Baryonium internal color transitions in the $L=0$ state

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Color transitions internal to diquark and antidiquark for baryonium in the $L=0$ state are explicitly considered. Significant color mixing is found, raising the possibility that baryonium states do not exist.

In recent papers, Weinstein, and Weinstein and Isgur[1] considered the possibility that internal color mixing of any multiquark state is so strong that such a hadron cannot exist. Their results are based on a harmonic-oscillator potential. This leaves the possibility that their conclusion is an artifact of the harmonic limit. In this paper their conclusion is substantiated for the $L=0$ state of baryonium by replacing the harmonic-oscillator potential by a harmonic-oscillator potential plus a nonharmonic potential $U(r)$. This potential is related to baryon data in first-order perturbation theory.

The Hamiltonian is

$$H=\sum_i m_i+H_0+H_{\text{hyp}}\ , \qquad (1)$$

where $m_i=m$ is the common constituent quark mass;

$$H_0=\sum_i P_i{}^2/2m+\sum_{i<j}\vec{\Lambda}_i\cdot\vec{\Lambda}_j[\tfrac{1}{2}kr_{ij}{}^2+U(r_{ij})]-\sum_i P_i{}^2\Big/\left[2\sum_i m_i\right]\ , \qquad (2)$$

where $\vec{\Lambda}_i(q)$ and $\vec{\Lambda}_j(q)$ are the color vectors of the quarks [for antiquarks $\Lambda_i(q)\rightarrow-\Lambda_i^*(\bar{q})$], r_{ij} are the interparticle distances, and $U(r_{ij})$ is some unknown potential which incorporates a short-range Coulomb-type piece and deviations from the harmonic-oscillator form at large distances;

$$H_{\text{hyp}}=\sqrt{2}A\sum_{i<j}\vec{\Lambda}_i\cdot\vec{\Lambda}_j\{-8\pi(\vec{S}_i\cdot\vec{S}_j)\delta^{(3)}(\vec{r}_{ij})+3r_{ij}{}^{-3}[3(\vec{S}_i\cdot\hat{r}_{ij})(\vec{S}_j\cdot\hat{r}_{ij})-\vec{S}_i\cdot\vec{S}_j]\}\ . \qquad (3)$$

The coefficient in Eq. (3) has been chosen for ease of evaluation (compare the baryon states described by Isgur and Karl[2]). U and H_{hyp} can be treated by first-order perturbation theory using harmonic-oscillator wave functions.

A baryonium system is now considered. This is made up of two quarks at positions $\vec{r}_1$ and $\vec{r}_2$ and two antiquarks at positions $\vec{r}_3$ and $\vec{r}_4$. Use of the relative coordinates

$$\begin{bmatrix}\vec{R}\\ \vec{\rho}_2\\ \vec{\rho}_3\\ \vec{\rho}_4\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0\\ 0 & 0 & -1/\sqrt{2} & 1/\sqrt{2}\\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}\end{bmatrix}\begin{bmatrix}\vec{r}_1\\ \vec{r}_2\\ \vec{r}_3\\ \vec{r}_4\end{bmatrix} \qquad (4)$$

will be made in subsequent calculations. The diquark-antidiquark system has a relative angular momentum L, which is equal to zero in the present case. Hence for states of zero total angular momentum quarks 1 and 2 have a relative angular momentum l_i which is the same as that of antiquarks 3 and 4.

Following Weinstein,[1] the wave functions for the T and M baryonium systems can be expressed as (summation is over repeated indices; $\alpha,\beta,\sigma=1,2,3$)

$$\begin{aligned}|E_3\rangle&=|T\rangle\\ &=\frac{1}{\sqrt{12}}\epsilon^{\sigma\alpha\alpha'}\epsilon^{\sigma\beta\beta'}|q^\alpha q^{\alpha'}\bar{q}^\beta\bar{q}^{\beta'}\rangle\\ &=\frac{1}{\sqrt{12}}(\delta_{\alpha\beta}\delta_{\alpha'\beta'}-\delta_{\alpha\beta'}\delta_{\alpha'\beta})|q^\alpha q^{\alpha'}\bar{q}^\beta\bar{q}^{\beta'}\rangle,\end{aligned} \qquad (5)$$

$$|E_6| = |M\rangle$$
$$= \frac{1}{\sqrt{24}}(\delta_{\alpha\beta}\delta_{\alpha'\beta'} + \delta_{\alpha\beta'}\delta_{\alpha'\beta})\,|q^\alpha q^{\alpha'}\bar{q}^\beta \bar{q}^{\beta'}\rangle\ . \tag{6}$$

Here $\epsilon^{\alpha\beta\gamma}$ is the Levi-Civita totally antisymmetric tensor. $|q^\alpha q^{\alpha'}\bar{q}^\beta\bar{q}^{\beta'}\rangle$ are the direct-product color wave functions for the $(qq\bar{q}\bar{q})$ baryonium.

There are six terms in the sums of Eqs. (5) and (6), corresponding to the six two-body forces between the four particles consisting of quarks and antiquarks. From Eq. (2), it is seen that one needs to use potentials of the form

$$\sum_{i<j} \vec{\Lambda}_i \cdot \vec{\Lambda}_j V(r_{ij})\ .$$

First, the wave functions given by Eqs. (5) and (6) will be used to evaluate the color factors $\langle\,|\,\vec{\Lambda}_i\cdot\vec{\Lambda}_j\,|\,\rangle$:

$$\langle E_6\,|-\vec{\Lambda}_1\cdot\vec{\Lambda}_3^*\,|E_6\rangle = \langle E_6\,|\,\vec{\Lambda}_2\cdot\vec{\Lambda}_4^*\,|E_6\rangle = -\tfrac{5}{6}\ ,$$
$$\langle E_6\,|\,\vec{\Lambda}_1\cdot\vec{\Lambda}_2\,|E_6\rangle = \langle E_6\,|\,\vec{\Lambda}_3^*\cdot\vec{\Lambda}_4^*\,|E_6\rangle = +\tfrac{1}{3}\ , \tag{7}$$
$$\langle E_6\,|-\vec{\Lambda}_1\cdot\vec{\Lambda}_4^*\,|E_6\rangle = \langle E_6\,|-\vec{\Lambda}_2\cdot\vec{\Lambda}_3^*\,|E_6\rangle = -\tfrac{5}{6}\ ,$$

$$\langle E_3\,|-\vec{\Lambda}_1\cdot\vec{\Lambda}_3^*\,|E_3\rangle = \langle E_3\,|-\vec{\Lambda}_2\cdot\vec{\Lambda}_4^*\,|E_3\rangle = -\tfrac{1}{3}\ ,$$
$$\langle E_3\,|\,\vec{\Lambda}_1\cdot\vec{\Lambda}_2\,|E_3\rangle = \langle E_3\,|\,\vec{\Lambda}_3\cdot\vec{\Lambda}_4^*\,|E_3\rangle = -\tfrac{2}{3}\ , \tag{8}$$
$$\langle E_3\,|-\vec{\Lambda}_1\cdot\vec{\Lambda}_4\,|E_3\rangle = \langle E_3\,|-\vec{\Lambda}_2\cdot\vec{\Lambda}_3^*\,|E_3\rangle = -\tfrac{1}{3}\ .$$

(The above results are found to be in agreement with those calculated in Ref. 3).

Using the coordinate system defined by Eq. (4), it follows that

$$_6V_6^{\text{HO}} = \sum_{i<j}\langle E_6\,|\,\vec{\Lambda}_i\cdot\vec{\Lambda}_j\,|E_6\rangle \tfrac{1}{2}kr_{ij}^2$$
$$= \tfrac{1}{2}k(\rho_2^2+\rho_3^2+\tfrac{10}{3}\rho_4^2)$$
$$\equiv \tfrac{1}{2}K[\tfrac{3}{2}(\rho_2^2+\rho_3^2)+5\rho_4^2]\ . \tag{9}$$

Similarly,

$$_3V_3^{\text{HO}} = \tfrac{1}{2}k(2\rho_2^2+2\rho_3^2+\tfrac{4}{3}\rho_4^2)$$
$$\equiv \tfrac{1}{2}K[3(\rho_2^2+\rho_3^2)+2\rho_4^2]\ . \tag{10}$$

According to Weinstein and Isgur,[1] in addition to the terms (9) and (10), one should also consider internal color transitions between the T and M baryonia as shown in Fig. 1.

The color factors relevant to the internal color transitions are

$$\langle E_6\,|-\vec{\Lambda}_1\cdot\vec{\Lambda}_3^*\,|E_3\rangle = \langle E_6\,|-\vec{\Lambda}_2\cdot\vec{\Lambda}_4^*\,|E_3\rangle$$
$$= -1/\sqrt{2}\ ,$$
$$\langle E_6\,|\,\vec{\Lambda}_1\cdot\vec{\Lambda}_2\,|E_3\rangle = \langle E_6\,|\,\vec{\Lambda}_3^*\cdot\vec{\Lambda}_4^*\,|E_3\rangle$$
$$= 0\ , \tag{11}$$
$$\langle E_6\,|\,\vec{\Lambda}_1\cdot\vec{\Lambda}_4^*\,|E_3\rangle = \langle E_6\,|-\vec{\Lambda}_2\cdot\vec{\Lambda}_3^*\,|E_3\rangle$$
$$= +1/\sqrt{2}\ ,$$

which yield

$$_6V_3^{\text{HO}} = \tfrac{1}{2}k(-2\sqrt{2}\vec{\rho}_2\cdot\vec{\rho}_3)$$
$$= {}_3V_6^{\text{HO}}\ (\text{using } \langle E_3|\quad|E_6\rangle = \langle E_6|\quad|E_3\rangle)\ . \tag{12}$$

The off-diagonal matrix elements as given by (12) will be used to take into account color mixing. Neglecting color mixing one can immediately write down energies and corresponding eigenstates of the potentials given by Eqs. (9) and (10). For the ground state, it has been shown that[3]

$$\psi_{L,l} = \psi_{0,0}(\vec{\rho}_2,\vec{\rho}_3,\vec{\rho}_4)$$
$$= \frac{\alpha_1^3\alpha_2^{3/2}}{\pi^{9/4}}\exp[-\tfrac{1}{2}(\alpha_1^2\rho_2^2+\alpha_1^2\rho_3^2+\alpha_2^2\rho_4^2)]\ , \tag{13}$$

where

$$\alpha^2 = (3mK/\hbar^2)^{1/2},\quad \omega = (3K/m)^{1/2}\ , \tag{14}$$
$$\alpha_1^2 = \alpha s_1^2,\quad \omega_1 = \omega s_1\ , \tag{15}$$
$$\alpha_2^2 = \alpha^2 s_2,\quad \omega_2 = \omega s_2\ , \tag{16}$$

so that one can write for T or M baryonia

$$E_{0,0} = (3\hbar\omega_1 + \tfrac{3}{2}\hbar\omega_2)\ , \tag{17}$$

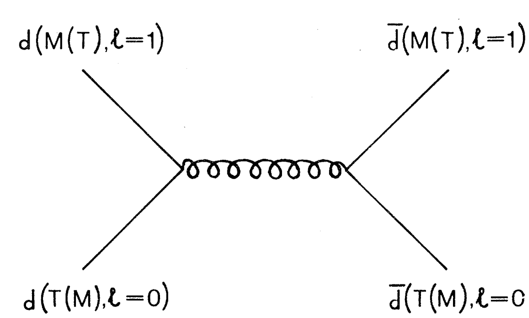

FIG. 1. Internal color transition accompanied by a change in the angular momentum (l) of the diquark and of the antidiquark.

where for T

$$(s_1,s_2)=(1,(\tfrac{2}{3})^{1/2})$$

and for M

$$(s_1,s_2)=((\tfrac{1}{2})^{1/2},(\tfrac{5}{3})^{1/2}) . \tag{18}$$

By using the value $\omega=250$ MeV from the baryon fit of Isgur and Karl,[2] one obtains for the energies $E_{L,l}^{T(M)}$ of T and M baryonia from Eqs. (15)–(18)

$$E_{0,0}^{T}=1056 \text{ MeV and } E_{0,0}^{M}=1014 \text{ MeV} . \tag{19}$$

For $l=1$, the wave functions and energies for zero total angular momentum are

$$\psi_{0,1}^{T}=\frac{2^{11/18}}{3^{7/8}}\frac{\alpha^{13/2}}{\pi^{9/4}}(\vec{\rho}_2\cdot\vec{\rho}_3)\exp\left\{-\frac{\alpha^2}{2}[\rho_2{}^2+\rho_3{}^2+(\tfrac{2}{3})^{1/2}\rho_4{}^2]\right\} , \tag{20}$$

$$\psi_{0,1}^{M}=\frac{(5)^{3/8}}{(3)^{7/8}}\frac{\alpha^{13/2}}{2^{1/4}\pi^{9/4}}(\vec{\rho}_2\cdot\vec{\rho}_3)\exp\left\{-\frac{\alpha^2}{2}[(\tfrac{1}{2})^{1/2}\rho_2{}^2+(\tfrac{1}{2})^{1/2}\rho_3{}^2+(\tfrac{5}{3})^{1/2}\rho_4{}^2]\right\} , \tag{21}$$

$$E_{0,1}=5\hbar\omega_1+\tfrac{3}{2}\hbar\omega_2,\quad E_{0,1}^{T}=1556 \text{ MeV},\quad E_{0,1}^{M}=1368 \text{ MeV} . \tag{22}$$

Moreover, from Eqs. (12), (13), (20), and (21) it follows that

$$\begin{aligned}\langle \psi_{0,0}^{T} \,|\, {}_3V_6 \,|\, \psi_{0,1}^{M}\rangle &= \langle \psi_{0,0}^{M} \,|\, {}_3V_6 \,|\, \psi_{0,1}^{T}\rangle \\ &= -0.5466\hbar\omega \\ &= -137 \text{ MeV} .\end{aligned} \tag{23}$$

The Hamiltonian excluding the hyperfine interactions is given by $(H_{\rm HO}+\hat{U})$, where

$$\begin{aligned}{}_A\hat{U}_B &= \sum_{i<j}^{4}\langle A \mid \vec{\Lambda}_i\cdot\vec{\Lambda}_j \mid B\rangle U(r_{ij}) , \\ |A\rangle, |B\rangle &= |E_3\rangle, |E_6\rangle ,\end{aligned} \tag{24}$$

and the color factors $\langle A \mid \vec{\Lambda}_i\cdot\vec{\Lambda}_j \mid B\rangle$ are as found in Eqs. (7), (8), and (11). For $\psi_{L,l}$ given by Eqs. (13), (20), or (21), the contribution of $\hat{U}$ to the energy is just $(\psi_{L,l},\hat{U}\psi_{L',l'})$. For transition between T and M, the contribution in the ground state is $(\psi_{0,0}^{T},\hat{U}\psi_{0,1}^{M}) = (\psi_{0,0}^{M},\hat{U}\psi_{0,1}^{T})$, where $\psi_{0,0}^{T}$ and $\psi_{0,0}^{M}$ are given by Eq. (13), $\psi_{0,1}^{T}$ is given by Eq. (20), and $\psi_{0,1}^{M}$ by Eq. (21). Using the same evaluation procedure as in Ref. 1 it then follows that

$$\langle L=l=0 \,|\, {}_3\hat{U}_3 \,|\, L=l=0\rangle = -832 \text{ MeV} , \tag{25}$$

$$\langle L=l=0 \,|\, {}_6\hat{U}_6 \,|\, L=l=0\rangle = -919 \text{ MeV} , \tag{26}$$

and

$$\langle L=0,l=1 \,|\, {}_3\hat{U}_3 \,|\, L=0,l=1\rangle = -974 \text{ MeV} , \tag{27}$$

$$\langle L=0,l=1 \,|\, {}_6\hat{U}_6 \,|\, L=0,l=1\rangle = -1083 \text{ MeV} , \tag{28}$$

and

$$\begin{aligned}\langle L=0,l=1 \,|\, {}_3\hat{U}_6 \,|\, L=0,l=0\rangle &= \langle L=0,l=1 \,|\, {}_6\hat{U}_3 \,|\, L=0,l=0\rangle \\ &= \langle L=0,l=0 \,|\, {}_3\hat{U}_6 \,|\, L=0,l=1\rangle \\ &= \langle L=0,l=0 \,|\, {}_6\hat{U}_3 \,|\, L=0,l=1\rangle = 101 \text{ MeV} .\end{aligned} \tag{29}$$

The difference in the signs of the values of Eqs. (25)–(28) and that of Eq. (29) arises from the color factors. This has already been seen earlier for the case of purely harmonic-oscillator potential [compare the signs of values of Eqs. (19)–(22) and that of Eq. (23)].

It is now possible to calculate the confinement energies for the various states without taking into account the color mixing. These values are listed in Table I, using Eqs. (19), (25), and (26) (the required value of $w=250$ MeV, as mentioned earlier).

In principle, one should calculate the hyperfine splitting. Since it will be seen later that the baryonic states do *not* exist at the confinement level, it is only of academic interest to take into account the hyperfine perturbations at this point in the discussion. (For interested readers they are given in the Appendix.)

The color mixing is now taken into account. It arises from the $\vec{\rho}_2\cdot\vec{\rho}_3$ part of the ${}_3V_6$ term in the harmonic-oscillator part of the Hamiltonian [Eq. (12)] and from ${}_3\hat{U}_6$ part of the nonharmonic term [Eq. (24)]. This corresponds to color transitions

TABLE I. Strength of internal color mixing for the confinement energies of ground state ($L=0$) baryonium levels considering only harmonic-oscillator term and harmonic-oscillator plus U term.

State	Energy before mixing (MeV) Harmonic oscillator only	Harmonic oscillator plus U term	Mixing strength Harmonic oscillator only	Harmonic oscillator + U term
T	2456	1624	-0.44	-0.59
M	2414	1495	-0.25	-0.07

characterized by changes in the internal angular momentum (l) of both the diquark and the antidiquark (Fig. 1). Then, from Eqs. (23) and (29) the following is noted:

$$\begin{aligned} {}_3V_6 &+ \langle L=0,l=1 \,|\, \hat{U} \,|\, L=0,l=0\rangle \\ &= {}_3V_6 + \langle L=0,l=0 \,|\, \hat{U} \,|\, L=0,l=1\rangle \\ &= {}_6V_3 + \langle L=0,l=1 \,|\, \hat{U} \,|\, L=0,l=0\rangle \\ &= {}_6V_3 + \langle L=0,l=0 \,|\, \hat{U} \,|\, L=0,l=1\rangle \\ &= -36 \text{ MeV} . \end{aligned} \tag{30}$$

The mixing between $|T\rangle$ and $|M\rangle$ states can be expressed by the following matrix:

$$\begin{bmatrix} \langle 3|H|3\rangle & \langle 3|H|6\rangle \\ \langle 6|H|3\rangle & \langle 6|H|6\rangle \end{bmatrix} . \tag{31}$$

Its strength is thus described by the ratio

$$R = \frac{\text{off diagonal}(l=0 \leftrightarrow l=1)}{\text{diagonal}(l=1) - \text{diagonal}(l=0)} . \tag{32}$$

For comparison purposes the values of mixing strength R are listed in Table I for the two cases of (i) the simple-harmonic oscillator term and (ii) the simple-harmonic plus U term. The signs and magnitudes are found to be consistent with the calculations of Weinstein and Isgur[1] implying that for the present case of $L=0$ the baryonium spontaneously splits up into two mesons. In other words baryonia do *not* exist at the confinement level for $L=0$. As far as $L\neq 0$ values are concerned, one could make a definitive statement only after making explicit calculation for these cases. However, on the basis of the present calculations, it appears *plausible* that baryonia also do not exist at the confinement level for $L\neq 0$.

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APPENDIX

In this appendix the values given in Table I are refined by also taking into account the hyperfine interaction.

After truncation in the orbital-angular-momentum space, the Fermi contact term, Eq. (3), can be reduced to

$$\begin{aligned} H_{\text{cont}} = &-8\sqrt{2}A\pi\langle L,l \,|\, \delta(r_{13}) \,|\, L',l'\rangle \sum_{i<j} (\vec{S}_i\cdot\vec{S}_j)(\vec{\Lambda}_i\cdot\vec{\Lambda}_j) \\ &- 8\sqrt{2}A\pi[\langle L,l \,|\, \delta(r_{12}) \,|\, L',l'\rangle - \langle L,l \,|\, \delta(r_{13}) \,|\, L',l'\rangle] \, [(\vec{S}_1\cdot\vec{S}_2)(\vec{\Lambda}_1\cdot\vec{\Lambda}_2) + (\vec{S}_3\cdot\vec{S}_4)(\vec{\Lambda}_3\cdot\vec{\Lambda}_4)] , \end{aligned} \tag{A1}$$

where $\vec{S}_k$ and $\vec{\Lambda}_k$ are the spin and color vectors for the kth quark, and r_{ij} is the distance between the ith and jth quark. In writing Eq. (A1) the fact that the matrix elements of $\delta(r_{ij})$ are the same for $ij=13$, 23, 14, 24, and for $\delta(r_{ij})$ when $ij=12$, 34 has been used. The tensor contribution, Eq. (3), can be expressed as

$$H_{\text{ten}} = 3\sqrt{2}A r_{ij}^{-3}[3(\vec{S}_i\cdot\hat{r}_{ij})(\vec{S}_j\cdot\hat{r}_{ij}) - \vec{S}_i\cdot\vec{S}_j]\vec{\Lambda}_i\cdot\vec{\Lambda}_j . \tag{A2}$$

The coefficients in Eqs. (A1) and (A2) have been chosen for ease of evaluation (compare the baryon states as described by Isgur and Karl[3]). The parameter A can be evaluated from the results of Ref. 2:

$$\Delta-N=4A\alpha^3\pi^{-1/2}\simeq 300\text{ MeV} . \tag{A3}$$

Using Eqs. (4) and (13),

$$\langle L=0,l=0\,|\,\delta(\vec{r}_{12})\,|\,L=0,l=0\rangle=2^{-3/2}\langle\delta(\vec{\rho}_2)\rangle=s_1{}^{3/2}\alpha^3/(2\pi)^{3/2} . \tag{A4}$$

Similarly one obtains

$$\langle L=0,l=0\,|\,\delta(\vec{r}_{13})\,|\,L=0,l=0\rangle=\left[\frac{2s_1s_2}{s_1+s_2}\right]^{3/2}\frac{\alpha^3}{(2\pi)^{3/2}} , \tag{A5}$$

$$\langle L=0,l=1\,|\,\delta(\vec{r}_{12})\,|\,L=0,l=1\rangle=0 , \tag{A6}$$

$$\langle L=0,l=1\,|\,\delta(\vec{r}_{13})\,|\,L=0,l=1\rangle=\tfrac{1}{16}[\alpha^3/(2\pi)^{3/2}](s_1{}^5s_2{}^{3/2})\times(5s^{-7/2}s_1{}^{-3/2}-12s^{-5/2}s_1{}^{-5/2}+20s^{-3/2}s_1{}^{-7/2}) , \tag{A7}$$

and

$$\langle L=l=0\,|\,\delta(\vec{r}_{12})\,|\,L=0,l=1\rangle=\langle L=0,l=1\,|\,\delta(\vec{r}_{12})\,|\,L=l=0\rangle=0 , \tag{A8}$$

$$\langle L=l=0\,|\,\delta(\vec{r}_{13})\,|\,L=0,l=1\rangle=\langle L=0,l=1\,|\,\delta(\vec{r}_{13})L=l=0\rangle$$
$$=\frac{\alpha^3}{(2\pi)^{3/2}}\frac{3^{5/8}(s)^{5/2}}{[89-2\sqrt{30}]^{1/2}}(t)^{-3/2}\left[\frac{1+(\frac{1}{2})^{1/2}}{2}\right]^{-3/2}\left[-t^{-1}+\left[\frac{1+(\frac{1}{2})^{1/2}}{2}\right]^{-1}\right] , \tag{A9}$$

where

$$s=[(\tfrac{1}{2})^{1/2}+(\tfrac{5}{3})^{1/2}]/2,\quad s'=[1+(\tfrac{2}{3})^{1/2}]/2,\quad t=(s+s')/2 .$$

In order to find the eigenvalues of H_{cont}, one starts with a set of basis functions in the color-spin space. A good choice seems to be the eigenvectors of the first term of Eq. (A1). It is noted[4] that products $S^k\Lambda^\alpha$ contained in this first term are among the generators of $\text{SU}(6)_{cs}$. Hence[4]

$$-4\sum_{\alpha}\sum_{i>j}(\vec{S}_i\cdot\vec{S}_j)(\vec{\Lambda}_i\cdot\vec{\Lambda}_j)=2N+2C_6(\text{tot})-S_{\text{tot}}(S_{\text{tot}}+1)/3$$
$$+C_3(Q)+2S_Q(S_Q+1)/3-4C_6(Q)+C_3(\bar{Q})+2S_{\bar{Q}}(S_{\bar{Q}}+1)/3-4C_6(\bar{Q}) , \tag{A10}$$

where C_6, C_3, and $S(S+1)$ are, respectively, the Casimir operators of $\text{SU}(6)_{cs}$, $\text{SU}(3)_c$, and $\text{SU}(2)_s$; the labels Q, $\bar{Q}$, and tot refer, respectively, to the diquark (made up of the quarks 1 and 2), the antidiquark (made up of the antiquarks 3 and 4), and the entire baryonium system, while the subscripts c and s refer to the color and spin subspaces.

To use Eq. (A10), it is necessary to construct

TABLE II. Diquark-antidiquark combinations formed into color-spin $\text{SU}(6)_{cs}$ representations. n_{cs}, n_c, and n_s are the dimensions of the $\text{SU}(6)_{cs}$, $\text{SU}(3)_c$, and $\text{SU}(2)_s$ representations.

State	Origin $[n_{cs}]\otimes[\bar{n}_{cs}']$	S	I	$[n_{cs}]$	Origin as a combination of $[n_c,n_s]\otimes(\bar{n}_c,n_s)$ factors
C	$[21]\otimes[\overline{21}]$	1	0	[35]	$(6,3)\otimes(\bar{6},3)$
D	$[21]\otimes[\overline{21}]$	2	0	[405]	$(6,3)\otimes(\bar{6},3)$
G	$[15]\otimes[\overline{15}]$	1	0,1,2	[35]	$(\bar{3},3)\otimes(3,3)$
H	$[15]\otimes[\overline{15}]$	2	0,1,2	[189]	$(\bar{3},3)\otimes(3,3)$

TABLE III. Strength of internal color mixing for the ground-state ($L=0$) baryonium levels. The quantum numbers J, P, and I refer to the total angular momentum, parity, and isospin of the whole system. l and n_c refer to orbital angular momentum and $SU(3)_c$ representations of the diquark and antidiquark.

State	J^P	I	l	Origin as a combination of $(n_c)\otimes(\bar{n}_c)$ factors	Energy before mixing (MeV)	Mixing strength
C	1^+	0	0	$(6)\otimes(\bar{6})$	1247	-0.05
G	1^+	0,1,2	1	$(\bar{3})\otimes(3)$	1914	
G	1^+	0,1,2	0	$(\bar{3})\otimes(3)$	1639	$+0.23$
C	1^+	0	1	$(6)\otimes(\bar{6})$	1495	
D	2^+	0	0	$(6)\otimes(\bar{6})$	1684	-0.11
H	2^+	0,1,2	1	$(\bar{3})\otimes(3)$	2050	
H	2^+	0,1,2	0	$(\bar{3})\otimes(3)$	1809	-0.59
D	2^+	0	1	$(6)\otimes(\bar{6})$	1875	

wave functions which span specific $SU(6)_{cs}$ representations. These have already been constructed by Kalman, Hall, and Misra.[3] Of the many states contained in that paper, for illustration purposes, we consider only the states C, D, G, H; these are reproduced in Table II. For them there is no contribution to the energy from the tensor term. Also, there is no mixing by H_{cont} between color states with the same value of l. In Table III, the total energies of these states, before color mixing but including the contributions from the hyperfine interaction described by Eqs. (A1), (A2)–(A7), and (A10), are as given.

The terms ${}_3V_6$ and ${}_3\hat{U}_6$ will thus produce the mixing of the following states:

$$C(l=0)\leftrightarrow G(l=1)\,, \tag{A11}$$

$$G(l=0)\leftrightarrow C(l=1)\,, \tag{A12}$$

$$D(l=0)\leftrightarrow H(l=1)\,, \tag{A13}$$

$$H(l=0)\leftrightarrow D(l=1)\,. \tag{A14}$$

From Eqs. (A1), (A3), and (A8)–(A10) the contributions to the energy due to hyperfine interactions between the state with different values of l are

$$\langle C\,|\,H_{\text{hyp}}\,|\,G\rangle = 3\ \text{MeV}\,, \tag{A15}$$

$$\langle D\,|\,H_{\text{hyp}}\,|\,H\rangle = -3\ \text{MeV}\,. \tag{A16}$$

The total energy due to internal $C\leftrightarrow G$ color transition for the part of the Hamiltonian $H_1 = {}_3V_6 + {}_3U_6 + H_{\text{hyp}}$ is then [Eqs. (30) and (A15)]

$$\langle 3\,|\,H_1\,|\,6\rangle = \langle 6\,|\,H_1\,|\,3\rangle = -33\ \text{MeV} \tag{A17}$$

and for the $D\leftrightarrow H$ system [Eqs. (30) and (A16)]

$$\langle 3\,|\,H_1\,|\,6\rangle = \langle 6\,|\,H_1\,|\,3\rangle = -39\ \text{MeV}\,. \tag{A18}$$

In order to take into account completely the contribution of internal color transitions, one also needs $\langle 3\,|\,H_2\,|\,3\rangle$ and $\langle 6\,|\,H_3\,|\,6\rangle$, where [Eqs. (1), (9), and (10)]

$$H_2 = \sum_i P_i^2/2m + {}_3V_3^{\text{HO}} + {}_3U_3 + H_{\text{hyp}}, \tag{A19}$$

$$H_3 = \sum_i P_i^2/2m + {}_6V_6^{\text{HO}} + {}_6U_6 + H_{\text{hyp}}. \tag{A20}$$

It is noted that H_1, H_2, and H_3 are obtained by truncation in the color space. In Table III are given the energies before mixing and the corresponding value of R [Eq. (32)].

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