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Neutral currents in E_6

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Electroweak theories based on gauge groups $SU(2)_L \times U(1) \times U(1) \times U(1)$ arising from the spontaneous breakdown of E_6 are examined. The neutral-current interactions at low Q^2 in two such three-Z models are derived. One neutral boson with a mass a few percent lighter than the standard-model Z^0 mass and two additional Z's as light as 300 GeV$-$1 TeV are allowed.

I. INTRODUCTION

The standard $SU(2)_L \times U(1)$ model[1] of the electroweak interactions has been remarkably successful in describing all low-energy weak-interaction phenomena. Many models based on extended gauge groups $SU(2)_L \times U(1) \times G$ have been considered[2–18] which reproduce the low-energy predictions of the standard model but which differ from it at sufficiently high energies. One class of such theories[2–6] finds an effective neutral-current (NC) interaction at low Q^2 of the form

$$\mathscr{H}^{\rm NC} = \frac{4G_F}{\sqrt{2}}[(J_{3L} - x_L J_{\rm EM})^2 + C J_{\rm EM}{}^2] , \quad (1.1)$$

where the constant C depends on the masses of the neutral bosons, i.e., measures the spectral functions of the weak quanta. This form is consistent with a result of Bjorken[7] which predicts a NC interaction of the type (1.1) on the basis of quite general arguments not necessarily in the context of gauge theories.

Another class of extended electroweak models is based on left-right symmetry,[8] e.g.,

$$\begin{aligned} &SU(2)_L \times SU(2)_R \times U(1)_{L+R} , \\ &SU(2)_L \times SU(2)_R \times U(1)_{B-L} . \end{aligned} \quad (1.2)$$

Here the effective NC interaction is often given by

$$\mathscr{H}^{\rm NC} = \frac{G_F}{\sqrt{2}}(J_{3L} - x_L J_{\rm EM})^2 + \frac{G_F'}{\sqrt{2}}(J_{3R} - x_R J_{\rm EM})^2 . \quad (1.3)$$

A naive extension of the Bjorken result with separate left and right sectors would suggest such a form along with an additional $J_{\rm EM}{}^2$ term whose coefficient probes the structure of the spectral functions of both the left- and right-handed quanta. For a two-Z model such as (1.2) such an extra term would vanish. Models based on left-right symmetry with groups larger than (1.2) would then test such speculations. Shafi and Wetterich[9] and Elias, Pati, and Salam[10] have considered manifestly left-right-symmetric models based on

$$SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R \quad (1.4)$$

and Costa, D'Anna, and Marcolungo[11] have shown that the additional piece of $\mathscr{H}^{\rm NC}$ beyond (1.3) is *not* a $J_{\rm EM}{}^2$ term in such theories.

Motivated by the success of grand unified theories based on SU(5), electroweak theories based on breakdowns of the grand unification group SO(10) (Refs. 12–18) have also been investigated both for their effects on current and future electroweak experiments and for the possibility of testing remnant grand unification structure at accessibly low energies. The authors of Ref. 18, for example, find that for one breakdown of SO(10) via $SU(2)_L \times SU(2)_R \times SU(4)$ the NC coupling have the form (1.3) as expected. They also find that same form for a very different breakdown scheme through an $SU(5) \times U(1)$ symmetry. They then discuss under what conditions two different breakdown patterns of a grand unified theory can give rise to the same effective NC interactions.

In this paper we will examine two different electroweak theories based on $SU(2)_L \times U(1) \times U(1) \times U(1)$ symmetries that can arise in the spontaneous symmetry breakdown of the grand unification group E_6. This will allow us to further test the ideas of Ref. 18 on the similarity in form of the NC couplings in different breakdown schemes of the same unification group.

The first succession of breakdowns we will examine will be

$$E_6 \to SO(10) \times U(1)_\psi , \quad (1.5)$$

$$SO(10) \to SU(5) \times U(1)_\chi , \quad (1.6)$$

$$SU(5)\rightarrow SU(3)_C\times SU(2)_L\times U(1)_{Y_W} . \quad (1.7)$$

This will be compared with the breakdown chain

$$E_6\rightarrow SU(3)_L\times SU(3)_R\times SU(3)_C , \quad (1.8)$$

$$SU(3)_{L,R}\rightarrow SU(2)_{L,R}\times U(1)_{L1,R1} , \quad (1.9)$$

$$SU(2)_R\rightarrow U(1)_R . \quad (1.10)$$

This second case will also allow us to test the form of the NC interaction in left-right models beyond (1.3).

In Sec. II we briefly review the structure of E_6 and extend the geometric language used for SO(10) representations and generators in Ref. 18 to E_6. We also discuss the symmetry-breakdown patterns that lead to $SU(3)_C\times SU(2)_L\times U(1)\times U(1)\times U(1)$ theories and discuss the Higgs structure used to break them to $SU(3)_C\times U(1)_{EM}$. In Sec. III we derive the effective NC interactions at low Q^2 for the two cases and examine the constraints that current experiments place on the parameters and gauge-boson masses in these three-Z models.

We find that the forms of the NC interaction in the two different breakdown schemes are very similar. They are in fact identical in the limit where various gauge couplings are equal in accordance with a theorem proved in Ref. 18.

The ratio of charged-current to neutral-current effective couplings is naturally the same as in the standard model as are all neutrino NC interactions. Constraints imposed on the model by measurements of parity-violating effects in heavy atoms and $e^+e^-\rightarrow\mu^+\mu^-$ allow two heavy Z's in addition to the standard-model Z^0 in the range 300 GeV$-$1 TeV.

II. STRUCTURE OF E_6

A. Fermion representations

The exceptional group E_6 has been considered by several authors[19] as a candidate for grand unification. In fact, if we associate SU(5) with "E_4" and SO(10) with "E_5" (as suggested by Dynkin-diagram language) we are led to E_6 as the next member in the sequence. It is the only exceptional group that is automatically anomaly-free in all representations and also allows for complex representations. The extra rank of E_6 beyond SO(10) gives an additional neutral and colorless gauge boson as the decomposition of the adjoint $\underline{78}$ of E_6 under SO(10),

$$\underline{78}=\underline{45}+\underline{16}+\underline{16}^*+\underline{1} , \quad (2.1)$$

shows. The singlet is the gauge boson which yields (after mixing) the third Z; we will examine its contribution to the neutral-current interactions and constraints on its mass.

The fermions are assigned to the fundamental $\underline{27}$ of E_6 with the branching to SO(10),

$$\underline{27}=\underline{16}+\underline{10}+\underline{1} . \quad (2.2)$$

The $\underline{16}$ is the usual SO(10) generation while the $\underline{10}$ contains a $Q=-\frac{1}{3}$ quark h and a charged lepton E^-, and their antiparticles along with corresponding neutral leptons ν_E and $\bar{N}_E$.[19] The SO(10) singlet is one component of a neutral lepton n^0. As is well known, two $\underline{27}$'s give a six-quark model with the h quark associated with the b, the E^- with the τ, and no top quark. Such models with the b quark in an $SU(2)_L$ singlet predict[20] fewer nonleptonic decays for B mesons and more $B\rightarrow Xl^+l^-$ than the standard-model assignment with the b in a sequential doublet. The measured values of the branching ratios for $B\rightarrow Xl^-\nu$ and $B\rightarrow Xl^+l^-$ (Ref. 21) almost completely rule out such an assignment however. Thus three E_6 generations ($\underline{27}$'s) must be used giving a nine-quark model. The additional quarks and leptons, those not in SO(10) $\underline{16}$'s, are assumed to acquire a heavy (possibly superheavy) mass.[19]

The breakdown $E_6\rightarrow SO(10)\times U(1)_\psi$ defines a new charge Q_ψ and Slansky[22] has given the relative value of this quantum number for many E_6 representations. Normalizing Q_ψ in the same way as all other U(1) charges in E_6 then gives

$$Q_\psi(\underline{16})=\frac{1}{\sqrt{24}},\quad Q_\psi(\underline{10})=\frac{-2}{\sqrt{24}} ,$$
$$Q_\psi(\underline{1})=\frac{4}{\sqrt{24}} \quad (2.3)$$

for members of the $\underline{27}$.

Under the restriction of E_6 to $SU(3)_L\times SU(3)_R\times SU(3)_C$ we have

$$\underline{27}=(\underline{3};\underline{1};\underline{3})+(\underline{3}^*;\underline{3};\underline{1})+(\underline{1};\underline{3}^*;\underline{3}^*)$$
$$=(u,d,h)_L+(\text{leptons})_L+(\bar{u},\bar{d},\bar{h})_L . \quad (2.4)$$

The quantum numbers $I_{L1,R1}$ associated with the Abelian group factors in the subsequent breakdown $SU(3)_{L,R}\rightarrow SU(2)_{L,R}\times U(1)_{L1,R1}$ can be similarly defined and will be discussed in the next section.

B. Generators and charges

A particularly convenient language for dealing with SO($2n$) algebras has been introduced[23] and was used extensively in Ref. 18 to aid in identify-

TABLE I. Members of the 10-dimensional representation of SO(10) contained in the E_6 27. The components in the five-dimensional weight space of SO(10) are shown; the sixth component is $-1/\sqrt{3}$.

SU(5) representation	$2\sqrt{10}$	Particle	Signs in weight vector $(\pm1,0,0,0,0)$	I_{3L}	Q
			(−0000)		
5*	−2	$\bar{h}_L$	(0−000)	0	$\frac{1}{3}$
			(00−00)		
		$(\nu_E)_L$	(0000−)	$\frac{1}{2}$	0
		E_L^-	(000−0)	$-\frac{1}{2}$	−1
			(+0000)		
5	2	h_L	(0+000)	0	$-\frac{1}{3}$
			(00+00)		
		$(\bar{N}_E)_L$	(0000+)	$-\frac{1}{2}$	0
		E_L^+	(000+0)	$\frac{1}{2}$	1

ing members of SO(10) fermion representations and various charge operators. We will use this same language and extend it to E_6.

The members of an SO($2n$) spinor can be represented by vectors in an n-dimensional Cartesian vector space by

$$\underline{2}^{n-1} \leftrightarrow \tfrac{1}{2}(\epsilon_1,\epsilon_2,\ldots,\epsilon_n) \tag{2.5}$$

with $\epsilon_i=\pm1$ and $\prod_{i=1}^{n}\epsilon_i=-1$. The members of the SO(10) 16 generation were identified in this way in Table I of Ref. 18.

The vector representation of SO($2n$) is given in this same space by

$$\underline{2n} \leftrightarrow \begin{bmatrix} (\epsilon,0,\ldots,0) \\ (0,\epsilon,\ldots,0) \\ \cdots \\ (0,0,\ldots,\epsilon) \end{bmatrix} \tag{2.6}$$

with $\epsilon=\pm1$. The members of the vector 10 in the 27 of E_6 can be represented this way and the resulting assignments in this language are listed in Table I of this work.

The SO(10) singlet is of course given by

$$\underline{1} \leftrightarrow (0,0,0,0,0)\ . \tag{2.7}$$

To embed this structure in E_6 we add an additional Cartesian component whose value essentially measures the ψ charge of the SO(10) multiplets. We have in the resulting six-dimensional space[24]

$$\underline{16} \leftrightarrow \tfrac{1}{2}\left[\pm1,\pm1,\pm1,\pm1,\pm1;\frac{1}{\sqrt{3}}\right] \tag{2.8}$$

(with an odd number of minus signs),

$$\underline{10} \leftrightarrow \left[\begin{matrix} \left[\pm1,0,0,0,0;\frac{-1}{\sqrt{3}}\right] \\ \cdots \\ \left[0,0,0,0,\pm1;\frac{-1}{\sqrt{3}}\right] \end{matrix}\right], \tag{2.9}$$

$$\underline{1} \leftrightarrow \left[0,0,0,0,0;\frac{2}{\sqrt{3}}\right]\ . \tag{2.10}$$

In this language the various diagonal generators are also given by vectors whose scalar product with the members of the fermion representations give the value of the corresponding charge. Extended trivially to the six-dimensional space the generator of weak isospin is

$$V(I_{3L})=\tfrac{1}{2}(0,0,0,1,-1;0) \tag{2.11}$$

while the hypercharge (normalized as all other operators) is

$$V(Y_W)=\frac{1}{\sqrt{15}}(-2,-2,-2,3,3;0)\ . \tag{2.12}$$

The normalization is $V(A)\cdot V(B)=\frac{1}{2}\delta_{AB}$. The χ and ψ charge operators are then

$$V(\chi)=\frac{1}{\sqrt{10}}(1,1,1,1,1;0)\ , \tag{2.13}$$

$$V(\psi)=\frac{1}{\sqrt{2}}(0,0,0,0,0;1)\ . \tag{2.14}$$

For the left-right breakdown we will need

$$V(I_{3R})=\tfrac{1}{2}(0,0,0,1,1;0) \tag{2.15}$$

and

$$V(I_{L1,R1})=\frac{1}{2\sqrt{3}}(-1,-1,-1,0,0;\pm\sqrt{3})\ . \tag{2.16}$$

[Equation (2.16) then determines all of the necessary $I_{L1,R1}$ quantum numbers.]

The set of generators useful in the breakdown through $SU(5)\times U(1)_\chi\times U(1)_\psi$, (Y_W,χ,ψ), is related to the set used in the breakdown via $SU(3)_L\times SU(3)_R\times SU(3)_C$, (I_{3R},I_{L1},I_{R1}). We find

$$\begin{bmatrix} Y_W \\ \chi \\ \psi \end{bmatrix} = \begin{bmatrix} \left[\frac{3}{5}\right]^{1/2} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \left[\frac{2}{5}\right]^{1/2} & -\left[\frac{3}{10}\right]^{1/2} & -\left[\frac{3}{10}\right]^{1/2} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} I_{3R} \\ I_{L1} \\ I_{R1} \end{bmatrix}\ . \tag{2.17}$$

The electric-charge operator is given by

$$\tilde{V}(Q)=(-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},1,0;0) \tag{2.18}$$

and we have

$$Q=I_{3L}+\left[\frac{5}{3}\right]^{1/2}Y_W \tag{2.19}$$

and

$$Q=I_{3L}+I_{3R}+\frac{1}{\sqrt{3}}I_{L1}+\frac{1}{\sqrt{3}}I_{R1}\ . \tag{2.20}$$

[Note that the electric-charge generator $\tilde{V}(Q)$ is not (and need not) be normalized in the same way as the other generators which are embedded in the grand unified theories.]

C. Symmetry breakdown and Higgs fields

In order to allow the various desired U(1) factors to remain unbroken down to low energies in our first scheme we demand that E_6 (SO(10), SU(5)) be broken down via an adjoint $\underline{78}$ ($\underline{45}$,$\underline{24}$) of Higgs fields. The resulting symmetry below the SU(5) mass scale M_X is then

$$S(A)=SU(3)_C\times SU(2)_L\times U(1)_{Y_W}\times U(1)_\chi\times U(1)_\psi \tag{2.21}$$

with couplings g, g_Y, g_χ, and g_ψ for the electroweak factors. We have the relations

$$\frac{e^2}{g^2}=\sin^2\theta_W,\quad \frac{e^2}{g_Y{}^2}=\tfrac{3}{5}\cos^2\theta_W\ . \tag{2.22}$$

Because the $U(1)_\chi$ and $U(1)_\psi$ are broken off at mass scales at least as large as the M_X of SU(5) the usual renormalization arguments, give

$$g_\psi{}^2\le g_\psi{}^2\le g_Y{}^2=\frac{5}{3}\frac{e^2}{\cos^2\theta_W} \tag{2.23}$$

with equality if $E_6\to S(A)$ all at once.

We imagine a similar series of breakdowns for the other case we discuss. For example, we can arrange for the ($\underline{8}$;$\underline{1}$;$\underline{1}$) and ($\underline{1}$;$\underline{8}$;$\underline{1}$) contained in the adjoint $\underline{78}$ to develop VEV's (possibly different) in the appropriate direction in group space in order to break $SU(3)_{L,R}$ down to $SU(2)_{L,R}\times U(1)_{L,R}$. Thus we also examine the possibility that the symmetry

$$S(B)=SU(3)_C\times SU(2)_L\times U(1)_R\times U(1)_{L1}\times U(1)_{R1} \tag{2.24}$$

might be valid at TeV energies and below. The corresponding couplings for the electroweak factors are g_L, g_R, g_{L1}, and g_{R1} and we have the identity $e^2/g_L{}^2=\sin^2\theta_W$. Using familiar renormalization-group arguments we can relate the couplings g_R, g_{L1}, and g_{R1} to the mass scales at which the various symmetries are broken but we allow these remaining couplings to be arbitrary in what follows for generality.

Several breakdowns of E_6 through its maximal subgroup $SU(2)\times SU(6)$ can also lead to $SU(2)_L\times U(1)\times U(1)\times U(1)$ electroweak theories at low energies. We will not consider these cases.

In Ref. 18 the two neutral and colorless fields in the SO(10) $\underline{16}$ were given nonzero (and different) vacuum expectation values (VEV's) in order to break the $SU(2)_L\times U(1)\times U(1)$ symmetries discussed there, i.e.,

$$\phi_1\in\underline{5}^*,\quad \langle\phi_1\rangle=\frac{v_1}{\sqrt{2}}\ , \tag{2.25}$$

$$\phi_2\in\underline{1},\quad \langle\phi_2\rangle=\frac{v_2}{\sqrt{2}} \tag{2.26}$$

with $v_1{}^{-2}=\sqrt{2}G_F$. This ensured that the ratio of neutral-current to charged-current effective couplings was naturally the same as in the standard model. The ϕ_2 Higgs field was chosen because under the breakdown of SO(10) to $SU(5)\times U(1)_\psi$ it is a singlet under everything except the χ charge.

We follow the same pattern here and choose as an additional Higgs field the SO(10) singlet contained in the E_6 27 which transforms nontrivially only under the Q_ψ generator in the breakdown of E_6 to $SO(10)\times U(1)_\psi$.

In the Cartesian language the Higgs fields used to break both the S(A) and S(B) symmetries are then

$$\phi_1 \leftrightarrow \tfrac{1}{2}\left[1,1,1,1,-1;\frac{1}{\sqrt{3}}\right]$$

$$\langle\phi_1\rangle = \frac{v}{\sqrt{2}}\,, \tag{2.27}$$

$$\phi_2 \leftrightarrow \tfrac{1}{2}\left[-1,-1,-1,-1,-1;\frac{1}{\sqrt{3}}\right],$$

$$\langle\phi_2\rangle = \frac{V}{\sqrt{2}}\,, \tag{2.28}$$

$$\phi_3 \leftrightarrow \left[0,0,0,0,0;\frac{2}{\sqrt{3}}\right],$$

$$\langle\phi_3\rangle = \frac{\widetilde{V}}{\sqrt{2}}\,, \tag{2.29}$$

with $v^{-2}=\sqrt{2}G_F$.

The two neutral and colorless fields in the SO(10) 10 can also be given nonzero VEV's and still leave an $SU(3)_C\times U(1)_{EM}$ symmetry intact. We will not consider this possibility here.

III. NEUTRAL-CURRENT COUPLINGS AND CONSTRAINTS

With the Higgs sector specified in (2.27)–(2.29) it is now straightforward to derive the mass matrix μ^2 for the neutral vector bosons. The photon, corresponding to zero eigenvalue, is obtained as always. Using the method of Georgi and Weinberg[25] one can invert a submatrix of μ^2 and immediately find the NC couplings at $Q^2\simeq 0$. For case S(A) we find

$$\mathscr{H}^{\rm NC} = \frac{2}{v^2}(J_{3L}-\sin^2\theta_W J_{\rm EM})^2 + \frac{2}{V^2}\left[\left(\frac{3}{5}\right)^{1/2} J_Y + \left(\frac{2}{5}\right)^{1/2} J_\chi - \tfrac{3}{5}\cos^2\theta_W J_{\rm EM}\right]^2$$
$$+\frac{1}{8\widetilde{V}^2}\left[J_{3L} - \left(\frac{3}{5}\right)^{1/2} J_Y - \left(\frac{2}{5}\right)^{1/2} J_\chi - \sqrt{6}J_\psi + \left(\frac{3-8\sin^2\theta_W}{5}\right) J_{\rm EM}\right]^2 . \tag{3.1}$$

Using (2.17) we can rewrite this as

$$\mathscr{H}^{\rm NC} = \frac{4G_F}{\sqrt{2}}\left[(J_{3L}-xJ_{\rm EM})^2 + \frac{1}{R}(J_{3R}-\widetilde{x}J_{\rm EM})^2 + \frac{1}{16\widetilde{R}}(J_{3L}-J_{3R}-\sqrt{6}J_\psi+\delta J_{\rm EM})^2\right], \tag{3.2}$$

where $x\equiv\sin^2\theta_W$, $\widetilde{x}\equiv 3\cos^2\theta_W/5$, $\delta\equiv(3-8x)/5=\widetilde{x}-x$, $R\equiv(V^2/v^2)$, and $\widetilde{R}\equiv(\widetilde{V}^2/v^2)$.

The first two terms of (3.2) are just those derived in Ref. 18 for the case $SO(10)\to SU(5)\times U(1)_\chi$ and give the same NC interactions for neutrinos as the standard model since $I_{3R}(\nu_L)=Q(\nu_L)=0$. The last term in (3.2) does not contribute to neutrino NC's either since $I_{3L}(\nu_L)=\sqrt{6}Q_\psi(\nu_L)$. We will briefly discuss the limits imposed on R and $\widetilde{R}$ by parity-violating interactions not involving neutrinos in a moment.

For case S(B) we similarly find

$$\mathscr{H}^{\rm NC} = \frac{4G_F}{\sqrt{2}}\left[(J_{3L}-x_L J_{\rm EM})^2 + \frac{1}{R}(J_{3R}-x_R J_{\rm EM})^2 + \frac{1}{16\widetilde{R}}[J_{3L}-J_{3R}-\sqrt{3}(J_{L1}-J_{R1})+\delta' J_{\rm EM}]^2\right], \tag{3.3}$$

where $x_L=e^2/g_L{}^2=\sin^2\theta_W$, $x_R\equiv e^2/g_R{}^2$, and $\delta'\equiv x_R-x_L+x_{L1}-x_{R1}$ with $x_{L1,R1}\equiv e^2/g_{L1,R1}{}^2$. Since by (2.17), $J_\psi=(J_{L1}-J_{R1})/\sqrt{2}$, the forms (3.2) and (3.3) are very similar as expected. We know from the discussions in the Appendix of Ref. 18 that the two interactions will be identical when the couplings of the neutral generators in the two schemes that "mix" [as in Eq. (2.17)] are equal. For this discussion that condition is

$$x=\sin^2\theta_W=x_L\,, \tag{3.4}$$

$$\widetilde{x}=\tfrac{3}{5}\cos^2\theta_W=x_R=x_{L1}=x_{R1}\,, \tag{3.5}$$

and (3.2) and (3.3) are indeed equal in that case.

As mentioned earlier, we have not considered any cases arising from breakdowns of E_6 through an $SU(2)\times SU(6)$ symmetry stage. We would expect, however, that *any* $SU(2)_L\times U(1)\times U(1)\times U(1)$ electroweak theory arising from E_6 and broken with the Higgs sector (2.27)–(2.29) would have its NC interactions described by the forms (3.2) or (3.3). The only difference among various breakdown chains would then be in the precise definitions of $\tilde{x}$ (or x_R) and δ (or δ') in terms of coupling constants.

Costa, D'Anna, and Marcolungo[11] have obtained an expression similar to (3.3) for an $SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model (not specifically derived from a grand unified theory) with manifest left-right symmetry ($g_L \equiv g_R$, $g_{L1} \equiv g_{R1}$) in which case $\delta'=0$.

In neither S(A) nor S(B) schemes is the additional piece of $\mathscr{H}^{\rm NC}$ beyond (1.3) solely a $J_{\rm EM}{}^2$ term. It would be of interest to generalize the arguments of Bjorken[7] to left-right models to see if the terms we find can be derived naturally under his general assumptions.

The couplings relevant for parity violation in eN and $e^+e^- \to \mu^+\mu^-$ experiments can be read from the interactions (3.2) and (3.3). We consider only the S(A) case explicitly as an example and find that

$$\tilde{\alpha}=(-1+2x)+\frac{1}{R}(1-2\tilde{x})-\frac{\delta}{2\tilde{R}}\,, \tag{3.6}$$

$$\tilde{\gamma}=\frac{2}{3}x-\frac{2\tilde{x}}{3R}-\frac{\delta}{6\tilde{R}}\,, \tag{3.7}$$

$$h_{AA}=\frac{1}{4}\left[1+\frac{1}{R}+\frac{1}{\tilde{R}}\right], \tag{3.8}$$

$$h_{VV}=(-\tfrac{1}{2}+2x)^2+\frac{1}{R}(-\tfrac{1}{2}+2\tilde{x})^2+\frac{\delta^2}{4\tilde{R}}\,. \tag{3.9}$$

(In manifestly left-right-symmetric models where $\delta'=0$, the additional term in $\mathscr{H}^{\rm NC}$ can be tested only by measurements of h_{AA}.)

Parity-violation experiments on bismuth[26] and thallium[27] give the constraints

$$-135\pm17.5=Q_W(\mathrm{Bi})=43\tilde{\alpha}-627\tilde{\gamma}\,, \tag{3.10}$$

$$-155\pm63=Q_W(\mathrm{Tl})=42\tilde{\alpha}-612\tilde{\gamma}\,. \tag{3.11}$$

Measurements of the asymmetry in polarized-electron–deutron scattering from SLAC (Ref. 28) give

$$-1.8\pm0.48=3\tilde{\alpha}+\tilde{\gamma}\,. \tag{3.12}$$

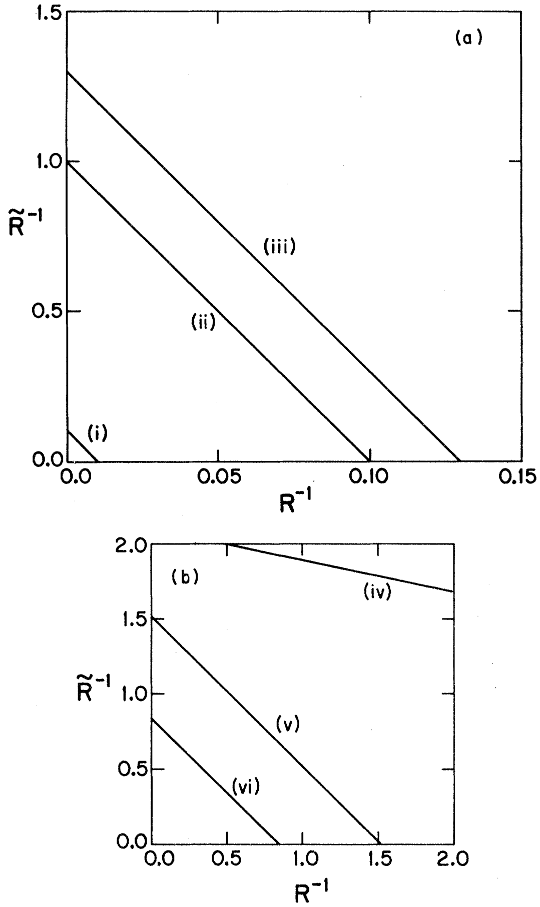

FIG. 1. Constraints on R^{-1} and $\tilde{R}^{-1}$ from various measurements of parity-violating effects in eN and $e^+e^- \to \mu^+\mu^-$ experiments. The triangular region between the axes and each line is allowed in each case. Parameters for case S(A) are used with $\sin^2\theta_W=0.23$. (a) (i) Novosibirsk bismuth experiment (1σ) (Ref. 26). (ii) Same as (i) at 2σ level. (iii) Berkeley thallium experiment (1σ) (Ref. 27). (b) (iv) SLAC ed experiment (1σ) (Ref. 28). (v) TASSO measurement of h_{AA} (1σ) (Ref. 30). (vi) JADE measurement of h_{AA} (1σ) (Ref. 29).

Two recent measurements[29,30] of the forward-backward charge asymmetry in $e^+e^- \to \mu^+\mu^-$ find

$$h_{AA}=\begin{cases}0.36\pm0.10 & (\text{Ref.}\,29)\,, \quad (3.13)\\ 0.53\pm0.10 & (\text{Ref.}\,30)\,. \quad (3.14)\end{cases}$$

The constraints (3.10)–(3.14) are all of the schematic form

$$\text{experimental value}\pm\text{error}=\text{standard-model value}+cR^{-1}+\tilde{c}\tilde{R}^{-1}\,. \tag{3.15}$$

If we demand that our values lie within 1σ of the quoted results, each measurement restricts us to a region of $(R^{-1},\tilde{R}^{-1})$ space. The resulting constraints on R^{-1} and $\tilde{R}^{-1}$ are plotted in this way in

TABLE II. Masses of the three bosons in selected cases in S(A). Masses are given relative to the mass of the standard model Z^0, M_0. We assume that $R=\tilde{R}$ and $g_\chi{}^2=g_\psi{}^2=5e^2/(3\cos^2\theta_W)$ for simplicity.

$R=\tilde{R}$	$\frac{M_1}{M_0}$	$\frac{M_2}{M_0}$	$\frac{M_3}{M_0}$
3	0.89	1.7	2.0
5	0.93	2.1	2.5
10	0.98	2.8	3.6
20	0.99	4.0	5.1
50	0.995	6.2	8.0

Figs. 1(a) and 1(b).

As in Ref. 18 the experiments on parity violation in heavy atoms, especially the Novosibirsk experiment,[26] prove the most restrictive; they force $R,\tilde{R}\gtrsim 10$. The other data allow, R, $\tilde{R}$ to be as small as $2-3$.

While we will not present any explicit forms for the masses of the three gauge bosons we list in Table II the masses for some selected cases as an example of the range allowed as R and $\tilde{R}$ are varied. We see that deviations of a few percent in the mass of the lightest Z from its standard-model value and additional heavy neutral bosons in the range 300 GeV $-$ 1 TeV are certainly allowed within the accuracy of the current data.

One should note that by letting either R or $\tilde{R}$ (but not both) tend to infinity drives one Z mass to infinity leaving an effective two-Z theory; specifically, if $\tilde{R}\to\infty$ we recover the results of Ref. 18.

IV. CONCLUSIONS

We have extended the analysis of electroweak theories based on extended gauge groups $SU(2)_L\times U(1)\times U(1)$ found in SO(10) to groups based on $SU(2)_L\times U(1)\times U(1)\times U(1)$ derived from two breakdowns of the grand unification group E_6.

The ratio of effective charged-current to neutral-current couplings is naturally the same as in the standard model and the neutrino neutral-current interactions are also identical. Constraints from parity-violating effects in eN and $e^+e^-\to\mu^+\mu^-$ interactions allow two additional heavy Z's beyond the standard-model Z^0 in the range 300 GeV $-$ 1 TeV. The lightest Z is within a few percent of its standard-model mass.

The same similarity in form of the NC couplings in different breakdown chains which was noticed in SO(10) models is also observed here.

The simple extension of the general arguments of Bjorken of left-right models seemingly does not reproduce the form of the extra term in $\mathscr{H}^{NC}$ beyond (1.3) in gauge models based on left-right symmetries larger then $SU(2)_L\times SU(2)_R\times U(1)_{L+R}$.

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[1]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); Phys. Rev. D 5, 1412 (1972); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367; S. L. Glashow, Nucl. Phys. 22, 579 (1961); S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[2]V. Barger, W. Y. Keung, and E. Ma, Phys. Rev. Lett. 44, 1169 (1980); Phys. Rev. D 22, 727 (1980).

[3]V. Barger, E. Ma, and K. Whisnant, Phys. Rev. Lett. 46, 1501 (1981).

[4]V. Barger, K. Whisnant, and W. Y. Keung, Phys. Rev. D 25, 291 (1982).

[5]E. H. deGroot, G. J. Gounaris, and D. Schildknecht, Phys. Lett. 85B, 399 (1979); Z. Phys. C 5, 127 (1980); E. H. de Groot, D. Schildknecht, and G. J. Gounaris, Phys. Lett. 90B, 427 (1980); E. H. de Groot and D. Schildknecht, Phys. Lett. 95B, 128 (1980); Z. Phys. C 10, 139 (1981).

[6]V. S. Berezinsky and A. Yu. Smirnov, Phys. Lett. 94B, 505 (1980).

[7]J. D. Bjorken, Phys. Rev. D 19, 335 (1979); E. H. De Groot and D. Schildknecht, Z. Phys. C 10, 55 (1981).

[8]J. C. Pati and A. Salam, Phys. Rev. Lett. 31, 661 (1973); Phys. Rev. D 10, 275 (1974); R. N. Mohapatra and J. C. Pati, *ibid.* 11, 566, 2258 (1975); G. Senjanović and R. N. Mohapatra, *ibid.* 12, 1502 (1975); R. N. Mohapatra and D. P. Sidhu, *ibid.* 18, 856 (1978); D. P. Sidhu, *ibid.* 22, 1158 (1980).

[9]Q. Shafi and Ch. Wetterich, Phys. Lett. 73B, 65 (1978).

[10]V. Elias, J. C. Pati, and A. Salam, Phys. Lett. 73B, 451 (1978).

[11]G. Costa, M. D'Anna, P. Marcolungo, Phys. Lett. 79B, 287 (1978).

[12]T. G. Rizzo and G. Senjanović, Phys. Rev. Lett. 46, 1315 (1981), Phys. Rev. D 24, 704 (1981), *ibid.* 25, 235 (1982).

[13]N. G. Deshpande and D. Iskandar, Phys. Rev. Lett.

42, 20 (1979); Phys. Lett. 87B, 383 (1979).
[14]M. Yasué, Prog. Theor. Phys. (Kyoto) 61, 269 (1979).
[15]A. Yu. Smirnov, Nuovo Cimento 64A, 297 (1981).
[16]A. Masiero, Phys. Lett. 93B, 295 (1981).
[17]F. Del Aguila and A. Mendez Nucl. Phys. B189, 212 (1981).
[18]R. W. Robinett and J. L. Rosner, Phys. Rev. D 25, 3036 (1982).
[19]F. Gürsey and P. Sikivie, Phys. Rev. Lett. 36, 775 (1976); Phys. Rev. D 16, 816 (1977); F. Gürsey and M. Serdaroğlu, Lett. Nuovo Cimento 21, 28 (1978); Y. Achiman and B. Stech, Phys. Lett. 77B, 389 (1978); Q. Shafi, Phys. Lett. 79B, 301 (1979); F. Gürsey and M. Serdaroğlu, Nuovo Cimento 65, 337 (1981).
[20]V. Barger and S. Pakvasa, Phys. Lett. 81B, 195 (1979); G. Branco and H. P. Nilles, Nucl. Phys. B151, (1979); G. L. Kane, Michigan Report No. UMHE 80-18 (unpublished); M. Gorn and E. A. Paschos, Phys. Lett. 101B, 255 (1981).
[21]C. Bebek *et al.*, Phys. Rev. Lett. 46, 84 (1981); K. Chadwick *et al.*, *ibid.* 46, 88 (1981).
[22]R. Slansky, Phys. Rep. 79C, 1 (1981).
[23]F. Wilczek and A. Zee, Phys. Lett. 88B, 311 (1979); Phys. Rev. D 25, 553 (1982); M. Yasué, Phys. Lett. 91B, 85 (1980); F. Mansouri, *ibid* 100B, 25 (1981).
[24]J. L. Rosner (private communication).
[25]H. Georgi and S. Weinberg, Phys. Rev. D 17, 275 (1978).
[26]L. M. Barkov and M. S. Zolotorev, Pis'ma Zh. Eksp. Teor. Fiz. 27, 379 (1978); 28, 544 (1978) [JETP Lett. 27, 357 (1978); 28, 503 (1978)]; Phys. Lett. 85B, 308 (1979).
[27]R. Conti *et al.*, Phys. Rev. Lett. 42, 343 (1979); P. H. Bucksbaum, E. D. Commins, and L. R. Hunter, *ibid.* 46, 640 (1981); Phys. Rev. D 24, 1134 (1981).
[28]C. Y. Prescott *et al.*, Phys. Lett. 77B, 347 (1978); 84B, 524 (1979).
[29]W. Bartel *et al.*, Phys. Lett. 108B, 140 (1982).
[30]R. Brandelik *et al.*, Phys. Lett. 110B, 173 (1982).