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Numerical studies of Wilson loops in SU(3) gauge theory in four dimensions

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Monte Carlo simulations are used to calculate Wilson loops for pure SU(3) gauge theory on a 6^4 lattice. Previous measurements of the scale parameter Λ_0 are improved.

The gauge group SU(3) has been examined in several recent Monte Carlo studies.[1-3] In Ref. 1 for instance most of the data were generated on 4^4 lattices with one data point generated on a 6^4 lattice. Since SU(3) is the gauge group of quantum chromodynamics (QCD), it is reasonable to improve our data sample and hence make a more accurate determination of the Λ_0 scale parameter. In the present paper, we report Monte Carlo simulations on a 6^4 lattice at 57 values of the inverse temperature and determine all Wilson loops up to size 3×3.

We work in a hypercubical lattice in four Euclidean dimensions.[4,5] On the link $\{ij\}$ joining nearest-neighbor lattice sites signified by i and j sits an $N\times N$ unitary-unimodular matrix U_{ij} of the group SU(N), with the condition that

$$U_{ji}=(U_{ij})^{-1}\ .$$

We define our partition function by

$$Z(\beta)=\int\left[\prod_{\{i,j\}}dU_{ij}\right]\exp(-\beta S[U])\ ,$$

where β is the inverse temperature given by $\beta=2N/g_0^2$ with g_0 the bare coupling constant. The measure in the above integral is the SU(N) normalized invariant Haar measure. The action S is defined as the sum over all unoriented plaquettes $\square$ such that

$$S[U]=\sum_{\square}S_{\square}=\sum_{\square}\left[1-\frac{1}{N}\mathrm{Re\,Tr}\,U_{\square}\right]\ .$$

Here $U_{\square}$ is the parallel transporter around a plaquette. Periodic boundary conditions were used throughout our calculations and the lattice was put in equilibrium by the method of Metropolis *et al.*[6] From now on we specialize to $N=3$.

We define the rectangular Wilson loops[7] by the expectation value

$$W(I,J)=\tfrac{1}{3}\langle\mathrm{Re\,Tr}\,U_C\rangle\ ,$$

where the I by J closed rectangular contour is denoted by C and U_C is the parallel transporter or product of link variables around C. The leading-order high-temperature expansion for the Wilson loop is

$$W(I,J)=(\beta/18)^{IJ}\ , \tag{1}$$

while the leading-order low-temperature expansion

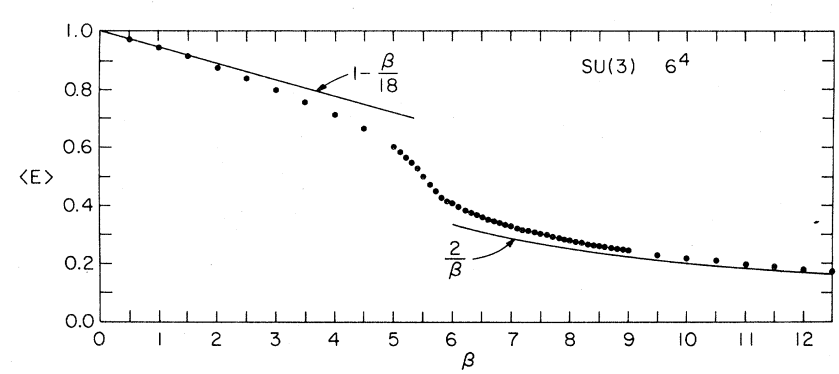

FIG. 1. The average action per plaquette $\langle E\rangle$ for pure SU(3) gauge theory on a 6^4 lattice as a function of the inverse temperature β. The curves represent the leading-order high- and low-temperature expansions of Eqs. (1) and (2), respectively.

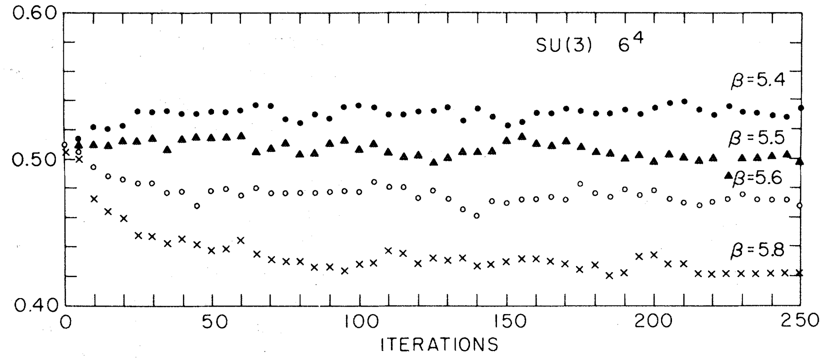

FIG. 2. The evolution of the average action per plaquette $\langle E\rangle$ for pure SU(3) gauge theory on a 6^4 lattice as a function of the number of iterations through the lattice for mixed-phase starting lattices for various values of the inverse temperature β.

for the average action per plaquette is

$$\langle E\rangle = 1 - W(1,1) = 2/\beta + O(\beta^{-2}) \ . \quad (2)$$

For asymptotically large Wilson loops we expect

$$W \sim \exp(-A - K \times \text{area} - C \times \text{perimeter}) \ ,$$

where for a given β, A, K, and C are constants. When the asymptotic behavior applies, we extract the string tension K by evaluating the quantity

$$\chi(I,J) = -\ln\left[\frac{W(I,J)\,W(I-1,J-1)}{W(I,J-1)\,W(I-1,J)}\right] \ .$$

The leading-order high-temperature expansion for the string tension is given by

$$\chi(I,J) = -\ln(\beta/18) + O(\beta^2) \ . \quad (3)$$

Asymptotic freedom determines how the lattice spacing varies with bare coupling for a continuum limit. This introduces a scale parameter Λ_0 defined by

$$\Lambda_0 = \lim_{a\to 0} \frac{1}{a}[\gamma_0 g_0^2(a)]^{(-\gamma_1/2\gamma_0^2)} \exp\left[\frac{1}{2\gamma_0 g_0^2(a)}\right] \ , \quad (4)$$

where, for SU(3), we have

$$\gamma_0 = \frac{11}{16\pi^2} \quad \text{and} \quad \gamma_1 = \frac{51}{128\pi^4} \ ,$$

and a is the lattice spacing.

In Fig. 1 we show the average action per plaquette $\langle E\rangle$ as a function of the inverse temperature on a 6^4 lattice. In carrying out these calculations, we first performed 200 iterations through the 6^4 lattice with 20 Monte Carlo updates per link. This resulted in the space-time lattice being in equilibrium. We then averaged over the next 100 iterations through the lattice. We used disordered starting lattices for $\beta \leqq 5.5$, mixed-phase[5] starting lattices for $5.5 < \beta < 9.0$, and ordered starting lattices for $\beta > 9.0$. Our results in Fig. 1 agree well with the leading-order high- and low-temperature expansions of Eqs. (1) and (2),

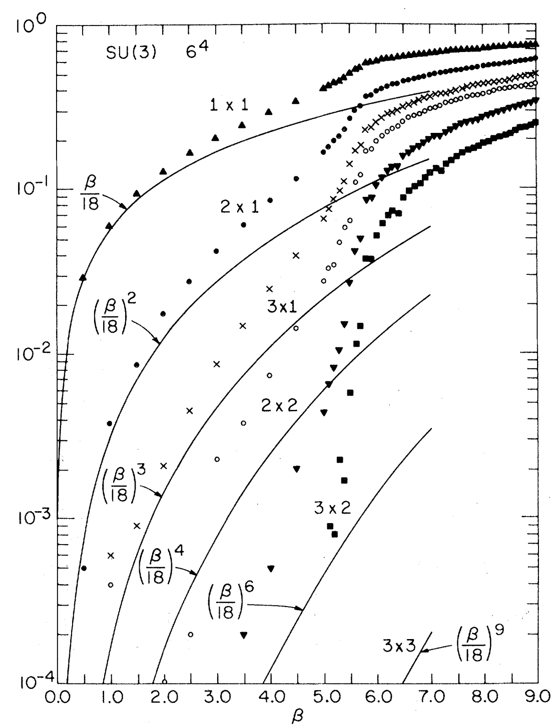

FIG. 3. The Wilson loops $W(I,J)$ for pure SU(3) gauge therory on a 6^4 lattice as a function of the inverse temperature β. The upward triangles represent $I=J=1$, the solid circles represent $I=2$, $J=1$, the crosses represent $I=J=2$, the downward triangles represent $I=3$, $J=2$, and the squares represent $I=J=3$. The curves represent the leading-order high-temperature expansion of Eq. (1).

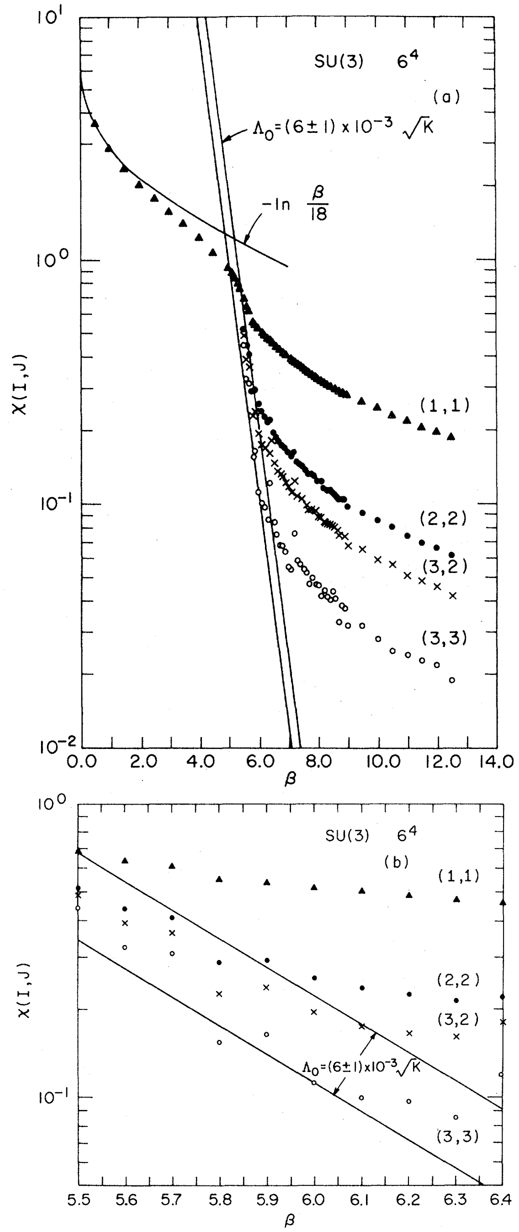

FIG. 4. The string tension $\chi(I,J)$ for pure SU(3) gauge theory on a 6^4 lattice as a function of the inverse temperature β. The triangles represent $I=J=1$, the solid circles represent $I=J=2$, the crosses represent $I=3$, $J=2$, and the open circles represent $I=J=3$. Also shown in the diagram is the leading-order high-temperature expansion of Eq. (3).

respectively. Figure 2 shows some of the mixed-phase runs for the average action per plaquette in the vicinity of the crossover between the high- and low-temperature regions. In Fig. 3 we show the Wilson loops up to size 3×3. The leading-order high-temperature expansions are also shown for comparison.

The logarithmic ratios $\chi(I,J)$ for $(I,J)=(1,1)$, (2,2), (3,2), and (3,3) are shown as a function of the inverse temperature β in Fig. 4(a). Our results agree with the leading-order high-temperature expansion of Eq. (3) up to $\beta\simeq1.0$. Obviously, higher-order terms are needed to bring about agreement with the Monte Carlo data in a larger range in β.

In the figure we show a band corresponding to the behavior of Eq. (4) with

$$\Lambda_0=(6\pm1)\times10^{-3}\sqrt{K}\ .$$

As in our previous analysis, the error is a subjective estimate. Putting in the Hasenfratz-Hasenfratz[8] factor relating Λ_0 to the parameter Λ^{MOM} characterizing the momentum-space three-point vertex in the Feynman gauge

$$\Lambda^{\text{MOM}}/\Lambda_0=83.5\ ,$$

we obtain

$$\Lambda^{\text{MOM}}=(0.5\pm0.1)\sqrt{K}\ .$$

This represents about 200 MeV if we use the Regge slope to estimate K.

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