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Spontaneous breakdown and the scalar nonet

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In the context of the QCD quark model and on the basis of dynamical Bethe-Salpeter ladder graphs, we suggest that (i) the existence of the scalar $\bar{q}q$ hadron multiplet, like the pseudoscalar $\bar{q}q$ multiplet, is a direct consequence of dynamical spontaneous breakdown of chiral symmetry with a chiral-limiting nonstrange mass scale of $m_{\sigma \mathrm{NS}}^{\mathrm{CL}} = 2m_{\mathrm{dyn}} \approx 630$ MeV, (ii) the lifting of the nonstrange σ-δ degeneracy is expected from the s-wave quark-gluon annihilation diagram, and (iii) the observed σ-S^* mixing follows from the existence of the p-wave scalar quark-annihilation diagram. The resulting *predicted* 0^{++} $\bar{q}q$ nonet is then σ(750 MeV), κ(800), S^*(980), and δ(985), in agreement with data for the resonant masses, the mixing angle, and also decay widths except for the κ(800).

I. INTRODUCTION

Spontaneous breakdown is certainly one of the most elegant and profound concepts in physics; it describes such diverse physical phenomena as superconductivity,[1] the massless pion in strong interactions,[2,3] and the implied supermassive W and Z bosons in weak interactions.[4] In the latter two cases the chiral-symmetric σ-model Lagrangian[5] and the gauge-symmetric electroweak Lagrangian[4] predict the existence of two additional massive scalar particles, the 0^{++} σ meson and the Higgs boson, but their experimental verification has proved quite elusive. Since such Lagrangian theories do not constrain the masses of these scalar bosons, their nonexistence becomes a distinct possibility. In a recent paper,[6] however, it has been shown that dynamical spontaneous breakdown of chiral symmetry for the QCD quark theory in fact suggests the nonstrange (NS) (and nonmixed) σ-meson mass to be fixed in the chiral limit (CL) to $m_{\sigma \mathrm{NS}}^{\mathrm{CL}} = 2m_{\mathrm{dyn}} \approx 630$ MeV.

In this work we extend the QCD theory of spontaneous breakdown to the entire scalar nonet. Since the $I=0$ σ meson is (primarily) composed of nonstrange quarks like the ω meson, it has long been anticipated that the $I=1$ nonstrange scalar partner of the σ should be degenerate in mass with σ, similar to the ω-ρ mass degeneracy. Indeed the observed $I=1$ scalar δ(980) resonance at 980 MeV, significantly above the $I=0$ ϵ(700) resonance region, thus violating the above σ-δ mass degeneracy, has been the compelling reason for the conjecture[7] that the lowest-lying scalar multiplet may be composed of four quarks, $\bar{q}\bar{q}qq$, rather than the simpler two-quark $\bar{q}q$ configuration. We shall attempt to attempt to demonstrate, however, that the spontaneous-breakdown origin of the scalar multiplet in fact suggests that the σ-δ mass degeneracy is lifted. Furthermore, we also include the small isoscalar σ-S^* mixing according to quark-gluon annihilation graphs which have proved so successful in explaining η-η' pseudoscalar and ω-ϕ vector mixing[8,9] and even electromagnetic π^0-η and ρ^0-ω mixing.[9,10] The predicted $\bar{q}q$ scalar nonet mass spectrum is then σ(750), κ(800), S^*(980), and δ(985), the latter two being in complete agreement with experiment.[11]

Since the crucial input to this analysis is dynamical spontaneous breakdown of chiral symmetry, in Sec. II we review the theory of dynamical mass generation of the nonstrange σ meson[6] based on the equivalence between the 0^- and 0^+ Bethe-Salpeter binding equations at invariant four-momentum transfers $q^2=0$ and $q^2=4m_{\mathrm{dyn}}^2$, respectively. Then in Sec. III we show that this chiral $(\sigma_{\mathrm{NS}}, \vec{\pi})$ equivalence breaks down for the chiral pair $(\vec{\delta}, \eta_{\mathrm{NS}})$ because the nonstrange η (η_{NS}) develops a non-Goldstone component due to the s-wave quark-annihilation graph.[8,9] Nevertheless, we suggest that the δ mass must also be shifted upward by an amount preserving the chiral-invariant mass difference $(2m_{\mathrm{dyn}})^2$ according to the symmetry relation

$$m_{\sigma \mathrm{NS}}^2 - m_\pi^2 = m_\delta^2 - m_{\eta \mathrm{NS}}^2 = (2m_{\mathrm{dyn}})^2 \approx 21 m_\pi^2 \ . \tag{1}$$

We shall discuss the dynamical analog of (1) in Sec. III. Then, combining (1) with the pseudoscalar quark-annihilation analysis of Ref. 9, which gives $m_{\eta \mathrm{NS}}^2 \approx 30 m_\pi^2$ we are led to $m_\delta^2 \approx 51 m_\pi^2$ or $m_\delta \approx 985$ MeV, in agreement with experiment.

Encouraged by this success, in Sec. IV we parallel the η-η' mixing scheme with a σ-S^* mixing analysis which predicts that $\sigma_{\rm NS}$ shifts up from 645 MeV to σ(750 MeV) and $\sigma_S = \bar{s}s$ shifts up from 930 MeV to the observed value of S^*(980 MeV) when the 0^+ quark-annihilation graph is turned on. Then in Sec. V we survey the phenomenology of the scalar decay modes $\sigma \to \pi\pi$; $\kappa \to K\pi$; $\delta \to \eta\pi$, $K\bar{K}$; $S^* \to \pi\pi, K\bar{K}$. We deduce that not only the SU(3) coupling g_{SPP} remains approximately invariant over all of these decay modes, but the σ-S^* nonstrange-strange mixing angle $\phi_S = 19° \pm 3°$ is consistent with $\phi_S \approx 16°$ as predicted from the p-wave quark-annihilation graph of Sec. IV. Finally, in Sec. VI we summarize the situation and stress that this scalar nonet of σ(750), κ(800), S^*(980), and δ(985) is every bit as significant a signature in nature as the pseudoscalar nonet for the spontaneous breakdown of chiral symmetry.

II. SPONTANEOUS BREAKDOWN AND THE NONSTRANGE σ MESON

The dynamical approach to the spontaneous breakdown of chiral symmetry was first considered by Nambu and Jona-Lasinio in the context of the simple four-fermion model.[2] The possibility of extending this idea to Yukawa-type coupling models such as QED or (Abelian) QCD was proposed quite some time ago,[12] but only recently resolved satisfactorily by the use of axial-vector Ward identities.[13] The upshot of this analysis can be summarized by the symbolic equation

$$\mathrm{DE} = \mathrm{PBE}\,|_{q\to 0}\ , \tag{2}$$

where DE represents the equations that dress a quark, giving it *all* of its mass, say by chiral-invariant gluon self-energy-type graphs. On the other hand, $\mathrm{PBE}|_{q\to 0}$ in (2) represents the s-wave pseudoscalar Bethe-Salpeter binding equation at vanishing four-momentum transfer. This chiral-symmetry identity is depicted in part by the spin-zero γ_5 coupling graphs of Fig. 1. Alternatively, as anticipated earlier by Nambu,[1,2] the identity (2) is the off-shell dynamical version of the kinematically deduced axial-vector-current vertex for $m \neq 0$,

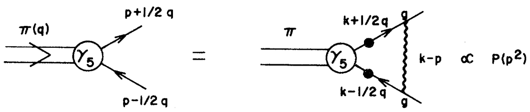

FIG. 1. Spinless component of the 0^- pseudoscalar Bethe-Salpeter bound-state equation for $P(p^2)$.

$$\Gamma_{\mu 5} \propto \bar{q}\left[\gamma_\mu - \frac{2mq_\mu}{q^2}\right]\gamma_5 q\ , \tag{3}$$

which is conserved on the fermion mass shell. Here the pole of (3) at $q^2=0$ signals the existence of a massless Nambu(-Goldstone) pseudoscalar meson, and the fermion mass m in (3) takes on the meaning of the running dynamically generated quark mass of QCD,[14] $m \to m_{\rm dyn}(p^2)$.

The key observation concerning the 0^+ $\bar{q}q$ p-wave bound states is that their associated spin-zero Bethe-Salpeter equation depicted in Fig. 2, when evaluated at $q^2=4m^2$, is *identical* to the spin-zero s-wave binding equation of Fig. 1 evaluated at $q \to 0$:

$$\mathrm{SBE}\,|_{q^2=4m^2} = \mathrm{PBE}\,|_{q\to 0}\ . \tag{4}$$

This relation was formally noted in the four-fermion model of Nambu and Jona-Lasinio,[2] but of course at that pre—quark-model time no further statement could be made.

Within the context of the QCD quark model and the quark-gluon graphs of Figs. 1 and 2, statement (4) again holds in the chiral limit.[6] More specifically, for a pseudoscalar form factor (i.e., wave function) $P(p^2,q^2)$ and scalar form factor $U(p^2,q^2)$ as indicated in Figs. 1 and 2, we first compute the discontinuities

$$\mathrm{Im}P(p^2,q^2) = 2g^2 \int d\rho_2(k)(k-p)^{-2}(-q^2)P(k^2,q^2=4m^2-4k^2)\ , \tag{5a}$$

$$\mathrm{Im}U(p^2,q^2) = 2g^2 \int d\rho_2(k)(k-p)^{-2}(4m^2-q^2)U(k^2,q^2)=4m^2-4k^2)\ , \tag{5b}$$

where $d\rho_2$ represents two-body phase space and $(k \pm \frac{1}{2}q)^2 = m^2$ requires $k \cdot q = 0$ and $q^2 = 4m^2 - 4k^2$. For simplicity we work in the Feynman gauge in (5), neglecting the momentum variation of the quark-gluon coupling g, and ignoring the non-Abelian color indices for the gluon-quark couplings. Also, the spinless nature of the π and σ means that we can drop a possible $p \cdot q$ dependence in $P(p^2,q^2)$ and $U(p^2,q^2)$ since a dispersive integral over such invariant functions with $k \cdot q = 0$ cannot generate a $p \cdot q$ term. Next, we construct (unsubtracted) dispersion relations for P and U, respectively, evaluated at $q^2=0$ and $q^2=4m^2$ so that the $-q^2$ and $4m^2-q^2$ factors in (5a) and (5b) are eliminated:

$$P(p^2,q^2=0)=\frac{1}{\pi}\int \frac{\mathrm{Im}P(p^2,q'^2)dq'^2}{q'^2-q^2}\Bigg|_{q^2=0} = -\frac{2g^2}{\pi}\int dq'^2 \int \frac{d\rho_2(k)}{(k-p)^2}P(k^2,q'^2=4m^2-4k^2) , \tag{6a}$$

$$U(p^2,q^2=4m^2)=\frac{1}{\pi}\int \frac{\mathrm{Im}U(p^2,q'^2)dq'^2}{q'^2-q^2}\Bigg|_{q^2=4m^2} = -\frac{2g^2}{\pi}\int dq'^2 \int \frac{d\rho_2(k)}{(k-p)^2}U(k^2,q'^2=4m^2-4k^2) . \tag{6b}$$

The crucial observation concerning (6a) and (6b) is that $P(p^2,q^2)$ and $U(p^2,q^2)$ have the same dynamical structure and are therefore proportional to one another *providing* both are independent of q^2. In QCD the asymptotic-freedom property with $\alpha_S(p^2)\propto(\ln p^2/\Lambda^2)^{-1}$ assures us of the q^2 independence of $P(p^2)$ and $U(p^2)$ in the high-momentum region with $p^2 \gg \Lambda^2$. Since, however, it is now accepted that the QCD energy scale is rather low,[15]

$$\Lambda \approx 150 \pm 50 \text{ MeV} , \tag{7}$$

we can expect $P(p^2)\propto U(p^2)$ for p^2 values as low as the spontaneous quark mass generation region of ~ 300 MeV, where still $p^2 \gg \Lambda^2$. Furthermore, since the QCD coupling is expected to freeze out[16] for all $p^2 \lesssim 1$ GeV, the proportionality $P(p^2)\propto U(p^2)$ and hence the chiral relation (4) should be valid even if Λ were as high as 400 MeV. In summary, we stress that the relation (4) is a model-dependent statement which appears to hold for chiral-invariant field theories that generate a weak q^2 dependence in the pseudoscalar and scalar wave functions. The asymptotic-freedom property enables QCD to be included in this class. While we simplified the derivation of (4) to the ladder graphs of Figs. 1 and 2, the result can be extended to include some vertex corrections as in the case[13] of the chiral identity (2).

Given (4) with $m \to m_{\rm dyn}$, we have found elsewhere from chiral-breaking QCD masses and "neutral PCAC" (partial conservation of axial-vector current) considerations that[13,17]

$$m_{\rm dyn}=m_{\rm dyn}(p^2=m_{\rm dyn}{}^2)\approx 315 \text{ MeV} . \tag{8}$$

Thus we may assume, from the $q^2=4m^2$ limit on the scalar binding equation in (4), that for $m_{\rm dyn}\neq 0$, when a massless pion must exist by virtue of (2), then a scalar nonstrange σ meson should also exist by virtue of (2) and (4) with chiral-limiting (but nonmixed) mass

$$m_{\sigma \rm NS}^{\rm CL}=2m_{\rm dyn}\approx 630 \text{ MeV} . \tag{9}$$

If instead of (8) we take Cornwall's estimate[18] of $m_{\rm dyn}$ as implied by the Richardson linearly rising potential, then $m_{\rm dyn}\approx 300$ MeV and $m_{\sigma \rm NS}^{\rm CL}\approx 600$ MeV, only slightly lower than (8) or (9).

We conclude that although Lagrangian chiral-symmetry theories such as the σ model also require a σ meson with an undetermined mass, the approach of dynamical symmetry breakdown suggests the additional chiral relation (4), which in turn constrains the chiral-limiting nonstrange σ mass to about 600–630 MeV.

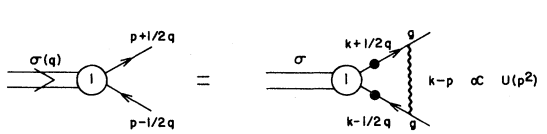

FIG. 2. The 0^+ scalar Bethe-Salpeter bound-state equation for $U(p^2)$.

III. LIFTING OF THE $\sigma_{\rm NS}$-δ MASS DEGENERACY

It is expected that the 0^+ U(3) scalar nonet should approximately mix "magically" as does the 1^- vector nonet. This suggests the nonstrange $I=0$ $\sigma_{\rm NS}$ and $I=1$ δ states to be degenerate in mass, similar to the ω-ρ mass degeneracy. There is, however, one loophole in this 0^+-1^- analogy: while the 1^- nonet follows the SU(6) constituent-quark additive mass pattern of the heavier (nonrelativistic) hadron $\frac{1}{2}^+$, $\frac{3}{2}^+$, 2^+, etc., multiplets, the $\bar{q}q$ 0^+ nonet like the 0^- nonet owes its existence to spontaneous breakdown and is composed of relativistic quarks which do not follow the additive constituent-quark mass pattern.

Thus if we are to understand the nondegeneracy of the $\sigma_{\rm NS}$ and δ masses, we must first review this nondegeneracy for the 0^- π and $\eta_{\rm NS}$ masses, whose origin seven years ago was referred to in part as the "U(1) problem."[19] It is now clear that the U(1) axial-vector "anomalous" divergence of QCD,[20]

$$\partial^\mu j^{i=0}_{\mu 5} = \frac{\sqrt{6}g^2}{64\pi^2}\epsilon^{\alpha\beta\gamma\delta}G^a_{\alpha\beta}G^a_{\gamma\delta} \neq 0 , \qquad (10)$$

where $G^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + f^{abc}V^b_\mu V^c_\nu$, is the origin of the π-$\eta_{\rm NS}$ mass splitting. In particular the U(1) singlet η_0 is not a Nambu-Goldstone boson[21,22] in spite of the total divergence structure of (10). In our language of dynamical symmetry breakdown, the fundamental relation (2) is easily generalized to the eight SU(3)-flavor currents and pseudoscalar binding equations at $q^2=0$. It *cannot*, however, be extended to the singlet flavor because (1) in effect requires[22]

$$\mathrm{DE} \neq \mathrm{U(1)PBE}\,|_{q\to 0} . \qquad (11)$$

Rather than continue with this $\partial \cdot j_5$ formalism to compute the pseudoscalar masses, it is more convenient to work directly with the Hamiltonian density and calculate the pseudoscalar mass according to the standard (infinite-momentum frame) formula

$$\langle P\,|\,\mathscr{H}\,|\,P\rangle = \langle P\,|\,\mathscr{H}_0 + \mathscr{H}'\,|\,P\rangle = 2m_p^{\,2} . \qquad (12)$$

Here the $\langle P\,|\,\mathscr{H}_0\,|\,P\rangle$ correspond to quark-scattering graphs of Fig. 3(a); they vanish for the Nambu-Goldstone bosons π, K, η_8. The chiral-breaking contributions to the 0^- masses $\langle P\,|\,\mathscr{H}'\,|\,P\rangle$ are given by model-dependent matrix elements of the *current*-quark-mass matrix [i.e., strong PCAC (Ref. 23) or neutral PCAC (Ref. 24)] which vanish in the chiral limit. However, since the analysis of the present paper is independent of such perturbative chiral-symmetry-breaking considerations, we must reexpress the current-quark-mass contributions to (12) back in terms of the observed π and K masses. A final contribution to the isoscalar nonstrange and strange states derives from the quark-annihilation graphs of Fig. 3(b), which, when "pinched together on one side" correspond to the vacuum to pseudoscalar matrix elements of (10). Since in fact Fig. 3(b) links $\bar{u}u$ to $\bar{u}u$, $\bar{d}d$, or $\bar{s}s$ states,[8] the final form[9] of the pseudoscalar mass matrix contains off-diagonal quark-annihilation contributions in the u,d,s basis of U(3)-invariant strength β_P:

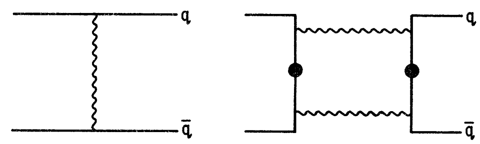

FIG. 3. (a) The $\bar{q}q$ scattering diagram with (at least) one gluon exchange. This graph contributes a vanishing $\langle P\,|\,\mathscr{H}_0\,|\,P\rangle$ to the pseudoscalar masses. (b) The $\bar{q}q$ $I=0$ quark-annihilation graph with (at least) two gluons exchanged. This graph has the respective strengths β_P, β_S, and β_V for the pseudoscalar, scalar, and vector $\bar{q}q$ bound states.

$$M_P = \begin{bmatrix} m_\pi^{\,2}+\beta_P & \beta_P & \beta_P \\ \beta_P & m_\pi^{\,2}+\beta_P & \beta_P \\ \beta_P & \beta_P & (2m_K^{\,2}-m_\pi^{\,2})+\beta_P \end{bmatrix} . \qquad (13)$$

Eliminating the π^0 state from the space spanned by M_P and expressing the remaining submatrix in the nonstrange-strange basis with $|\eta_{\rm NS}\rangle = (\bar{u}u + \bar{d}d)/\sqrt{2}$ and $|\eta_S\rangle = |\,\bar{s}s\,\rangle$, we obtain a 2×2 isoscalar matrix which can be rediagonalized to obtain the observed η and η' masses:

$$\begin{bmatrix} m_\pi^{\,2}+2\beta_P & \sqrt{2}\beta_P \\ \sqrt{2}\beta_P & (2m_K^{\,2}-m_\pi^{\,2})+\beta_P \end{bmatrix} \to \begin{bmatrix} m_\eta^{\,2} & 0 \\ 0 & m_{\eta'}^{\,2} \end{bmatrix} . \qquad (14)$$

The significant point about (14) is that *one* value of β_P must set the scale for the *two* masses $m_\eta^{\,2}$ and $m_{\eta'}^{\,2}$. Indeed, this is almost the case because the trace and determinant of (14), respectively, give[9] $\beta_P \approx 12.7m_\pi^{\,2}$ and $\beta_p \approx 14.5m_\pi^{\,2}$ for $m_\pi \equiv 138$ MeV. In fact, if we include slight (20%) dynamical (multiplicative) SU(3)-symmetry-breaking effects in (14) due to the difference between the nonstrange and strange constituent quark masses in the loop of Fig. 3(b), then we obtain the unique pseudoscalar quark-annihilation strength of[9]

$$\beta_P = \frac{(m_{\eta'}^{\,2}-m_\pi^{\,2})(m_\eta^{\,2}-m_\pi^{\,2})}{4(m_K^{\,2}-m_\pi^{\,2})} \approx 14.7m_\pi^{\,2} . \qquad (15)$$

Finally then, this result (15) can be translated by (14) into an effective mass splitting of the nonstrange η and π states:

$$m_{\eta{\rm NS}}^{\,2} - m_\pi^{\,2} = 2\beta_P \approx 29m_\pi^{\,2} . \qquad (16)$$

Now we are prepared to discuss the central issue of this section—the lifting of the $\sigma_{\rm NS}$-δ mass degeneracy. First working in the chiral limit, we know from (9) that

$$(m_{\sigma\mathrm{NS}}^{\mathrm{CL}})^2=(2m_{\mathrm{dyn}})^2\approx 21m_\pi^2 , \tag{17}$$

which is the analog $\langle S\,|\,\mathscr{H}_0\,|\,S\rangle$ scalar contribution to (12) since the Bethe-Salpeter graph of Fig. 2 is the "pinched down" version of the scattering graph of Fig. 3(a). In order to find the $I=1$ chiral-limiting mass formula which parallels (17), we note that the spontaneous breakdown relation (4) for the chiral pair $(\sigma_{\mathrm{NS}},\vec{\pi})^{\mathrm{CL}}$ *cannot* be extended to the chiral pair $(\vec{\delta},\eta_{\mathrm{NS}})^{\mathrm{CL}}$ precisely because η_{NS} is *not* a Nambu-Goldstone boson, i.e., β_P in (16) is nonvanishing in the chiral limit. Indeed, since (2) and (4) imply $\mathrm{SBE}|_{q^2=4m^2}=\mathrm{DE}$ which differs from the singlet $\mathrm{PBE}\,|_{q\to 0}$ by a term proportional to β_P according to (10), (11), and $\langle 0\,|\,\partial\cdot j_5^{\mathrm{NS}}\,|\,\eta_{\mathrm{NS}}\rangle\propto\beta_P$, we may infer the $I=1$ analog to (17) in the chiral limit to be

$$(m_{\delta}^{\mathrm{CL}})^2=(2m_{\mathrm{dyn}})^2+2\beta_P\approx 50m_\pi^2 . \tag{18}$$

Note that the factor of $2\beta_P$ in (18) is the same factor appearing in the pseudoscalar nonstrange diagonal matrix element of (14). This $2\beta_P$ term in (18) does not represent a quark-annihilation graph for the $I=1$ δ state; no such graph exists because the two-gluon intermediate state of Fig. 3(b) always transforms as an isoscalar. Instead we suggest that for the chiral pair $(\vec{\delta},\eta_{\mathrm{NS}})$, (4) is altered in such a way that the $I=0$ pseudoscalar quark-annihilation contribution of $2\beta_P$ appears in the $I=1$ scalar scattering graph and in (18) all the same as a dynamical consequence of chiral symmetry. To verify (18) rigorously even at the level of the leading order Bethe-Salpeter graphs (6) is quite difficult. Nevertheless we appeal to the simplicity of (18) as the natural $I=1$ dynamical analog of (9).

In the chiral-broken world the $\langle S\,|\,\mathscr{H}\,|\,S\rangle$ contribution to $2m_S^2$ in (12) must be again computed in terms of current-quark masses, but as in the pseudoscalar case, it too can be reexpressed back in terms of m_π^2 and m_K^2. In fact, in the σ model,[5,25] we know that m_σ^2-m_π^2 represents the chiral-limiting form of $(m_{\sigma\mathrm{NS}}^{\mathrm{CL}})^2$, or equivalently,

$$\begin{aligned}\langle\sigma_{\mathrm{NS}}\,|\,\mathscr{H}\,|\,\sigma_{\mathrm{NS}}\rangle&=\langle\delta\,|\,\mathscr{H}\,|\,\delta\rangle=\langle\pi\,|\,\mathscr{H}\,|\,\pi\rangle\\&=\langle\eta_{\mathrm{NS}}\,|\,\mathscr{H}\,|\,\eta_{\mathrm{NS}}\rangle=2m_\pi^2 .\end{aligned} \tag{19}$$

Thus, just as $m_{\eta\mathrm{NS}}^2$ is shifted upwards from $2\beta_P$ by an amount m_π^2 according to (16), we suggest from (17)–(19) that

$$m_{\sigma\mathrm{NS}}^2=(2m_{\mathrm{dyn}})^2+m_\pi^2\approx 22m_\pi^2 ,\quad m_{\sigma\mathrm{NS}}\approx 645\ \mathrm{MeV} , \tag{20a}$$

$$m_\delta^2=m_{\sigma\mathrm{NS}}^2+2\beta_P\approx 51m_\pi^2 ,\quad m_\delta\approx 985\ \mathrm{MeV} . \tag{20b}$$

This then is the dynamical analog of the chiral symmetry relation (1). We observe that m_δ in (20b) is perfectly consistent with the experimental value of[11] $m_\delta=981\pm 3$ MeV.

IV. SCALAR NONET AND σ-S^* MIXING

Just as the η_{NS} and η_S masses have off-diagonal terms proportional to the pseudoscalar quark-annihilation strength β_P which must be diagonalized to obtain the η and η' masses in (14), so now the σ_{NS} and σ_S (the latter the 0^+ $\bar{s}s$ state) masses have off-diagonal terms when we turn on the small scalar quark-annihilation strength β_S (we cannot of course turn off β_S even in the chiral limit). Such a scalar mass matrix analogous to (14) can be diagonalized to

$$\begin{bmatrix}(2m_{\mathrm{dyn}})^2+m_\pi^2+2\beta_S & \sqrt{2}\beta_S\\ \sqrt{2}\beta_S & (2m_{\mathrm{dyn}})^2+(2m_K^2-m_\pi^2)+\beta_S\end{bmatrix}\to\begin{bmatrix}m_\sigma^2 & 0\\ 0 & m_{S^*}^2\end{bmatrix} , \tag{21}$$

where the $I=0$ scalar $S^*(980)$ has mass[11] 980 ± 10 MeV. Unlike the pseudoscalar case where the diagonalization process is overdetermined, since m_σ is unknown in (21) we must input $m_{S^*}\approx 980$ MeV, i.e., $m_{S^*}^2\approx 50m_\pi^2$ along with $(2m_{\mathrm{dyn}})^2\approx 21m_\pi^2$ and $(2m_K^2-m_\pi^2)\approx 25m_\pi^2$ and then determine β_S and m_σ. The trace and determinant of (21) then lead uniquely to $m_\sigma\approx 720$ MeV and $\beta_S\approx 3m_\pi^2$. If we instead required the SU(3) breaking to be 20% as is the case for both the 0^- and 1^- nonets, then the 0^+ analog of Eq. (17) of Ref. 9 predicts $m_\sigma\approx 750$ MeV. The 0^+ analog of (15) is then a p-wave quark-annihilation graph of strength

$$\beta_S=\frac{[m_{S^*}^2-(2m_{\mathrm{dyn}})^2-m_\pi^2][m_\sigma^2-(2m_{\mathrm{dyn}})^2-m_\pi^2]}{4(m_K^2-m_\pi^2)}\approx 4.5m_\pi^2 . \tag{22}$$

In order to appreciate the relative scale of the 0^+ quark-annihilation strength β_S, we recall that the 1^- vector mesons also admit a similar ω-ϕ diagonalization procedure[9] yielding $\beta_V \approx 1 m_\pi^2$. The argument for the relative scales

$$\beta_P:\beta_S:\beta_V \approx 14.7:4.5:1 \tag{23}$$

is that the two-gluon 0^- and 0^+ graphs with $C(\bar{q}q)=(-)^{l+S}=1$ are, respectively, in s wave and p wave, the latter then being dynamically suppressed. Furthermore, the 1^- graph has $C=-1$ which corresponds to a three-gluon quark-annihilation graph, thus suppressing[8] β_V relative to β_P and β_S by one power of the QCD coupling α_S.

Next we convert the diagonalization result $\beta_S \approx 4.5 m_\pi^2$ to an equivalent mixing angle. Returning first to the pseudoscalar case we define ϕ_P as the η-η' mixing angle relative to the $\eta_{\rm NS}$-η_S quark basis:

$$|\eta\rangle = \cos\phi_P\,|\eta_{\rm NS}\rangle - \sin\phi_P\,|\eta_S\rangle\ , \tag{24a}$$

$$|\eta'\rangle = \sin\phi_P\,|\eta_{\rm NS}\rangle + \cos\phi_P\,|\eta_S\rangle\ . \tag{24b}$$

The angle ϕ_P, i.e., $\tan 2\phi_P$, is directly proportional to the strength $\beta_P \approx 14.7 m_\pi^2$ in

$$\tan 2\phi_P \propto \beta_P (m_{\eta S}^2 - m_{\eta\rm NS}^2)^{-1}\ .$$

Alternatively the formula for ϕ_P which incorporates the 20% SU(3) breaking is[9]

$$\tan^2\phi_P = \frac{(m_{\eta'}^2 - 2m_K^2 + m_\pi^2)(m_\eta^2 - m_\pi^2)}{(2m_K^2 - m_\pi^2 - m_\eta^2)(m_{\eta'}^2 - m_\pi^2)}\ ,$$

$$\phi_P \approx 42^\circ\ . \tag{25}$$

As noted by Isgur,[8] ϕ_P near 45° is a statement of the dominance of pseudoscalar mixing over mass breaking. The reverse is true for the 1^- vectors, with[9] $\phi_V \approx 3^\circ$ a statement of the (Okubo) dominance of mass breaking over mixing. Stated in the more conventional language of the mixing angle θ relative to the $1-8$ basis, we have $\theta = \phi - tan^{-1}\sqrt{2}$ or $\theta_P \approx -13^\circ$ and $\theta_V \approx -52^\circ$.

On the other hand, for the 0^+ case we define ϕ_S according to

$$|\sigma\rangle = \cos\phi_S\,|\sigma_{\rm NS}\rangle - \sin\phi_S\,|\sigma_S\rangle\ , \tag{26a}$$

$$|S^*\rangle = \sin\phi_S\,|\sigma_{\rm NS}\rangle + \cos\phi_S\,|\sigma_S\rangle\ , \tag{26b}$$

so that by analogy with (25), $\beta_S \approx 4.5 m_\pi^2$ with 20% SU(3) breaking leads to

$$\tan^2\phi_S = \frac{[m_{S^*}^2 - (2m_{\rm dyn})^2 - 2m_K^2 + m_\pi^2][m_\sigma^2 - (2m_{\rm dyn})^2 - m_\pi^2]}{[2m_K^2 - m_\pi^2 - m_\sigma^2 + (2m_{\rm dyn})^2][m_{S^*}^2 - (2m_{\rm dyn})^2 - m_\pi^2]}\ , \tag{27a}$$

$$\phi_S \approx 16^\circ\ . \tag{27b}$$

The 0^-, 0^+, and 1^- mixing angles relative to the quark basis then obey the pattern

$$\phi_P,\ \phi_S,\ \phi_V \approx 42^\circ,\ 16^\circ,\ 3^\circ\ . \tag{28}$$

In the next section we shall have more to say about the phenomenological determination of these angles.

V. PHENOMENOLOGY OF THE SCALAR NONET

A. General considerations

For the U(3)-invariant $0^+0^-0^-$ (*SPP*) coupling $\mathscr{H} = g_\sigma d_{abc} S^a P^b P^c$, the general $S \to PP$ decay width is

$$\Gamma_{\rm SPP} = \left[\sum |d_{abc}|^2\right] \frac{p}{2m_S^2}\frac{g_\sigma^2}{4\pi}\ , \tag{29a}$$

where p is the c.m. decay momentum and the U(3)-symmetry coefficients d_{abc} are tabulated in the nonstrange-strange basis in the Appendix. In the SU(2) σ model the *SPP* coupling has the form[5,25] $g_\sigma = (m_\sigma^2 - m_\pi^2) g_{\pi NN}/2m_N$ where now $m_\sigma = m_{\sigma\rm NS}$. But since from (20a) we know that $m_{\sigma\rm NS}^2 - m_\pi^2 = (2m_{\rm dyn})^2 \approx 21 m_\pi^2$, we may deduce that

$$g_\sigma^2/4\pi \approx 0.64\ {\rm GeV}^2\ . \tag{29b}$$

We shall continue to employ (29b) not only for the σ, but also for the other particles in the scalar nonet because U(3) invariance of the SPP coupling makes most sense when the (squared) mass term is expressed in terms of the above chiral invariant $(2m_{\rm dyn})^2$.

B. The σ meson

We have seen that spontaneous breakdown of chiral symmetry predicts $m_{\sigma\rm NS}^{\rm CL} \approx 630$ MeV, $m_{\sigma\rm NS} \approx 645$ MeV, and a diagonal physical σ mass

$m_\sigma \approx 750$ MeV. The corresponding chiral theory $\sigma \to \pi^+\pi^-, \pi^0\pi^0$ total decay width for $p \approx 349$ MeV is found from (29) to be

$$\Gamma_{\sigma\pi\pi} = \frac{3p}{4m_\sigma^2}\cos^2\phi_S \frac{g_\sigma^2}{4\pi} \approx 280 \text{ MeV}, \qquad (30)$$

where the slight deviation of $\cos\phi_S \approx 0.96$ from unity measures the $\bar{s}s$ contamination in the predominantly nonstrange σ. This latter factor derives from (26a) and $d_{33NS} = 1$ as tabulated in the Appendix.

Such a σ mass and large width are in fact consistent with simultaneous phase-shift fits to $\pi\pi$ and $K\bar{K}$ data which yields the typical value[26]

$$m_\sigma^{\text{exp}} = 750 \pm 100 \text{ MeV}, \quad \Gamma_\sigma^{\text{exp}} = 800 \pm 400 \text{ MeV}. \qquad (31)$$

An indirect test of m_σ can be obtained in the σ model with the phenomenologically deduced πN scattering σ term of[27] $\sigma_{\pi N} = 65 \pm 5$ MeV implying

$$m_{\sigma\text{NS}}^2 = \frac{(m_\pi g_A)^2}{\sigma_{\pi N}} m_N \approx 23 m_\pi^2, \quad m_{\sigma\text{NS}} \approx 660 \text{ MeV}. \qquad (32)$$

Finally, the ${}^3S_1$ NN force in nuclear physics requires a 0^+ 2π isobar exchange of mass $500-600$ MeV. Both of these results are compatible with our analysis.

C. The δ meson

Our spontaneous-breakdown prediction of $m_\delta \approx 985$ MeV combined with (29) yields the $\delta \to \eta\pi$ width for $p \approx 320$ MeV,

$$\Gamma_{\delta\eta\pi} = \frac{P}{2m_\delta^2}\cos^2\phi_P \frac{g_\sigma^2}{4\pi} \approx 58 \text{ MeV}, \qquad (33)$$

which is perfectly consistent with the observed values[11]

$$m_\delta^{\text{exp}} = 981 \pm 3 \text{ MeV}, \quad \Gamma_{\delta\eta\pi}^{\text{exp}} = 52 \pm 8 \text{ MeV}. \qquad (34)$$

This is a very strong indication that δ indeed originates from the spontaneous breakdown of chiral symmetry.

Note that $\cos^2\phi_P \approx 0.55$ in (33) replaces $\cos^2\phi_S$ in (30); the former completely accounts for η-η' mixing via (24). This 55% mixing-factor reduction in (33) is not only predicted by our quark-annihilation–graph theory from (25), i.e., $\theta_P \approx -13°$, but it also follows independently from phenomenology. (For example, the strong decay $A_2 \to \eta\pi/K\bar{K}$ branching ratio predicts $\theta_P = -12° \pm 2°$.)

A somewhat more tenuous prediction for δ decays follows from an observation[28] of the $\delta \to K\bar{K}/\eta\pi$ branching ratio

$$\frac{\Gamma_{\delta K\bar{K}}}{\Gamma_{\delta\eta\pi}} = \frac{p_{K\bar{K}}}{2p_{\eta\pi}\cos^2\phi_P} = 0.25 \pm 0.08. \qquad (35)$$

The difficulty in (35) is determining $p_{K\bar{K}}$ since the δ(985) particle is below the threshold for this decay. If we assume that the 70-MeV total δ width gives the δ an effective-mass fraction above the $K\bar{K}$ threshold, then folding in a Breit-Wigner form[29] requires $p_{K\bar{K}}^{\text{eff}} \approx 50$ MeV. Then the left-hand side of (35) becomes 0.14, again reasonably consistent with the data. The drawback is a second measurement of (35) which is over twice as large but with a much bigger error.[11]

D. The S^* meson

A direct test of the ϕ_S mixing angle follows from the recently measured $S^* \to \pi^+\pi^-, \pi^0\pi^0$ total decay rate[30] with $p \approx 470$ MeV,

$$\Gamma_{S^*\pi\pi} = \frac{3p}{4m_{S^*}^2}\sin^2\phi_S \frac{g_\sigma^2}{4\pi} = 24 \pm 8 \text{ MeV}. \qquad (36)$$

Applying the chiral-symmetry coupling (29b) to (36), we find at $m_{S^*} \approx 980$ MeV,

$$\phi_S = 19° \pm 3°, \qquad (37)$$

which is in excellent agreement with our theoretical value of $\phi_S \approx 16°$ obtained from (27).

Another test of ϕ_S follows from the $S^* \to K\bar{K}/\pi\pi$ branching ratio. Since in fact this ratio is substantially greater than unity,[31] the angle ϕ_S is again predicted to be small.

E. The κ meson

The strange-particle analog of (20a) is obviously (see, e.g., Ref. 25)

$$m_\kappa^2 = (2m_{\text{dyn}})^2 + m_K^2 \approx 34 m_\pi^2, \quad m_\kappa \approx 800 \text{ MeV}. \qquad (38)$$

The associated theoretical $\kappa \to K\pi$ width following from (29) is, for $p \approx 220$ MeV,

$$\Gamma_{\kappa K\pi} = \frac{3p}{8m_\kappa^2}\frac{g_\sigma^2}{4\pi} \approx 80 \text{ MeV}. \qquad (39)$$

Recall that theoretical analyses of a decade ago assumed a κ in this mass region; the κ-meson width (39) is relatively narrow compared to the σ width, and such a κ perhaps was seen.[32] Unfortunately it no longer appears in the Particle Data Group tables.[11] Nevertheless there is some amplitude activity in the $K\pi$ phase-shift 800-MeV region[33] when the phase shift is $\sim 60°$. Perhaps the problem is due to the complicated coupled-channel analysis required in the $K\pi$ case. In any case, the present absence of a phenomenological $\kappa(800)$ is the only evidence that we are aware of which is counter to our theoretical predictions.

F. Radial excitations

Since in our model the ground state $l=1$, 0^{++} $\bar{q}q$ configuration corresponds to $\sigma(750)$, $\kappa(800)$, $S^*(980)$, and $\delta(985)$, the observed[11] higher resonances $\epsilon(1300)$ and $\kappa(1500)$ are presumably radial excitations of $\sigma(750)$ and $\kappa(800)$, respectively. Just as the radially excited $\rho'(1600)$ is ~ 800 MeV above the $\rho(776)$, it is not surprising that $\epsilon(1300)$ is ~ 600 MeV above $\sigma(750)$ and $\kappa(1500)$ is ~ 700 MeV above $\kappa(800)$. We find this picture simpler and more appealing than a low-lying $\bar{q}\,\bar{q}qq$ four-quark bound nonet[7] accompanied by a higher $\bar{q}q$ nonet with two of the four members as yet undetected and with a ϕ_S mixing angle unrelated to any underlying quark-annihilation-graph picture.

VI. SUMMARY

In this paper we have tried to present a complete theoretical and phenomenological picture underlying the dynamics of the $\bar{q}q$ scalar nonet $\sigma(750)$, $\kappa(800)$, $S^*(980)$, and $\delta(985)$. Except for the κ meson, dynamical spontaneous breakdown of chiral symmetry completely accounts for the observed masses, branching ratios, decay widths, and σ-S^* mixing angle of the nonet. Of particular significance is the correct prediction for the lifting of the nonstrange $\sigma_{\rm NS}$-δ mass degeneracy. By way of contrast, the nonrelativistic SU(6) constituent-quark model cannot account for this $\sigma_{\rm NS}$-δ mass splitting unless one speculates that this lowest-lying scalar nonet is somehow composed of four-quark states, unlike any other meson multiplet so far detected. With hindsight, the observed existence and measured dynamical parameters of the scalar nonet provide further evidence that spontaneous breakdown of chiral symmetry actually is operative in nature.

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APPENDIX: THE CARTESIAN U(3) STRUCTURE CONSTANTS d_{ijk} IN THE NONSTRANGE-STRANGE BASIS

Given the phase assignments

$$\pi^{\pm}=P^{(1\pm i2)/(2)^{1/2}},\quad \pi^0=P^3,$$
$$K^{\pm}=P^{(4\pm i5)/(2)^{1/2}},$$
$$K^0,\bar{K}^0=P^{(6\pm i7)/(2)^{1/2}},$$

and the $I=0$ states

$$|\eta_{\rm NS}\rangle=(\sqrt{2}\,|\eta_0\rangle+|\eta_8\rangle)/\sqrt{3}$$
$$=(|\bar{u}u\rangle+|\bar{d}d\rangle)/\sqrt{2}\;;$$
$$|\eta_S\rangle=(|\eta_0\rangle-\sqrt{2}\,|\eta_8\rangle)/\sqrt{3}=|\bar{s}s\rangle\;,$$

the symmetric structure constants obey

$$d_{S,{\rm NS},j}=d_{3,3,S}=d_{{\rm NS},{\rm NS},3}=0\;,$$
$$d_{S,S,S}=\sqrt{2}\;,\quad d_{3,3,{\rm NS}}=d_{{\rm NS},{\rm NS},{\rm NS}}=1\;,$$
$$d_{K,K,{\rm NS}}=\tfrac{1}{2}\;,\quad d_{K,K,S}=1/\sqrt{2}\;,$$
$$d_{{\rm NS},{\rm NS},8}=1/\sqrt{3}\;,\quad d_{{\rm NS},8,8}=\tfrac{1}{3}\;,$$
$$d_{S,S,8}=-2/\sqrt{3}\;,\quad d_{S,8,8}=2\sqrt{2}/3\;.$$

Likewise the nonvanishing antisymmetric structure constants f_{ijk} are

$$f_{4,5,{\rm NS}}=f_{6,7,{\rm NS}}=\tfrac{1}{2}$$

and

$$f_{4,5,S}=f_{6,7,S}=-1/\sqrt{2}\;.$$

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