PHYSICAL REVIEW D VOLUME 26, NUMBER 9 1 NOVEMBER 1982

P-wave baryons in a consistent quark model with hyperfine interactions

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(Received 5 March 1982; revised manuscript received 3 May 1982)

Masses of the various negative-parity baryons are calculated using a single consistent parameter set as described by Kalman and Hall. Reasonable fits to the masses of most of the ground-state and negative-parity baryons are obtained.

I. INTRODUCTION

De Rújula, Georgi, and Glashow[1] introduced a framework for models based on quantum chromodynamics (QCD) in low-energy applications. This consists of a short-range Coulomb-type vector force as expected from one-gluon exchange, and a scalar confining potential. The remainder of the forces are represented by one-gluon-exchange terms analogous to the one-photon-exchange terms familiar in atomic spectroscopy. The interactions are flavor independent except for the usual dependence on the quark masses. Within this framework Isgur and Karl[2–5] introduced a harmonic-oscillator confining potential to describe the spectra and decay couplings of the ground and low-lying excited baryon states; here each set of levels of alternating parity has been analyzed with a different parameter set. In particular, in examining the excited positive-parity baryons Isgur and Karl modify the harmonic-oscillator potential by the introduction of an interaction term $U(r_{ij})$ which incorporates the effect of a Coulomb-type force derived from QCD and deviations from the harmonic-oscillator form at large distance. A major triumph[3] of the model of Isgur and Karl is the correct prediction in sign and magnitude of the observed inversion of $\Lambda\frac{5}{2}^-(1830)$ and $\Sigma\frac{5}{2}^-(1765)$ relative to the ground state. However, if the parameters obtained in their fit to the positive-parity baryons are applied to the problem of the negative-parity baryons the mass difference between $\Lambda\frac{5}{2}^-$ and $\Sigma\frac{5}{2}^-$ will be reduced from 50 to 15 MeV; this is no longer consistent with experiment.[6]

Kalman and Hall[6] noted that the key to the resolution of this difficulty is the method of calculation of the nonharmonic part of the potential. Isgur and Karl[4] obtain the value of the contribution of this term $[U(r_{ij})]$ in the SU(3) limit $(m_s=m_u)$. By using a technique introduced by Kalman, Hall, and Misra,[7] the effect of this term is calculated by Kalman and Hall[6] in a manner which properly takes into account the difference in mass between the strange and nonstrange quarks. Kalman and Hall[6] show that in such a consistent model, the mass difference between $\Lambda\frac{5}{2}^-$ and $\Sigma\frac{5}{2}^-$ is restored to a value in agreement with experiment. In the present work a full calculation of the masses of all the negative-parity baryons is carried out.

II. THE ISGUR-KARL MODEL

The model employs a Hamiltonian of the form

$$H=\sum_{i=1}^{3} m_i+H_0+H_{\text{hyp}}\ , \tag{2.1}$$

where m_i are the quark masses and

$$H_0=\sum_i P_i^2/2m_i+\sum_{i<j} V^{ij}-\left[\sum_i P_i^2\right]\bigg/\left[2\sum_i m_i\right], \tag{2.2a}$$

where

$$V^{ij}=[\tfrac{1}{2}kr_{ij}^2+U(r_{ij})]\ . \tag{2.2b}$$

$U(r_{ij})$ is some unknown potential which incorporates an attractive potential at short range (a Coulomb-type piece derived from QCD) and deviations from the harmonic-oscillator interaction at large distances:

$$H_{\text{hyp}}=\sum_{i<j}^{3}\frac{2\alpha_s}{3m_im_j}\left\{\frac{8\pi}{3}\delta^3(\vec r_{ij})(\vec s_i\cdot\vec s_j)+\frac{1}{r_{ij}^3}\left[\frac{3(\vec s_i\cdot\vec r_{ij})(\vec s_j\cdot\vec r_{ij})}{r_{ij}^2}-\vec s_i\cdot\vec s_j\right]\right\}. \tag{2.3}$$

$\vec{r}_{ij}$ is the separation between a pair of quarks. In terms of Jacobi relative coordinates

$$\vec{\rho}=\frac{1}{\sqrt{2}}(\vec{r}_1-\vec{r}_2) , \quad (2.4a)$$

$$\vec{\lambda}=\frac{1}{\sqrt{6}}(\vec{r}_1+\vec{r}_2-2\vec{r}_3) \quad (2.4b)$$

in the $U=H_{\rm hyp}=0$ limit, Eqs. (2.2) decouple into a description of two independent harmonic oscillators with the same spring constant k:

$$H_0\rightarrow\widetilde{H}_0=\frac{P_\rho{}^2}{2m_\rho}+\frac{P_\lambda{}^2}{2m_\lambda}+\tfrac{3}{2}k(\rho^2+\lambda^2) . \quad (2.5)$$

The contribution of the Hamiltonian $\widetilde{H}_0$ of Eq. (2.5) to the total energy is then

$$\widetilde{E}_0=\tfrac{3}{2}(\omega_\rho+\omega_\lambda) , \quad (2.6)$$

where

$$\omega_{\rho,\lambda}=(3k/m_{\rho,\lambda}) . \quad (2.7)$$

Here m_ρ and m_λ are the reduced mass of the ρ and λ oscillators, respectively. Now in the approximation $m_u\simeq m_d$ at least two of the quark masses are equal. We will employ the convention that in all cases the quarks comprising the ρ oscillator have the same mass. Hence

$$m_\rho=m_1 , \quad (2.8a)$$

$$m_\lambda=\frac{3m_1m_3}{2m_1+m_3} , \quad (2.8b)$$

where in the various strangeness (S) sectors,

$$m_1=m_u,\ \ S=0,-1 , \quad (2.9a)$$

$$=m_s,\ \ S=-2,-3 , \quad (2.9b)$$

$$m_3=m_u,\ \ S=0,-2 , \quad (2.10a)$$

$$=m_s,\ \ S=-1,-3 . \quad (2.10b)$$

Applying Eqs. (2.3)–(2.10) to Eq. (2.6), we see that if we write

$$x=m_u/m_s \quad (2.11)$$

and

$$\widetilde{E}_0\equiv 3\omega,\ \ S=0 \quad (2.12a)$$

for the ground state, then for the ground state in the other strangeness sectors:

$$\widetilde{E}_0=\tfrac{3}{2}\omega\{1+[(2x+1)/3]^{1/2}\},\ \ S=-1 , \quad (2.12b)$$

$$=\tfrac{3}{2}\omega\{x^{1/2}+[(2+x/3)]^{1/2}\},\ \ S=-2 , \quad (2.12c)$$

$$=3\omega x^{1/2},\ \ S=-3 . \quad (2.12d)$$

For the $N=1$ (negative-parity) baryons, the ρ and λ oscillators can be separately excited. Thus in the $S=-1,-2$ sectors the degeneracy is lifted. The exact energies are as follows:

$$\widetilde{E}_0=4\omega,\ \ S=0 , \quad (2.13a)$$

$$\widetilde{E}_0^\rho=\omega\{\tfrac{5}{2}+\tfrac{3}{2}[(2x+1)/3]^{1/2}\},\ \ S=-1 , \quad (2.13b)$$

$$\widetilde{E}_0^\lambda=\omega\{\tfrac{3}{2}+\tfrac{5}{2}[(2x+1)/3)]^{1/2}\},\ \ S=-1 , \quad (2.13c)$$

$$\widetilde{E}_0^\rho=\omega\{\tfrac{5}{2}x^{1/2}+\tfrac{3}{2}[(2+x)/3]^{1/2}\},\ \ S=-2 , \quad (2.13d)$$

$$\widetilde{E}_0^\lambda=\omega\{\tfrac{3}{2}x^{1/2}+\tfrac{5}{2}[(2+x)/3]^{1/2}\},\ \ S=-2 , \quad (2.13e)$$

$$\widetilde{E}_0=4\omega x^{1/2},\ \ S=-3 . \quad (2.13f)$$

The splitting in energy in the $S=-1$ sector, Eqs. (2.13b) and (2.13c), contributes to making the $\Lambda\frac{5}{2}^-$ heavier than $\Sigma\frac{5}{2}^-$ in reversal of the situation in the ground state. In accordance with Eqs. (2.13d) and (2.13e) a similar splitting occurs in the $S=-2$ sector. Similar equations can be given for the energies corresponding to the $N=2$ (positive-parity) baryons.

When U is not zero in the $S=0$ sector its effect on the spectrum can be given in terms of three parameters E_0, Ω, and Δ:

$$E(S_s)=E(56,0^+)=E_0 , \quad (2.14a)$$

$$E(P_m)=E(70,1^-)=E_0+\Omega , \quad (2.14b)$$

$$E(S_s')=E(56',0^+)=E_0+2\Omega-\Delta , \quad (2.14c)$$

$$E(S_m)=E(70,0^+)=E_0+2\Omega-\tfrac{1}{2}\Delta , \quad (2.14d)$$

$$E(D_s)=E(56,2^+)=E_0-\tfrac{2}{5}\Delta+2\Omega , \quad (2.14e)$$

$$E(D_m)=E(70,2^+)=E_0-\tfrac{1}{5}\Delta+2\Omega , \quad (2.14f)$$

$$E(P_A)=E(20,1^+)=E_0+2\Omega . \quad (2.14g)$$

To obtain the masses of the baryons in each case the hyperfine matrix elements are also calculated and added to the result from the rest of the Hamiltonian. Each set of baryons, $N=0,1,2$ are treated as separate problems. That is, a different parameter set is used is in each case.[6]

III. CONSISTENT QUARK MODEL

As seen in the previous section, Isgur and Karl calculate the effect of the nonharmonic part of the potential [see Eq. (2.2b)] in the SU(3) limit. That is they take no account of the difference between

$$\alpha_\rho=(3km_\rho)^{1/4} \tag{3.1}$$

and

$$\alpha_\lambda=(3km_\lambda)^{1/4} . \tag{3.2}$$

If the parameters used by Isgur and Karl in describing the positive-parity baryons[2,4] are applied to the negative-parity baryons[3] the mass difference between $\Lambda\frac{5}{2}^-$ and $\Sigma\frac{5}{2}^-$ is reduced from 50 to 15 MeV and is apparently no longer consistent with experiment.[6]

Calculations of the contributions from the nonharmonic part of the potential for different values of α have been discussed by Kalman, Hall, and Misra.[7] Based on this work, we set

$$a(t)=(3\alpha^3t^{3/2}/\pi^{3/2}) \times \int d^3\rho\, U(\sqrt{2}\rho)\exp(-t\alpha^2\rho^2) , \tag{3.3a}$$

$$b(t)=(3\alpha^5t^{5/2}/\pi^{3/2}) \times \int d^3\rho\, U(\sqrt{2}\rho)\rho^2\exp(-t\alpha^2\rho^2) , \tag{3.3b}$$

and

$$c(t)=(3\alpha^7t^{7/2}/\pi^{3/2}) \times \int d^3\rho\, U(\sqrt{2}\rho)\rho^4\exp(-t\alpha^2\rho^2) . \tag{3.3c}$$

In terms of $a=a(1)$, $b=b(1)$, and $c=c(1)$, Isgur and Karl have shown that[8] if one sets

$$E_0=3m_u+3\omega+a , \tag{3.4}$$

$$\Omega=\omega-a/2+b/3 , \tag{3.5}$$

$$\Delta=-\tfrac{5}{4}a+\tfrac{5}{3}b+c/3 , \tag{3.6}$$

then the equations for application of the nonharmonic potential U, in the $S=0$ sector [Eqs. (2.14)] immediately follow. Kalman and Hall[6] used Eqs. (3.3) to obtain the equations for calculation of the nonharmonic potential U in the $S=-1$ sector. A generalization of their results to all values $S\neq0$ yields

$$E(s)=2m_1+m_3+\tfrac{3}{2}(\omega_\rho+\omega_\lambda)+\langle U\rangle_s , \tag{3.7a}$$

$$E(\rho)=2m_1+m_3+\tfrac{5}{2}\omega_\rho+\tfrac{3}{2}\omega_\lambda+\langle U\rangle_\rho , \tag{3.7b}$$

$$E(\lambda)=2m_1+m_3+\tfrac{3}{2}\omega_\rho+\tfrac{5}{2}\omega_\lambda+\langle U\rangle_\lambda , \tag{3.7c}$$

where

$$\langle U\rangle_s=\tfrac{1}{3}a(r)+\tfrac{2}{3}a(rt) , \tag{3.8a}$$

$$\langle U\rangle_\rho=\tfrac{2}{9}b(r)+\alpha_\rho^2ta(rt)/2\alpha_\lambda^2+(t/9)b(rt) , \tag{3.8b}$$

$$\langle U\rangle_\lambda=\tfrac{1}{3}a(r)+(t/6)a(rt)+\alpha_\rho^2tb(rt)/3\alpha_\lambda^2 , \tag{3.8c}$$

TABLE I. Preliminary calculations of the masses of the ground-state baryons.

Particle	Mass (MeV) Experiment	Mass (MeV) Calculation	% deviation from experiment
N	938.9	933.4	0.59
Δ	1232	1241	0.73
Λ	1115.6	1113.0	0.23
Σ	1193.4	1191.5	0.16
Σ^*	1385 $\pm$1	1390	0.36
Ξ	1318.1$\pm$0.7	1324.3	0.45
Ξ^*	1533.4$\pm$0.9	1525.1	0.54

where

$$r=1,\ S=0,-1 , \tag{3.9a}$$

$$=x^{-1/2},\ S=-2,-3 , \tag{3.9b}$$

$$t=1,\ S=0,-3 , \tag{3.9c}$$

$$=4/[1+3(\alpha_\rho/\alpha_\lambda)^2],\ S=-1,-2 . \tag{3.9d}$$

To complete the calculations of the energy spectrum, the hyperfine matrix elements for the $S=0$ and -1 sectors are found in Isgur and Karl[3] and Kalman and Hall.[6] For the $S=-2$ and -3 baryons, the same matrix elements can be used by making the interchange $m_u\leftrightarrow m_s$ everywhere.

IV. CALCULATIONS

There are seven parameters to be calculated, namely, x, m_s, ω, a, b, c, and

$$\delta=\frac{4\alpha_s(m_u\omega)^{3/2}}{3(\pi)^{1/2}m_u{}^2 2^{1/2}} . \tag{4.1}$$

Neglecting the hyperfine interactions, the parameter δ can be ignored. In this case consider the ground-state baryons; N and Δ have a common mass $M_0(N,\Delta)$; Λ, Σ, and Σ^* have a common mass $M_0(\Lambda,\Sigma)$; Ξ and Ξ^* have a common mass $M_0(\Xi)$; and Ω has a mass $M_0(\Omega)$. Isgur and Karl[2] note that to fit the masses of the ground-state masses, the mixing between the ground-state and the first excited positive-parity baryons caused by the hyperfine interaction must be included. In this work the compositions and masses of the excited positive-parity baryons obtained by Isgur and Karl[4] are used. The best fit to the ground-state baryons occurs for $x=0.58$ and $\delta=265$, $M_0(N,\Delta)=1132$, $M_0(\Lambda,\Sigma)=1301.5$, $M_0(\Xi)=1453$. (Here as elsewhere in this section

TABLE II. Predicted masses of the ground-state and negative-parity baryons.

Particle	Mass (MeV) Experiment	Calculation
N	938.9	933.4
Δ	1232	1241
Λ	1115.6	1113
Σ	1193.4	1191.5
Σ^*	1385 ± 1	1390
Ξ	1318.1$\pm$0.7	1329.7
Ξ^*	1533.4$\pm$0.9	1530.7
$N\frac{1}{2}^-$	1520 $-$1560	1487
	1620 $-$1680	1635
$N\frac{3}{2}^-$	1510 $-$1530	1526
	1670 $-$1730	1716
$N\frac{5}{2}^-$	1650 $-$1685	1645
$\Delta\frac{1}{2}^-$	1600 $-$1650	1661
$\Delta\frac{3}{2}^-$	1630 $-$1740	1661
$\Lambda\frac{1}{2}^-$	1405 ± 5	1576
	1660 $-$1680	1671
	1700 $-$1850	1801
$\Lambda\frac{3}{2}^-$	1519 ± 1.5	1577
	1690 ± 10	1708
	?	1873
$\Lambda\frac{5}{2}^-$	1810 $-$1830	1809
$\Sigma\frac{1}{2}^-$	1608 $-$1633	1678
	1703 $-$1870	1774
	1730 $-$1820	1813
$\Sigma\frac{3}{2}^-$	1675 ± 10	1706
	?	1810
	?	1829
$\Sigma\frac{5}{2}^-$	1810 $-$1830	1830
$\Xi\frac{1}{2}^-$	?	1832
	?	1935
	?	1952
$\Xi\frac{3}{2}^-$	1823 ± 6?	1849
	?	1945
	?	1998
$\Omega\frac{1}{2}^-$	?	2072
$\Omega\frac{3}{2}^-$	?	2072

all parameters and masses are in MeV except δ which is dimensionless.) Corresponding masses are found in Table I. The maximum percent deviation occurs for Δ. Values of $M_0(\Omega)$ between 1602 and 1624 yield a calculated mass of Ω within this percent deviation from the experimental values. A variation of $M_0(\Lambda,\Sigma)$ and $M_0(\Xi)$ up to about 5 MeV would also keep the calculated masses of Λ, Σ, Σ^*, Ξ, and Ξ^* within this percent deviation. Using these values of $M_0(N,\Delta)$, $M_0(\Lambda,\Sigma,\Sigma^*)$, $M_0(\Xi,\Xi^*)$, $M_0(\Omega)$ and also the mass of $\Sigma\frac{5}{2}^-(1765)$, the remaining parameters m_s, ω, a, b, and c were obtained. For this purpose the mass of $\Sigma\frac{5}{2}^-(1765)$ was constrained to lie within the experimental limits (1774$\pm$7). The exact values were fixed by comparison with $\Lambda\frac{5}{2}^-(1830)$, $N\frac{5}{2}^-(1670)$, and $\Delta\frac{7}{2}^+(1950)$. The final results were $M_0(N,\Delta)=1131.9$, $M_0(\Lambda,\Sigma,\Sigma^*)=1301.5$, $M_0(\Xi,\Xi^*)=1458.75$, $M_0(\Omega)=1605$, $m_s=665$, $\omega=274$, $a=-847.1$, $b=-705.5$, and $c=-1313.3$. Using the calculated values of x, δ, m_s, ω, a, b, and c the masses of the remaining negative-parity baryons were obtained. The final results for all the baryons are found in Table II.

V. DISCUSSION OF THE RESULTS

As in Isgur and Karl,[3] a major problem is the value of the mass of the lowest $\Lambda\frac{1}{2}^-$. As the fit is even worse in the consistent model, this is indeed problematic. Aside from this, the model is quite successful. The entire spectrum of strange and nonstrange negative-parity baryons is fit by parameters derived from the positive-parity baryon spectrum.

ACKNOWLEDGMENTS

The author is grateful to the Natural Sciences and Engineering Research Council of Canada for partial financial support (Grant No. A0358). The author is also grateful to Professor S. K. Misra for a critical reading of the manuscript and useful comments.

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[8]The notation (definition of E_0 and Ω) used here is slightly different from that of Isgur and Karl in Ref. 4.