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Model for the structure functions of hadrons

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Multijet production at extremely high energy, generated by iteration of the standard hard-parton-scattering mechanism, in the framework of quantum chromodynamics, is assumed to be processed by low-p_T hadron multiplicities satisfying Koba-Nielsen-Olesen scaling. This assumption is argued to lead to a specific structure in the angular-momentum plane, which gives rise to long-range effects in hadron collisions, and in particular to a rising component in the hadron cross section. This component is due to hard parton subcollisions and can be correlated with the hadron structure function. The model is shown to be consistent with the πp, Kp, and pp total-cross-section data, as well as with the π, K, and p structure-function data.

I. INTRODUCTION

Since quantum chromodynamics (QCD) is only applicable for hadronic processes involving the production of at least one high-transverse-momentum quark or gluon, materializing itself as a jet ("hard" processes), one has been accustomed during the past few years to treating such processes as distinct from those involving the production of only low-transverse-momentum hadrons.[1] The latter processes, generally brought under the name "soft" processes, are analyzed within the framework of Reggeon field theory, where short-rapidity-range interactions at high energy are described by bare-Pomeron or meson exchange, while long-range correlations originate from higher-order corrections (cuts). If one were able to solve the confinement problem, all soft physics would emerge out of the QCD Lagrangian as a result of multigluon and quark exchanges.[2] On the other hand, hard processes involve parton subcollisions with large submomentum transfers, for which asymptotically free QCD can be solved within the framework of perturbation theory, which now requires only few-vector-gluon exchange: Such hard interactions can generally be of long rapidity range.[3] Therefore, long-range correlations in hadron production can originate from both the soft and the hard sector of the theory.

In the present work we further study the consequences of the requirement[4] that the contribution of hard-scattering processes to long-rapidity-range phenomena[3] should be consistent with Koba-Nielsen-Olesen (KNO) scaling,[5] the universal relevance of which to hadron production is rather well established experimentally.[6] Specifically, we require that the low-p_T hadron multiplicities resulting by iteration of the standard jet-production mechanism (see Fig. 1) according to the general factorization law

$$\langle n(n-1)\cdots(n-q+1)\rangle\sigma^h(Y) = \frac{q!}{2\pi i}\int_{c-i\infty}^{c+i\infty} dJ\, e^{(J-1)Y}[\sigma^h(J)]^{q+1}\,, \quad (1.1)$$

where $Y = \ln s/s_0$ ($s_0 \sim 1$ GeV^2) is the rapidity of the collision, satisfy KNO scaling. The unique structure on the angular-momentum plane allowing for this property is[7]

$$\sigma^h(J) = [\exp b(J-1)^{1-\eta} - 1]^{-1} \quad (1.2a)$$

$$\simeq \exp[-b(J-1)^{1-\eta}] + \exp[-2b(J-1)^{1-\eta}] + \cdots\,, \quad (1.2b)$$

$$0<\eta<1\,,\ b>0$$

which, despite its factorizability, gives rise to long-range correlations and, in particular, provides a rising component

$$\sigma^h(Y) \underset{Y\to 0}{\sim} [\kappa(\kappa-1)a/2\pi]^{1/2} Y^{-(\kappa+1)/2} \times \exp(-aY^{1-\kappa}) , \quad (1.3)$$

where

$$a = [(1-\eta)b]^\kappa/(\kappa-1) , \quad \kappa = 1/\eta > 1 , \quad (1.4)$$

(see also the Appendix) to hadron cross sections, which is consistent with the data.[8] Our requirement means that the lowest-order graph of Fig. 1, which gives the contribution of hard parton scattering to the total cross section, exhibits the singularity structure (1.2), which should then be traced in the hadron structure function $F^h(x,Q^2)$. We should thus have

$$F^h(x,Q^2) \underset{x\to 1}{\sim} \exp\left[-\frac{a}{2}\left[2\ln\frac{1}{x}\right]^{-\kappa+1}\right] , \quad (1.5)$$

which is the main result of this paper (Sec. II) and is found to be consistent with the available experimental data (Sec. III).

One expects that the probability to produce 4, 6,. . .jets in this model must be quickly decreasing. Indeed, at present energies, the probability to produce even 4 jets with the mechanism of Fig. 1 turns out to be negligible. Specifically, one finds[7,8] that the threshold behavior (1.3) represents the inverse Mellin transform of (1.2) with accuracy better than 1% in the energy range covered by present and future experiments (see also the Appendix); this behavior corresponds to the first term of the "jet expansion" (1.2b).

Now the inclusive jet yield obtained in hadronic collisions is quantitatively given as a folding integral over the parton distributions $G_a^h(x,Q^2)$ and the parton-parton differential scattering cross section $d\hat{\sigma}^{ab}/d\hat{t}$, namely,[1,9]

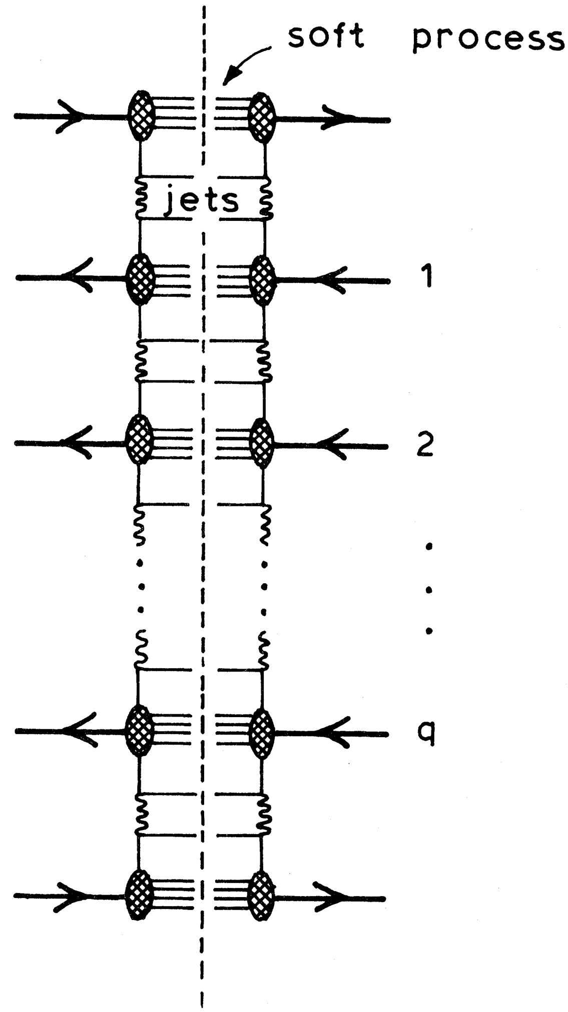

FIG. 1. Mueller graph corresponding to the iteration of the standard mechanism for two-jet production via one-gluon exchange, with simultaneous soft production of low-p_T hadrons, q of which are observed inclusively [formula (1.1)].

$$E_j\frac{d^3\sigma^{hp}}{dp_j{}^3} = \sum_{a,b} \int dx_a \int dx_b G_a^h(x_a,Q^2) G_b^p(x_b,Q^2) \frac{\hat{s}}{\pi}\delta(\hat{s}+\hat{t}+\hat{u})\frac{d\hat{\sigma}^{ab}}{d\hat{t}}(\hat{s},\hat{t}) , \quad (1.6)$$

where the indices a,b label quarks, antiquarks, or gluons. The careted Mandelstam invariants $\hat{s},\hat{t},\hat{u}$ refer to the hard subprocess $ab\to jk$. Quark masses are neglected throughout this work. Although Eq. (1.6) has been introduced in the parton-model framework, it has been argued to hold true in QCD, at least in the leading-logarithm approximation, the infrared and mass singularities of $d\hat{\sigma}^{ab}/d\hat{t}$ being factored out and absorbed by the bare parton distributions.[10]

To find the contribution of jet production to the total hadron-scattering cross section, which, according to our model, is identified with (1.3), one has to integrate (1.6) over the phase space of both of the produced jets. Since the average multiplicity of these jets is obviously 2 and $E_j{}^{-1}dp_j{}^3 = \pi\hat{s}^{-1}d\hat{t}\,d\hat{u}$, neglecting $\hat{u}$-channel ($\hat{t}$-channel) gluon exchange in the forward (backward) hemisphere of the center-of-mass of the subprocess, the contribution of incoherent elastic quark and/or antiquark scattering with gluon exchange in the $\hat{t}$ channel or $\hat{u}$ channel to the total hadron-scattering cross section is given by

$$\sigma_j^{hp}(s)=\int_{s_0/s}^{1}\frac{dx_a}{x_a}\int_{s_0/sx_a}^{1}\frac{dx_b}{x_b}\int_{-\hat{s}/2}^{-s_0/2}d\hat{t}\,F^h(x_a,Q^2)F^p(x_b,Q^2)\frac{d\hat{\sigma}_{qq}}{d\hat{t}}(\hat{s},\hat{t})\ , \tag{1.7}$$

where[11]

$$\frac{d\hat{\sigma}_{qq}}{d\hat{t}}(\hat{s},\hat{t})=\frac{4\pi}{9}\alpha_s{}^2(Q^2)\frac{\hat{s}^2+\hat{u}^2}{\hat{s}^2\hat{t}^2}\ . \tag{1.8}$$

The coupling strength $\alpha_s(Q^2)$ of QCD is

$$\alpha_s(Q^2)=\frac{12\pi}{33-2N_f}\frac{1}{\ln Q^2/\Lambda^2}\ , \tag{1.9}$$

where N_f is the number of flavors considered. We have defined

$$F^h(x,Q^2)\equiv x\sum_a G_a^h(x,Q^2)\ , \tag{1.10}$$

where $h=\pi,K,p$ and the index a runs over all quarks and antiquarks. The cutoff parameter s_0 is introduced so that $\hat{s}=sx_ax_b>s_0$ and $-\hat{t}>s_0/2$. This is of the order of a few GeV^2, and since it refers to the hard parton subcollision it has to be process-independent. To obtain formula (1.7) we neglected the interference between $\hat{t}$-channel and $\hat{u}$-channel gluon exchange in the case of identical quark or antiquark scattering, as well as all $\hat{s}$-channel gluon exchanges, and the interference between $\hat{s}$-channel and $\hat{t}$-channel gluon exchange in the case of quark-antiquark elastic scattering. These terms, together with antiquark-quark annihilation into gluons, are checked to contribute, collectively, to the total cross section no more than 10% of (1.7). In the framework of the model we are going to develop in the next section, we shall justify that quark-gluon and gluon-gluon subcollisions need not be considered in the right-hand side of (1.7) if it is identified with (1.3). With these simplifications, we have no ambiguity in interpreting $Q^2\equiv-\hat{t}$ (or $Q^2\equiv-\hat{u}$) in relations (1.7) and (1.8).

According to our model, the component (1.7) of the total hadron-scattering cross section is identified with (1.3) and gives the observed rise of this cross section for $s\gtrsim 100\ \mathrm{GeV}^2$. We note at this point that, although the inclusive jet yield obtained in hadron collisions falls off very rapidly as a function of the transverse momentum p_T, its contribution to total-cross-section rises is appreciable,[12] while its reflection to the average-transverse-momentum rise of the produced hadrons can be suppressed, as observed experimentally.[13] The last statement can qualitatively be made plausible by plain numerology, without explicit reference to (1.7). Indeed, the inclusive charged-pion yield in pp collisions (as an example) can, very roughly, be parametrized as

$$E\frac{d^3\sigma}{dp^3}=\begin{cases}Ae^{-\lambda p_T} & \text{if } p_T<p_T^{\min}\ ,\\ \dfrac{B(s)}{(p_T/p_T^{\min})^{N(s)}} & \text{if } p_T>p_T^{\min}\ ,\end{cases} \tag{1.11}$$

where $B(s)$ and $N(s)$ depend slightly on the center-of-mass energy and the cutoff parameter $p_T^{\min}$ is of the order of 1 GeV. The relative contribution of "high-p_T" events to the pp total cross section and the average transverse momentum $\langle p_T\rangle$ of a charged pion are then $\epsilon(s)$ and $\mu\epsilon(s)$, respectively, where

$$\begin{aligned}\epsilon(s)&\approx\frac{\lambda^2}{A}\frac{B(s)}{N(s)-2}(p_T^{\min})^2\ ,\\ \mu&\approx\tfrac{1}{2}\lambda p_T^{\min}-1\ .\end{aligned} \tag{1.12}$$

To give agreement with experiment, $\epsilon(s)$ should rise to about 10% at $\sqrt{s}\approx 60$ GeV while μ should remain as small as about 0.5 there. The second requirement is met by choosing $p_T^{\min}\approx\frac{3}{2}\langle p_T\rangle$, while the first is automatically fulfilled. Although $p_T^{\min}\approx 0.5$ GeV may seem a rather small cutoff value, within the framework of the present qualitative estimation it can be acceptable.

II. STRUCTURE FUNCTIONS

In this section we quantitatively identify the contribution (1.7) of incoherent elastic quark and antiquark subcollisions to the total hadron-scattering cross section with the component (1.3), whose iterated convolution produces low-p_T hadron multiplicities with KNO scaling according to the law (1.1) (see the factorizable graph of Fig. 1) and gives rise to long-rapidity-range phenomena, as discussed in the Introduction. We thus write

$$\sigma_j^{hp}(\ln s/s_0)\equiv\sigma^h(Y)\ . \tag{2.1}$$

It is apparent that this identification suggests a particular model for the x dependence of the structure functions of hadrons, which we shall now develop.

Since there is no experimental information, as yet, about the Q^2 dependence of the pion and kaon structure functions, we suppress this dependence to a single point $\bar{Q}$ (in the case of proton as well) which we do not show as an argument any more.

In the case of proton's structure function, we have checked[4] that the Q^2 dependence, required by lowest-order QCD, can be taken into account in a standard fashion,[14] leading to quite satisfactory agreement with the neutrino-production data. In this paper, we concentrate on the x dependence of the structure functions of hadrons, which is the crucial prediction of the present model.

Introducing the moments of the sum (1.10) of all quark and antiquark distributions within hadron h, namely,

$$F^h(p)=\int_0^1 x^{p-1}F^h(x)dx \;, \tag{2.2}$$

the Mellin transform of (2.1) assumes the form

$$F^h(p)\cdot\hat{\sigma}(p)\cdot F^p(p)=g_h g_p \exp(-b_{hp}p^{1-\eta}) \;, \tag{2.3}$$

where

$$\hat{\sigma}(p)=\int_{s_0}^{\infty}\left[\frac{\hat{s}}{s_0}\right]^{-p-1}\hat{\sigma}(\hat{s})\frac{d\hat{s}}{s_0} \;, \tag{2.4}$$

and

$$\hat{\sigma}(\hat{s})=\int_{-\hat{s}/2}^{-s_0/2} d\hat{t}\frac{d\hat{\sigma}_{qq}}{d\hat{t}}(\hat{s},\hat{t}) \;. \tag{2.5}$$

The parameter g_h represents the coupling of the factorizable singularity (1.2) to hadron h. The exponent η is restricted in the range $0<\eta<1$, since for $\eta<0$ positivity is violated,[15] whereas for $\eta\geq 1$ it is easily seen that we have a constant average multiplicity and no KNO scaling. In the framework of the Feynman-Wilson gas analogy[16] this exponent is interpreted as the inverse of the critical index δ, which is defined in statistical mechanics[17] to control the relation between an order parameter and the corresponding ordering field near a critical point. Thus, in the spirit of universality,[18] the exponent η should be process independent. On the other hand, the parameter b should, in general, be different for various hadronic processes. Isospin conservation requires

$$b_{hp}=b_{hn}\equiv b_h \;,\quad g_p=g_n \;. \tag{2.6}$$

Moreover, since the sum (1.10) is the same for a particle and its antiparticle, we have

$$b_h=b_{\bar{h}} \;,\quad g_h=g_{\bar{h}} \;. \tag{2.7}$$

Note that in Eq. (2.1) we naturally identified the hard-process cutoff s_0 with the corresponding energy unit. This results in having the essential singularity of $F^h(x)$ at exactly $x=1$, as expected by general arguments.

In the case of pp scattering, Eq. (2.3) gives

$$F^p(p)=g_p[\hat{\sigma}(p)]^{-1/2}\exp(-\tfrac{1}{2}b_p p^{1-\eta}) \;, \tag{2.8}$$

which is our prediction for the large-order moments of the proton's singlet structure function. The inversion of (2.8) in closed form is facilitated by the presence of the exponential factor. We employ a standard stationary-phase approximation technique,[19] which gives the exact result in the limit $x\to 1$ (see the Appendix). Since a singularity of the form (1.2) controls the inverse Mellin transform of (2.8), the asymptotic behavior of $F^p(x)$ for $x\to 1$, related to the asymptotic behavior of (2.8) for $p\to\infty$, holds far away from $x=1$. This is also the case with the threshold behavior (1.3), which, as explained in the Introduction, holds in the whole energy range for which total-cross-section data exist. Therefore, giving up the exact form of $F^p(x)$ for x close to zero which is, in any case, seriously affected by higher-order QCD effects, it suffices to consider the asymptotic behavior of (2.4) for $p\to\infty$. In the same approximation, the contribution of gluon interactions in the right-hand side of (1.7) is neglected: indeed, for $p\to\infty$ the mixing between quark singlet and gluon operators becomes unimportant and we have $G(p)/F^p(p)\to 0$, where $G(p)$ are the moments of the gluon distribution.

From (1.8), (2.4), and (2.5), for large p we find

$$\hat{\sigma}(p)=\frac{\sigma_0}{p^2}+O\left[\frac{1}{p^3}\right] \;, \tag{2.9}$$

where

$$\sigma_0=\left[\frac{4\pi}{5}\right]^2\frac{\pi}{5}s_0^{-1}(\ln s_0/\Lambda^2)^{-1} \;, \tag{2.10}$$

and (2.8) becomes

$$F^p(p)=\frac{5g_p}{4\pi}\left[\frac{5s_0}{\pi}\right]^{1/2}\ln\frac{s_0}{\Lambda^2}\left[1+O\left[\frac{1}{p}\right]\right]p$$
$$\times\exp(-\tfrac{1}{2}b_p p^{1-\eta}) \;. \tag{2.11}$$

In the Appendix we show that the inverse Mellin transform of this expression has the representation

$$F^p(x)\equiv\mathscr{F}(a_p,g_p;x) \;, \tag{2.12}$$

where

$$\mathscr{F}(a,g;x)=\frac{25g}{8\pi^2}\left[\frac{2\kappa s_0}{5}\right]^{1/2}\left[\frac{(\kappa-1)a}{2^\kappa}\right]^{3/2}\ln\frac{s_0}{\Lambda^2}\left[\ln\frac{1}{x}\right]^{3\kappa/2-1/2}$$
$$\times\left\{1-\frac{(\kappa+1)}{(\kappa-1)a}\left[2\ln\frac{1}{x}\right]^{\kappa-1}+O\left[\left[\ln\frac{1}{x}\right]^{2(\kappa-1)}\right]\right\}\exp\left[-2^{-\kappa}a\left[\ln\frac{1}{x}\right]^{1-\kappa}\right] \quad (2.13)$$

and

$$a_h=[(1-\eta)b_h]^\kappa/(\kappa-1) \quad (h=\pi,K,p)\ , \quad (2.14)$$

which is an asymptotic expansion for x near unity. However, we calculated numerically the inverse Mellin transform of (2.11) and checked that the first two terms of the expansion, given in (2.13), are sufficient to represent this transform, with more than 5% accuracy, for essentially all x. We also checked that the nonasymptotic terms of order $1/p$ and higher, in the brackets of Eq. (2.11), give rise to correction terms in (2.13), which are at most 10% of the total contribution for $x\simeq 0.3$ and become quickly zero for $x\to 1$.

It is now straightforward to obtain our predictions for the x dependence of the sum of all quark and antiquark distributions in the pion and kaon. Applying (2.3) in the cases of πp and pp scattering, we find

$$F^\pi(p)=g_\pi[\hat{\sigma}(p)]^{-1/2}\exp[-\tfrac{1}{2}(2b_\pi-b_p)p^{1-\eta}]\ . \quad (2.15)$$

We invert this expression, which has the same analytic form as (2.8), with the method previously developed, to find

$$F^\pi(x)=\mathscr{F}(\bar{a}_\pi,g_\pi;x)\ , \quad (2.16)$$

where

$$\bar{a}_h=(2a_h^\eta-a_p^\eta)^\kappa \quad (h=\pi,K)\ . \quad (2.17)$$

Similarly, applying (2.3) for Kp and pp scattering we find

$$F^K(x)=\mathscr{F}(\bar{a}_K,g_K;x)\ . \quad (2.18)$$

We thus succeeded to obtain relatively tractable expressions for the structure functions of the nucleon, pion, and kaon, which involve an essential singularity at $x=1$, contrary to the popular parametrization, $F^h(x)\propto(1-x)^{d_h}$. This singularity is correlated with the J-plane singularity (1.2), which produces the total hadron-scattering cross-section rise in our model. The phenomenological consequences of this correlation are studied in the next section.

III. COMPARISON WITH EXPERIMENTAL DATA

To confront our model with experiment, we simultaneously fit the predictions (2.12), (2.16), and (2.18) for the hadrons's structure functions and the prediction (1.3) for the total hadron-scattering cross-section rise to the corresponding experimental data. In the latter case, the rising component (1.3), which in our model is due to hard quark subcollisions, is superposed on standard soft background, described by the exchange of a Pomeron trajectory (P) with unit intercept, and an exchange-degenerate meson trajectory (M), with intercept $\alpha_{hp}<1$ ($h=\pi^\pm,K^\pm,p^\pm$). We thus have

$$\sigma_t^{hp}(s)=g_h^Mg_p^M(s/s_0)^{-1+\alpha_{hp}}+g_h^Pg_p^P+g_hg_p[\kappa(\kappa-1)a_h/2\pi]^{1/2}(\ln s/s_0)^{-(\kappa+1)/2}\exp[-a_h(\ln s/s_0)^{1-\kappa}]\ , \quad (3.1)$$

where

$$g_h^P=g_{\bar{h}}^P\ . \quad (3.2)$$

Formula (3.1) is fitted to six sets of data, for the processes $\pi^\pm p$, $K^\pm p$, and $p^\pm p$.[20] Cosmic-ray data[21] are also included in the pp case. Although g_h^M, g_h^P, and α_{hp} are used as free parameters in the fit, they simply fix the soft Regge-pole background under the long-rapidity-range hard contribution. The values obtained for these parameters are consistent with the results of previous analyses.[13] The crucial parameters for the present model are a_h and κ, which determine the shapes of the rising parts of the total hadron-scattering cross sections and correlate them with the shapes of the structure functions of hadrons as x moves away from $x=1$. On the other hand, the parameters g_h and Λ correlate the absolute magnitudes of the hadron structure functions with the corresponding total-cross-section rises. Finally, the energy scale is deter-

TABLE I. Least values of χ^2/DF, obtained in a simultaneous fit of the total-cross-section (Refs. 20 and 21) and structure-function (Refs. 22–24) ($0.3 < x < 0.9$) data to the model described in the text, with the corresponding values and errors of the parameters.

f^π Data	Newman (Ref. 23)	Badier 150 GeV/c (Ref. 24)	Badier 200 GeV/c (Ref. 24)
χ^2/DF	335/333	349/335	357/336
κ	1.326 ± 0.001	1.315 ± 0.001	1.331 ± 0.001
s_0 (GeV^2)	1.527 ± 0.002	1.199 ± 0.006	1.630 ± 0.008
a_p	28.14 ± 0.08	28.95 ± 0.07	27.61 ± 0.07
a_K	24.09 ± 0.06	24.48 ± 0.05	24.42 ± 0.06
a_π	23.72 ± 0.06	24.04 ± 0.05	24.07 ± 0.06
g_p (μb$^{1/2}$)	9603	13 298	9975
g_K (μb$^{1/2}$)	1567	1650	1627
g_π (μb$^{1/2}$)	1215	1630	1235
$\Lambda(p)$ (GeV)	0.14 ± 0.01	0.08 ± 0.01	0.17 ± 0.01
$\Lambda(K)$ (GeV)	0.45 ± 0.02	0.13 ± 0.01	0.22 ± 0.01
$\Lambda(\pi)$ (GeV)	0.43 ± 0.01	0.18 ± 0.01	0.19 ± 0.01

mined by the parameter s_0.

Neglecting sea-quark distributions for $x > 0.3$, where predictions (2.12), (2.16), and (2.18) are valid, simultaneously with (3.1) we fitted Eq. (2.12) to the nucleon structure-function data obtained in electroproduction experiments,[22] i.e.,

$$\nu W_2^{eN}(x,\bar{Q}^2)=\tfrac{5}{9}F^p(x)\ , \tag{3.3}$$

where we averaged these data over Q^2, Eq. (2.16) to the pion structure-function data[23,24] obtained in muon-pair production experiments, i.e.,

$$f^\pi(x)=\tfrac{1}{2}F^\pi(x)\ , \tag{3.4}$$

and the ratio of Eqs. (2.18) and (2.16) to the kaon-over-pion structure-function ratio measured in $\mu^+\mu^-$ production experiments.[24] Since the parameter Λ need not be process independent in lowest-order QCD,[14] we allowed three different Λ's in

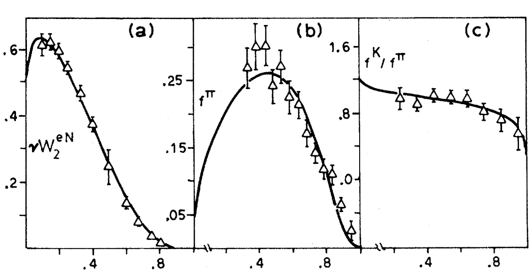

FIG. 2. Fit to selected structure-function data with the model described in the text. (a) Electroproduction data (Ref. 22) for the nucleon structure function $\nu W_2^{eN}(x)$. These data are averaged over Q^2. (b) Data (Ref. 23) for the structure function of the pion. (c) Data (Ref. 24) for the ratio of the structure function of the kaon and pion.

predictions (2.12), (2.16), and (2.18), in order to obtain an optimum fit to the data. To exclusively test our model, we consider π and K structure-function data points with $x < 0.9$, since very close to $x = 1$ higher-twist corrections can be appreciable.[25] Nonetheless, we obtain an equally satisfactory fit for all x with, e.g.,

$$f^\pi(x)=\tfrac{1}{2}F^\pi(x)+\tfrac{2}{9}\langle k_T{}^2\rangle/Q^2\ , \tag{3.5}$$

where $\frac{2}{9}\langle k_T{}^2\rangle/Q^2\approx0.05$ is a constant, which simulates higher-twist effects.[25] Since we wish to concentrate on the x dependence of the structure functions predicted by the present model and not on their Q^2 dependence, to avoid extra degrees of freedom, we do not include such a constant in the analysis.

The results of our best fit, performed by the standard CERN routine MINUIT, are summarized in Table I and Figs. 2 and 3. The quality of this fit is very good. Three different sets of data, obtained by Newman *et al.*[23] and by Badier *et al.*[24] have been used for the pion structure function. Our model gives best agreement with the data of Ref. 23. Note that these sets of data differ not only in absolute normalization (perhaps this could be accounted for by a change of Λ), but also in shape. We should stress that, since we have many more total-cross-section than structure-function data points, the fitted shapes of structure functions (Fig. 2) are almost exclusively (save Λ) determined, through the χ^2-minimization process, by the shapes of the corresponding total-cross-section rises (Fig. 3). In this sense, the curves of Fig. 2 should be considered predictions, rather than fits. The values of the parameters corresponding to minimum χ^2

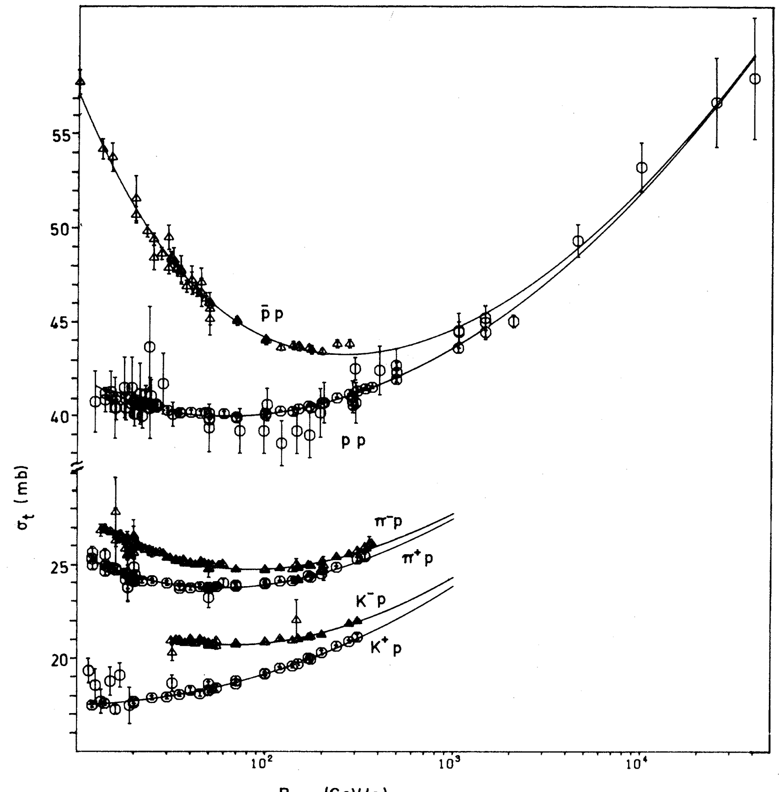

FIG. 3. Fit of the expressions (3.1) to the hadron- ($\pi^{\pm},K^{\pm},p^{\pm}$) proton total-cross-section data (Refs. 20 and 21).

are seen in Table I to be quite satisfactory. In particular, the parameters obtained in the proton case are consistent with the values obtained in previous fits, where either Q^2 dependence was allowed in the proton structure function,[4] or, besides the total cross section, the contributions of singularity (1.2) to the rise of the inclusive plateau (Feynman-scaling breaking), as well as to the steep rise of the average multiplicity and the integrated correlation function were taken into account.[8]

In view of the results on particle production at extremely high energy, which will soon be available, we have extrapolated our fits to the pp and $\bar{p}p$ total-cross-section data up to equivalent laboratory momentum $p_L \approx 1.5\times 10^5$ GeV/c, where we predict about 50% higher values than the corresponding ones in the Fermilab energy range. For example, the pp total cross section is predicted to be as large as $\sim$65 mb at $\sqrt{s} = 540$ GeV. On the other hand, for $p_L \gtrsim 10^3$ GeV/c the antiparticle-particle cross sections are predicted to differ by less than $\sim$3%.

IV. CONCLUSIONS

We studied the consequences of the hypothesis that multijet production at extremely high energies, generated by iteration of the standard hard-scattering mechanism (Fig. 1), is accompanied by low-p_T hadron production with KNO scaling. Although high-p_T partons are involved in this study, since we only consider inclusive low-p_T hadron production, S-matrix techniques, adopted for multiperipheral models with longitudinal phase space, are appropriate. In our particular model, in which a reasonable amount of analyticity is implicit, together with the assumption that the joint distribution of quarks a,b with momentum fractions x_a,x_b in hadron h has the form $G_a^h(x_a)G_b^h(x_b)$, the hypothesis stated above implies a branch-point singularity at $J=1$, in the angular-momentum plane. This singularity is responsible for long-rapidity-range phenomena in hadron processes and, in particular, for a rising component in the total hadron-scattering cross section. The same singularity manifests itself in hadron structure functions, at the point $x=1$. We are thus led to a particular correlation between the energy dependence of the total hadron-scattering cross-section rises and the x dependence of the structure functions of hadrons. Our analysis shows that this correlation is in remarkable agreement with both categories of data (Figs. 2 and 3). The Q^2 dependence of structure functions, required by QCD, can easily be incorporated in the model. In the case of the structure function of the nucleon, it has been checked[4] that this incorporation leads to good agreement with the neutrino-production data. Finally, we should stress that, quite independent of our particular structure-function model, this work shows that hard quark subcollisions, in the QCD framework, with quantitatively correct structure functions, can saturate the rises of all πp, Kp, and pp total cross sections.

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APPENDIX

In this appendix, we find an asymptotic expansion for an inverse Mellin transform of the form

$$F(Y)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} p^{\alpha}\exp(pY-bp^{1-\eta})dp\ ,$$

$$\alpha\geq 0\ ,\ b>0\ ,\ 1>\eta>0\ , \qquad \text{(A1)}$$

which is useful in the limit $Y\equiv \ln 1/x\rightarrow 0$. Following the standard stationary-phase technique,[19] we expand p^{α} as well as the function

$$pY-bp^{1-\eta}\equiv f(p) \qquad \text{(A2)}$$

in Taylor series around the point of stationary phase of the exponential, given by

$$p_0=[(1-\eta)b/Y]^{\kappa}\ ,\ \ \kappa=1/\eta\ , \qquad \text{(A3)}$$

to obtain

$$F(Y)=\frac{p_0^{\alpha}e^{f(p_0)}}{2\pi}\int_{-\infty}^{+\infty}dx\left[1+i\alpha\frac{x}{p_0}-\frac{\alpha(\alpha-1)}{2!}\frac{x^2}{{p_0}^2}+\cdots\right]\exp\left[-\tfrac{1}{2}f''(p_0)x^2-\frac{i}{3!}f^{(3)}(p_0)x^3+\cdots\right]\ , \qquad \text{(A4)}$$

where we have chosen an integration contour passing through the point p_0. Now, throughout the region where the quadratic term in the exponential of (A4) is appreciable, defined by

$$|\tfrac{1}{2}f''(p_0)x^2|<E\ ,\ \ E>0\ , \qquad \text{(A5)}$$

the cubic term is very small, for $p_0\rightarrow\infty$, namely,

$$\left|\frac{1}{3!}f^{(3)}(p_0)x^3\right|<\frac{(2E)^{3/2}}{3!}\frac{1+\eta}{[\eta(1-\eta)b]^{1/2}}\frac{1}{p_o^{(1-\eta)/2}} \qquad \text{(A6)}$$

and we set

$$\exp\left[-\frac{i}{3!}f^{(3)}(p_0)x^3\right]=1-\frac{i}{3!}f^{(3)}(p_0)x^3 \qquad \text{(A7)}$$

to find

$$F(Y)=\frac{p_0^{\alpha}e^{f(p_0)}}{[2\pi f''(p_0)]^{1/2}}\left[1-\frac{\alpha(\alpha+\eta)}{2\eta(1-\eta)b}\frac{1}{{p_0}^{1-\eta}}+O\left[\frac{1}{{p_0}^{2-2\eta}}\right]\right]\ , \qquad \text{(A8)}$$

whence (2.13) and (1.3) follow.

The integral (A1) can be transformed to an integral of a real function along the positive real axis, namely,

$$F(Y)=\frac{1}{\pi}\int_0^{\infty}p^{\alpha}\sin(\gamma p^{1-\eta}-\pi\alpha)\times\exp(-pY-\beta p^{1-\eta})dp\ , \qquad \text{(A9)}$$

where

$$\beta=b\cos(1-\eta)\pi\ ,$$
$$\gamma=b\sin(1-\eta)\pi\ . \qquad \text{(A10)}$$

Calculating (A9) numerically, we checked that the first two (one, for $\alpha=0$) terms of the asymptotic expansion of $F(Y)$ in powers of ${p_0}^{-1+\eta}$, given in (A8), are sufficient to give better than 5% (1%, for $\alpha=0$) accuracy for essentially all Y.

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