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Flavor-changing electromagnetic transitions

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We compute the $q_i q_j \gamma$ transition amplitude for off-shell and on-shell quarks with flavors $i \neq j$, in the standard weak-interaction model. The results we present are valid for all quark and photon momenta and for all external and internal quark masses. We discuss the Feynman rules and Ward-Takahashi identities in the 't Hooft—Feynman gauge. Our results can be applied to a variety of phenomena such as the electric dipole moment of the neutron, the process $e^+e^- \to t\bar{c}$, and flavor-changing transitions with either gluon or photon emission.

I. INTRODUCTION

The unified theory of weak and electromagnetic interactions[1] has been so far extremely successful. Apart from the direct observation of the intermediate gauge bosons, an unequivocal experimental verification is still missing for the predicted triple-vector couplings such as γWW, and for higher-order corrections which are all so essential for the theory.

A systematic investigation of the effects of the full structure of the theory in the leptonic sector as applied to e^+e^- collisions was recently carried out.[2] Only small deviations from quantum-electrodynamic predictions were found up to the production of Z bosons, with similar conclusions for the weak corrections to the anomalous magnetic moment of the muon.[3]

In this paper we set the ground for a systematic investigation of weak corrections in the quark sector for processes in which one of the quarks undergoes a flavor change accompanied with the emission of a real or a virtual photon or gluon. The transition considered here is $q_i q_j \gamma$, where the corresponding transition $q_i q_j g$, where g stands for a gluon, will be trivially obtained from part of the previous one. Our calculation is the first complete one since we (1) do not neglect internal or external quark masses with respect to M_W, the charged-vector-boson mass, (2) do not neglect external momenta with respect to M_W, (3) present out results for off-shell as well as for on-shell external quarks, and (4) perform our calculations in the 't Hooft—Feynman gauge, thus avoiding divergence problems which are typical to the calculation of high-order corrections in the unitary gauge. The on-shell results are of course gauge independent.

The reader may at this point wonder why one should embark on such a general calculation. The relevance of the general flavor-changing vertex becomes clear from a discussion of some possible applications. Consider D_n, the electric dipole moment of the neutron in the Kobayashi-Maskawa (KM) model.[4] Let us view the neutron as built from three constituent quarks and neglect strong-interaction corrections.[5] It is obvious that the electric dipole moment of a single quark d_q, vanishes in the lowest order, i.e., at the one-loop level. Although estimates were made for the order-$G_F{}^2$ contributions to d_q,[6] it was subsequently shown that $d_q = 0$ at the two-loop level.[7] It was then suggested that CP violation in the $ud \to du + \gamma$ "weak scattering" (a W is exchanged in the crossed channel) with the third quark acting as a spectator, gives the dominant contribution to D_n.[8,9] Therefore the calculation of D_n requires as an intermediate step the knowledge of the one-loop $q_i \to q_j + \gamma$ vertex with one external quark off-shell. The t quark, for which the assumption $m_t{}^2 << M_W{}^2$ clearly breaks down, can appear either inside the loop or as an external quark in the $q_i q_j \gamma$ vertex. Small external quark and photon momenta can be assumed throughout the calculation of D_n.

Another example is the process $e^+e^- \to t\bar{c}$, where an assumption of small photon momentum in the $tc\gamma$ vertex is clearly unjustified, while the quarks in the loop do satisfy $m^2 << M_W{}^2$.

Similarly the $sd\gamma$[10] and sdg[11] vertices, responsible for the CP violation in $K \to \pi e^+e^-$ and $\Lambda \to n\gamma$, and for the $\Delta I = \frac{1}{2}$ rule and CP violation in nonleptonic K decays, respectively, involve an intermediate t quark, whose mass is not negligible

TABLE I. The coefficients of the decomposition of the seven parts which add to the complete renormalized vertex $V^{\mu}_{\rm ren}$. The A_i and B_i are defined in Eq. (51). Summation over integral quarks $\sum_l U_{il} U^{\dagger}_{lj}$ is always assumed.

	$R^{\mu}_{W1,{\rm ren}}$	$R^{\mu}_{G,{\rm ren}}$	$R^{\mu}_{S,{\rm ren}}$
A_1	$-\dfrac{4\alpha_p(1-\alpha_p)}{X}$	0	$\dfrac{-2m^2\alpha_p(1-\alpha_p)}{X}$
A_2	$-\dfrac{4\alpha_k(1-\alpha_p)}{X}$	$\dfrac{2(\alpha_p+\alpha_k-1)}{X}$	$\dfrac{m^2(1-\alpha_p)(1-2\alpha_k)}{X}$
A_3	$-\dfrac{2(1-\alpha_p-2\alpha_p\alpha_k)}{X}$	$\dfrac{-2\alpha_k}{X}$	$\dfrac{2m^2\alpha_p\alpha_k}{X}$
A_4	$-\dfrac{2\alpha_k(1-2\alpha_k)}{X}$	0	$\dfrac{-m^2\alpha_k(1-2\alpha_k)}{X}$
A_5	$\dfrac{2(1-\alpha_p)}{X}$	$\dfrac{(1-\alpha_p)}{X}$	0
A_6	$-2\left[\dfrac{(m_i^2-m_j^2)\alpha_p\alpha_k}{X_0} - \dfrac{m_i^2\alpha_p(1-\alpha_p)}{X_0} - \ln\dfrac{X}{X_0}\right]$	$-\dfrac{2m^2}{X}+4\ln\dfrac{X}{4\pi\mu^2}$	$-[m^2(m_i^2-m_j^2)\alpha_p\alpha_k$ $+m^2\alpha_p(\alpha_p m_i^2+m_j^2)$ $-m_j^2 m_i^2\alpha_p(1-\alpha_p)](1/X_0)$ $+m^2\ln(X/X_0)$
A_7	0	$\dfrac{\alpha_k(2\alpha_k-1)}{X}$	0
A_8	$\dfrac{2(1-\alpha_p)}{X}$	$\dfrac{2(1-\alpha_p-\alpha_k+2\alpha_p\alpha_k)}{X}$	0
A_9	0	$\dfrac{-2\alpha_p(1-\alpha_p)}{X}$	0
A_{10}	0	$\dfrac{-m_j(1-\alpha_p)}{X}$	0
A_{11}	0	$\dfrac{m_j\alpha_k}{X}$	0
A_{12}	0	$\dfrac{2m_j(1-\alpha_p)}{X}$	$\dfrac{2m_j m^2\alpha_p}{X}$
A_{13}	0	$\dfrac{-2m_j\alpha_k}{X}$	$\dfrac{-m_j m^2(1-2\alpha_k)}{X}$

TABLE I. (*Continued.*)

	$R^{\mu}_{W1,\mathrm{ren}}$	$R^{\mu}_{G,\mathrm{ren}}$	$R^{\mu}_{S,\mathrm{ren}}$
B_1	0	0	$\frac{-2m_i m_j \alpha_p(1-\alpha_p)}{X}$
B_2	0	0	$\frac{m_i m_j(1-\alpha_p)(1-2\alpha_k)}{X}$
B_3	0	0	$\frac{2m_i m_j \alpha_p \alpha_k}{X}$
B_4	0	0	$\frac{-m_i m_j \alpha_k(1-2\alpha_k)}{X}$
B_5	0	0	0
B_6	$\frac{2m_i m_j \alpha_p(1-\alpha_p)}{X_0}$	0	$-m_i m_j\{[(m_i^2-m_j^2)\alpha_p\alpha_k+m^2\alpha_p(1+\alpha_p)$ $-m_i^2\alpha_p(1-\alpha_p)](1/X_0)$ $-\ln(X/X_0)\}$
B_7	0	0	0
B_8	0	0	0
B_9	0	0	0
B_{10}	0	$\frac{m_i(1-\alpha_p)}{X}$	0
B_{11}	0	$\frac{-m_i\alpha_k}{X}$	0
B_{12}	0	0	$\frac{2m_i m^2\alpha_p}{X}$
B_{13}	0	0	$\frac{-m_i m^2(1-2\alpha_k)}{X}$
A common factor	$\frac{ig^2 e_W}{32\pi^2}\int_0^1 d\alpha_p \int_0^{1-\alpha_p} d\alpha_k$	$\frac{ig^2 e_W}{32\pi^2}\int_0^1 d\alpha_p \int_0^{1-\alpha_p} d\alpha_k$	$\frac{ig^2 e_W}{32\pi^2 M_W^2}\int_0^1 d\alpha_p \int_0^{1-\alpha_p} d\alpha_k$

TABLE I. (*Continued.*)

	$\Lambda^{\mu}_{W,\mathrm{ren}}$	$\Lambda^{\mu}_{S,\mathrm{ren}}$	$\Sigma^{\mu}_{1,\mathrm{ren}}$	$\Sigma^{\mu}_{2,\mathrm{ren}}$
A_1	$\frac{4\alpha_p^2}{Y}$	$\frac{2m^2\alpha_p^2}{Y}$	0	0
A_2	$\frac{4\alpha_p(\alpha_k-1)}{Y}$	$\frac{2m^2\alpha_p\alpha_k}{Y}$	0	0
A_3	$\frac{4\alpha_p(\alpha_k-1)}{Y}$	$\frac{2m^2\alpha_p\alpha_k}{Y}$	0	0
A_4	$\frac{4\alpha_k(\alpha_k-1)}{Y}$	$\frac{2m^2\alpha_k(\alpha_k-1)}{Y}$	0	0
A_5	$\frac{2\alpha_p}{Y}$	$\frac{-m^2\alpha_p}{Y}$	0	0
A_6	$2m^2\left[\frac{1}{Y}-\frac{1}{Y_0}\right]$ $-2\ln\frac{Y}{Y_0}$	$m^4\left[\frac{1}{Y}-\frac{1}{Y_0}\right]-m^2\ln\frac{Y}{Y_0}$ $-[\alpha_p{}^2m_i{}^2m_j{}^2$ $-m^2(m_i{}^2+m_j{}^2)\alpha_p]/Y_0$	$\bar{f}+C$ $+\frac{[\bar{\phi}+B+m_i(\bar{f}+C)]m_i}{(p-k)^2-m_i{}^2}$	$f+C$ $+\frac{[\psi+A+m_j(f+C)]m_j}{p^2-m_j{}^2}$
A_7	$\frac{-2\alpha_k(\alpha_k-1)}{Y}$	$\frac{-m^2\alpha_k(\alpha_k-1)}{Y}$	0	0
A_8	$\frac{-4\alpha_p(\alpha_k-1)}{Y}$	$\frac{-2m^2\alpha_p\alpha_k}{Y}$	0	0
A_9	$\frac{-2\alpha_p{}^2}{Y}$	$\frac{-m^2\alpha_p{}^2}{Y}$	0	0
A_{10}	0	0	$-\frac{\bar{\psi}+A+m_i(\bar{h}+D)}{(p-k)^2-m_i{}^2}$	$\frac{\psi+A+m_j(f+C)}{p^2-m_j{}^2}$
A_{11}	0	$\frac{-m^2m_j}{Y}$	$\frac{\bar{\psi}+A+m_i(\bar{h}+D)}{(p-k)^2-m_i{}^2}$	0
A_{12}	0	$\frac{-2m^2m_j\alpha_p}{Y}$	$2\frac{\bar{\psi}+A+m_i(\bar{h}+D)}{(p-k)^2-m_i{}^2}$	0
A_{13}	0	$\frac{2m^2m_j(1-\alpha_k)}{Y}$	$-2\frac{\bar{\psi}+A+m_i(\bar{h}+D)}{(p-k)^2-m_i{}^2}$	0

TABLE I. (*Continued.*)

	$\Lambda^{\mu}_{W,\text{ren}}$	$\Lambda^{\mu}_{S,\text{ren}}$	$\Sigma^{\mu}_{1,\text{ren}}$	$\Sigma^{\mu}_{2,\text{ren}}$
B_1	0	$\frac{2m_i m_j \alpha_p{}^2}{Y}$	0	0
B_2	0	$\frac{2m_i m_j \alpha_p \alpha_k}{Y}$	0	0
B_3	0	$\frac{2m_i m_j \alpha_p \alpha_k}{Y}$	0	0
B_4	0	$\frac{2m_i m_j \alpha_k(\alpha_k-1)}{Y}$	0	0
B_5	0	$\frac{-m_i m_j \alpha_p}{Y}$	0	0
B_6	$\frac{-2m_i m_j \alpha_p{}^2}{Y_0}$	$m_i m_j \left\{ m^2 \left[\frac{1}{Y} - \frac{1}{Y_0} \right] -\ln\frac{Y}{Y_0} + \frac{\alpha_p(2-\alpha_p)}{Y_0} \right\}$	$\bar{h}+D + \frac{[\bar{\psi}+A+m_i(\bar{h}+D)]m_i}{(p-k)^2-m_i{}^2}$	$h+D + \frac{[\phi+B+m_j(h+D)]m_j}{p^2-m_j{}^2}$
B_7	0	$\frac{-m_i m_j \alpha_k(\alpha_k-1)}{Y}$	0	0
B_8	0	$\frac{-2m_i m_j \alpha_p \alpha_k}{Y}$	0	0
B_9	0	$\frac{-m_i m_j \alpha_p{}^2}{Y}$	0	0
B_{10}	0	0	$-\frac{\bar{\phi}+B+m_i(\bar{f}+C)}{(p-k)^2-m_i{}^2}$	$\frac{\phi+B+m_j(h+D)}{p^2-m_j{}^2}$
B_{11}	0	$\frac{-m^2 m_i}{Y}$	$\frac{\bar{\phi}+B+m_i(\bar{f}+C)}{(p-k)^2-m_i{}^2}$	0
B_{12}	0	$\frac{-2m^2 m_i \alpha_p}{Y}$	$2\frac{\bar{\phi}+B+m_i(\bar{f}+C)}{(p-k)^2-m_i{}^2}$	0
B_{13}	0	$\frac{2m^2 m_i(1-\alpha_k)}{Y}$	$-2\frac{\bar{\phi}+B+m_i(\bar{f}+C)}{(p-k)^2-m_i{}^2}$	0
A common factor	$\frac{ig^2 e_l}{32\pi^2} \int_0^1 d\alpha_p \int_0^{1-\alpha_p} d\alpha_k$	$\frac{ig^2 e_l}{32\pi^2 M_W{}^2} \int_0^1 d\alpha_p \int_0^{1-\alpha_p} d\alpha_k$	$-e_i$	$-e_j$

in comparison with M_W.

It is therefore important to have a complete calculation of the electromagnetic flavor-changing vertex which will involve no approximations. The final result will be expressed in terms of a double integral the value of which will depend on the physical application at hand.

Our presentation will be as pedagogical as possible and will include the Feynman rules,[12] the renormalization procedure, and the Ward-Takahashi identities in the 't Hooft–Feynman gauge for the quark sector of the theory. We adopt the dimensional regularization procedure[13] and present, in a compact new form, the general n-dimensional loop integral. Applications of the vertex function calculated here will be discussed elsewhere.[14]

In Sec. II we summarize the relevant Feynman rules in the 't Hooft–Feynman gauge. In Sec. III we compute the off-diagonal self-energy of a quark and renormalize it. The unrenormalized off-diagonal proper vertex is calculated in Sec. IV and in Sec. V it is renormalized; Sec. VI contains the main result of our paper: the renormalized off-diagonal vertex function, which is given in Table I. In Sec. VII we summarize our results. In Appendix A we present some useful identities including a compact form for the general n-dimensional integral. In Appendix B all the integrals which appear in the self-energy diagrams are explicitly evaluated. In Appendix C we discuss the absence of counterterms for the part of the vertex which does not obey a Ward-Takahashi identity. In Appendix D we present the consequences of the Ward-Takahashi identity and emphasize the difference between the identity in the 't Hooft–Feynman gauge and in the unitary gauge. In Appendix E we place one quark on-shell, and then put two quarks on-shell and present the form of the vertex functions for these cases.

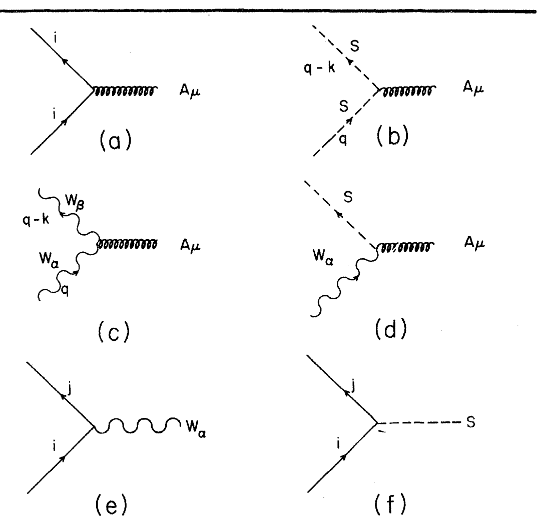

FIG. 1. Feynman vertices relevant to our calculation. i,j are quark flavors, curly, wavy, dashed, and solid lines stand for photon, charged W, charged S, and quarks, respectively. The rules are in Eq. (1).

II. FEYNMAN RULES IN THE 't HOOFT–FEYNMAN GAUGE

The relevant elementary vertices are given in Fig. 1. A, W, and S denote photon, charged W boson, and charged unphysical scalar, respectively, while i, j stand for quarks with flavors i and j, and the photon momentum is k. Although the 't Hooft–Feynman gauge discussed here contains the unphysical scalars $S^{\pm}$, the W-boson propagator is simple. This has to be contrasted with the unitary gauge for which no unphysical scalars are necessary but the $q_\mu q_\nu/M_W{}^2$ term in the W propagator induces severe divergences.

The Feynman rules for the elementary vertices are[15]

$$ie_q\bar{u}_i\gamma_\mu u_i \quad (ii\gamma)\,, \tag{1a}$$

$$ie_S(2q-k)_\mu \quad (SS\gamma)\,, \tag{1b}$$

$$-ie_W[g_{\alpha\beta}(2q-k)_\mu-g_{\mu\alpha}(q+k)_\beta-g_{\mu\beta}(q-2k)_\alpha] \quad (WW\gamma)\,, \tag{1c}$$

$$-i\,|\,e_W\,|\,M_W g_{\alpha\mu} \quad (WS\gamma)\,, \tag{1d}$$

$$-\frac{ig}{\sqrt{2}}\bar{u}_j\gamma_\alpha\frac{(1-\gamma_5)}{2}u_i \quad (ijW)\,, \tag{1e}$$

$$\frac{ig\sqrt{2}}{M_W}I_3^j\left[m_j\bar{u}_j\frac{(1-\gamma_5)}{2}u_i-m_i\bar{u}_j\frac{(1+\gamma_5)}{2}u_i\right] \quad (ijS)\,, \tag{1f}$$

where I_3^j is the third component of the weak isospin for quark j ($\frac{1}{2}$ for u and $-\frac{1}{2}$ for d and similarly for other weak doublets) and $g^2/8M_W{}^2=G_F/\sqrt{2}$. Note that while the first vertex changes sign when the sign of e_q is reversed, and the second and third vertices change sign as e_W ($=e_S$) is flipped, the sign of the $WS\gamma$ vertex is invariant under a sign change of e_W.

If the photon is replaced by a gluon, the vertices

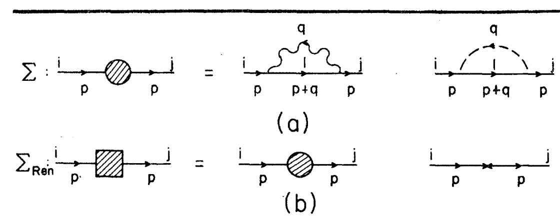

FIG. 2. (a) The unrenormalized off-diagonal self-energy; it is given in Eqs. (7) and (10). (b) Renormalization of the off-diagonal self-energy [see Eq. (13)].

(1b), (1c), and (1d) do not exist, and the vertex (1a) is multiplied with the appropriate SU(3) generator.

The relevant propagators are

$$\frac{i}{p^2-M_W{}^2} \quad (S)\,, \tag{2a}$$

$$\frac{-ig_{\mu\nu}}{p^2-M_W{}^2} \quad (W)\,, \tag{2b}$$

$$\frac{i}{\not p-m} \quad (\text{quark})\,. \tag{2c}$$

III. Σ: THE OFF-DIAGONAL SELF-ENERGY AND ITS RENORMALIZATION

In this section we calculate the off-diagonal self-energy diagrams depicted in Fig. 2(a) and then renormalize them as shown graphically in Fig. 2(b). Denoting by Σ^W,Σ^S the contributions of a virtual W, S, respectively, the

$$\Sigma(p)=\Sigma^W(p)+\Sigma^S(p)\,. \tag{3}$$

A. Σ^W

Define the left and right projection operators

$$L=\frac{1-\gamma_5}{2},\quad R=\frac{1+\gamma_5}{2}\,. \tag{4}$$

We perform our calculations in n dimensions and use $\epsilon=4-n$. A scale parameter μ^2 is introduced; the physical results are of course independent of μ^2. Then from Fig. 2(a)

$$\Sigma^W=\left[\frac{-ig}{\sqrt{2}}\right]^2(\mu^2)^{\epsilon/2}\int\frac{d^nq}{(2\pi)^n}\gamma_\alpha L\frac{i}{\not p+\not q-m}\frac{(-i)g^{\alpha\beta}}{q^2-M_W{}^2}\gamma_\beta L$$

$$=\frac{-g^2}{2}(\mu^2)^{\epsilon/2}\int\frac{d^nq}{(2\pi)^n}\frac{(\epsilon-2)(\not p+\not q)L}{[(p+q)^2-m^2](q^2-M_W{}^2)}\,, \tag{5}$$

where m is the mass of the internal quark with flavor l and the last step follows from n-dimensional Dirac algebra (see Appendix A). The final result in the KM model requires a summation over intermediate quark,

$$\sum_l U_{il}U_{lj}^\dagger\,, \tag{6}$$

where U_{il} are elements of the KM mixing matrix. We will omit this summation but assume it implicitly in all our calculations. From Eq. (5) we find—using Feynman parametrization and n-dimensional integration (see Appendix A)—that

$$\Sigma^W(p)=\frac{-ig^2}{16\pi^2}\int_0^1 d\alpha\,\alpha\not p\left[-\frac{2}{\epsilon}+\ln\frac{\Delta}{4\pi\mu^2}-\Gamma'+1\right]L\,, \tag{7}$$

where $\Gamma'=\Gamma'(1)$, and

$$\Delta=M_W{}^2\alpha+m^2(1-\alpha)-p^2\alpha(1-\alpha)\,. \tag{8}$$

B. Σ^S

From the right-hand-side of Fig. 2(a) we can write

$$\Sigma^S=\left[\frac{ig}{M_W{}^2\sqrt{2}}\right]^2(\mu^2)^{\epsilon/2}\int\frac{d^nq}{(2\pi)^n}(m_jL-mR)\frac{i}{\not p+\not q-m}\frac{i}{q^2-M_W{}^2}(-mL+m_iR)$$

$$=\frac{g^2}{2M_W{}^2}(\mu^2)^{\epsilon/2}\int\frac{d^nq}{(2\pi)^n}\frac{m^2(\not p+\not q-m_j)L+m_i[m_j(\not p+\not q)-m^2]R}{[(p+q)^2-m^2](q^2-M_W{}^2)}\,. \tag{9}$$

Therefore

$$\Sigma^S(p)=\frac{ig^2}{32\pi^2M_W{}^2}\int_0^1 d\alpha[m^2(\not p\alpha-m_j)L+m_i(m_j\not p\alpha-m^2)R]\left[\frac{2}{\epsilon}-\ln\frac{\Delta}{4\pi\mu^2}+\Gamma'\right]. \tag{10}$$

Thus by adding Eqs. (7) and (10) we obtain the unrenormalized off-diagonal self-energy.

C. Renormalization of Σ

The renormalized $\Sigma_{\rm ren}$ [Fig. 2(b)] is obtained by adding a counterterm to Σ subject to the on-shell conditions

$$\Sigma_{\rm ren}u_i=0,\ \ p^2=m_i^2 \ \text{ and } \ \bar{u}_j\Sigma_{\rm ren}=0,\ \ p^2=m_j^2\ . \tag{11}$$

The $1/\epsilon$ divergence in Σ will of course disappear once the counterterm is added.

To this end we define four functions of p^2 [$f(p^2)$, $h(p^2)$, $\psi(p^2)$, and $\phi(p^2)$] such that

$$\Sigma=f\not{p}L+h\not{p}R+\psi L+\phi R\ . \tag{12}$$

We have to find the four constants (i.e., independent of p^2) A, B, C, D in

$$\Sigma_{\rm ren}=(f+C)\not{p}L+(h+D)\not{p}R+(\psi+A)L+(\phi+B)R \tag{13}$$

from the on-shell conditions in Eq. (11). The four coefficients in $\Sigma_{\rm ren}$ are then finite and given by

$$f+C=\frac{m_j^2(f-f_j)-m_i^2(f-f_i)+m_im_j(h_i-h_j)+m_j(\psi_i-\psi_j)+m_i(\phi_i-\phi_j)}{m_j^2-m_i^2}\ , \tag{14a}$$

$$h+D=\frac{m_j^2(h-h_j)-m_i^2(h-h_i)+m_im_j(f_i-f_j)+m_i(\psi_i-\psi_j)+m_j(\phi_i-\phi_j)}{m_j^2-m_i^2}\ , \tag{14b}$$

$$\psi+A=\frac{m_j^2(\psi-\psi_i)-m_i^2(\psi-\psi_j)-m_im_j(\phi_i-\phi_j)-m_i^2m_j(f_i-f_j)-m_im_j^2(h_i-h_j)}{m_j^2-m_i^2}\ , \tag{14c}$$

$$\phi+B=\frac{m_j^2(\phi-\phi_i)-m_i^2(\phi-\phi_j)-m_im_j(\psi_i-\psi_j)-m_im_j^2(f_i-f_j)-m_i^2m_j(h_i-h_j)}{m_j^2-m_i^2}\ , \tag{14d}$$

where $f_i=f(p^2=m_i^2)$, etc. From Eqs. (7) and (10),

$$f=\frac{ig^2}{32\pi^2}\int_0^1 d\alpha\,\alpha\left[\frac{4}{\epsilon}-2\ln\frac{\Delta}{4\pi\mu^2}+2\Gamma'-2+\frac{m^2}{M_W^2}\left[\frac{2}{\epsilon}-\ln\frac{\Delta}{4\pi\mu^2}+\Gamma'\right]\right], \tag{15a}$$

$$h=\frac{ig^2m_im_j}{32\pi^2M_W^2}\int_0^1 d\alpha\,\alpha\left[\frac{2}{\epsilon}-\ln\frac{\Delta}{4\pi\mu^2}+\Gamma'\right], \tag{15b}$$

$$\psi=-\frac{ig^2m_jm^2}{32\pi^2M_W^2}\int_0^1 d\alpha\left[\frac{2}{\epsilon}-\ln\frac{\Delta}{4\pi\mu^2}+\Gamma'\right], \tag{15c}$$

$$\phi=-\frac{ig^2m_im^2}{32\pi^2M_W^2}\int_0^1 d\alpha\left[\frac{2}{\epsilon}-\ln\frac{\Delta}{4\pi\mu^2}+\Gamma'\right] \tag{15d}$$

with Δ given in Eq. (8). The combinations $f-f_j$, etc. in Eq. (14) are of course finite. The explicit forms of all the terms in the numerators of Eq. (14) for any external momenta and internal masses are given in Appendix B.

We have thus completed the calculation of the renormalized off-diagonal self-energy $\Sigma_{\rm ren}$, and we can move on to the off-diagonal $ij\gamma$ vertex itself. First we calculate those terms in the vertex which do not involve Σ (the proper vertex; see Fig. 3).

IV. Γ^μ: THE UNRENORMALIZED OFF-DIAGONAL PROPER VERTEX

All the contributions to $\Gamma^\mu(p)$, the off-diagonal proper vertex, are shown in Fig. 3. We first calculate the unrenormalized Γ^μ and then find the counterterm needed to renormalize the proper vertex.

Let us define R^μ as the set of diagrams in Figs. 3(a)–3(d), and Λ_μ as the diagrams 3(e) and 3(f). Then

$$\Gamma^\mu=R^\mu+\Lambda^\mu\ . \tag{16}$$

Note that for the ij gluon vertex $R^\mu=0$ while Λ^μ survives.

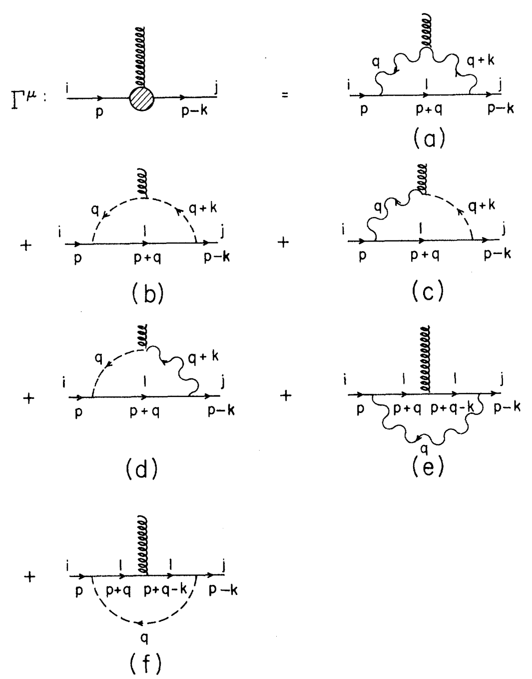

FIG. 3. The unrenormalized proper vertex (see Sec. III).

Before calculating R^{μ} and Λ^{μ}, we discuss the separation of the vertices in Eq. (1) into parts that obey Ward-Takahashi identities and those that do not. We also define the renormalization scheme that we adopt.

A. Renormalization of vertices in the 't Hooft–Feynman gauge

Unlike a physical gauge like the unitary gauge which has only observed particles, the 't Hooft–Feynman gauge involves unphysical degrees of freedom. A consequence is that the proper vertex Γ_{μ} does not obey the Ward-Takahashi identity $k^{\mu}\Gamma^{ij}_{\mu}=e_i[\Sigma_{ij}(p-k)-\Sigma_{ij}(p)]$.[16] A quick glance at the photon vertices in Eq. (1) reveals that only the diagrams arising from Eqs. (1a) and (1b) obey the Ward-Takahashi (WT) identity, while diagrams from Eqs. (1c) and (1d) do not. Actually the vertex 1(c) can be divided for convenience into two parts, $-ie_W[g_{\alpha\beta}(2q+k)_{\mu}-g_{\mu\alpha}k_{\beta}+g_{\mu\beta}k_{\alpha}]$ which satisfies the identity, and $-ie_W[-g_{\mu\alpha}(q+k)_{\beta}-g_{\mu\beta}q_{\alpha}]$ which does not. Thus, in one-loop approximation, Λ_{μ} and part of R_{μ} obey the WT identity, and those parts arising from Eq. (1d) and the second part of the Eq. (1c), which we call R^{G}_{μ}, do not obey the identity. Thus $\hat{\Gamma}_{\mu}\equiv\Gamma_{\mu}-R^{G}_{\mu}$ satisfies the WT identity. The renormalized vertex is defined by $\Gamma^{\mu}_{\rm ren}=\hat{\Gamma}^{\mu}_{\rm ren}+R^{\mu}_{G}$, where $\hat{\Gamma}^{\mu}_{\rm ren}$ satisfies the WT identity for the *renormalized* quantities.[17] Thus $k_{\mu}\hat{\Gamma}^{\mu}_{\rm ren}=e_i[\Sigma^{ij}_{\rm ren}(p-k)-\Sigma^{ij}_{\rm ren}(p)]$. From the relationship of $\Sigma^{ij}_{\rm ren}$ to Σ^{ij} defined in Eq. (11), it is obvious that $\hat{\Gamma}^{\mu}_{\rm ren}\equiv\hat{\Gamma}^{\mu}+T_L\gamma_{\mu}L+T_R\gamma_{\mu}R$, where T_L and T_R are constants related to constants C and D in Eq. (13). (See Appendix D for the exact relation.) We call T_L and T_R the counterterms for the vertex. From the definition of $\Sigma^{ij}_{\rm ren}$, it follows that $k_{\mu}\Gamma^{\mu}_{\rm ren}|_{\text{on-shell}}=0$. This is the quickest way to determine the constants T_L and T_R and the one we adopt. R^{G}_{μ} does not obey the WT identity and no counterterms are possible for this quantity. We verify that $k^{\mu}R^{G}_{\mu}|_{\text{on-shell}}=0$, thus the renormalized proper vertex $\Gamma^{\rm ren}_{\mu}$ satisfies current conservation. The power of this renormalization scheme is that it is not necessary to calculate anything more for the on-shell flavor-changing electromagnetic vertex. For generality we shall present the off-shell vertex also. We shall discuss in the Appendix how this renormalization scheme is equivalent to the usual scheme where many unrenormalized diagrams are added.

B. R^{μ}_{W1}: The part of Fig. 3(a) obeying a Ward-Takahashi identity

The on-shell current-conserving part of the $WW\gamma$ vertex in Eq. (1c) is

$$-ie_W[g_{\alpha\beta}(2q+k)_{\mu}-g_{\mu\alpha}k_{\beta}+g_{\mu\beta}k_{\alpha}]$$

and therefore

$$\begin{aligned}R^{\mu}_{W1}=&\left[\frac{-ig}{\sqrt{2}}\right]^2(-ie_W)(\mu^2)^{\epsilon/2}\\&\times\int\frac{d^nq}{(2\pi)^n}\gamma_{\alpha}L\frac{(-i)}{(q+k)^2-M_W{}^2}[g^{\alpha\beta}(2q+k)^{\mu}-g^{\mu\alpha}k^{\beta}+g^{\mu\beta}k^{\alpha}]\frac{(-i)}{q^2-M_W{}^2}\frac{i}{\not{p}+\not{q}-m}\gamma_{\beta}L\\=&\frac{g^2e_W}{2}(\mu^2)^{\epsilon/2}\int\frac{d^nq}{(2\pi)^n}[(\epsilon-2)(2q+k)^{\mu}(\not{p}+\not{q})-\gamma^{\mu}(\not{p}+\not{q})\not{k}+\not{k}(\not{p}+\not{q})\gamma^{\mu}]L\\&\times\frac{1}{(q^2-M_W{}^2)[(q+k)^2-M_W{}^2][(p+q)^2-m^2]}\,,\end{aligned}\tag{17}$$

where the last step follows from n-dimensional Dirac algebra (Appendix A). After Feynman parametrization and n-dimensional integration (Appendix A), we obtain

$$R^{\mu}_{W1}=\frac{-ig^2e_W}{32\pi^2}\int_0^1 d\alpha_p\int_0^{1-\alpha_p}d\alpha_k\left\{\frac{-2[-2p^\mu\alpha_p+k^\mu(1-2\alpha_k)][\not p(1-\alpha_p)-\not k\alpha_k]}{X}+\frac{(\not p\gamma^\mu\not k-\not k\gamma^\mu\not p)(1-\alpha_p)}{X}+2\gamma^\mu\left[\frac{2}{\epsilon}-\ln\frac{X}{4\pi\mu^2}+\Gamma'-1\right]\right\}L\,, \tag{18}$$

where

$$X=M_W{}^2(1-\alpha_p)+m^2\alpha_p-p^2\alpha_p(1-\alpha_p)+k^2\alpha_k(\alpha_k-1)+2p\cdot k\alpha_p\alpha_k\,, \tag{19}$$

m is the mass of the internal quark with flavor l, and a summation $\sum_l U_{il}U_{lj}^\dagger$ is assumed here and in all the subsequent expressions for R and Λ.

The result in Eq. (18) is typical to all the diagrams in Fig. 3. The α_k integration is simple, however the α_p integration for any external momenta and internal and external masses involves Spence[18] functions and although straightforward, it is very lengthy and will not be given here. At various limits relevant for specific physical applications as discussed in the Introduction, the α_p integrals become simple too.

C. R^{μ}_{W2}: The part of Fig. 3(a) violating the Ward-Takahashi identity

The part of the $WW\gamma$ vertex in Eq. (1c) violating on-shell current conservation is $-ie_W[-g_{\mu\alpha}(q+k)_\beta-g_{\mu\beta}q_\alpha]$. Thus

$$R^{\mu}_{W2}=\left[\frac{-ig}{\sqrt{2}}\right]^2(-ie_W)(\mu^2)^{\epsilon/2}\times\int\frac{d^nq}{(2\pi)^n}\gamma_\alpha L\frac{(-i)}{(q+k)^2-M_W{}^2}[-g^{\mu\alpha}(q+k)^\beta-g^{\mu\beta}q^\alpha]\frac{(-i)}{q^2-M_W{}^2}\frac{i}{\not p+\not q-m}\gamma_\beta L$$
$$=\frac{g^2e_W}{2}(\mu^2)^{\epsilon/2}\int\frac{d^nq}{(2\pi)^n}[-\gamma^\mu(\not p+\not q)(\not q+\not k)-\not q(\not p+\not q)\gamma^\mu]L\times\frac{1}{(q^2-M_W{}^2)[(q+k)^2-M_W{}^2][(p+q)^2-m^2]} \tag{20}$$

and following the by-now familiar steps

$$R^{\mu}_{W2}=\frac{-ig^2e_W}{32\pi^2}\int_0^1 d\alpha_p\int_0^1 d\alpha_k\left\{\frac{-\gamma^\mu[\not p(1-\alpha_p)-\not k\alpha_k][-\not p\alpha_p+\not k(1-\alpha_k)]}{X}+\frac{(\not p\alpha_p+\not k\alpha_k)[\not p(1-\alpha_p)-\not k\alpha_k]\gamma^\mu}{X}+2\gamma^\mu\left[\frac{4}{\epsilon}-2\ln\frac{X}{4\pi\mu^2}+2\Gamma'-1\right]\right\}L\,, \tag{21}$$

where X is defined by Eq. (19).

D. R_S^μ: Fig. 3(b)

R_S^μ obeys a Ward-Takahashi identity and is given by

$$R_S^\mu=\left[\frac{ig}{M_W\sqrt{2}}\right]^2 ie_S(\mu^2)^{\epsilon/2}\int\frac{d^nq}{(2\pi)^n}(m_jL-mR)\frac{i}{(q+k)^2-M_W^2}(2q+k)^\mu\frac{i}{q^2-M_W^2}\frac{i}{\not p+\not q-m}(-mL+m_iR)$$

$$=\frac{-g^2e_W}{2M_W^2}(\mu^2)^{\epsilon/2}\int\frac{d^nq}{(2\pi)^n}[-m^2(2q+k)^\mu(m_jL+m_iR)$$

$$+(2q+k)^\mu(\not p+\not q)(m^2L+m_im_jR)]$$

$$\times\frac{1}{(q^2-M_W^2)[(q+k)^2-M_W^2][(p+q)^2-m^2]}\,. \tag{22}$$

Therefore

$$R_S^\mu=\frac{ig^2e_W}{32\pi^2M_W^2}\int_0^1 d\alpha_p\int_0^{1-\alpha_p}d\alpha_k\left\{\frac{-m^2[2p^\mu\alpha_p+k^\mu(1-2\alpha_k)](m_jL+m_iR)}{X}\right.$$
$$+\frac{[-2p^\mu\alpha_p+k^\mu(1-2\alpha_k)][\not p(1-\alpha_p)-\not k\alpha_k](m^2L+m_im_jR)}{X}$$
$$\left.-\gamma_\mu\left[\frac{2}{\epsilon}-\ln\frac{X}{4\pi\mu^2}+\Gamma'\right](m^2L+m_im_jR)\right\}. \tag{23}$$

E. R_{SW}^μ: Figs. 3(c) and 3(d)

We now calculate the contribution of diagrams 3(c) and (d) which include the $WS\gamma$ vertex. This contribution, denoted by R_{SW}^μ, will be later combined with R_{W2}^μ to yield the complete part R_G^μ of Γ^μ which does not obey a Ward-Takahashi identity.

By adding the diagrams in Figs. 3(c) and 3(d) we obtain

$$R_{SW}^\mu=\left[\frac{ig}{M_W\sqrt{2}}\right]\left[\frac{-ig}{\sqrt{2}}\right]ie_W(\mu^2)^{\epsilon/2}\int\frac{d^nq}{(2\pi)^n}\left[(m_jL-mR)\frac{i}{(q+k)^2-M_W^2}g^{\beta\mu}M_W\frac{(-i)}{q^2-M_W^2}\frac{i}{\not p+\not q-m}\gamma_\beta L\right.$$
$$+\gamma_\beta L\frac{(-i)}{(q+k)^2-M_W^2}$$
$$\left.\times g^{\beta\mu}M_W\frac{i}{q^2-M_W^2}\frac{i}{\not p+\not q-m}(-mL+m_iR)\right]$$

$$=\frac{-g^2e_W}{2}(\mu^2)^{\epsilon/2}\int\frac{d^nq}{(2\pi)^n}[m_j(\not p+\not q)\gamma^\mu L+\gamma^\mu m_i(\not p+\not q)R-2m^2\gamma^\mu L]$$
$$\times\frac{1}{(q^2-M_W^2)[(q+k)^2-M_W^2][(p+q)^2-m^2]}\,, \tag{24}$$

which reduces to

$$R_{SW}^\mu=\frac{ig^2e_W}{32\pi^2}\int_0^1 d\alpha_p\int_0^{1-\alpha_p}d\alpha_k\left\{\frac{m_j[\not p(1-\alpha_p)-\not k\alpha_k]\gamma^\mu L}{X}+\frac{\gamma^\mu m_i[\not p(1-\alpha_p)-\not k\alpha_k]R}{X}-\frac{2m^2\gamma_\mu L}{X}\right\}. \tag{25}$$

We first note that unlike all the other contributors to Γ^μ, R^μ_{SW} is finite. Like all the other parts in R^μ, R^μ_{SW} changes sign when e_W changes sign. While for all other parts of R^μ the sign change results from reversing the sign of e_W $(=e_S)$ in the photon coupling, R^μ_{SW} flips its sign since the $WS\gamma$ coupling is invariant but the single coupling of S with the quarks reverses its sign.

We now turn to calculate the diagrams in Figs. 3(e) and 3(f) denoted by Λ^μ_W and Λ^μ_S, respectively. It is easy to see that each of the Λ^μ diagrams obeys a Ward-Takahashi identity.

F. Λ^μ_W: Fig. 3(e)

We have

$$\Lambda^\mu_W=\left[\frac{-ig}{\sqrt{2}}\right]^2 ie_l(\mu^2)^{\epsilon/2}\int\frac{d^nq}{(2\pi)^n}\gamma_\alpha L\frac{i}{\not p+\not q-\not k-m}\gamma^\mu\frac{i}{\not p+\not q-m}\frac{(-i)}{q^2-M_W{}^2}\gamma^\alpha L$$
$$=\frac{g^2e_l}{2}(\mu^2)^{\epsilon/2}\int\frac{d^nq}{(2\pi)^n}[-2(\not p+\not q)\gamma^\mu(\not p+\not q-\not k)-2\gamma^\mu m^2+\epsilon\not q\gamma^\mu\not q]L$$
$$\times\frac{1}{(q^2-M_W{}^2)[(p+q)^2-m^2][(p+q-k)^2-m^2]}\,. \tag{26}$$

Therefore

$$\Lambda^\mu_W=\frac{-ig^2e_l}{16\pi^2}\int_0^1 d\alpha_p\int_0^{1-\alpha_p}d\alpha_k\left\{\frac{-(\not p\alpha_p+\not k\alpha_k)\gamma^\mu[\not p\alpha_p+\not k(\alpha_k-1)]-\gamma_\mu m^2}{Y}\right.$$
$$\left.+\gamma^\mu\left[-\frac{2}{\epsilon}+2-\Gamma'+\ln\frac{Y}{4\pi\mu^2}\right]\right\}L\,, \tag{27}$$

where

$$Y=M_W{}^2\alpha_p+m^2(1-\alpha_p)+p^2\alpha_p(\alpha_p-1)+k^2\alpha_k(\alpha_k-1)+2p\cdot k\alpha_p\alpha_k\,. \tag{28}$$

G. Λ^μ_S: Fig. 3(f)

The last diagram in Fig. 3 is

$$\Lambda^\mu_S=\left[\frac{ig}{M_W\sqrt{2}}\right]^2 ie_l(\mu^2)^{\epsilon/2}\int\frac{d^nq}{(2\pi)^n}(m_jL-mR)\frac{i}{\not p+\not q-\not k-m}\gamma^\mu\frac{i}{\not p+\not q-m}\frac{i}{q^2-M_W{}^2}(-mL+m_iR)$$
$$=\frac{-g^2e_l}{2M_W{}^2}(\mu^2)^{\epsilon/2}\int\frac{d^nq}{(2\pi)^n}\{-m^2[\gamma^\mu(\not p+\not q)+(\not p+\not q-k)\gamma^\mu](m_jL+m_iR)$$
$$+[(p+q-\not k)\gamma^\mu(\not p+\not q)+m^2\gamma^\mu](m^2L+m_im_jR)\}$$
$$\times\frac{1}{(q^2-M_W{}^2)[(p+q)^2-m^2][(p+q-k)^2-m^2]}\,, \tag{29}$$

which becomes

$$\Lambda^\mu_S=\frac{ig^2e_l}{32\pi^2M_W{}^2}\int_0^1 d\alpha_p\int_0^{1-\alpha_p}d\alpha_k\left\{\frac{\{[\not p\alpha_p+\not k(\alpha_k-1)]\gamma^\mu(\not p\alpha_p+\not k\alpha_k)+m^2\gamma^\mu\}(m^2L+m_im_jR)}{Y}\right.$$
$$-\frac{m^2[2p^\mu\alpha_p+2k^\mu\alpha_k-\not k\gamma^\mu](m_jL+m_iR)}{Y}$$
$$\left.-\gamma_\mu\left[-\frac{2}{\epsilon}+1-\Gamma'+\ln\frac{Y}{4\pi\mu^2}\right](m^2L+m_im_jR)\right\}\,. \tag{30}$$

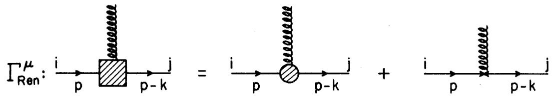

FIG. 4. The unrenormalized proper vertex [see Eq. (35)].

We note that $R^\mu \propto e_W$, $\Lambda^\mu \propto e_l = e_W + e_i$ and therefore the expressions for R^μ and Λ^μ are valid for the flavor-changing electromagnetic vertices of both charge $\frac{2}{3}$ and $-\frac{1}{3}$ quarks.

V. $\Gamma^\mu_{\rm ren}$: RENORMALIZING THE PROPER VERTEX

Γ^μ as calculated in Sec. IV contains $1/\epsilon$ and has to be renormalized. The counterterm added to Γ^μ (see Fig. 4) will also ensure that for the renormalized proper vertex current conservation holds for on-shell transitions, i.e., $k_\mu \Gamma^\mu_{\rm ren}=0$ when both

$$\not p u_i = m_i u_i \text{ and } \bar u_j m_j = \bar u_j(\not p - \not k) \;. \tag{31}$$

For each term in Γ^μ one should find two constants (i.e., independent of external momenta) T_L and T_R such that

$$\Gamma^\mu_{\rm ren} = \Gamma^\mu + T_L \gamma^\mu L + T_R \gamma^\mu R \;, \tag{32}$$

subject to the on-shell condition in Eq. (31). We will compute T_L and T_R for each term of the proper vertex separately and find that the counterterm for R^μ_G—i.e., that part of Γ^μ which does not obey the Ward-Takahashi identity—is zero in the KM model.

The technique we employ to determine counterterms is rather straightforward because of current conservation in the $ij\gamma$ vertex. Alternately, T_L and T_R can be obtained from the off-diagonal self-energy by employing the Ward-Takahashi identity. We explain this in Appendix D.

For the total KM current which involves the summation $\sum_l U_{il} U^\dagger_{lj}$, some of the $1/\epsilon$ terms in Γ^μ will drop out since their coefficient is independent of the intermediate mass m. Nevertheless a counterterm will be needed (except for $R^\mu_G \equiv R^\mu_{W2} + R^\mu_{SW}$) to ensure on-shell current conservation.

The counterterms are computed by multiplying each term in Γ^μ by the photon momentum k_μ, and by imposing the on-shell condition in Eq. (31) at $k^2=0$ (T_L and T_R are momentum independent). We denote by $T_L^{R_{W1}}$, $T_R^{R_{W1}}$ the constants in

$$R^\mu_{W1,\rm ren} = R^\mu_{W1} + T_L^{R_{W1}} \gamma^\mu L + T_R^{R_{W1}} \gamma^\mu R \;, \tag{33}$$

etc. We define the on-shell, $k^2=0$ values of X and Y from Eqs. (19) and (28) as

$$X_0 = M_W{}^2(1-\alpha_p) + m^2\alpha_p - m_i{}^2\alpha_p(1-\alpha_p) + (m_i{}^2 - m_j{}^2)\alpha_p\alpha_k \;, \tag{34a}$$

$$Y_0 = M_W{}^2\alpha_p + m^2(1-\alpha_p) - m_i{}^2\alpha_p(1-\alpha_p) + (m_i{}^2 - m_j{}^2)\alpha_p\alpha_k \;, \tag{34b}$$

and obtain

$$T_R^{R_{W1}} = \frac{ig^2 e_W}{16\pi^2}\int_0^1 d\alpha_p \int_0^{1-\alpha_p} d\alpha_k \frac{m_i m_j(1-\alpha_p)}{X_0} \;, \tag{35a}$$

$$T_L^{R_{W1}} = -\frac{ig^2 e_W}{16\pi^2}\int_0^1 d\alpha_p \int_0^{1-\alpha_p} d\alpha_k \left[\frac{(m_i{}^2-m_j{}^2)\alpha_p\alpha_k - m_i{}^2\alpha_p(1-\alpha_p)}{X_0} - \frac{2}{\epsilon} + \ln\frac{X_0}{4\pi\mu^2} - \Gamma' + 1 \right] , \tag{35b}$$

$$T_R^{R_G} = 0 \;, \tag{35c}$$

$$T_L^{R_G} = -\frac{ig^2 e_W}{32\pi^2}\int_0^1 d\alpha_p \int_0^{1-\alpha_p} d\alpha_k \left[\frac{2m_i{}^2(1-\alpha_p)(1-\alpha_p-\alpha_k) + 2m_j{}^2(1-\alpha_p)\alpha_k - 2m^2}{X_0} - 2\left[\frac{4}{\epsilon} - 2\ln\frac{X_0}{4\pi\mu^2} + 2\Gamma' - 1\right]\right] , \tag{35d}$$

$$T_R^{R_S} = -\frac{ig^2 e_W}{32\pi^2 M_W{}^2}\int_0^1 d\alpha_p \int_0^{1-\alpha_p} d\alpha_k \left\{ \frac{m_i m_j[(m_i{}^2-m_j{}^2)\alpha_p\alpha_k + m^2\alpha_p(1+\alpha_p) - m_i{}^2\alpha_p(1-\alpha_p)]}{X_0} - m_i m_j \left[\frac{2}{\epsilon} - \ln\frac{X_0}{4\pi\mu^2} + \Gamma'\right]\right\} , \tag{35e}$$

$$T_L^{R_S}=-\frac{ig^2e_W}{32\pi^2M_W{}^2}\int_0^1 d\alpha_p\int_0^{1-\alpha_p}d\alpha_k\left[\frac{m^2(m_i{}^2-m_j{}^2)\alpha_p\alpha_k+m^2\alpha_p(\alpha_p m_i{}^2+m_j{}^2)}{X_0}-\frac{m_j{}^2m_i{}^2\alpha_p(1-\alpha_p)}{X_0}-m^2\left[\frac{2}{\epsilon}-\ln\frac{X_0}{4\pi\mu^2}+\Gamma'\right]\right], \tag{35f}$$

$$T_R^{A_W}=-\frac{ig^2e_l}{16\pi^2}\int_0^1 d\alpha_p\int_0^{1-\alpha_p}d\alpha_k\frac{m_im_j\alpha_p{}^2}{Y_0}, \tag{35g}$$

$$T_L^{A_W}=-\frac{ig^2e_l}{16\pi^2}\int_0^1 d\alpha_p\int_0^{1-\alpha_p}d\alpha_k\left[\frac{m^2}{Y_0}+\frac{2}{\epsilon}-2+\Gamma'-\ln\frac{Y_0}{4\pi\mu^2}\right], \tag{35h}$$

$$T_R^{A_S}=-\frac{ig^2e_l}{32\pi^2M_W{}^2}\int_0^1 d\alpha_p\int_0^{1-\alpha_p}d\alpha_k m_im_j\left[\frac{m^2(1-\alpha_p)^2}{Y_0}+\frac{2}{\epsilon}-1+\Gamma'-\ln\frac{Y_0}{4\pi\mu^2}\right], \tag{35i}$$

$$T_L^{A_S}=-\frac{ig^2e_l}{32\pi^2M_W{}^2}\int_0^1 d\alpha_p\int_0^{1-\alpha_p}d\alpha_k\left[\frac{\alpha_p{}^2m_i{}^2m_j{}^2+m^4-m^2(m_i{}^2+m_j{}^2)\alpha_p}{Y_0}+m^2\left[\frac{2}{\epsilon}-1+\Gamma'-\ln\frac{Y_0}{4\pi\mu^2}\right]\right]. \tag{35j}$$

We now turn to the counterterm of R_G^μ. From Eq. (35c), $T_R^{R_G}=0$ even before summation over intermediate quarks. The counterterm $T_L^{R_G}$ multiplying $\gamma^\mu L$ in R_G^μ vanishes too, but only after summation over intermediate quarks. The proof is rather lengthy; in Appendix C we present a proof that

$$\sum_l U_{il}U_{lj}^\dagger T_L^{R_G}=0 \tag{36}$$

for $m_i=m_j$. Therefore

$$k_\mu R_G^\mu=0 \text{ when } \not p u_i=m_iu_i \text{ and } \bar u_jm_j=\bar u_j(\not p-\not k) \tag{37a}$$

and

$$R_{G,\text{ren}}^\mu=R_G^\mu\ . \tag{37b}$$

The infinite part of R_G^μ [coming from the $1/\epsilon$ term in R_{W2}^μ; see Eq. (21)] vanishes once we sum over all intermediate states.

We have now completed the computation of Γ_{ren}^μ, the renormalized proper vertex which is shown in Fig. 4. The complete off-shell flavor-changing electromagnetic vertex V_{ren}^μ is given as a sum (see Fig. 5)

$$V_{\text{ren}}^\mu=\Gamma_{\text{ren}}^\mu+\Sigma_{1,\text{ren}}^\mu+\Sigma_{2,\text{ren}}^\mu\ , \tag{38}$$

where $\Sigma_{1,\text{ren}}^\mu$ and $\Sigma_{2,\text{ren}}^\mu$ are shown in Figs. 5(b) and 5(c), respectively. In the next section we compute V_{ren}^μ.

VI. V_{ren}^μ: THE COMPLETE FLAVOR-CHANGING VERTEX

The complete flavor-changing electromagnetic vertex is given in Eq. (38) (see Fig. 5) with $\Sigma_{1,\text{ren}}^\mu$, $\Sigma_{2,\text{ren}}^\mu$ still to be computed.

A. $\Sigma_{1,\text{ren}}^\mu$ and $\Sigma_{2,\text{ren}}^\mu$

$\Sigma_{1,\text{ren}}^\mu$ and $\Sigma_{2,\text{ren}}^\mu$ are the parts of the vertex which include self-energy diagrams [see Figs. 5(b) and 5(c)]. When quarks i and j are off-shell, both $\Sigma_{1,\text{ren}}^\mu$ and $\Sigma_{2,\text{ren}}^\mu$ contribute to V_{ren}^μ. When both i and j are on-shell (e.g., $e^+e^-\to t\bar c$), there is no contribution from $\Sigma_{1,\text{ren}}^\mu$ and $\Sigma_{2,\text{ren}}^\mu$ since each term vanishes when one quark is on shell,

$$\bar u_j\Sigma_{1,\text{ren}}^\mu[(p-k)^2=m_j{}^2]=0\ , \tag{39a}$$

$$\Sigma_{2,\text{ren}}^\mu(p^2=m_i{}^2)u_i=0\ . \tag{39b}$$

From Figs. 5(b) and 5(c)

$$\Sigma_{1,\text{ren}}^\mu(p)=-e_i\Sigma_{\text{ren}}(p-k)(\not p-\not k+m_i)\gamma^\mu\times\frac{1}{(p-k)^2-m_i{}^2}\ , \tag{40a}$$

$$\Sigma_{2,\text{ren}}^\mu(p)=-e_j\gamma^\mu(\not p+m_j)\Sigma_{\text{ren}}(p)\frac{1}{p^2-m_j{}^2}\ , \tag{40b}$$

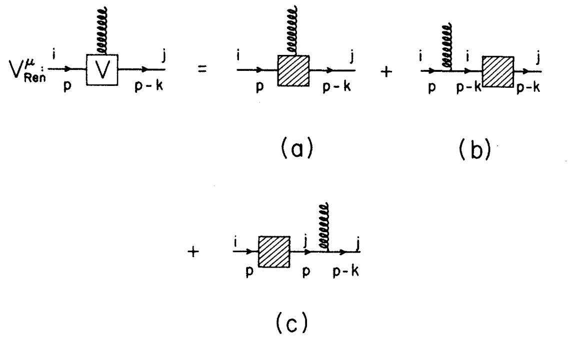

FIG. 5. The complete renormalized vertex function [see Eq. (39)].

where $\Sigma_{\rm ren}(p)$ is given in Eq. (13). For $\Sigma_{\rm ren}(p-k)$ we have the same expression as in Eq. (13) but with

$$\bar{f}\equiv f((p-k)^2),\quad \bar{h}\equiv h((p-k)^2),$$
$$\bar{\psi}\equiv\psi((p-k)^2),\quad \bar{\phi}=\phi((p-k)^2) \tag{41}$$

substituted for $f(p)$, $h(p)$, $\psi(p)$, $\phi(p)$. It is convenient to rewrite Eq. (13) for the renormalized self-energy as follows:

$$\begin{aligned}\Sigma_{\rm ren}(p)=&(p\!\!\!/-m_j)(f+C)L+(p\!\!\!/-m_j)(h+D)R\\&+[\psi+A+m_j(f+C)]L\\&+[\phi+B+m_j(h+D)]R\end{aligned} \tag{42}$$

with $f+C$, $h+D$ as given in Eqs. (14a) and (14b) and [again from Eq. (14)]

$$\psi+A+m_j(f+C)=m_j(f-f_j)+\psi-\psi_j\ , \tag{43a}$$

$$\phi+B+m_j(h+D)=m_j(h-h_j)+\phi-\phi_j\ . \tag{43b}$$

$f-f_j$, $\psi-\psi_j$, $h-h_j$, and $\phi-\phi_j$ were calculated in Appendix B. Similarly

$$\begin{aligned}\Sigma_{\rm ren}(p-k)=&(\bar{f}+C)R(p\!\!\!/-k\!\!\!/-m_i)\\&+(\bar{h}+D)L(p\!\!\!/-k\!\!\!/-m_i)\\&+[\bar{\psi}+A+m_i(\bar{h}+D)]L\\&+[\bar{\phi}+B+m_i(\bar{f}+C)]R\ ,\end{aligned} \tag{44}$$

where $\bar{f}+C$, $\bar{h}+D$ are given in Eqs. (14a) and (14b) with $\bar{f}-f_j$, $\bar{f}-f_i$, etc. substituted for $f-f_j$, $f-f_i$, etc. respectively. $\bar{f}-f_j$, $\bar{f}-f_i$, etc., are given in Eq. (B4) and the integrals I and J are calculated with

$$\bar{\Delta}=M_W{}^2\alpha+m^2(1-\alpha)-(p-k)^2\alpha(1-\alpha) \tag{45}$$

instead of Δ defined in Eq. (8). The net effect of calculating $\Sigma_{\rm ren}(p-k)$ with the help of the formulas in Appendix B is then to substitute $\bar{b}=(p-k)^2/M_W{}^2$ instead of $b=p^2/M_W{}^2$ defined in Eq. (B7). The equations analogous to Eq. (43), but now for $\Sigma_{\rm ren}(p-k)$ are

$$\bar{\psi}+A+m_i(\bar{h}+D)=m_i(\bar{h}-h_i)+\bar{\psi}-\psi_i\ , \tag{46a}$$

$$\bar{\phi}+B+m_i(\bar{f}+C)=m_i(\bar{f}-f_i)+\bar{\phi}-\phi_i\ . \tag{46b}$$

The vertex functions which involve the self-energy diagrams are then, from Eqs. (40a) and (44),

$$\begin{aligned}\Sigma^{\mu}_{1,\rm ren}=-e_i\Bigg\{&(\bar{f}+C)R+(\bar{h}+D)L+\frac{[\bar{\psi}+A+m_i(\bar{h}+D)]L(p\!\!\!/-k\!\!\!/+m_i)}{(p-k)^2-m_i{}^2}\\&+\frac{[\bar{\phi}+B+m_i(\bar{f}+C)]R(p\!\!\!/-k\!\!\!/+m_i)}{(p-k)^2-m_i{}^2}\Bigg\}\gamma^{\mu}\end{aligned} \tag{47}$$

and from Eqs. (40b) and (42)

$$\begin{aligned}\Sigma^{\mu}_{2,\rm ren}=-e_j\gamma^{\mu}\Bigg\{&(f+C)L+(h+D)R+\frac{[\psi+A+m_j(f+C)]L(p\!\!\!/+m_j)}{p^2-m_j{}^2}\\&+\frac{[\phi+B+m_j(h+D)]R(p\!\!\!/+m_j)}{p^2-m_j{}^2}\Bigg\}\ ,\end{aligned} \tag{48}$$

from which one can readily verify the on-shell conditions in Eq. (39).

At this point we have the complete renormalized off-shell vertex $V^{\mu}_{\rm ren}$ which will be now written in a compact form.

B. Collecting all the parts of $V^{\mu}_{\rm ren}$

$V^{\mu}_{\rm ren}$ is given by

$$V^{\mu}_{\rm ren}=\Gamma^{\mu}_{\rm ren}+\Sigma^{\mu}_{1,\rm ren}+\Sigma^{\mu}_{2,\rm ren}=R^{\mu}_{W1,\rm ren}+R^{\mu}_{G,\rm ren}+R^{\mu}_{S,\rm ren}+\Lambda^{\mu}_{W,\rm ren}+\Lambda^{\mu}_{S,\rm ren}+\Sigma^{\mu}_{1,\rm ren}+\Sigma^{\mu}_{2,\rm ren}\;. \tag{49}$$

It is of course finite and obeys

$$k_\mu V^{\mu}_{\rm ren}=0 \text{ when both } \not p u_i=m_i u_i \text{ and } \bar u_j m_j=\bar u_j(\not p-\not k)\;. \tag{50}$$

Each of its seven parts [Eq. (49)] obeys the same on-shell current conservation condition as in Eq. (50). To unify the notation and to put all the parts of $V^{\mu}_{\rm ren}$ in a form useful for future applications, we decompose each part of $V^{\mu}_{\rm ren}$ as follows:

$$\begin{aligned}&(A_1\not p p^\mu+A_2\not p k^\mu+A_3\not k p^\mu+A_4\not k k^\mu+A_5\not k\gamma^\mu\not p+A_6\gamma^\mu+A_7\gamma^\mu k^2\\&+A_8\gamma^\mu k\cdot p+A_9\gamma^\mu p^2+A_{10}\gamma^\mu\not p+A_{11}\gamma^\mu\not k+A_{12}p^\mu+A_{13}k^\mu)L\\&+(B_1\not p p^\mu+B_2\not p k^\mu+B_3\not k p^\mu+B_4\not k k^\mu+B_5\not k\gamma^\mu\not p+B_6\gamma^\mu+B_7\gamma^\mu k^2\\&\quad+B_8\gamma^\mu k\cdot p+B_9\gamma^\mu p^2+B_{10}\gamma^\mu\not p+B_{11}\gamma^\mu\not k+B_{12}p^\mu+B_{13}k^\mu)R\;,\end{aligned} \tag{51}$$

where A_i and B_i are functions of the incoming momentum p, the photon momentum k, and the external (m_i,m_j) and intermediate (m) quark masses.

We make a few comments about A_i and B_i before presenting them.

(1) For all terms in $\Gamma^{\mu}_{\rm ren}$, only A_6 and B_6 are different from their unrenormalized value [see Eq. (32)].

(2) There are some relations among different A_i,B_i as a result of the Ward-Takahashi identities and the on-shell current-conservation conditions. We used these relations to check our computation. In Appendix D we investigate the relations for those parts in $\Gamma^{\mu}_{\rm ren}$ which obey the Ward-Takahashi identities; an obvious observation is that $A_{10}=0$ and $B_{10}=0$ for all parts in $\Gamma^{\mu}_{\rm ren}$ except for R^{μ}_{G}.

In Table I we summarize A_i and B_i as defined in Eq. (51) for all the seven renormalized parts of $V^{\mu}_{\rm ren}$. To avoid lengthy entries in the tables we pull out a common factor, separately for each term in $V^{\mu}_{\rm ren}$. Thus A_1 for R^{μ}_{W1} is equal to

$$-\frac{ig^2e_W}{32\pi^2}\sum_l U_{il}U^{\dagger}_{lj}\int_0^1 d\alpha_p\int_0^{1-\alpha_p}d\alpha_k\frac{4\alpha_p(1-\alpha_p)}{X}\;,$$

etc. Note that m which appears for example in X in the above term is an intermediate quark mass and it therefore depends on l. Note also that for the Λ terms $e_l=e_W+e_i$. In $R^{\mu}_{G,\rm ren}$ we leave in the table the $\ln(4\pi\mu^2)$ term for A_6, which of course cancels out after the summation over intermediate quarks

All A_i and B_i in the proper vertex part of $V^{\mu}_{\rm ren}$ are given in terms of a double integral. The α_k integration is simple but the final integral will require a lengthy presentation. For any specific physical process a limiting procedure turns the integration into a simple matter.[14] Our results can be then applied to all problems which require the electromagnetic or strong transition $i\to j$.

In many applications one or both the external quarks are on-shell, and we will present in Appendix E the general form of $V^{\mu}_{\rm ren}$ for these cases. When one quark is on-shell the number of coefficients in $V^{\mu}_{\rm ren}$ is reduced from 26 to 16, and further to 6 when both quarks are on-shell (see Appendix E).

VII. SUMMARY

We have calculated the off-diagonal electromagnetic vertex $V^{\mu}_{\rm ren}$ for quarks. The results are summarized in Table I, where $V^{\mu}_{\rm ren}$ is the sum of all the terms as given in Eqs. (49) and (51). This is the first such calculation which does not assume on-shell external quarks masses and is valid for all external momenta; it is therefore applicable to a wide range of phenomena.

A similar calculation can be carried out for the flavor-changing ijZ vertex, where Z is the neutral gauge boson.[19] For the flavor-changing strong vertex ij-gluon, the results are identical to the $ij\gamma$ vertex presented here except that $R^\mu=0$, and a common group-theoretic factor appears in the elementary quark-quark-gluon vertex.

It is sometimes stated[9] that the contribution of unphysical scalars in the 't Hooft—Feynman gauge is negligible when quark masses are small as compared to M_W. This is incorrect, as is evident from inspection of our results for R^μ_{SW} [Figs. 3(c) and 3(d)] in Eq. (25). Furthermore, while the calculation is easier than in the unitary gauge, where the W-boson propagator is $(-g_{\mu\nu}+q_\mu q_\nu/M_W{}^2)/(q^2-M_W{}^2)$, the Ward-Takahashi identities are obeyed by only a part of the proper vertex. It is a piece of a diagram with a W boson which causes violation of the identity for the proper vertex.

Our calculation comes at a time when the mass of the top quark—if it exists—is clearly not negligible with respect to M_W, and one can investigate processes which are sensitive to it.[10] Furthermore, tests of loop corrections to the unified theory of weak and electromagnetic interactions of quarks are becoming feasible; these corrections as tested through an electromagnetic (or a strong) probe, where the subject of our investigation. Applications of our general result to physical processes will be published elsewhere.[14]

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APPENDIX A: n-DIMENSIONAL ALGEBRA AND INTEGRALS

The Dirac algebra in n dimensions ($\epsilon=4-n$) is as follows:

$$\begin{aligned}&\gamma_\mu\gamma^\mu=n\underline{1}\,,\\&\gamma_\mu\gamma_\nu\gamma^\mu=(\epsilon-2)\gamma_\nu\,,\\&\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\lambda\gamma^\mu=-2\gamma_\lambda\gamma_\alpha\gamma_\nu+\epsilon\gamma_\nu\gamma_\alpha\gamma_\lambda\,,\\&\{\gamma_5,\gamma_\mu\}=0\quad(\mu=0,1,\ldots,n-1)\,;\end{aligned}\tag{A1}$$

for discussion of the last identity see Ref. 20.

We also use

$$\begin{aligned}&\Gamma\left[\frac{\epsilon}{2}\right]=\frac{2}{\epsilon}+\Gamma'(1)+O(\epsilon)\,,\\&A^{-\epsilon/2}=1-\frac{\epsilon}{2}\ln A+O(\epsilon^2)\,,\end{aligned}\tag{A2}$$

and the Feynman parametrization

$$\frac{1}{a_1a_2\cdots a_N}=\Gamma(N)\int_0^1 d\alpha_1\int_0^{\alpha_1}d\alpha_2\cdots\int_0^{\alpha_{N-2}}d\alpha_{N-1}\times\frac{1}{[a_1\alpha_{N-1}+a_2(\alpha_{N-2}-\alpha_{N-1})+\cdots a_N(1-\alpha_1)]^N}\,.\tag{A3}$$

We will now calculate the general n-dimensional loop integral,

$$I(N,\alpha)\equiv\int\frac{d^nq}{(2\pi)^n}\frac{q_{\mu_1}q_{\mu_2}\cdots q_{\mu_N}}{(q^2+2q\cdot P-M^2)^\alpha}\,.\tag{A4}$$

In fact we use it here only for $\alpha=2,3$ and $N=0,1,2$. The integrals for these values appear in numerous places in the literature. Nevertheless, we present the general result for readers who would like to verify our calculation in the unitary gauge where integrals up to $I(6,\alpha)$ are needed. The results for up to $N=4$ can be found in Passarino and Veltman (Ref. 2), and it agrees with our general compact form in which the pole structure is transparent [see Eq. (A9)].

We first use the identity

$$\int \frac{d^nq}{(2\pi)^n}\frac{q_{\mu_1}q_{\mu_2}\cdots q_{\mu_N}}{(q^2+2q\cdot P-M^2)^\alpha}=\frac{(-1)^\alpha i\pi^{n/2}}{(2\pi)^n\Gamma(\alpha)}\int_0^\infty \frac{d\rho}{(2\rho)^n}\rho^{\alpha-1-n/2}\left[\frac{\partial}{\partial P}\right]^N e^{-\rho\Delta}, \tag{A5}$$

where

$$\Delta=M^2+P^2 \tag{A6}$$

and

$$\left[\frac{\partial}{\partial P}\right]^N=\frac{\partial}{\partial P_{\mu_1}}\frac{\partial}{\partial P_{\mu_2}}\cdots\frac{\partial}{\partial P_{\mu_N}}. \tag{A7}$$

Then, generalizing the derivative relation

$$\left[\frac{d}{dx}\right]^N e^{ax^2}=(2ax)^N e^{ax^2}\sum_{j=0}\frac{N(N-1)\cdots(N-2j+1)}{j!(4ax^2)^j}, \tag{A8}$$

we finally obtain

$$I(N,\alpha)=\frac{(-1)^\alpha i\pi^{n/2}(-1)^N\Gamma(\alpha-n/2)}{(2\pi)^n\Gamma(\alpha)\Delta^{\alpha-n/2}}\sum_{j=0}^{[N/2]}\frac{(-1)^j}{(\alpha-n/2-1)\cdots(\alpha-n/2-j)}\left[\frac{\Delta}{2}\right]^j\{g^jP^{N-2j}\}_{\mu_1\cdots\mu_N} \tag{A9}$$

where $[N/2]=0$ for $N=0,1$ and 1 for $N=2,3$ etc. and

$$\{g^jP^{N-2j}\}_{\mu_1\cdots\mu_N}=\text{all permutations of } g\cdots gP\cdots P, \tag{A10}$$

where there are j g's and $(N-2j)$ P's. For a fixed N and j there are $\binom{N}{2j}(2j-1)!!$ term in $\{g^jP^{N-2j}\}_{\mu_1\cdots\mu_N}$.

APPENDIX B: EVALUATION OF INTEGRALS IN Σ_{ren}

The renormalized off-diagonal self-energy is given in Eq. (13), with the coefficients given in Eq. (14). Using the functions f,h,ψ,ϕ as written in Eq. (15) we can now calculate all the terms in the numerators of Eq. (14).

We first define the integrals

$$I_i=\int_0^1 d\alpha\,\alpha\ln\frac{\Delta}{\Delta_i},\quad I_j=\int_0^1 d\alpha\,\alpha\ln\frac{\Delta}{\Delta_j},\quad I_{ij}=\int_0^1 d\alpha\,\alpha\ln\frac{\Delta_i}{\Delta_j}, \tag{B1a}$$

$$J_i=\int_0^1 d\alpha\ln\frac{\Delta}{\Delta_i},\quad J_j=\int_0^1 d\alpha\ln\frac{\Delta}{\Delta_j},\quad J_{ij}=\int_0^1 d\alpha\ln\frac{\Delta_i}{\Delta_j}, \tag{B1b}$$

where Δ is given in Eq. (8) and

$$\Delta_i=\Delta(p^2=m_i^2)=M_W^2\alpha+m^2(1-\alpha)-m_i^2\alpha(1-\alpha), \tag{B2a}$$

$$\Delta_j=\Delta(p^2=m_j^2)=M_W^2\alpha+m^2(1-\alpha)-m_j^2\alpha(1-\alpha). \tag{B2b}$$

Then we define

$$K=\frac{ig^2}{32\pi^2M_W^2}, \tag{B3}$$

and obtain

$$\begin{aligned}
&f-f_j=-2KM_W{}^2I_j-m^2KI_j,\ \ f-f_i=-2KM_W{}^2I_i-m^2KI_i,\ \ f_i-f_j=-2KM_W{}^2I_{ij}-m^2KI_{ij}\ ,\\
&h-h_j=-Km_im_jI_j,\ \ h-h_i=-Km_im_jI_i,\ \ h_i-h_j=-Km_im_jI_{ij}\ ,\\
&\psi-\psi_i=Km_jm^2J_i,\ \ \psi-\psi_j=Km_jm^2J_j,\ \ \psi_i-\psi_j=Km_jm^2J_{ij}\ ,\\
&\phi-\phi_j=Km_im^2J_j,\ \ \phi-\phi_i=Km_im^2J_i,\ \ \phi_i-\phi_j=Km_im^2J_{ij}\ .
\end{aligned} \tag{B4}$$

The integrals I and J are easily obtained in the $p^2,m_i{}^2,m_j{}^2,m^2 << M_W{}^2$ limit. We now present the complete expressions (i.e., no small momenta or masses assumed) for I and J

$$\begin{aligned}
I_i&=\frac{1}{2}\left[-x_1{}^2\ln\frac{1+x_1}{x_1}-x_2{}^2\ln\frac{1+x_2}{x_2}+x_1+x_2+x_{1,i}{}^2\ln\frac{1+x_{1,i}}{x_{1,i}}+x_{2,i}{}^2\ln\frac{1+x_{2,i}}{x_{2,i}}-x_{1,i}-x_{2,i}\right],\\
I_j&=\frac{1}{2}\left[-x_1{}^2\ln\frac{1+x_1}{x_1}-x_2{}^2\ln\frac{1+x_2}{x_2}+x_1+x_2+x_{1,j}{}^2\ln\frac{1+x_{1,j}}{x_{1,j}}+x_{2,j}{}^2\ln\frac{1+x_{2,j}}{x_{2,j}}-x_{1,j}-x_{2,j}\right],\\
I_{ij}&=\frac{1}{2}\left[-x_{1,i}{}^2\ln\frac{1+x_{1,i}}{x_{1,i}}-x_{2,i}{}^2\ln\frac{1+x_{2,i}}{x_{2,i}}+x_{1,i}+x_{2,i}+x_{1,j}{}^2\ln\frac{1+x_{1,j}}{x_{1,j}}+x_{2,j}{}^2\ln\frac{1+x_{2,j}}{x_{2,j}}-x_{1,j}-x_{2,j}\right],
\end{aligned} \tag{B5}$$

$$\begin{aligned}
J_i&=x_1\ln\frac{1+x_1}{x_1}+x_2\ln\frac{1+x_2}{x_2}-x_{1,i}\ln\frac{1+x_{1,i}}{x_{1,i}}-x_{2,i}\ln\frac{1+x_{2,i}}{x_{2,i}}\ ,\\
J_j&=x_1\ln\frac{1+x_1}{x_1}+x_2\ln\frac{1+x_2}{x_2}-x_{1,j}\ln\frac{1+x_{1,j}}{x_{1,j}}-x_{2,j}\ln\frac{1+x_{2,j}}{x_{2,j}}\ ,\\
J_{ij}&=x_{1,i}\ln\frac{1+x_{1,i}}{x_{1,i}}+x_{2,i}\ln\frac{1+x_{2,i}}{x_{2,i}}-x_{ij}\ln\frac{1+x_{1,j}}{x_{1,j}}-x_{2,j}\ln\frac{1+x_{2,j}}{x_{2,j}}
\end{aligned}$$

with

$$x_1=\frac{1-a-b+[(1-a-b)^2-4ab]^{1/2}}{2b},\ \ x_2=\frac{1-a-b-[(1-a-b)^2-4ab]^{1/2}}{2b}\ ,$$

$$x_{1,i}=\frac{1-a-b_i+[(1-a-b_i)^2-4ab_i]^{1/2}}{2b_i},\ \ x_{2,i}=\frac{1-a-b_i-[(1-a-b_i)^2-4ab_i]^{1/2}}{2b_i}\ , \tag{B6}$$

$$x_{1,j}=\frac{1-a-b_j+[(1-a-b_j)^2-4ab_j]^{1/2}}{2b_j},\ \ x_{2,j}=\frac{1-a-b_j-[(1-a-b_j)^2-4ab_j]^{1/2}}{2b_j}\ , \tag{B6}$$

and

$$a=\frac{m^2}{M_W{}^2},\ \ b=\frac{p^2}{M_W{}^2},\ \ b_i=\frac{m_i{}^2}{M_W{}^2},\ \ b_j=\frac{m_j{}^2}{M_W{}^2}\ . \tag{B7}$$

From Eqs. (B4) and (B5) the coefficients in Eq. (14) are determined and $\Sigma_{\rm ren}$ is obtained without approximations.

APPENDIX C: R_G^{μ} HAS NO COUNTERTERMS

In this appendix we prove that in the KM model R_G, the part of the proper flavor-changing electromagnetic vertex which does not obey the Ward-Takahashi identity in the 't Hooft–Feynman gauge, has no counterterms. In Eq. (35c) $T_R^{R_G}=0$ and there is no right-handed counterterm. To prove that there is no left-handed counterterm we have to show that $T_L^{R_G}$ [Eq. (35d)] is independent of the intermediate mass m and therefore vanishes when summing over intermediate quarks: $\sum_l U_{il}U_{lj}^{\dagger}$. We prove it here for $m_i{}^2=m_j{}^2$ (but with $i \neq j$ for the flavor of the external quarks). The proof for general masses follows similar steps but is longer and will not be presented here.

We have to show that [integrate Eq. (35d) over α_k for $m_i^2=m_j^2$ and drop m^2-independent terms]

$$\int_0^1 d\alpha_p \left\{ \frac{m_i^2(1-\alpha_p)^3-m^2(1-\alpha_p)}{M_W^2(1-\alpha_p)+m^2\alpha_p-m_i^2\alpha_p(1-\alpha_p)} + 2(1-\alpha_p)\ln[M_W^2(1-\alpha_p)+m^2\alpha_p - m_i^2\alpha_p(1-\alpha_p)] \right\} \quad \text{(C1)}$$

is independent of m^2. Define

$$a=\frac{m^2}{M_W^2}, \quad b_i=\frac{m_i^2}{M_W^2} \quad \text{(C2)}$$

and change variable to $x=1-\alpha_p$. Then we have to show that

$$\int_0^1 dx \left\{ \frac{b_i x^3-ax}{x+a(1-x)-bx(1-x)} + 2x\ln[x+a(1-x)-b_i x(1-x)] \right\} \quad \text{(C3)}$$

is independent of $m^2=aM_W^2$, where the m^2-independent term $2x\ln M_W^2$, was dropped from the integrand. By defining x_1 and x_2 as the roots of the polynomial $x+a(1-x)-bx(1-x)$, the integral in Eq. (C3) can be rewritten as

$$\int_0^1 dx \left[\left[x^3-\frac{a}{b_i}x \right] \left[\frac{1}{x-x_1}-\frac{1}{x-x_2} \right] \frac{1}{x_1-x_2} + 2x\ln b_i+2x\ln(x-x_1)+2x\ln(x-x_2) \right] \quad \text{(C4)}$$

which is equal to $-\frac{1}{2}$, thus completing the proof.

APPENDIX D: CONSTRAINTS FROM WARD-TAKAHASHI IDENTITIES

In the unitary gauge the W-boson propagator is

$$(-g_{\mu\nu}+q_\mu q_\nu/M_W^2)/(q^2-M_W^2) ,$$

there are no unphysical scalars and there is one R^μ diagram [Fig. 3(a)] and one Λ^μ diagram [Fig. 3(e)]. Furthermore, the Ward-Takahashi identity is obeyed, i.e., $k_\mu\Gamma^\mu$ is proportional to $\Sigma(p-k) -\Sigma(p)$, where Γ^μ is the unrenormalized proper vertex.

In the 't Hooft—Feynman gauge only part of the unrenormalized vertex obeys the Ward-Takahashi identity. There is a part of Γ^μ denoted by R_G^μ in the text, which comes from adding a part of the diagram in Fig. 3(a) (R_{W2}^μ in the text) to Figs. 3(c) and 3(d) (denoted by R_{SW}^μ), that causes Γ^μ to violate the identity. We will now consider the constraints imposed on

$$\hat{\Gamma}^\mu=\Gamma^\mu-R_G^\mu , \quad \text{(D1)}$$

which satisfies

$$k_\mu\hat{\Gamma}^\mu=e_i[\Sigma(p-k)-\Sigma(p)] . \quad \text{(D2)}$$

This equation is actually equivalent to two equations since both $R_{W1}^\mu + \Lambda_W^\mu$ and $R_S^\mu + \Lambda_S^\mu$ satisfy an equation similar to Eq. (D2), but with an index W, S on $\Sigma(p-k)$ and $\Sigma(p)$, respectively. Let us write

$$\hat{\Gamma}^\mu=\hat{R}^\mu+\Lambda^\mu , \quad \text{(D3)}$$

where

$$\hat{R}^\mu=R^\mu-R_G^\mu \quad \text{(D4)}$$

and decompose $\hat{R}^\mu$ and Λ^μ as specified in Eq. (51) with coefficients A_i^R, B_i^R for $\hat{R}^\mu$ and A_i^Λ, B_i^Λ for Λ^μ ($i=1,2,\ldots,13$). Remember that these are unrenormalized coefficients, but that all—except those with $i=6$ (i.e., the coefficients of $\gamma^\mu L$ and $\gamma^\mu R$)—are equal to the renormalized coefficients. Now since $R^\mu \propto e_W$ and $\Lambda^\mu \propto e_l=e_i+e_W$, it is convenient to define the reduced coefficients

$$\alpha_i^R=e_W^{-1}A_i^R, \quad \beta_i^R=e_W^{-1}B_i^R , \qquad \alpha_i^\Lambda=(e_i+e_W)^{-1}A_i^\Lambda, \quad \beta_i^\Lambda=(e_i+e_W)^{-1}B_i^\Lambda . \quad \text{(D5)}$$

Then using Eq. (51) we obtain

$$\begin{aligned} k_\mu\hat{\Gamma}^\mu=\{ & [(e_i+e_W)\alpha_1^\Lambda+e_W\alpha_1^R]k\cdot p\not{p}+[(e_i+e_W)(\alpha_2^\Lambda+\alpha_5^\Lambda)+e_W(\alpha_2^R+\alpha_5^R)]\not{p}k^2 \\ & +[(e_i+e_W)(\alpha_3^\Lambda+\alpha_8^\Lambda)+e_W(\alpha_3^R+\alpha_8^R)]\not{k}p\cdot k+[(e_i+e_W)(\alpha_{11}^\Lambda+\alpha_{13}^\Lambda)+e_W(\alpha_{11}^R+\alpha_{13}^R)]k^2 \\ & +[(e_i+e_W)\alpha_6^\Lambda+e_W\alpha_6^R]\not{k}+[(e_i+e_W)(\alpha_4^\Lambda+\alpha_7^\Lambda)+e_W(\alpha_4^R+\alpha_7^R)]\not{k}k^2 \\ & +[(e_i+e_W)\alpha_9^\Lambda+e_W\alpha_9^R]\not{k}p^2+[(e_i+e_W)\alpha_{10}^\Lambda+e_W\alpha_{10}^R]\not{k}\not{p} \\ & +[(e_i+e_W)\alpha_{12}^\Lambda+e_W\alpha_{12}^R]k\cdot p\ \}L+\{\ \alpha_i^\Lambda\to\beta_i^\Lambda,\ \alpha_i^R\to\beta_i^R\ \}R . \end{aligned} \quad \text{(D6)}$$

From Eqs. (D2) and (12) we have

$$k_\mu \hat{\Gamma}^\mu = e_i[(\bar{f}-f)\not{p}L + (\bar{h}-h)\not{p}R - \bar{f}\not{k}L - \bar{h}\not{k}R + (\psi-\bar{\psi})L + (\bar{\phi}-\phi)R] , \quad \text{(D7)}$$

where $f(p^2)$, $h(p^2)$, $\psi(p^2)$, $\phi(p^2)$ are given in Eq. (15), and $\bar{f}=f[(p-k)^2]$, $\bar{h}=h[(p-k)^2]$, $\bar{\psi}=\psi[(p-k)^2]$, $\bar{\phi}=\phi[(p-k)^2]$.

Therefore, (1) A_{10} and B_{10} are zero for all the parts of Γ^μ, except for R_G^μ, since there is no $\not{k}\not{p}$ term in Eq. (D7). (2) Only A_6 and B_6 require renormalization, since only the coefficients of $\not{k}L$ and $\not{k}R$ are infinite in Eq. (D7). (3) By comparing Eqs. (D6) and (D7) and by equating the coefficients of e_i and e_W, we find that the reduced coefficients obey the following constraints:

$$\begin{aligned}
&\alpha_1^\Lambda k\cdot p + (\alpha_2^\Lambda+\alpha_5^\Lambda)k^2 = -\alpha_1^R k\cdot p - (\alpha_2^R+\alpha_5^R)k^2 = \bar{f}-f , \\
&\beta_1^\Lambda k\cdot p + (\beta_2^\Lambda+\beta_5^\Lambda)k^2 = -\beta_1^R k\cdot p - (\beta_2^R+\beta_5^R)k^2 = \bar{h}-h , \\
&(\alpha_3^\Lambda+\alpha_8^\Lambda)k\cdot p + \alpha_6^\Lambda + (\alpha_4^\Lambda+\alpha_7^\Lambda)k^2 + \alpha_9^\Lambda p^2 = -(\alpha_3^R+\alpha_8^R)k\cdot p - \alpha_6^R - (\alpha_4^R+\alpha_7^R)k^2 - \alpha_9^R p^2 = -\bar{f} , \\
&(\beta_3^\Lambda+\beta_8^\Lambda)k\cdot p + \beta_6^\Lambda + (\beta_4^\Lambda+\beta_7^\Lambda)k^2 + \beta_9^\Lambda p^2 = -(\beta_3^R+\beta_8^R)k\cdot p - \beta_6^R - (\beta_4^R+\beta_7^R)k^2 - \beta_9^R p^2 = -\bar{h} , \\
&(\alpha_{11}^\Lambda+\alpha_{13}^\Lambda)k^2 + \alpha_{12}^\Lambda k\cdot p = -(\alpha_{11}^R+\alpha_{13}^R)k^2 - \alpha_{12}^R k\cdot p = \bar{\psi}-\psi , \\
&(\beta_{11}^\Lambda+\beta_{13}^\Lambda)k^2 + \beta_{12}^\Lambda k\cdot p = -(\beta_{11}^R+\beta_{13}^R)k^2 - \beta_{12}^R k\cdot p = \bar{\phi}-\phi .
\end{aligned} \quad \text{(D8)}$$

These equations hold for the reduced coefficients of $\hat{R}^\mu$ and Λ_W^μ, and of R_S^μ and Λ_S^μ. In these cases we have in Eq. (D8) for the W contribution

$$\begin{aligned}
&\bar{f} = \frac{ig^2}{16\pi^2}\int_0^1 d\alpha\,\alpha\left[\frac{2}{\epsilon} - \ln\frac{\bar{\Delta}}{4\pi\mu^2} - \Gamma' + 1\right] , \\
&\bar{h} = 0 \\
&\bar{f}-f = \frac{ig^2}{16\pi^2}\int_0^1 d\alpha\,\alpha \ln\frac{\Delta}{\bar{\Delta}} , \\
&\bar{h}-h = \bar{\psi}-\psi = \bar{\phi}-\phi = 0 ,
\end{aligned} \quad \text{(D9)}$$

and for the S contribution

$$\begin{aligned}
&\bar{f} = \frac{ig^2 m^2}{32\pi^2 M_W{}^2}\int_0^1 d\alpha\,\alpha\left[\frac{2}{\epsilon} - \ln\frac{\bar{\Delta}}{4\pi\mu^2} + \Gamma'\right] , \\
&\bar{h} = \frac{ig^2 m_i m_j}{32\pi^2 M_W{}^2}\int_0^1 d\alpha\,\alpha\left[\frac{2}{\epsilon} - \ln\frac{\bar{\Delta}}{4\pi\mu^2} + \Gamma'\right] , \\
&\bar{f}-f = \frac{ig^2 m^2}{32\pi^2 M_W{}^2}\int_0^1 d\alpha\,\alpha \ln\frac{\Delta}{\bar{\Delta}} , \\
&\bar{h}-h = \frac{ig^2 m_i m_j}{32\pi^2 M_W{}^2}\int_0^1 d\alpha\,\alpha \ln\frac{\Delta}{\bar{\Delta}} , \\
&\bar{\psi}-\psi = -\frac{ig^2 m_j m^2}{32\pi^2 M_W{}^2}\int_0^1 d\alpha \ln\frac{\Delta}{\bar{\Delta}} , \\
&\bar{\phi}-\phi = -\frac{ig^2 m_i m^2}{32\pi^2 M_W{}^2}\int_0^1 d\alpha \ln\frac{\Delta}{\bar{\Delta}} ,
\end{aligned} \quad \text{(D10)}$$

where Δ, $\bar{\Delta}$ are defined in Eqs. (8) and (45), respectively.

We now relate the counterterms in the proper vertex function to the self-energy:

$$\Gamma_{\mu,\mathrm{ren}} = \Gamma_\mu + T_L\gamma_\mu L + T_R\gamma_\mu R . \quad \text{(D11)}$$

We define

$$\hat{\Gamma}_{\mu,\mathrm{ren}} = \Gamma_{\mu,\mathrm{ren}} - R_\mu^G . \quad \text{(D12)}$$

This part of the proper vertex satisfies the Ward-Takahashi identity for the renormalized quantities, i.e.,

$$k^\mu \hat{\Gamma}_{\mu,\mathrm{ren}} = e_i[\Sigma_{\mathrm{ren}}(p-k) - \Sigma_{\mathrm{ren}}(p)] . \quad \text{(D13)}$$

From (D13) and Eq. (13), we find

$$T_L = -e_i C \quad \text{(D14)}$$

and

$$T_R = -e_i D , \quad \text{(D15)}$$

where C and D are defined in Eqs. (14a) and (14b).

If we separate the contributions to self-energy from W and S so that

$$C = C^W + C^S , \quad \text{(D16)}$$

$$D = D^W + D^S , \quad \text{(D17)}$$

we then obtain

$$\begin{aligned}
&T_L^{R_{W1}}/e_W = -T_L^{\Lambda_W}/e_l = C^W , \\
&T_L^{R_S}/e_W = -T_L^{\Lambda_S}/e_l = C^S ,
\end{aligned} \quad \text{(D18)}$$

$$T_R^{R_{W1}}/e_W = -T_R^{\Lambda_W}/e_l = D^W ,$$

$$T_R^{R_S}/e_W = -T_R^{\Lambda_S}/e_l = D^S .$$

These relations have been verified as a check on our calculation.

We now show the equivalence of results of our renormalization scheme to those computed by adding only unrenormalized diagrams. A flavor-changing electromagnetic transition $q_i \to q_j + \gamma$ is given by the unrenormalized vertex:

$$V_\mu^{ij} = \Gamma_\mu^{ij} + \Sigma_{\mu,1}^{ij} + \Sigma_{\mu,2}^{ij} , \tag{D19}$$

where

$$\Sigma_{\mu,1}^{ij} = -e_i \Sigma^{ij}(p-k)(\not p - \not k - m_i)^{-1}\gamma_\mu , \tag{D20}$$

$$\Sigma_{\mu,2}^{ij} = -e_j \gamma_\mu (\not p - m_j)^{-1} \Sigma^{ij}(p) , \tag{D21}$$

Rather lengthy but straightforward algebra shows that

$$V_{\mu,\text{ren}}^{ij}\,|_{\text{on-shell}} \equiv [\Gamma_{\mu,\text{ren}}^{ij} + \Sigma_{\mu,1}^{\text{ren}} + \Sigma_{\mu,2}^{\text{ren}}]_{\text{on-shell}} = \Gamma_{\mu,\text{ren}}^{ij}\,|_{\text{on-shell}} = V_\mu^{ij}\,|_{\text{on-shell}} . \tag{D22}$$

Thus $\Gamma_{\mu,\text{ren}}^{ij}$ on-shell is equivalent to the sum of a rather large set of diagrams involving proper vertex diagrams of Fig. 3 as well as self-energy diagrams with photon emission on the external legs. Computation of $\Gamma_{\mu,\text{ren}}^{ij}$ is rather simple because of the constraint $k^\mu \Gamma_\mu^{\text{ren}} = 0$ on-shell. Note that both V_μ and $V_{\mu,\text{ren}}$ off-shell are gauge-dependent quantities, but on-shell they are gauge independent.

APPENDIX E: THE ON-SHELL DECOMPOSITION OF THE VERTEX FUNCTIONS

In Eq. (51) we defined a general decomposition valid for each term [see Eq. (49)] in V_{ren}^μ. It is easy to show that when the quark i (the incoming quark) is on-shell, then the decomposition into 26 functions turns into a 16-functions decomposition as follows:

$$\begin{aligned}
&[A_3 \not k p^\mu + A_4 \not k k^\mu + (A_6 + B_{10} m_i + A_9 m_i^2)\gamma^\mu + A_7 \gamma^\mu k^2 + A_8 \gamma^\mu k\cdot p \\
&\quad + (-B_5 m_i + A_{11})\gamma^\mu \not k + (B_1 m_i + A_{12}) p^\mu + (B_2 m_i + 2B_5 m_i + A_{13}) k^\mu] L \\
&\qquad + [B_3 \not k p^\mu + B_4 \not k k^\mu + (B_6 + A_{10} m_i + B_9 m_i^2)\gamma^\mu + B_7 \gamma^\mu k^2 + B_8 \gamma^\mu k\cdot p \\
&\qquad\quad + (-A_5 m_i + B_{11})\gamma^\mu \not k + (A_1 m_i + B_{12}) p^\mu + (A_2 m_i + 2A_5 m_i + B_{13}) k^\mu] R .
\end{aligned} \tag{E1}$$

When the outgoing quark j is on-shell we obtain

$$\begin{aligned}
&[(A_1 + A_3 + 2A_5)\not k p^\mu + (A_2 + A_4)\not k k^\mu + (A_6 + A_9 m_j^2 - A_{10} m_j)\gamma^\mu \\
&\quad + (A_5 + A_7 - A_9)\gamma^\mu k^2 + (-2A_5 + A_8 + 2A_9)\gamma^\mu k\cdot p + (-m_j A_5 + A_{10} + A_{11})\gamma^\mu \not k \\
&\quad + (A_1 m_j + 2A_{10} + A_{12}) p^\mu + (m_j A_2 + 2m_j A_5 - 2A_{10} + A_{13}) k^\mu] L \\
&\qquad + [(B_1 + B_3 + 2B_5)\not k p^\mu + (B_2 + B_4)\not k k^\mu + (B_6 + B_9 m_j^2 - B_{10} m_j)\gamma^\mu \\
&\qquad\quad + (B_5 + B_7 - B_9)\gamma^\mu k^2 + (-2B_5 + B_8 + 2B_9)\gamma^\mu k\cdot p + (-m_j B_5 + B_{10} + B_{11})\gamma^\mu \not k \\
&\qquad\quad + (B_1 m_j + 2B_{10} + B_{12}) p^\mu + (m_j B_2 + 2m_j B_5 - 2B_{10} + B_{13}) k^\mu] R .
\end{aligned} \tag{E2}$$

When both i and j are on-shell, we find a decomposition into six functions multiplying p_μ, k_μ, and γ_μ for the left and the right part of the vertex,

$$\begin{aligned}
&\Bigg\{ [(B_1 + B_3 + 2B_5) m_i - A_3 m_j + A_{12} - 2A_{11}] p^\mu + [(B_2 + B_4) m_i - A_4 m_j + A_{13} + 2A_{11}] k^\mu \\
&\quad + \left[(-B_5 m_j + B_{10} + B_{11}) m_i + (-A_5 + A_9) m_i^2 + A_6 + A_{11} m_j + A_7 k^2 + \frac{A_8}{2}(m_i^2 - m_j^2 + k^2) \right] \gamma^\mu \Bigg\} L \\
&\qquad + \Bigg\{ [(A_1 + A_3 + 2A_5) m_i - B_3 m_j + B_{12} - 2B_{11}] p^\mu + [(A_2 + A_4) m_i - B_4 m_j + B_{13} + 2B_{11}] k^\mu \\
&\qquad\quad + \Bigg[(-A_5 m_j + A_{10} + A_{11}) m_i + (-B_5 + B_9) m_i^2 + B_6 \\
&\qquad\qquad + B_{11} m_j + B_7 k^2 + \frac{B_8}{2}(m_i^2 - m_j^2 + k^2) \Bigg] \gamma^\mu \Bigg\} R .
\end{aligned} \tag{E3}$$

We can reexpress the last equation in terms of γ^μ, $\sigma^{\mu\nu}k_\nu$, k^μ instead of γ^μ, k^μ, p^μ by using

$$p^\mu L = \frac{k^\mu}{2}L + \frac{i\sigma^{\mu\nu}k_\nu}{2}L + \frac{\gamma^\mu m_i}{2}R + \frac{\gamma^\mu m_j}{2}L \ ,$$
$$p^\mu R = \frac{k^\mu}{2}R + \frac{i\sigma^{\mu\nu}k_\nu}{2}R + \frac{\gamma^\mu m_i}{2}L + \frac{\gamma^\mu m_j}{2}R \ . \qquad \text{(E4)}$$

In the limit of small external masses and momenta one recovers the results of Ref. 10.

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