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Relativistic harmonic-oscillator quark model and $K \to \pi\pi$ decays

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It is shown how the theoretical description of $K \to \pi\pi$ decays depends on the particular quark model. The relativistic MIT bag model is compared with the nonrelativistic harmonic-oscillator model and with two relativistically corrected versions of the harmonic-oscillator model. The relativistic effects in quark dynamics seem to be crucial for the understanding of $K \to \pi\pi$ transitions.

I. INTRODUCTION

Quark models have played an important role in recent analyses of nonleptonic weak decays.[1–6] They lead to theoretical results which are in reasonable agreement with a number of experimental data. Moreover, there are indications that some of the theoretical results are independent of dynamical details underlying a particular quark model.[4,5] A decisive input seems to be the symmetry of hadron states built up of valence quarks. In particular, nonleptonic hyperon decays and Ω^- decays seem to be equally describable by the MIT bag model and the harmonic-oscillator (HO) quark model. In the nonrelativistic HO model, the missing small components of the quark wave functions are replaced by the proper renormalization of the model wave functions.[7] It is interesting to investigate whether this parallel extends to $K \to \pi\pi$ decays.

In this paper we are concerned mostly with a comparison of quark models. We do not investigate other important questions, such as hard-[8] and soft-gluonic[9] corrections or the best form of the effective weak Hamiltonian. We discuss only briefly the importance of the analytic continuation of decay amplitudes from an unphysical point reached by current algebra to their on-mass-shell values.

There are two marked differences between the MIT model and the HO model. The HO model is entirely nonrelativistic, but allows for the separation of the center-of-mass (c.m.) motion. Its meson dynamics is due to the motion of the quark and the antiquark relative to each other. The MIT model is ultrarelativistic, but without a possibility for separating the c.m. motion. Its quarks move relative to the center of the coordinate system in which the bag is fixed.

The nonrelativistic character of the HO model is not satisfactory. It is well known that the momentum of a quark in a hadron is not negligible.[10] This means that small components ought to be taken into account even when using the HO model. This can be estimated by replacing quark Pauli spinors by Dirac spinors.[11]

To investigate the importance of the c.m. motion, we study a quark model which closely parallels the MIT model. In this model, quarks move relative to the harmonic-oscillator potential located (or fixed) about the center of the coordinate system.

We first discuss the calculations of $K \to \pi\pi$ amplitudes based on the MIT model and those based on the nonrelativistic HO model. Next we consider a modification of the HO model. Section IV deals with the relativistic corrections to the HO model. In the concluding section we summarize all results and make a comparison.

II. $K \to \pi\pi$ DECAYS DESCRIBED IN THE MIT BAG MODEL AND THE HO QUARK MODEL

Current algebra (CA) is the only way by which quark models can be introduced into the calculation of weak meson decay amplitudes. However, the momentum dependence of physical decay amplitudes can be misrepresented in the soft-pion limit. The remedies for that are usually based on a general decomposition of the decay amplitude to the leading order in meson momenta:[3,5,12,13]

$$A(M\to M_1+M_2)=L_0+p_1\cdot(p_2+p)L_1+p_2\cdot(p_1+p)L_2+p\cdot(p_2-p_1)L_3\ . \qquad (2.1)$$

The physical amplitude depends only on three combinations of L components:

$$A(M\to M_1+M_2)_{\rm phys}=L_0+(L_1-L_3)(m^2-m_2{}^2)+(L_2+L_3)(m^2-m_1{}^2)\ . \qquad (2.2)$$

It follows from the well-known theorem[14] that $L_0=0$. The rest can be fixed in the CA limit:

$$\lim_{p_1\to 0} A=A_1^{\rm CA}=(L_2+L_3)(p\cdot p_2)\ ,$$
$$\lim_{p_2\to 0} A=A_2^{\rm CA}=(L_1-L_3)(p\cdot p_1)\ . \qquad (2.3)$$

Reference 3 used $p\cdot p_i=\frac{1}{2}(m^2+m_i{}^2)$. For $m_i << m$ ($m_\pi << m_K$), the value obtained was close to the value which is obtained when all mesons are on the mass shell, i.e., $p\cdot p_j=\frac{1}{2}(m^2+m_j^2-m_i{}^2)$.[15] This expression gives for the physical amplitude

$$A(M+M_1+M_2)=2\left[\left(1-\frac{m_2{}^2}{m^2+m_2{}^2-m_1{}^2}\right)A_1^{\rm CA}+\left(1-\frac{m_1{}^2}{m^2+m_1{}^2-m_2{}^2}\right)A_2^{\rm CA}\right]\ . \qquad (2.4)$$

The amplitude (2.4) is just twice the value which would follow by the choice $p\cdot p_i=m^2$.[12] For $K\to\pi\pi$ decays, both mass factors are identical. Therefore, the CA amplitudes obtained by the Lehmann-Symanzik-Zimmermann reduction of both pions in succession are to be multiplied by 1.85 (1.72). These CA amplitudes are

$$A(K^+\to\pi^+\pi^0)=\frac{\widetilde{G}_F}{f_\pi}(6C_4)(a-3b)(4E_\pi E_K)^{1/2},\quad \widetilde{G}_F=G_F\cos\theta_C\sin\theta_C\ , \qquad (2.5a)$$

$$A(K^0\to\pi^+\pi^-)=\frac{\sqrt{2}}{f_\pi}\widetilde{G}_F\{[C_1-2(C_2+C_3+C_4)](a-3b)+(C_6+\tfrac{16}{3}C_5)(a+b)\}(4E_\pi E_K)^{1/2}\ , \qquad (2.5b)$$

$$A(K^0\to\pi^0\pi^0)=\frac{\sqrt{2}}{f_\pi}\widetilde{G}_F\{[C_1-2(C_2+C_3-2C_4)](a-3b)+(C_6+\tfrac{16}{3}C_5)(a+b)\}(4E_\pi E_K)^{1/2}\ . \qquad (2.5c)$$

There is a technical detail connected with the QCD-induced left-right operators O_5 and O_6.[8,1] In the CA procedure, an anomalous term appears in their commutators.[3] The contribution from this anomalous term is equivalent to the separable (i.e., factorizable or vacuum-insertion) approximation to the matrix element of the operators O_5 and O_6. It has been argued that the separable matrix elements of the left-left operators are already included in the CA decay amplitudes.[3,16] This is not necessarily so with the operators O_5 and O_6. However, the relevant difference seems to be rather small; for example,

$$\lim_{q_1\to 0}\left[\langle\pi_1^0\pi_2^0|O_5|K^0\rangle_{\rm CA}-\frac{i\sqrt{2}}{f_\pi}\tfrac{1}{2}\langle\pi_2^0|O_5|K^0\rangle-\langle\pi_1^0\pi_2^0|O_5|K^0\rangle_{\rm SEP}\right]=\tfrac{16}{9}\times1.3\times10^{-3}\ {\rm GeV}^2\ . \qquad (2.6a)$$

Here CA denotes the CA contribution with the anomalous term included and SEP denotes the separable contribution. The contribution (2.6a) should be compared with the standard CA contribution

$$\frac{-i\sqrt{2}}{f_\pi}\langle\pi_2^0|O_5|K^0\rangle=\tfrac{16}{9}\times22.3\times10^{-3}\ {\rm GeV}^2\ . \qquad (2.6b)$$

The contribution (2.6a) is small in comparison with

(2.6b). These estimates were obtained using the MIT model, but similar results are obtained using the relativized HO model. A somewhat different estimate can be found in Ref. 3. As we are interested mainly in a comparison of quark models, we have neglected both expression (2.6a) and the correction discussed in Ref. 16.

The QCD enhancement factors C_i in expressions (2.5) are defined in Refs. 1–5. The integrals over the quark wave functions a and b are defined for the MIT model in Refs. 1, 4, and 5 as follows:

$$a=\int (u_u{}^3 u_s + v_u{}^3 v_s) d^3r \ , \qquad (2.7a)$$

$$b=\int (u_u{}^2 v_u v_s + v_u{}^2 u_u u_s) d^3r \ . \qquad (2.7b)$$

Here u and v denote large and small components of the MIT-bag-model wave function, respectively. The numerics we use in the present calculation is based on the same set of model parameters as used in Refs. 1, 4, and 5:

$$R_u=R_s=R=3.26\text{GeV}^{-1} \ , \quad m(u)=m(d)=0, \quad m(s)=0.279 \text{ MeV} \ . \qquad (2.8)$$

In the nonrelativistic HO quark model, one obtains for the model wave functions a and b[4,5]

$$b\to 0 \ , \quad a\to I_{\pi K}=\left[\frac{\beta_\pi \beta_K}{\pi}\right]^{3/2} . \qquad (2.9)$$

The mesonic parameters appearing in expression (2.9) are connected with the baryonic parameters α_p and x of Ref. 17:

$$\beta_K{}^2=\left[\frac{2}{x+1}\right]^{1/2}\beta_\pi{}^2, \quad \beta_\pi{}^2=\frac{1}{\sqrt{3}}\alpha_p{}^2 \ ,$$
$$\alpha_p{}^2=(3\kappa m_u)^{1/2}, \quad x=m_u/m_s=0.6 \ , \qquad (2.10)$$
$$m_u=0.33 \text{ GeV}, \quad \kappa=0.0106 \text{ GeV}^3 \ .$$

Comparison of relations (2.5), (2.7), and (2.9) reveals the difference between the MIT model and the HO quark model. In the MIT model, formulas (2.5) depend on the combination $(a-3b)=-1.20\times10^{-3}$ GeV^3. In the HO model, this combination is replaced by a larger value, i.e., $(a-3b)\to I_{\pi K}=2.81\times10^{-3}$ GeV^3. The smallness of the combination $(a-3b)$, which is helicity-suppressed in the MIT model,[3] helps reproduce the $K^+\to\pi^+\pi^0$ amplitude. The cancellation due to the helicity suppression introduces an element of uncertainty in the value of $(a-3b)$. Small variations in the MIT-model parameters can introduce marked differences in the value of the $\Delta I=\frac{1}{2}$ contribution f_1 to decay amplitudes. For f_1 defined as

$$f_1=(C_1-2C_2-2C_3)(a-3b) + (C_6+\tfrac{16}{3}C_5)(a+b) \ , \qquad (2.11)$$

using our parameters, we obtain $f_1=0.039\times10^{-3}$ GeV^3. The parameters used in Ref. 3, which are slightly different from ours, lead to $f_1=0.618\times10^{-3}$ GeV^3. This justifies the introduction of the parameter ξ as follows:

$$(a-3b)\to(a-3b)\xi \ . \qquad (2.12)$$

Using $\xi=-0.3$ and $C_5=-0.3$, one can reproduce experimental decay amplitudes to a certain extent. In the framework of the nonrelativistic HO model, one has no motivation for introducing the parameter ξ. In this model, the small components of the quark wave function are absent and no helicity suppression appears. The penguin-operator strength C_5 is also increased by some suitable factor P.[3,18] This seems to be reasonable. The coefficients C_5 and C_6 depend strongly and crucially on an uncertain choice of the renormalization mass μ and on the other parameters in the QCD calculations.[8] In the HO model, the only way of repro-

TABLE I. $K\to\pi\pi$ decay amplitudes in the MIT bag model and the HO quark model, with continuation given by (2.4).

Amplitude (GeV)	MIT bag model $\xi=1$ $P=1$	$\xi=-0.3$ $P=1.9$	HO quark model C_4 $P=1$	$0.1C_4$ $P=0$	Expt.[a]
$10^8A(K^+\to\pi^+\pi^0)$	7.72	1.93	18.31	1.83	1.83
$10^8A(K^0\to\pi^+\pi^-)$	3.79	27.88	43.22	30.79	27.96
$10^8A(K^0\to\pi^0\pi^0)$	7.14	25.15	17.43	28.21	25.36

[a]Reference 5.

ducing the $K^+ \to \pi^+\pi^0$ amplitude correctly would be to suppress that part of the weak Hamiltonian which transforms as $\underline{84}$ in SU(4) flavor. This might be the result of various soft-gluonic effects.[9] At the same time, the representation $\underline{20}$ might be strengthened. However, further study of these important questions, which are especially connected with D-meson decays, is beyond the scope of the present investigations. We just want to emphasize that analogous problems would be encountered in any nonrelativistic quark model.[12] Table I lists some illustrative numerical values.

III. THE HARMONIC-OSCILLATOR SHELL MODEL

In this model, the harmonic-oscillator potential is fixed about the origin of the coordinate system. Such a situation corresponds closely to the square-well-potential "shell model," which in its turn parallels the MIT model.

As one is interested in general features and not in precise numerics, relativistic corrections will be introduced in an approximate way. The small components ϕ are connected with the large components as

$$\phi = \frac{\vec{\sigma}\cdot\vec{p}}{2m}\hat{u} = -i\vec{\sigma}\cdot\vec{r}\frac{1}{r}\frac{1}{2m}\frac{\partial}{\partial r}\hat{u} \; ,$$

$$\hat{u} = f(r) = N\frac{\beta^{3/2}}{\pi^{3/4}}\exp(-\tfrac{1}{2}\beta^2 r^2) \; , \tag{3.1}$$

$$\hat{v} = \frac{1}{2m}\frac{\partial}{\partial r}\hat{u}(r) \; .$$

The quantities $\hat{u}$ and $\hat{v}$ are analogous to the wave functions u and v used in the MIT model. They are normalized by the requirement

$$4\pi N_f^2 \int_0^\infty r^2 dr[\hat{u}_f^2(r) + \hat{v}_f^2(r)] = 1 \; ,$$

$$N_f = \left[1 + \frac{3}{8}\left[\frac{\beta_f}{m_f}\right]^2\right]^{-1/2} \; , \tag{3.2}$$

$$\beta_f^2 = (\kappa m_f)^{1/2} \; , \quad f = u,d,s,\ldots \; .$$

Once the norm is fixed, one can use all formulas appearing in Sec. II, just by making the replacements $u \to \hat{u}$ and $v \to \hat{v}$ everywhere.

In the HO "shell model" (HOS), helicity suppression is also present. This is not surprising because the spin and angular-momentum structure of the HOS model were constructed so as to be identical with the structure of the MIT model.

Such an arrangement follows naturally because in both models the center of mass is fixed at the origin and hadrons are static as if they had an infinite mass.

It is easy to evaluate the integrals a and b:

$$a = \left[\frac{2}{\pi}\right]^{3/2} N_u{}^3 N_s \beta_u{}^3 \left[3\frac{\beta_u}{\beta_s} + \frac{\beta_s}{\beta_u}\right]^{-3/2} \times \left[1 + \frac{15}{16}\frac{\beta_u{}^4}{m_u{}^3 m_s}\left[3\frac{\beta_u}{\beta_s} + \frac{\beta_s}{\beta_u}\right]^{-2}\right] , \tag{3.3}$$

$$b = \frac{3}{(2\pi^3)^{1/2}} N_u{}^3 N_s \frac{\beta_u{}^4 \beta_s}{m_u m_s}\left[3\frac{\beta_u}{\beta_s} + \frac{\beta_s}{\beta_u}\right]^{-5/2} \times \left[1 + \frac{m_s}{m_u}\frac{\beta_u{}^2}{\beta_s{}^2}\right] .$$

The numerical values depend on the choice of the HOS-model parameters. The choice (2.10) leads to

$$a = 7.3\times 10^{-4}\ \text{GeV}^3 \; , \quad b = 1.2\times 10^{-4}\ \text{GeV}^3 \; . \tag{3.4a}$$

It is obvious that the HO model ought to be relativized. If the small components of the Dirac four-spinors were neglected, one would obtain the following values instead of (3.4a):

$$a = 7.1\times 10^{-4}\ \text{GeV}^3 \; , \quad b = 0 \; . \tag{3.4b}$$

The combination $(a-3b) = 3.7\times 10^{-4}$ GeV^3 is much smaller in this model than in the HO model [i.e., $(a-3b) \to a = 2.81\times 10^{-3}$ GeV^3]. It is interesting to note that it would lead to the prediction

$$A(K^+ \to \pi^+\pi^0) = 1.31\times 10^{-8}\ \text{GeV} \; , \tag{3.5}$$

which is rather close to the experimental value 1.83×10^{-8} GeV. If the CA result (3.5) is multiplied by 1.72 in order to continue it to the mass shell, one obtains the value 2.25×10^{-8} GeV. However, neutral-kaon decay amplitudes come out much too small, and one would have to consider some strong enhancement of that part of the effective weak Hamiltonian which transforms as $\underline{20}$ (i.e., $\Delta I = \frac{1}{2}$) in SU (4).[9]

The results depends on the HO parameter β which was fixed on the basis of other experimental data.[17] For example, in the HO model, the ratio of pion-to-proton charge radii is

$$\frac{\sum_i \langle e_i r_i^2\rangle_{\pi^+}^{1/2}}{\sum_i \langle e_i r_i^2\rangle_p^{1/2}}=\frac{(\frac{3}{8})^{1/2}\beta^{-1}}{\alpha^{-1}}=0.81\ , \qquad (3.5a)$$

where one has

$$\alpha^2=\sqrt{3}\beta^2\ . \qquad (3.5b)$$

In the HOS model, expression (3.5a) remains unchanged. The matrix element of r^2 is diagonal, so the contributions from the small components are absorbed in the normalization (3.2). The ratio (3.5) is in good agreement with the experimental value ~ 0.88.[19]

However, changes in value of the parameter β can lead to a change in sign of the expression $(a-3b)$, whose value can even pass through zero. This can be parametrized through the HO-potential strength parameter κ which determines the values of β:

$$\beta_u=(\kappa m_u)^{1/4}\equiv\beta_\pi\ . \qquad (3.6)$$

If the parameter κ is increased eight times (i.e., the parameters β are increased 1.68 times), the values of a, b, and $(a-3b)$ are estimated to be 2.91×10^{-3}, 1.01×10^{-3}, and -0.12×10^{-3} GeV^3, respectively. In this way, one illustrates both the importance of the relativistic effects and the dependence of the combination $(a-3b)$ on the model parameters.

IV. THE HO MODEL WITH RELATIVISTIC CORRECTIONS

Even the problem of a two-body potential becomes a complex exercise when a full relativistic treatment is attempted.[20] We shall, therefore, content ourselves with a somewhat less ambitious task of calculating relativistic corrections to the nonrelativistic two-body HO model. The corrections turn out to be very large, so that one should not refer to corrections, but to a reformulated approach (i.e., the RHO model). The results we outline in this section can be considered mostly as a qualitative illustration.

CA determines $K\to\pi\pi$ amplitudes as matrix elements of four-quark operators of the type

$$O=\int d^4x\,\bar\psi(x)\Gamma^1_\mu\psi(x)\bar\psi(x)\Gamma^2_\mu\psi(x)$$
$$(\Gamma^1_\mu,\Gamma^2_\mu\sim\gamma_\mu,\gamma_\mu\gamma^5)\ . \qquad (4.1)$$

The quark field appearing in (4.1) can be decomposed in a standard way for a flavor f:

$$\psi_f(x)=\frac{1}{(2\pi)^{3/2}}\left[\frac{m}{E_f}\right]^{1/2}\int d^3q[u_f(q)e^{-iq\cdot x}b_f(q)+v_f(q)e^{iq\cdot x}d_f^\dagger(q)]\ , \qquad (4.2a)$$

with the following definition of the spinors u and v:

$$u(q)=\left[\frac{E+m}{2m}\right]^{1/2}\begin{bmatrix}\chi\\ \frac{\vec\sigma\cdot\vec q}{E+m}\chi\end{bmatrix},\quad v(q)=\left[\frac{E+m}{2m}\right]^{1/2}\begin{bmatrix}\frac{\vec\sigma\cdot\vec q}{E+m}\\ \chi\end{bmatrix}\ . \qquad (4.2b)$$

This approximation closely resembles that used in Ref. 11. In this reference, the SU(6) harmonic-oscillator wave function was used, with Pauli spinors being replaced by Dirac spinors.

In the HO model, the operator (4.1) is to be sandwiched between meson states of the form[21]

$$|M_i(\vec k)\rangle=\int d^3p\,f_{M_i}(p)b_\alpha^\dagger\left[\frac{m_\alpha}{\mu_i}\vec k+\vec p\right]d_\beta^\dagger\left[\frac{m_\beta}{\mu_i}\vec k-\vec p\right]|0\rangle\ . \qquad (4.3)$$

Here $\vec k$ is the c.m. momentum, while $b^\dagger$ and $d^\dagger$ are the creation operators of the particle and the antiparticle, respectively. The function $f_M(p)$ is a solution of the HO potential (in momentum space). Since the HO model is nonrelativistic, one usually takes the nonrelativistic approximation for the operator 0. In our considerations, this would mean simply discarding the small components in expression (4.2b). In order to calculate the relativistic correction, the small components will be retained and their contributions calculated by averaging over the HO wave function f_M. For example,

$$A=\int d^4x\langle\pi^+|\bar{\psi}_s\gamma^\mu\gamma_5\psi_u\bar{\psi}_u\gamma_\mu\gamma^5\psi_d|K^+\rangle$$
$$=\int d^3p\,d^3q\left[\bar{v}\left[\frac{m_s}{\mu_k}\vec{k}_K-\vec{q}\right]\gamma^\mu\gamma_5 u\left[\frac{m_u}{\mu_K}\vec{k}_K+\vec{q}\right]\right.$$
$$\left.\times\,\bar{u}\left[\frac{m_u}{\mu_\pi}\vec{k}_\pi+\vec{p}\right]\gamma_\mu\gamma^5 v\left[\frac{m_d}{\mu_\pi}\vec{k}_\pi-\vec{p}\right]\right]f_\pi^*(p)f_K(q)\times\left[\frac{1}{\sqrt{6}}\right]^2\times 9\times 4\,, \tag{4.4}$$

where the factor $(1/\sqrt{6})^2\times 9\times 4$ comes from the color-flavor-spin dependence. Since this formalism is not relativistically covariant, one has to specify hadron momenta. The most reasonable choice is to take kaon at rest, which then, in the CA limit, implies $\vec{k}_\pi=0$. Furthermore, such a limit is implied by formulas (2.3) and it closely parallels the MIT and HOS models in which hadrons are static. Expression (4.4) will then be a complicated function of q^2 and p^2. This is because q^2 and p^2 appear under the square roots of the spinors u and v. In a reasonable approximation, all these q^2 and p^2 will be replaced by their average values, i.e.,

$$\langle\vec{p}_M{}^2\rangle=\frac{\int d^3p\,p^2 f_M(p)}{\int d^3p\,f_M(p)}=3\beta_M{}^2\,. \tag{4.5}$$

The small numerical inaccuracy introduced by such an approximation is not of great importance in view of the approximate character of the whole procedure. It allows one to write expression (4.4) in a compact analytical form,

$$A=6(a-3b)\,,$$
$$a=\alpha\left[\frac{\beta_\pi\beta_K}{\pi}\right]^{3/2},\quad \alpha=\left[\frac{(E_s+m_s)_q(E_u+m_u)_q(E_u+m_u)_p{}^2}{16(E_u)_p{}^2(E_u)_l(E_s)_l}\right]^{1/2},$$
$$b=\left[\frac{\beta_\pi{}^2}{(E_u+m_u)_p{}^2}+\frac{\beta_K{}^2}{(E_u+m_u)_q(E_s+m_s)_q}-\frac{3\beta_\pi{}^2\beta_K{}^2}{(E_u+m_u)_p{}^2(E_u+m_u)_q(E_s+m_s)_q}\right]a\,,$$
$$(E_i+m_i)_{p,q}=(m_i{}^2+3\beta_{\pi,K}{}^2)^{1/2}+m_i;\quad i=u,s\,. \tag{4.6}$$

Here a and b correspond exactly to the quantities appearing for the first time in formula (2.5). Table II lists some interesting numerical values. These values depend on the choice of the model parameters β_π and β_K defined by (2.10) and (3.6). These parameters are connected with the ratio of pion-to-proton charge radii in the same way as in the HO and HOS models. This statement is based on the same arguments as those used in deriving formula (3.5).[22]

Comparison with experiments depends strongly on the precise continuation of the CA results to the physical point. In order to illustrate that, let us first start with the continuation suggested by Ref. 12. Practically, this means using formulas (2.5) without the mass-dependent factors defined in (2.4).[23] It is obvious that only the strengthening of the penguin terms, in connection with the weakening of the 84-flavor contribution (i.e., $\Delta I=\frac{3}{2}$), can lead to agreement with experiment. This is evident

TABLE II. $K\to\pi\pi$ decay amplitudes in the RHO quark model, with the CA given by (2.5) and (4.7).

Amplitude (GeV)	$x=1$ $P=1$	$x=0.5$ $P=1$	$x=0.45$ $P=5$	$x=0.45$ $P=10$	Expt.
$10^8A\,(K^+\to\pi^+\pi^0)$	4.13	2.07	1.86	1.86	1.83
$10^8A\,(K^0\to\pi^+\pi^-)$	10.68	9.72	17.46	27.26	27.96
$10^8A\,(K^0\to\pi^0\pi^0)$	4.84	6.80	14.83	24.63	25.36

TABLE III. $K \to \pi\pi$ decay amplitudes in the RHO quark model, with the continuation given by (2.4).

Amplitude (GeV)	$x=1$ $P=1$	$x=0.25$ $P=1$	$x=0.1$ $P=2$	$x=0.24$ $P=4$	Expt.
$10^8 A(K^+ \to \pi^+\pi^0)$	7.60	1.90	1.82	1.82	1.83
$10^8 A(K^0 \to \pi^+\pi^-)$	19.66	16.96	20.54	27.76	27.96
$10^8 A(K^0 \to \pi^0\pi^0)$	8.89	14.28	17.96	25.18	25.36

from the following formulas:

$$\begin{aligned} 10^8 A(K^+ \to \pi^+\pi^0) &= 4.13x \ , \\ 10^8 A(K^0 \to \pi^+\pi^-) &= 6.78 + 1.95x \\ &\quad + 1.96P \ , \\ 10^8 A(K^0 \to \pi^0\pi^0) &= 6.78 - 3.90x \\ &\quad + 1.96P \ . \end{aligned} \tag{4.7}$$

Here $x=1$ and $P=1$ correspond to the standard QCD-enhanced weak Hamiltonian. The parameter x indicates the quenching of the operator O_4 (C_4), while P measures the increase in strength of the penguin terms. Only $x<0.5$ can reproduce the experimental values listed in Table II. The prescription of Ref. 12 requires that all amplitudes should be multiplied by 0.92 and then $x=0.45$ fits the $A(K^+ \to \pi^+\pi^0)$ amplitude.

If one used the factor 1.84, suggested by (2.4), one would encounter an entirely different situation, as illustrated in Table III. A very good fit of the experimental results is possible if there are penguin terms. The quenching of the O_4 contribution cannot work in the absence of penguin terms.[24]

To obtain an $(a-3b)$ combination which would be almost vanishing, one needs significant alterations in HO-model parameters. Such a situation is different from that encountered in the MIT model.

Although we are primarily interested in a comparison of the HO model and the MIT model, let us mention, for the sake of completeness, that many theoretical attempts have been made to understand the modification of QCD corrections (i.e., "hard" gluons) on the basis of soft-gluonic contributions.[9] In most of the attempts, the $\underline{20}$ contribution increases and the $\underline{84}$ contribution is quenched. The enhancement of the $\underline{20}$ contribution means that the value 6.78 in expression (4.7) should be increased; thus, a fit to experiments might be possible with smaller penguin terms. However, the quenching of the $\underline{84}$ contribution is never strong enough to reproduce the experimental amplitude for the $K^+ \to \pi^+\pi^0$ decay. The same situation is encountered in quark-density models. Therefore, it was suggested in Ref. 12 that some soft-gluonic mechanism should exist which would be responsible for the quenching of the $\underline{84}$ contribution.

V. DISCUSSION

Further discussion can be conveniently carried out by comparing the values of a and b calculated using the MIT model with those calculated in the RHO model.

In the MIT model, with $a=5.34\times10^{-3}$ GeV^3 and $b=2.18\times10^{-3}$ GeV^3, the value of the crucial combination is very small and (seems to be) negative, $(a-3b)_{\rm MIT}=-1.2\times10^{-3}$ GeV^3. Its precise value depends strongly on the values of the MIT-model parameters;[25] therefore, this value can, in a certain sense, be treated as a fitting parameter. In the MIT model, this value can easily be zero, or it can be small and positive; therefore, in Ref. 3 it is replaced by $(a-3b)\xi$ with $\xi=-0.3$. Thus, in principle, the MIT model can explain the smallness of the K^+ amplitude.

In the RHO model, the situation is completely different. In this model, $a=1.95\times10^{-3}$ GeV^3 and $b=0.252\times10^{-3}$ GeV^3, so that $(a-3b)$ is always positive and larger than the value calculated in the MIT model [$(a-3b)_{\rm RHO}=1.20\times10^{-3}$ GeV^3]. A similar situation is encountered in the nonrelativistic HO model [$(a-3b)_{\rm HO}=a_{\rm HO}=2.81\times10^{-3}$ GeV^3], but not in the HOS [$(a-3b)_{\rm HOS}=3.9\times10^{-4}$ GeV^3]. However, the results obtained in the HO model depend on the model parameters. The value based on the MIT model can be duplicated only with some sizable changes in parameter. One is thus inclined to conclude that both relativistic corrections (i.e., small components) and recoil corrections (i.e., c.m. motion) do play a role. Some of the difference must also be caused by the confining dynamics of the HO model; the dynamics of the HO model leads to the quark wave functions which behave in a markedly different way from the wave functions in the MIT model.

Thus, $K \to \pi\pi$ decays can test for the essential difference between quark models. In principle, such tests are always possible with quantities depending on the interference between small and large components of the quark-model wave functions. When the contributions from small and large components add, the result can be model independent to a large extent: it is determined mostly by the color-flavor-spin structure of hadron states (which is taken to be the same in all models), while the scale is fixed by the renormalization of wave functions. However, in practice, the theoretical analysis is rather involved, as outlined in Secs. II an IV, and, therefore, no firm conclusion is possible. It seems that only the MIT model can account for $K \to \pi\pi$ decay amplitudes with the standard QCD-enhanced effective weak Hamiltonian. Even this conclusion is really not satisfactory because it depends strongly on the very particular details: on the fortuitous cancellation in the $(a-3b)$ term and on the enhanced penguin terms.

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[16]This follows if the form factors appearing in the matrix elements $\langle \pi | V_\mu | K \rangle$ are estimated in the flavor-symmetry limit, $f_+ \sim 1$, $f_- = 0$. Symmetry breaking, for which $f_- \neq 0$, would introduce small corrections.

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averaging is performed over $\vec{r}_i^{\,2}$.

[23]Such an approach would influence the conclusions based on the MIT model in an unimportant way. The fit of the experimental amplitudes requires that P should be somewhat larger than the values listed in Table I.

[24]With $x=0.24$, $P=0$, one has $A(K^+\to\pi^+\pi^0)=1.82\times10^{-8}$ GeV, $A(K^0\to\pi^+\pi^-)=13.32\times10^{-8}$ GeV, and $A(K^0\to\pi^0\pi^0)=14.18\times10^{-8}$ GeV.

[25]Some interesting examples are listed in Ref. 33 of Ref. 3.