Supersymmetry breaking in a magnetic field

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The one-loop effective potential of an Abelian supersymmetric model in an environment provided by a constant external magnetic field is derived. It is shown that the magnetic field breaks supersymmetry and that the value of the resulting minimum potential is lower than that of the tree level. This could be relevant to the question of possible restoration of the symmetry at higher loops.

I. INTRODUCTION

Mechanisms for breaking supersymmetry spontaneously have been proposed by Fayet and Iliopoulos[1] who studied an Abelian supersymmetric model and by Slavnov[2] who applied his mechanism to a non-Abelian model and succeeded in giving different masses to all the particles involved. It is well known that symmetries which are spontaneously broken may make transitions from the broken phase to the unbroken one either in a high-temperature environment[3] or in an environment provided by a strong external magnetic field.[4] To investigate whether such transitions also occur in temperature-dependent supersymmetric systems, Das and Kaku[5] considered the Fayet-Iliopoulos model in such an environment and showed that once supersymmetry is spontaneously broken it cannot be restored at high temperatures at least in the one-loop approximation. Girardello, Grisaru, and Salomonson[6] later showed that the result of Das and Kaku was true not only at the one-loop approximation but to all orders, that is, finite temperature automatically breaks supersymmetry since it describes excitations about a ground state which is a statistical ensemble and such an ensemble treats bosons and fermions differently by means of Bose-Einstein and Fermi-Dirac distributions, respectively.

In this paper we shall consider the effect of an external magnetic field on a supersymmetric system. It is not immediately obvious to us how the method of Girardello *et al.* can be applied here and so we shall follow the approach of Das and Kaku by computing the one-loop quantum corrections to the effective potential for systems placed in a magnetic field $\vec{H}$. Salam and Strathdee[4] and, more recently, Midorikawa[7] and Shore[8] studied such problems in ordinary symmetries and it is our purpose to do the same for supersymmetry.

In Sec. II we shall discuss the Abelian supersymmetric model of Fayet.[9] The one-loop quantum corrections to the effective potential, including the term proportional to H^2, are computed in Sec. III. Section IV summarizes our conclusions.

II. FAYET'S U (1) MODEL

Let V and Φ_- denote the real Abelian superfield and the left-handed matter superfield, respectively. The component fields of V are C, χ, M, N, A_μ, λ, and D, where C, M,N, and D are (pseudo)scalars, χ and λ are Majorana spinors, and A_μ is a photon field. The component fields of Φ_- are represented by fields A_-, ψ_- and F_-.[10] Using the Wess-Zumino gauge some of the components of V vanish and the superfield can be expressed in the form

$$V(x,\theta)=\tfrac{1}{4}\bar{\theta}i\gamma_\mu\gamma_5\theta A_\mu(x)+\frac{1}{2\sqrt{2}}\bar{\theta}\theta\bar{\theta}\gamma_5\lambda(x)+\tfrac{1}{16}(\bar{\theta}\theta)^2D(x)\ . \tag{2.1}$$

In this gauge the superfield strength is then given by

$$W_{++}=\exp(\tfrac{1}{2}\bar{\theta}_+i\partial\theta_+)\left[\lambda_++\frac{i}{\sqrt{2}}(D+\tfrac{1}{2}\sigma_{\mu\nu}F_{\mu\nu})\theta_++\tfrac{1}{2}\bar{\theta}_-\theta_+(-i\partial\lambda_-)\right] \tag{2.2}$$

$F_{\mu\nu}$ being the electromagnetic field strength.

The most general Lagrangian density compatible with supersymmetry and gauge invariance of the action is given by

$$\mathscr{L}=\tfrac{1}{4}\int d^2\theta_+\overline{W}_{--}W_{++}+\text{H.c.}+\int d^2\theta_+d^2\theta_-[\Phi_-^+\exp(2eV)\Phi_-+\xi V]\ , \tag{2.3}$$

where $W_{--}=C\overline{W}_{++}$, ξ is a parity-violating parameter, and e, the coupling constant, will be assumed positive. In the Wess-Zumino gauge this reduces to

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{i}{2}\bar{\lambda}\partial\lambda+\tfrac{1}{2}D^2+i\bar{\psi}_-\nabla\psi_-+(\nabla_\mu A_-)^\dagger(\nabla^\mu A_-)$$
$$+F_-^\dagger F_--ie\sqrt{2}(\bar{\psi}_-\lambda A_--A_-^\dagger\bar{\lambda}\psi_-)+eDA_-^\dagger A_-+\xi D\ , \tag{2.4}$$

where ∇_μ is the covariant derivative $\nabla_\mu\psi_-=(\partial_\mu-ieA_\mu)\psi_-$. The classical potential of the system is thus given by

$$V_{\text{cl}}=F_-^\dagger F_-+\tfrac{1}{2}D^2\ , \tag{2.5}$$

where the auxiliary fields F_- and D are eliminated from the above Lagrangian by means of the equations of motion

$$F_-=0,\quad D+eA_-^\dagger A_-+\xi=0\ . \tag{2.6}$$

On substituting these values into (2.4) and (2.5), the Lagrangian and the potential reduce to

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{i}{2}\bar{\lambda}\partial\lambda+i\bar{\psi}_-\nabla\psi_-+(\nabla_\mu\phi)^\dagger(\nabla^\mu\phi)-ie\sqrt{2}(\bar{\psi}_-\lambda\phi-\phi^\dagger\bar{\lambda}\psi_-)-\tfrac{1}{2}(\xi+e\phi^\dagger\phi)^2\ , \tag{2.7}$$

$$V_{\text{cl}}=\tfrac{1}{2}(\xi+e\phi^\dagger\phi)^2\ , \tag{2.8}$$

where, instead of A_-, we have used the usual notation ϕ for the complex scalar field. This is the supersymmetric Higgs model proposed by Fayet.[9] Now from (2.8) we can draw a number of conclusions. If $\xi>0$, then the minimum for the potential occurs when the scalar field ϕ has zero expectation value; gauge invariance is conserved but supersymmetry is spontaneously broken. The particle spectrum is found to be a vector A_μ, two left-handed Dirac spinors ψ_-,λ_- all massless, and a complex scalar ϕ of mass $(\xi e)^{1/2}$. If $\xi<0$, then the minimum for the potential occurs when $\langle\phi^2\rangle=-\xi/e$; supersymmetry is conserved while gauge invariance is broken. Thus the symmetry breaking depends on the sign of the parameter ξ.

III. SUPERSYMMETRY AND MAGNETIC FIELD

The constant external magnetic field is introduced by adding to the classical action the external source term $A_\mu J_\mu^{\text{ext}}$, where $A_\mu^{\text{ext}}=-\tfrac{1}{2}F_{\mu\nu}x^\nu$ by an appropriate choice of gauge. The potential at the tree approximation therefore remains the same so that for $\xi>0$ supersymmetry remains broken. Also for $\xi<0$, we note from the supersymmetry transformations[5] that

$$\langle\delta\lambda_\pm\rangle=\sigma_3H\epsilon_\pm\neq 0 \tag{3.1}$$

for $H\neq 0$, ϵ being a constant Majorana spinor parameter. Therefore, supersymmetry is now broken in this case also.

To consider the effect of quantum corrections on the system we have to evaluate at least the one-loop contributions to the effective potential. Following Dolan and Jackiw,[3] let us define a new field ϕ' such that $\phi'=\phi-\phi_0$ with $\langle\phi'\rangle=0$ and ϕ_0 assumed real. Expressing ϕ' in terms of its real and imaginary parts, $\phi'=(1/\sqrt{2})(\phi_1'+i\phi_2')$, and substituting it in Eq. (2.7) we can write the relevant contributions to the shifted Lagrangian as

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}M_v{}^2A_\mu{}^2+\frac{i}{2}\bar{\lambda}\partial\lambda+i\bar{\psi}_-\partial\psi_--im_f(\bar{\psi}_-\lambda-\bar{\lambda}\psi_-)+\tfrac{1}{2}(\partial_\mu\phi_1')^2-\tfrac{1}{2}M_1{}^2\phi_1'^2$$
$$+\tfrac{1}{2}(\partial_\mu\phi_2')^2-\tfrac{1}{2}M_2{}^2\phi_2'^2-e\sqrt{2}\phi_0A_\mu\partial^\mu\phi_2'-\tfrac{1}{2}\alpha^{-1}(\partial_\mu A^\mu)^2+\mathscr{L}_{\text{int}}$$
$$=\mathscr{L}_0+\mathscr{L}_{\text{int}}\ , \tag{3.2}$$

where a gauge-fixing term has been added and

$$\mathscr{L}_{\rm int}=eA_\mu\phi_2'\overleftrightarrow{\partial}^\mu\phi_1'+\tfrac{1}{2}e^2A_\mu{}^2(\phi_1'^2+\phi_2'^2)+e^2\sqrt{2}\phi_0\phi_1'A_\mu{}^2+e\bar{\psi}_-A\!\!\!/\psi_-+\text{terms not involving } A_\mu\ , \tag{3.3}$$

$$M_v{}^2=m_f{}^2=2e^2\phi_0{}^2\ ,\quad M_1{}^2=e\xi+3e^2\phi_0{}^2\ ,\quad M_2{}^2=e\xi+e^2\phi_0{}^2\ . \tag{3.4}$$

On introducing the constant external field the effective action becomes a function of $A_\mu^{\rm ext}$ and ϕ_0 and the one-loop contribution can be expanded around $A_\mu^{\rm ext}=0$ for fixed ϕ_0 to give

$$S_1(A,\phi)=S_1(0,\phi)+\tfrac{1}{2}\int d^4x\,d_4x'\left.\frac{\delta^2S_1(A,\phi)}{\delta A_\mu(x)\delta A_\nu(x')}\right|_{A=0}A_\mu(x)A_\nu(x')+\cdots \tag{3.5}$$

with the linear term vanishing because of translation invariance. The first term in Eq. (3.5), $S_1(0,\phi)$, is independent of the magnetic field and represents the radiative corrections, while the second term is the correction due to the presence of the external electromagnetic field. We compute these two terms separately.

Consider then the first term. The corresponding one-loop contribution to the effective potential is given by

$$V_0=i\ln\int[dA_\mu][d\phi][d\bar{\psi}_-][d\psi_-][d\bar{\lambda}_+][d\lambda_+]\exp\left[i\int d^4x\,\mathscr{L}_0\right]$$

$$=-\frac{i}{2}\ln\det\Delta-\frac{i}{2}\ln\det\Delta_{11}+\frac{i}{2}\ln\det S\ , \tag{3.6}$$

where Δ_{11} is the propagator for the ϕ_1 field, S the propagator for the fermion field $\chi(=\psi_-+\lambda_+)$, and Δ is the 5×5 matrix given by

$$\Delta=\begin{bmatrix}\Delta_{\mu\nu} & \Delta_{\mu2}\\ \Delta_{2\nu} & \Delta_{22}\end{bmatrix} \tag{3.7}$$

with the various propagators given in momentum space to be

$$i\Delta_{\mu\nu}=\left\{-g_{\mu\nu}+\frac{p_\mu p_\nu[(\alpha-1)(-p^2-M_2{}^2)+\alpha M_v{}^2]}{p^2(p^2-M_2{}^2)+\alpha M_v{}^2M_2{}^2+i\epsilon}\right\}\frac{i}{p^2-M_v{}^2+i\epsilon}\ ,$$

$$i\Delta_{22}=\frac{i(p^2-\alpha M_v{}^2)}{p^2(p^2-M_2{}^2)+\alpha M_2{}^2M_v{}^2+i\epsilon},\quad i\Delta_{\mu2}=-\frac{\alpha\sqrt{2}\phi_0p_\mu}{p^2(p^2-M_2{}^2)+\alpha M_2{}^2M_v{}^2}=-i\Delta_{2\mu}\ , \tag{3.8}$$

$$i\Delta_{11}=\frac{i}{p^2-M_1{}^2+i\epsilon},\quad iS=\frac{i}{p\!\!\!/-m_f+i\epsilon}\ .$$

Substituting these in Eq. (3.6) yields the result

$$V_0=-i\int\frac{d^4K}{(2\pi)^4}[\tfrac{3}{2}\ln(-K^2+M_v{}^2)+\tfrac{1}{2}\ln(-K^2+\beta_1{}^2)+\tfrac{1}{2}\ln(-K^2+\beta_2{}^2)$$

$$+\tfrac{1}{2}\ln(-K^2+M_1{}^2)-2\ln(-K^2+m_f{}^2)]\ , \tag{3.9}$$

where $\beta_1{}^2$ and $\beta_2{}^2$ are the roots of the equation

$$x^2-xM_2{}^2+\alpha M_2{}^2M_v{}^2=0\ . \tag{3.10}$$

The integrals are evaluated using dimensional regularization to give

$$V_0=\frac{1}{64\pi^2}[-M_v{}^4\ln(M_v{}^2/\Lambda^2)+\beta_1{}^4\ln(\beta_1{}^2/\Lambda^2)+\beta_2{}^4\ln(\beta_1{}^2/\Lambda^2)+M_1{}^4\ln(M_1{}^2/\Lambda^2)]\ , \tag{3.11}$$

where we have used the mass relation $M_v{}^2 = m_f{}^2$ and have introduced the arbitrary renormalization mass Λ. Equation (3.11) represents the radiative correction to the effective potential.

The term corresponding to the second term in Eq. (3.5) is best computed using Feynman diagrams. The graphs which will contribute in an arbitrary gauge are as shown in Fig. 1 where the external lines carry zero momentum. The notations for the internal lines are summarized in Fig. 2. Since only gauge-invariant results are required not all the diagrams contribute. We shall also simplify the calculation of this contribution by working in the Landau gauge; we thus need compute only the first three diagrams of Fig. 1 using the propagators in Eq. (3.8). After some straightforward computation, we find the magnetic correction term to the effective potential to be

$$V_1 = -\frac{e^2H^2}{96\pi^2}[\ln(M_1{}^2/\Lambda^2)+\ln(M_2{}^2/\Lambda^2)+2\ln(M_v{}^2/\Lambda^2)+\tfrac{3}{2}F(M_v{}^2/M_2{}^2)] , \tag{3.12}$$

where

$$F(a)=\tfrac{2}{3}\int_0^1 dy\frac{y^3}{a+y}+\tfrac{1}{3}\int_0^1 dy\frac{(1-2y)^3}{a^{-1}+y} . \tag{3.13}$$

Combining Eqs. (3.11) and (3.12) with the classical potential (2.8), we thus obtain the effective potential of the system up to the one-loop level in the presence of the constant external magnetic field to be

$$V_{\text{eff}}=\tfrac{1}{2}(\xi+e\phi^2)^2+\frac{1}{64\pi^2}[(M_1{}^4-\tfrac{2}{3}e^2H^2)\ln(M_1{}^2/\Lambda^2)+(M_2{}^4-\tfrac{2}{3}e^2H^2)\ln(M_2{}^2/\Lambda^2)$$
$$-(M_v{}^4+\tfrac{4}{3}e^2H^2)\ln(M_v{}^2/\Lambda^2)-e^2H^2F(M_v{}^2/M_2{}^2)] . \tag{3.14}$$

The question of symmetry restoration can in principle be considered by first finding the turning points of the potential which should occur when

$$\frac{\partial V_{\text{eff}}}{\partial\phi}=2\phi\frac{\partial V_{\text{eff}}}{\partial(\phi^2)}=0 . \tag{3.15}$$

The resulting equation, however, is nonlinear and can only be solved using numerical techniques. Fortunately, for our purposes such a detailed analysis of the equation may be avoided since qualitative features of V_{eff} should be sufficient to allow us to draw the necessary conclusions.

As before we consider the two cases $\xi<0$ and $\xi>0$. Now if $\xi<0$, then ϕ_0 is nonzero and M_2 vanishes. While V_{cl} and V_0 vanish, V_1 does not so that the magnetic field does break supersymmetry which is conserved at the tree level. The $\xi>0$ case is perhaps of more interest. Choosing the arbitrary parameter Λ^2 to be $e\xi$, we sketch in the same graph the three functions V_{cl}, V_0, and V_1 (see Fig. 3). From this graph, it is seen that for ϕ small, V_1 dominates, while for ϕ large it is V_0 which dominates. Thus the effective potential V_{eff} has a minimum value which is smaller than the classical one. We can estimate the value $\phi_{\min}{}^2$ at which this occurs by considering small values of ϕ represent-

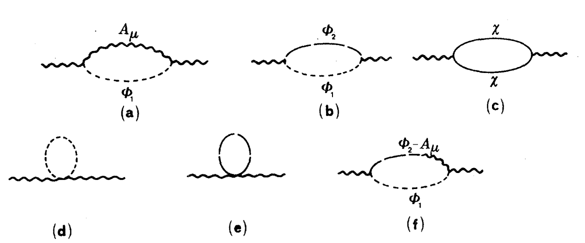

FIG. 1. The relevant one-loop diagrams in an arbitrary gauge. The external lines carry zero momentum.

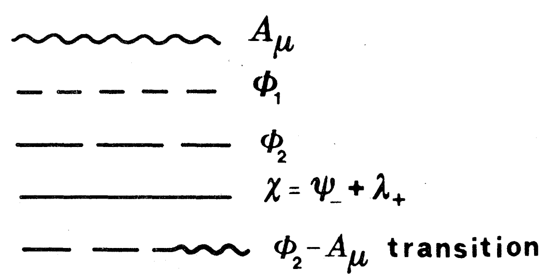

FIG. 2. Notations for the internal lines of Fig. 1.

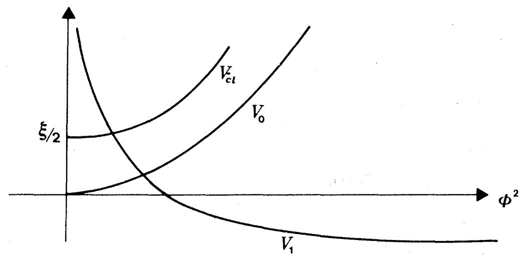

FIG. 3. Sketches of the potentials V_{cl}, V_0, and V_1 as a function of ϕ^2.

ing one-loop quantum correction to the classical value $\langle \phi \rangle = 0$. Therefore, expanding V_{eff} around $\phi=0$ and retaining terms up to ϕ^2, we get

$$\phi_{\min}{}^2 \sim \frac{1}{48\pi^2}\frac{eH^2}{\xi} . \tag{3.16}$$

Now as H increases, V_{eff} decreases and the potential may cross the ϕ^2 axis. The value of V_{eff} at that point is lower than at the classical minimum; this may indicate a tendency towards symmetry restoration although for a more definitive statement to be made one must solve Eq. (3.14) exactly or consider the vacuum expectation values, up to one loop, of $\delta\psi_-$ and $\delta\lambda$. It is nonetheless reasonable to conclude from this qualitative analysis that magnetic field breaks supersymmetry and that once broken supersymmetry remains broken, irrespective of the sign of the parameter ξ in agreement with the result obtained for temperature-dependent supersymmetric systems. Also, in the presence of the magnetic field, the classical minimum is reduced at least up to the one-loop level.

IV. CONCLUSION

We have examined the Fayet U(1) supersymmetric model in a magnetic environment, obtaining in the process the one-loop effective potential of the system. The minimum value of this potential is seen to be lower than that of the classical one. We deduce, using qualitative arguments, that generally a spontaneously broken supersymmetry remains broken in the presence of a magnetic field. This supports the earlier works in Refs. 5 and 6 that finite temperature always breaks supersymmetry. This may not be altogether a bad feature of the symmetry for an unbroken supersymmetric theory cannot be a candidate for describing nature—there is no degeneracy in nature among particles of different spin.

We have restricted ourselves in this paper to Fayet's U(1) model because there is, as yet, no satisfactory way of spontaneously breaking a non-Abelian supersymmetric theory. The Slavnov mechanism is generally considered as an explicit symmetry breaker so that the present method cannot be applied directly to it.

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