mass is different from zero has recently increased. A massive neutrino may develop a (anomalous) magnetic dipole moment[1] κ_ν, and indeed calculations based on extensions of the standard $SU(2)\times U(1)$ electroweak model show[2,3] that it may be (in units of Bohr magnetons $\mu_\nu = e\kappa_\nu/em_e$) of the order of $10^{-19}m_\nu$ (eV) to $10^{14}(m_l/m_e)$, where m_l is the mass of the charged lepton associated to the neutrino in question. On the other hand the analyses of neutrino scattering data imply the upper bound[1,2,4]

$$|\kappa_{\nu_e}| < 1.4\times10^{-9}\,,\quad |\kappa_{\nu_\mu}| < 8.1\times10^{-9}\,, \tag{1}$$

while astrophysical considerations[2,5] lead to $|\kappa_{\nu_l}| < 8.5\times10^{-11}$ (for $m_{\nu_l} < 10$ keV).

In this note we study the contribution of the neutrino magnetic moment to the muon anomalous magnetic moment. Our first objective is to show explicitly the smallness of the effects linear in κ_ν. The general rule is that effects are small because the magnetic coupling changes the neutrino helicity which is restored thanks to the (Dirac) mass of the neutrino. Thus the effects linear in κ_ν are at most of order $\kappa_\nu m_\nu/m$, where m is some typical mass of the problem under consideration. Given the above mentioned bounds for κ_ν and the bounds on the neutrino mass based on terrestrial experiments,[6]

$$\begin{aligned} &14\text{ eV} < m_{\nu_e} < 60\text{ eV}\,,\\ &m_{\nu_\mu} < 0.57\text{ MeV}\,,\\ &m_{\nu_\tau} < 250\text{ MeV}\,, \end{aligned} \tag{2}$$

we find that the contribution to the muon magnetic moment is negligibly small.

The present understanding of the muon magnetic moment $\mu_\mu = eg_\mu/2m_\mu$ is such that any new contribution to $a_\mu = (g_\mu - 2)/2$ must satisfy the bound[7]

$$-20\times10^{-9} < a_\mu < 26\times10^{-9}\,. \tag{3}$$

To take into account the contribution of the neutrino magnetic moment we should write the $\gamma(q) + \nu(p) \rightarrow \nu(p')$ coupling as $\kappa_\nu e\sigma_{\mu\lambda}q^\lambda f_2(q^2)/2m_e$, where $f_2(0)=1$, and we will assume an unsubtracted dispersion relation for the muon magnetic form factor $F_2(q^2)$ defined by

$$\langle \mu(p')|j_\rho(0)|\mu(p)\rangle = \bar{u}(p')[\gamma_\rho F_1(q^2) + i\sigma_{\rho\lambda}q^\lambda F_2(q^2)/2m_\mu + \gamma_5\gamma_\lambda(g_\rho^\lambda q^2 - q^\lambda q_\rho)G_1(q^2)]u(p)\,, \tag{4}$$

where j_ρ is the electromagnetic current, $F_1(0)=1$, $F_2(0)=a_\mu$, and $G_1(0)$ is the anapole moment. Next we saturate the imaginary part of $F_2(q^2)$ with a two-neutrino intermediate state (Fig. 1):

$$F_2(0) = \frac{1}{\pi}\int_{4m_\nu^2}^{\infty} dt\, \mathrm{Im}F_2(t)/t\,, \tag{5a}$$

$$\mathrm{Im}F_2(t) = \sum_n \hat{P}^\mu \langle \mu(p',s')\bar{\mu}(p,s)|S-1|n\rangle \times \langle n|j_\mu(0)|0\rangle\,, \tag{5b}$$

where $|n\rangle = |\bar{\nu}(q_2,\sigma_2)\nu(q_1,\sigma_1)\rangle$ and $\hat{P}^\mu$ is the operator:

$$\hat{P}^\mu = i2\frac{m_\mu^3(t+2m_\mu^2)}{q^2t} \times \sum_{s,s'}\bar{v}(p,s)\left[\frac{q^\mu}{q^2} - \frac{m_\mu}{t+2m_\mu^2}\gamma^\mu\right]u(p',s')\,, \tag{6}$$

$t=(p+p')^2$, and $q^\mu = (p'-p)^\mu$.

$\langle \mu(p',s')\bar{\mu}(p,s)|S-1|\bar{\nu}(q_2,\sigma_2)\nu(q_1,\sigma_1)\rangle$ is given, to lowest order in weak interactions, by a W-boson-exchange diagram (the exchange of a Z boson and that of a neutral scalar do not contribute to F_2). After putting everything together we obtain up to terms of order m_μ^2/M_W^2

$$F_2(0) = \frac{4}{(2\pi)^3}\frac{G_F}{\sqrt{2}}\frac{m_\nu m_\mu^2}{m_e}\kappa_\nu\int_{4m_\nu^2}^{\infty}dt\,\frac{f_2(t)}{t(4m_\mu^2-t)}[tm_\mu^2T_0 + 2(t-4M_W^2)T_1 + 8(t+2m_\mu^2)T_2/(4m_\mu^2-t)]\,, \tag{7}$$

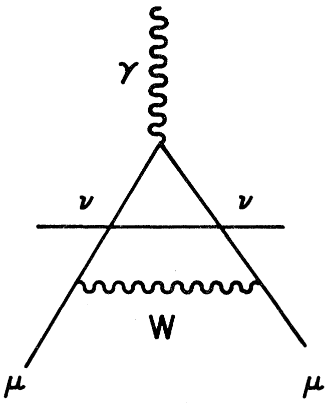

FIG. 1. Two-neutrino intermediate-state contribution to $\mathrm{Im}F_2(t)$

where

$$T_n=\int\frac{d^3q_1}{2E_1}\int\frac{d^3q_2}{2E_2}\frac{\delta^4(p+p'-q_1-q_2)}{(q_1-p_1')^2-M_W{}^2}(\vec{p}_1'\cdot\vec{q}_1)^n\,. \tag{8}$$

The term proportional to $M_W{}^2T_1$ in Eq. (7) arises from the $g_{\alpha\beta}$ part of the W propagator and, since $T_1\rightarrow(\pi/8)[2-\ln(t/M_W{}^2)]$ as $t\rightarrow\infty$, it gives a convergent contribution even if $f_2(t)=1$. The rest of the integrand in Eq. (7) arises from the $K_\alpha K_\beta/M_W{}^2$ part of the W propagator and, except for the term proportional to T_0, it gives a divergent contribution unless $f_2(t)$ decreases (as $t^{-\epsilon}$). We will assume that the finiteness of $F_2(0)$ is guaranteed by the behavior of $f_2(t)$ and we will adopt an *ad hoc* form for it:

$$f_2(t)=\frac{M^4+\Gamma^2M^2}{(t-M^2)^2+\Gamma^2M^2}\,. \tag{9}$$

We will also assume that $\Gamma/M \ll 0.1$ and that $M\gtrsim 20$ GeV which implies that the magnetic radius of the neutrino satisfies $r^2\lesssim 10^{-30}$ cm^2, a bound satisfied by the electric radius of the neutrino.[1,4,8] With these assumptions a good approximation for

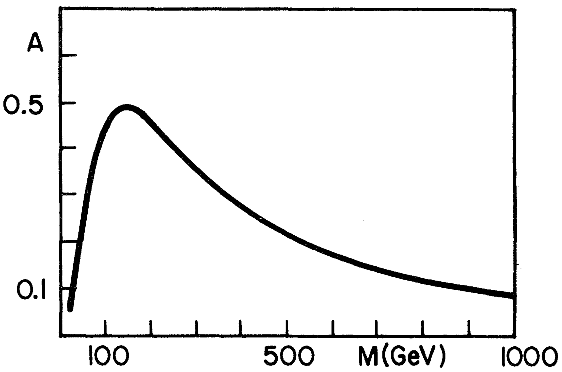

FIG. 2. Values of A in Eqs. (11)–(13).

$f_2(t)$ is

$$f_2(t)\simeq\pi\delta(t-M^2)M^3/\Gamma\,, \tag{10}$$

implying

$$F_2(0)\simeq-\frac{2}{(2\pi)^2}\frac{G_F}{\sqrt{2}}\frac{m_\nu}{m_e}m_\mu{}^2\kappa_\nu\frac{M}{\Gamma}A\,, \tag{11}$$

where

$$A=m_\mu{}^2T_0(M^2)+2\,\frac{(M^2-4M_W{}^2)}{M^2}\,T_1(M^2)-\frac{8}{M^2}T_2(M^2)\,. \tag{12}$$

In Fig. 2 we have displayed the values of A for M between 20 and 1000 GeV. One can say that A is of order 1. Since

$$F_2(0)=-8m_\nu(\mathrm{eV})\kappa_\nu\frac{M}{\Gamma}A\times10^{-12} \tag{13}$$

it is obvious that the contribution of a magnetic coupling of the neutrino to a_μ is negligibly small.

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