Compton scattering for circularly polarized light

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Angular momentum coupling theory indicates that some generalizations of the Klein-Nishina cross section for Compton scattering to circularly polarized light are invalid. A corrected expression is given. The analysis illustrates the extent to which a Feynman diagram may be interpreted naively as an angular momentum coupling diagram.

Standard works[1–4] give the differential scattering cross section for Compton scattering from unpolarized stationary electrons (Fig. 1) in the form

$$\frac{d\sigma}{d\Omega}=\left[\frac{e^2}{2mc^2}\right]^2\left[\frac{\omega_2}{\omega_1}\right]^2 \times\left[\frac{\omega_1}{\omega_2}+\frac{\omega_2}{\omega_1}-2+4\,|\epsilon_1\cdot\epsilon_2^*|^2\right]. \quad (1)$$

Here $\hbar\omega_1$ and $\hbar\omega_2$ are the energies of incoming and outgoing photons. Their respective polarization four-vectors are written as $\epsilon_i=(0,\vec{e}_i)$, $i=1,2$.

Other standard works explicitly assume linearly polarized beams and omit the asterisk in Eq. (1), which is then correct in this limit. The works cited above, however, imply incorrectly that Eq. (1) is the appropriate generalization to circularly polarized beams. Feynman[2] gives a detailed proof of Eq. (1), which contains a step

$$\not{\epsilon}\not{\epsilon}^*=-1 \quad (2)$$

which is not valid for elliptically polarized photons. The correct expression, using Feynman's method but avoiding Eq. (2), is

$$\frac{d\sigma}{d\Omega}=\left[\frac{e^2}{2mc^2}\right]^2\left[\frac{\omega_2}{\omega_1}\right]^2\left[\left[\frac{\omega_1}{\omega_2}+\frac{\omega_2}{\omega_1}\right](1+|\epsilon_1\cdot\epsilon_2^*|^2-|\epsilon_1\cdot\epsilon_2|^2)+2(|\epsilon_1\cdot\epsilon_2|^2+|\epsilon_1\cdot\epsilon_2^*|^2-1)\right]. \quad (3)$$

This equation seems to be new. It alters only that element M_{33} of the Mueller matrix corresponding to circularly polarized incoming and outgoing beams. Equation (3), but not Eq. (1), agrees with the result of Fano[5]:

$$M_{33}=\left[\frac{e^2}{2mc^2}\right]^2\left[\frac{\omega_2}{\omega_1}\right]^2\left[\frac{\omega_1}{\omega_2}+\frac{\omega_2}{\omega_1}\right]2\cos\theta\,. \quad (4)$$

Rose[6] and Berestetskii *et al.*[7] give a very different formulation of the polarization dependence of Compton scattering, which we have not analyzed; the simplicity of Eq. (3) makes it a superior formulation.

We found this correction by noting that Eq. (1) is incompatible with naive angular momentum theory, applied in analogy with many-body theory[8] to the diagrams of Fig. 1(a). These have been obtained by combining one of the QED diagrams for the amplitude of Compton scattering with a similar diagram for the complex-conjugate amplitude, with a summation over electron polarization implied as for the total cross section.

Biritz[9] has developed a useful diagram formalism. If j_1, j_2 label the finite irreducible representations (irreps) of the Lorentz group [locally $SO(3)\times SO(3)$], the coupling symmetry j in $j_1\times j_2$ may be identified with the intrinsic spin of the Dirac particle if the spinor is referred to the rest frame. Hence, to the extent that the finite final speed of the electron does not affect the algebraic structure of the cross-section formula, the angular momentum content of the QED diagrams is given by assigning to each propagator the intrinsic spin of the particle [Fig. 1(b)]. Simple symmetry arguments [e.g., Figs. 1(d) and 1(e)] help to reduce this

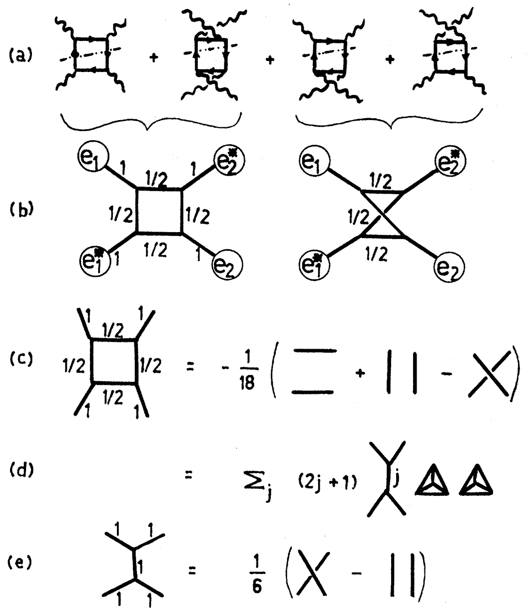

FIG. 1. (a) Diagrams for Compton scattering. (b) Corresponding angular momentum coupling diagrams. (c) Analysis of angular momentum diagram. (d) and (e) Proof of (c).

expression to Fig. 1(c). Hence the cross section should depend on mutual scalar products of the polarization vectors in the following combinations:

$$\begin{aligned}&(\vec{e}_1^{\,*}\cdot\vec{e}_1)(\vec{e}_2^{\,*}\cdot\vec{e}_2)+|\vec{e}_1^{\,*}\cdot\vec{e}_2|^2-|\vec{e}_1\cdot\vec{e}_2|^2\,,\\ &|\vec{e}_1\cdot\vec{e}_2|^2+|\vec{e}_1^{\,*}\cdot\vec{e}_2|^2-(\vec{e}_1^{\,*}\cdot\vec{e}_1)(\vec{e}_2^{\,*}\cdot\vec{e}_2)\,,\end{aligned} \tag{5}$$

corresponding to the two topologies of Fig. 1(b). This agrees with Eq. (3), but not Eq. (1) (note that the two topologies have differing energy dependence, and $\vec{e}_i\cdot\vec{e}_i^{\,*}=1$). We note that the right side of Fig. 1(c) appears in the well-known trace rule for the trace of the product of four Dirac γ^μ matrices, and has been written in this form recently by Kennedy.[10]

Photon scattering from a scalar particle in the Klein-Gordon theory may be analyzed similarly (Fig. 2). It is then a trivial consequence of angular

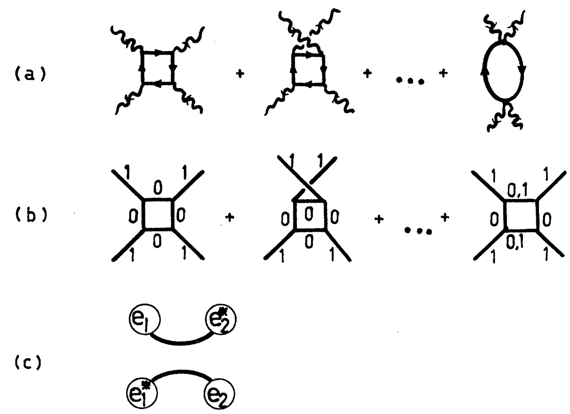

FIG. 2. Photon scattering from Klein-Gordon particles. (a) Feynman diagrams. (b) Angular momentum coupling diagrams. (c) Polarization dependence.

momentum conservation that the linear coupling cannot induce scattering. (Of course, this is no longer true when the initial particle is not at rest.[11]) The quadratic coupling on the same analysis will permit scalar products within each amplitude only, in agreement with the combination $|\vec{e}_1\cdot\vec{e}_2^{\,*}|^2$ appearing in the standard Thomson cross section (Ref. 12).

As in this example, the naive method of associating intrinsic spins with propagators fails when scalar products between polarization vectors and particle three-momenta appear in the cross section. These will not appear if the particle is stationary, if the particle is a photon corresponding to the polarization vector in question, or even for a different photon unless the gauge dependence introduced by such terms is countered by another term in the cross section. The angular momentum approach used here succeeded since the gauge-dependent terms have canceled from Eq. (3).

Insofar as only mutual scalar products of polarization vectors appear in a cross section, this method may give insight into such problems as the double Compton effect and light-light scattering.[13]

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[13]Reference 4, pp. 235 and 287.