Scaling of differential cross section and predictions for inelastic charge-exchange processes at higher energies

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Using available high-energy data and a scaling hypothesis proposed recently, the scaling functions are computed and the differential-cross-section values as functions of $|t|$ are predicted for higher energies for the processes $\pi^- p \to \pi^0 n$ and $\pi^- p \to \eta n$.

Exact results on scaling in hadronic collision processes have been criticized as being based upon assumptions without bearing any theoretical foundations.[1] Further, these results cannot be applied to explain the observed scaling phenomena in nondiffractive processes.[1,2] Since analyticity is universal to all hadronic collision processes, it is more logical to believe that a correct understanding of scaling may be achieved through analyticity, rather than unitarity. Using Mandelstam analyticity and convergent polynomial expansion (CPE), a new method of approach to scaling has been proposed[3] which has been generalized to include diffractive and nondiffractive processes.[2,4] In no other variable has the scaling of the cross-section-ratio data been exhibited in such a remarkable fashion for such a variety of processes.

Using the geometrical-scaling hypothesis Barger, Luthe, and Phillips[5] have made predictions on differential cross sections for some diffractive processes. Also, using the CPE approach to scaling such predictions for pp, $\pi^{\pm}p$, and K^+p scattering have been carried out recently.[6] In this paper, as further evidence in favor of our scaling hypothesis,[2–4] we compute scaling functions and predict differential cross sections as functions of $|t|$ for higher energies for the two inelastic nondiffractive processes, $\pi^- p \to \pi^0 n$ and $\pi^- p - \eta n$.

We follow notations and definitions of Ref. 4. The following CPE in terms of Laguerre polynomials in the scaling variable χ has been proposed as the scaling function for the differential-cross-section ratio

$$f(s,t) = e^{-\chi} \sum_n g_n L_n(2\chi) \tag{1}$$

with

$$\chi(s,t) = \alpha(s) Z(s,t) , \tag{2}$$

$$\alpha(s) = \sum_m d_m \eta^m . \tag{3}$$

It has been noted that the expression (1) is an optimized polynomial expansion (OPE) at asymptotic energies.[7] In principle, scaling in the variable χ can be tested by fitting the data on $f(s,t)$ for every fixed but large value of s by the series (1) and determining $\alpha(s)$. A plot of $f(s,t)$ against χ should then show scaling. But, instead, a simpler method of demonstration of scaling has been adopted[2,4] where it has been assumed that the first term in the expansion for $f(s,t)$ represents the data reasonably well in a very limited part of the forward-peak (extrapolated-forward-peak) region for diffractive (nondiffractive) processes, for $|t| << t_+ = 0.078$ GeV^2. This assumption[8] determines $\alpha(s)$ from the forward-slope-parameter data

$$b(s) = \left. \frac{-d\chi}{dt} \right|_{t=0} = \frac{\alpha(s)}{t_+} \left[1 + \frac{t_+}{s - \Sigma + t_-} \right] , \tag{4}$$

where t_+ (t_-) is the start of the right-hand cut in the $t(u)$ plane. Only the first two terms in $\alpha(s)$ occurring in Eqs. (3) and (4) are necessary to fit the slope-parameter data. This corresponds to the saturation of Regge behavior for large s and small $|t| << t_+$, and also makes $f(s,t)$ an entire function of χ for $s \to \infty$. Here, including the new high-energy slope-parameter data of Apel *et al.*[9]

for $P_{lab} \geq 15$ GeV/c and others[9] we obtain somewhat modified values of the parameters

$$d_0=0.45, \quad d_1=0.18 \tag{5}$$

for $\pi^- p \to \pi^0 n$, with χ^2/DOF=1.68 for 15 data points. Also for $\pi^- p \to \eta n$, a somewhat improved fit is obtained[10] for

$$d_0=0.018, \quad d_1=0.19 \tag{6}$$

with χ^2/DOF=0.35 as compared to the previous value[2] of 0.56. To obtain the scaling curve more accurately, necessary for the purpose of predictions for higher energies, we plot $f(s,t)$ against χ for the two processes by using the parameters given by Eqs. (5) and (6) and taking only the high-energy data on $f(s,t)$ for $p_{lab} \geq 40$ GeV/c as shown in Figs. 1 and 2. In these figures all the data appear to generate the corresponding scaling curves shown by the solid lines, obtained by joining the mean positions of the data points in the plots. We have fitted these curves by the series (1) and found that the scaling curves, within their error corridors, can be very well represented with the first four (six) terms of the series for $\pi^- p \to \pi^0 n$ ($\pi^- p \to \eta n$). The fits in both cases pass very close to the solid lines. The values of the parameters g_n's entering into the definition of the scaling function (1), are given in Table I. It is clear that the series converges, as suggested in Ref. 2, as we go over to higher-order terms. Our computation of scaling function as reported here is the first one in the literature for the two nondiffractive processes. Computation of scaling functions for diffractive processes have been reported elsewhere.[6]

From the definitions of χ and Z, and for all values of s and t such that $|(s+t_- -\Sigma)| >> |t|$, it is possible to write[2,4]

$$|t| = 4m_\pi^2[\sinh(\chi/\alpha(s))^{1/2}]^2 . \tag{7}$$

Since the available high-energy data on $f(s,t)$ have already appeared to have approached the limiting curve for the two processes in Figs. 1 and 2, it is

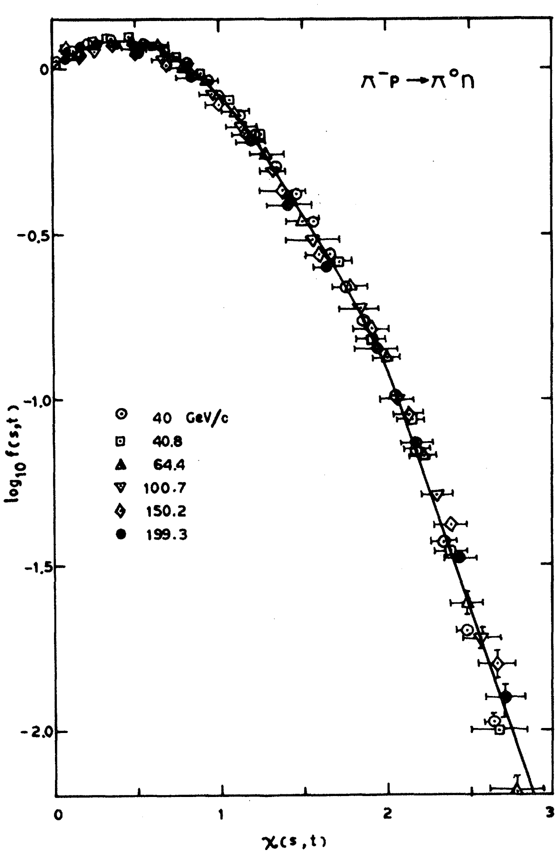

FIG. 1. Scaling of the high-energy data on the cross-section ratio for $\pi^- p \to \pi^0 n$. The solid line represents the scaling curve detained by joining the mean positions of the data points and passing very close to the curve given by the series (1) with the parameters of Table I.

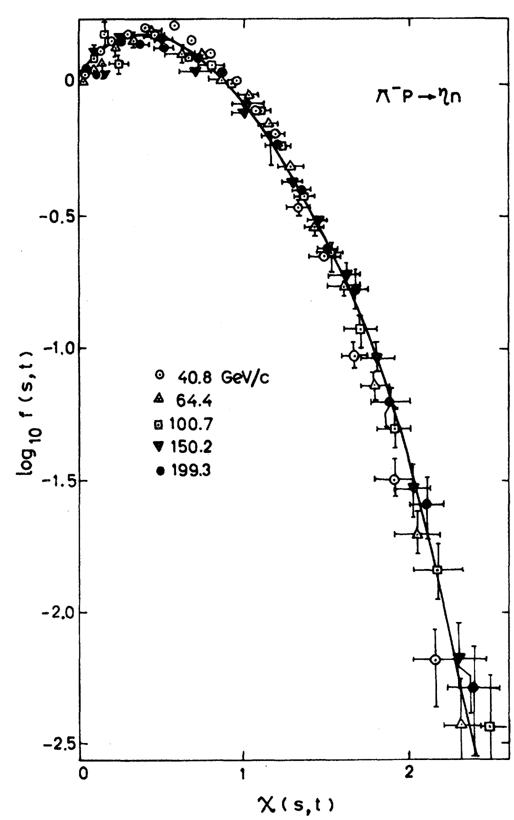

FIG. 2. Same as Fig. 1 but for $\pi^- p \to \eta n$.

TABLE I. Values of the coefficients in the series (1) defining the scaling function.

Scattering process	g_0	g_1	g_2	g_3	g_4	g_5	Range of χ
$\pi^- p \to \pi^0 n$	1.632	−0.117	−0.239	−0.366	0	0	0 to 2.7
$\pi^- p \to \eta n$	1.865	0.056	−0.446	−0.177	−0.497	−0.049	0 to 2.4

not unreasonable to suppose that the future data at higher energies would also fall on these curves. For the sake of predictions for higher energies, the coordinates of $f(s,t)$ and χ are noted for a large number of points on a scaling curve. The value of $\alpha(s)$ for the desired higher value of s is obtained using the parameters of Eqs. (5) and (6). This is equivalent to extrapolating the fit to the slope parameter on to higher energies. From the knowledge of $\alpha(s)$ at the desired higher value of s and the value of χ corresponding to any point on the scaling curve, the value of $|t|$ is computed for that point using formula (7). In this manner the image of a large number of points on the scaling curve are obtained in the $f(s,t)$ versus $|t|$ plot for a desired higher value of s for each process. Our predictions for the two processes are shown in Figs. 3 and 4 for $P_{\rm lab}=400$, 600, 800, and 1000 GeV/c.

The method of OPE by conformal mapping of the cuts[7] has been extensively used to obtain information on physical quantities by extrapolation onto

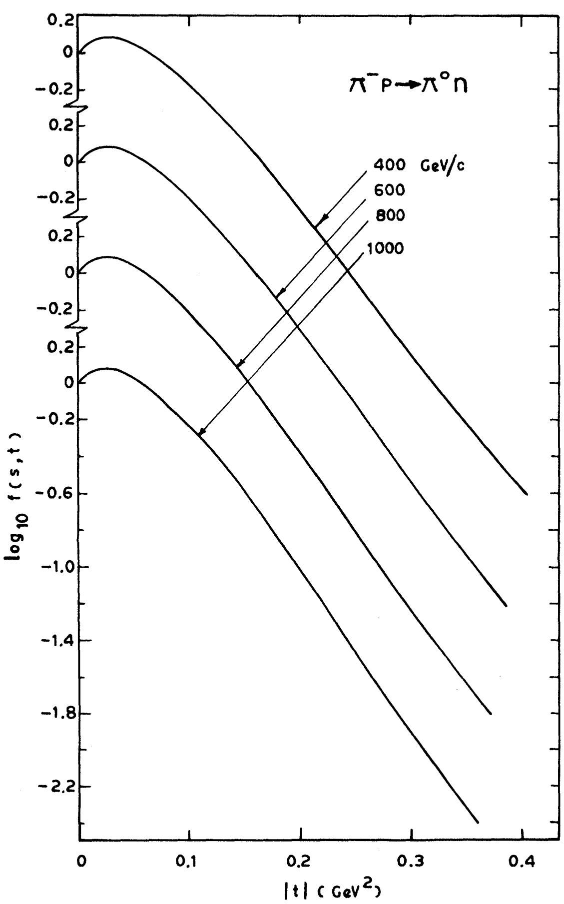

FIG. 3. Predictions of $f(s,t)$ as a function of $|t|$ for higher energies for $\pi^- p \to \pi^0 n$.

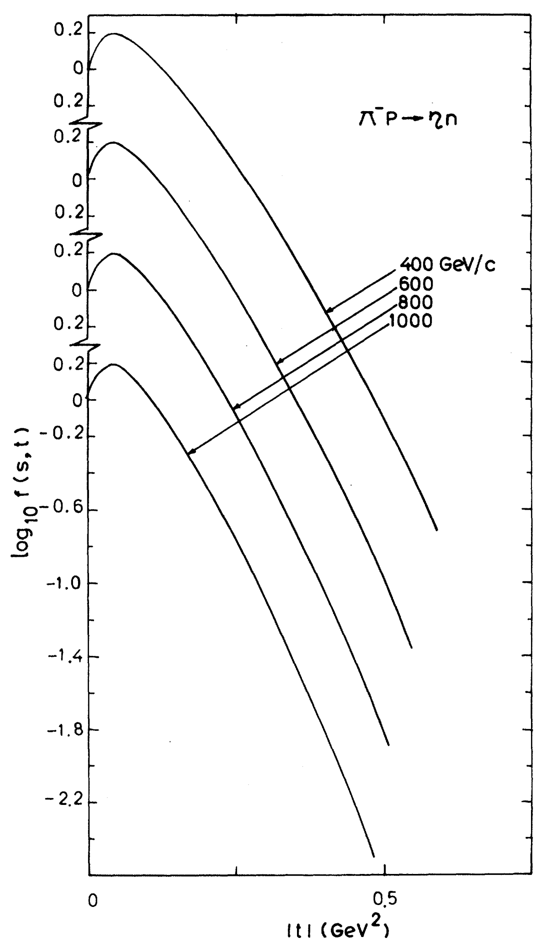

FIG. 4. Same as Fig. 3 but for $\pi^- p \to \eta n$.

regions which are either inaccessible to experiments or where the data do not exist. In this paper we have verified the proposed hypothesis[2–4] that the data on $f(s,t)$ can be represented by a convergent series in Laguerre polynomials in the scaling variable χ, even for nondiffractive processes. Cornille[11] has suggested that, only for diffraction scattering, a series in Laguerre polynomials in the variable $\tau = tb(s)$ could be the possible scaling function. We have used the method of analytic approximation by conformal mapping and exploited the excellent description of scaling of the available data to predict the cross-section ratio as a function of $|t|$ at higher energies. If the agreement of the predicted results is found to be reasonably good, there is a possibility that the present method may be adopted to compute the cross section at still higher energies, depending on the desired accuracy, as an alternative to highly expensive experiments with the accelerators.

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[1]P. P. Divakaran, Phys. Lett. 76B, 203 (1978).
[2]M. K. Parida and N. Giri, Phys. Rev. D 21, 2548 (1980).
[3]M. K. Parida, Phys. Rev. D 19, 150 (1979); 19, 164 (1979); 20, 820(E) (1979).
[4]M. K. Parida and N. Giri, Phys. Rev. D 21, 2528 (1980).
[5]V. Barger, J. Luthe, and R. J. N. Phillips, Nucl. Phys. B 88, 237 (1975).
[6]M. K. Parida, Phys. Rev. D 21, 2563 (1980); N. Giri and M. K. Parida, Pramana 16, 333 (1981); 17, 297 (1981).
[7]R. E. Cutkosky and B. B. Deo, Phys. Rev. 174, 1859 (1968); S. Ciulli, Nuovo Cimento 61A, 787 (1969).
[8]On the other hand, if the forward-slope-parameter data are fitted by retaining the whole series, the corresponding formula becomes

$$b(s) = \frac{\alpha(s)}{t_+}\left[1+\frac{t_+}{s-\Sigma+t_-}\right]\left[1-\frac{2\sum_n g_n L_n'(0)}{\sum_n g_n L_n(0)}\right].$$

Since, according to our scaling hypothesis, g_n's are independent of s, this formula differs from Eq. (4) by a constant factor. This does not alter the predicted results since $\chi/\alpha(s)$ remains unchanged.
[9]W. D. Apel *et al.*, Nucl. Phys. B 154, 192 (1979); A. V. Stirling *et al.*, Phys. Rev. Lett. 14, 763 (1965); A. V. Barnes *et al.*, *ibid.* 37, 76 (1976); O. I. Dahl *et al.*, *ibid.*, 37, 80 (1976).
[10]For an improved value of total χ^2, the errors in the data on $b(s)$ used in Ref. 2 were artificially reduced and values of the parameters given in Eq. (6) were obtained.
[11]H. Cornille, Phys. Rev. D 14, 1693 (1976).