PHYSICAL REVIEW D VOLUME 26, NUMBER 8 15 OCTOBER 1982

# Dyon-fermion dynamics

Curtis G. Callan, Jr.
*Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544*
(Received 19 May 1982)

We continue our study of the effect of light fermions on the charge degree of freedom of magnetic monopoles. Even though the gauge coupling is weak, the Fermi vacuum is strongly perturbed by its coupling to the charge degree of freedom of the monopole. To obtain a correct picture of the vacuum we concentrate on the lowest partial wave of the Fermi field about the monopole core. We find that this simplified system can be transformed to an equivalent one-dimensional *scalar* field theory in which the original fermions appear as sine-Gordon solitons and the monopole charge is determined by the expectation value of the scalar field at spatial infinity. The scalar theory, though not soluble, is sufficiently transparent for us to extract the qualitative physics of monopole charge in the presence of light fermions: the Witten formula for the dependence of monopole charge on vacuum angle, $Q_n = e(n-\theta/2\pi)$, is true no matter how small the Fermi mass $m$; the fractional charge is spread through the Fermi vacuum over a region size $m^{-1}$ and the excitation energy of a charged state is of order $m$; the existence of vacuum structure on such a small energy scale means that certain exotic fermion-monopole scattering processes have very large cross sections. In particular it appears that in grand unification theories monopoles will catalyze baryon decay at typical strong-interaction rates.

## I. INTRODUCTION

In a recent paper[1] we gave arguments that the existence of light fermions should have a major qualitative effect on the charge, or "dyon," degree of freedom[2] of gauge-theory magnetic monopoles.[3] We were able to give a detailed analysis of the *zero*-fermion-mass case, finding that the dyon electric field is completely screened and that the monopole is surrounded by an extended "halo" of chiral-symmetry-breaking condensate. The effect on this picture of nonzero fermion mass was not clear but we argued that, since quarks and leptons are in fact quite light, the zero-mass results should be closer to reality than the conventional picture based on pure gauge theory. Since Wilczek has recently presented general arguments that the total dyon charge is independent of the fermion mass[4] (and therefore presumably does *not* vanish in the zero-mass limit) it has become important to explore the precise nature of the transition between zero and finite fermion mass.

In this paper we shall show how to take explicit account of finite fermion mass. In a nutshell, our previous method was to reduce the fermion physics to that of a Schwinger model by discarding "inessential" degrees of freedom and then to exploit the fact that the *massless* Schwinger model can be solved exactly. The massive fermion theory similarly reduces to a massive Schwinger model which, however, cannot be solved. Instead, we use a variant of the "bosonization" trick[5] to convert it to an equivalent *boson* theory which is very much like the sine-Gordon model. Although this boson theory is also not soluble, its qualitative behavior is clear enough for us to draw conclusions about the true phenomenology of monopoles in theories with finite-mass fermions.

We find, in agreement with Wilczek, that the allowed net charge of the dyon is *unaffected* by the value of the fermion mass. In the simplest model the charge *always* satisfies Witten's relation $Q_n = (n-\theta/2\pi)e$ (Ref. 6) (where $\theta$ is the vacuum angle). On the other hand, the charge is spread out through the Fermi vacuum over a region of radius $\sim m^{-1}$ (where $m$ is the Fermi mass) and the excitation energies of charged states are $\sim m$. (In other words, the fermion vacuum carries fractional charge—that this is possible is a reflection of the chiral anomaly.) Because of this, the charge density and electric field at any *fixed* distance from the monopole core go to zero as the fermion mass goes to zero. It is this spatial nonuniformity of the zero-fermion-mass limit which reconciles the mass-independent dyon charge claimed here with the chargeless dyons of the strictly massless theory.

We also find that the allowed dyon charge depends on how the fermions get their mass. In particular if we adopt the Peccei-Quinn mechanism[7] (which is designed to produce a massive fermion but no "strong" *CP* violation) we can show that the dyon charges are strictly integral or half-integral (unless there is *explicit* weak *CP* violation in the theory, in which case *calculable* fractional dyon charge reappears). If the fermions get their mass through a coupling to the same Higgs field as breaks the gauge symmetry, Jackiw and Rebbi[8] found that there is a zero-energy eigenvalue of the Dirac equation and that the monopole is doubly degenerate, the two states being assigned fermionic charge $\pm\frac{1}{2}$. We not only find the same degeneracy but also find that these "half-fermion" states carry *half* the charge that an elementary fermion would carry.

Our most interesting results concern the scattering of fermions from the monopole. It has been realized for some time that external particles can exchange charge with the dyon degree of freedom[9] of the monopole and that, in grand unification theories, reactions in which a quark turns into a lepton (or *vice versa*) are possible.[10] It has also been assumed that the cross section for such charge-exchange processes is geometrical in nature and therefore, given the small physical size of gauge-theory monopoles, negligibly small.

Since in our treatment asymptotic fermions are just sine-Gordon solitons, it is rather easy to study their scattering from the monopole. We find that the relevant cross sections are not determined by the geometrical size of the monopole core and are in fact quite large. The most naive extension of our results to a phenomenologically "realistic" model suggests that under certain conditions a monopole will catalyze baryon decay at typical strong-interaction rates. The detailed phenomenology of this fascinating class of processes is at the moment obscured by uncertainties concerning the color and confinement properties of grand-unification monopoles.

Some of the above-mentioned issues are discussed in a recent paper by Wilczek.[4] He does not try to solve the problem of fermion vacuum dynamics directly, but relies on general kinematic arguments to reach conclusions about broad qualitative features of dyon physics. Where there is overlap, we are in rough agreement (except perhaps on the question of the rate of monopole-catalyzed baryon decay). Wilczek also discusses some interesting questions concerning the statistics of fractionally charged dyons that we did not think to ask. We have recently learned that Rubakov[11] has also proposed that the monopole can catalyze baryon decay at a large rate. We feel that the virtue of our treatment is that by dealing directly with the dynamics of the Fermi vacuum, we can make these suggestions of exotic physics more concrete and quantitative and can explore in greater detail the range of phenomena to be expected in different models of weak-interaction physics.

## II. THE MODEL AND AN APPROXIMATION

To keep things simple we shall study the monopole of the Georgi-Glashow model.[3] This model has an O(3) gauge field $\vec{A}_\mu$ interacting with an $I=1$ Higgs field $\vec{\phi}$ through the Lagrangian

$$L_{\rm YM}=\frac{1}{2e^2}\sum(\vec{F}_{\mu\nu})^2+\tfrac{1}{2}\sum(D_\mu\vec{\phi})^2-V(\vec{\phi}\cdot\vec{\phi}) \,. \tag{2.1}$$

$V$ is adjusted to produce a nonzero vacuum expectation value of $\vec{\phi}$ ($\langle\vec{\phi}^{\,2}\rangle=\phi_0^{\,2}$) which breaks the gauge symmetry down to U(1) and gives the charged vector meson a mass $m_w^{\,2}=e^2\phi_0^{\,2}$. The well-known monopole solution is

$$\begin{aligned} A_i^a&=\frac{1}{e}\epsilon_{aij}\hat{x}_j\frac{1}{r}A(r)\,,\\ A_0^a&=0\,,\\ \phi^a&=\hat{x}_aH(r)\phi_0\,, \end{aligned} \tag{2.2}$$

where $a=1,2,3$ is the isospin index and both $A$ and $H$ vanish at the monopole center and rapidly approach 1 for $r\gtrsim m_w^{-1}$ (i.e., outside the monopole core).

The dyon degree of freedom arises in the $A_0=0$ gauge (which we adopt) as follows. The minimum-energy gauge field configuration is not unique, but is parametrized by all possible gauge rotations which leave the Higgs field invariant:

$$A_i^\lambda=U_\lambda A_i U_\lambda^{\ -1}+iU_\lambda\nabla_i U_\lambda^{\ -1}\,, \tag{2.3}$$

where $U_\lambda=\exp\{i[\lambda(x,t)\hat{x}\cdot\vec{T}]\}$, $A_i=A_i^aT_a$, and $T_a=\tau_a/2$. Because of the underlying spherical symmetry of the problem we will restrict our attention to gauge functions, $\lambda$, which depend only on the radial coordinate. Motions of the system in this configuration space produce radial electric fields

$$E_r=\hat{x}_aE_r^a=-\dot{\lambda}'(r,t) \tag{2.4}$$

(the electric direction in isospin space is defined by the Higgs-field direction; the dot and the prime refer to differentiation with respect to $t$ and $r$). The action for such motions, including the effect of a $\theta$ (or vacuum angle) term, is easily read off from that part of the gauge field action which involves $E$:

$$L_\lambda=\frac{1}{2e^2}\int dt\, d^3x\, \vec{E}^2+\frac{\theta}{8\pi^2}\int dt\, d^3x\, \vec{E}\cdot\vec{B}$$
$$=\frac{1}{2e^2}\int dt \int_0^\infty 4\pi r^2 dr(\dot{\lambda}')^2+\frac{\theta}{2\pi}\int dt\int_0^\infty dr\dot{\lambda}' . \tag{2.5}$$

But for the geometrical factor of $4\pi r^2$ in the Coulomb energy term, this is the action for a one-dimensional electric field. Alternatively, we can think of this as a one-dimensional Coulomb action with a position-dependent coupling, $\bar{e}(r)=e/(4\pi r^2)^{1/2}$. Quantization of this action leads to the usual spectrum of dyon states[2] with Coulomb electric fields corresponding to charges $Q_n=(n-\theta/2\pi)e$ (Ref. 6) concentrated on the monopole core.

If light charged fermions are added to the system the space of low-energy configurations is enlarged and the problem must be rethought. We will couple an $I=\frac{1}{2}$ Dirac fermion, having the simplest possible mass term, to the gauge field system just described:

$$L_\psi=\bar{\psi}(i\gamma_\mu D_\mu^\lambda-m)\psi ,$$
$$D_\mu^\lambda\psi=(\partial_\mu-i\vec{A}_\mu^\lambda\cdot\vec{T})\psi . \tag{2.6}$$

We shall eventually consider two other options for giving the fermion a mass: (a) Yukawa coupling to the Higgs field, (b) Yukawa coupling to a new complex scalar field whose potential supports an extra U(1) symmetry (the Peccei-Quinn mechanism[7] for eliminating strong $CP$ violation).

The fermions now satisfy a Dirac equation which has been analyzed by Jackiw and Rebbi.[8] Because of the underlying spherical symmetry, an angular momentum $\vec{J}=\vec{L}+\vec{S}+\vec{T}$ is conserved and may be used to do a partial-wave analysis. The $J=0$ partial wave is the only one for which the fermions are not kept away from the monopole core by a centrifugal barrier, and the higher partial waves presumably decouple from the physics of the low-lying eigenstates of the monopole-fermion system. In what follows, then, we discard all but the $J=0$ part of the fermion field. Apart from the arguments already given in favor of this approximation, we can also point out that the chiral anomaly turns out to play a crucial role in this problem, and the anomaly is saturated by the $J=0$ partial wave. The connection between the anomaly and the $J=0$ fermion partial waves has been emphasized by Blaer, Christ, and Tang[13] who have also examined many aspects of the monopole plus the $J=0$ fermion system.

The mechanics of this reduction of the Fermi field have been worked out in our previous paper on this problem[1] and we will simply state the essential results. For each helicity we may write the $J=0$ piece of the Dirac field in terms of two radial functions $g$ and $p$,

$$\psi_{J=0}^{(\pm)}=\begin{bmatrix} \chi_\pm \\ \pm\chi_\pm \end{bmatrix} ,$$
$$\chi_\pm=\frac{1}{\sqrt{8\pi}r}(g_\pm+p_\pm\hat{x}\cdot\vec{\tau})\tau_2 , \tag{2.7}$$

where $\pm$ refer to helicity, $\chi$ is a $2\times2$ matrix, one index describing spin and the other isospin, and finally, $g$ and $p$ depend only on $r$ and $t$. We can forget about the monopole core and let $g$ and $p$ be defined on $0\le r\le\infty$ if we impose the boundary condition $p(r=0)=0$ at the origin. If we define a two-component field for each helicity by

$$X_\pm(r,t)=\begin{bmatrix} g_\pm(r,t) \\ \pm ip_\pm(r,t) \end{bmatrix} \tag{2.8}$$

the $J=0$ part of the full four-dimensional Fermi action may be written in the suggestive form

$$L_\psi=\int dt\int_0^\infty dr[\bar{X}_+\bar{\gamma}^\alpha(i\partial_\alpha-A_\alpha)X_+ +\bar{X}_-\bar{\gamma}^\alpha(i\partial_\alpha+A_\alpha)X_- +m(\bar{X}_+X_-+\bar{X}_-X_+)] , \tag{2.9}$$

where $\alpha=0,1$; $\partial_0=\partial_t$, $\partial_1=\partial_r$; $A_\alpha=\delta_{0\alpha}\lambda'/2$; $\bar{\gamma}^0=-\tau_3$, $\bar{\gamma}^1=i\tau$; and $\bar{X}=X^*\bar{\gamma}^0$. This is the action for two flavors of one-dimensional fermion living on the half line, interacting with an Abelian vector potential (representing the dyon electric field) and having a flavor-off-diagonal mass term. Note also that $X_+$ and $X_-$ interact with $A_\mu$ with opposite charges. (Since we now have one-dimensional Fermi fields in the game, we will have frequent occasion to use the $2\times2$ Dirac matrices $\bar{\gamma}_\mu$; they are to be distinguished from the $4\times4$ matrices $\gamma_\mu$.)

Our approximation to the monopole-fermion system is then defined by the action $L_{\rm MF}=L_\lambda+L_\psi$. This is very nearly a conventional massive Schwinger model. The key differences are the boundary condition at $r=0$ and the $r$-dependent coupling constant $\bar{e}(r)$ in the Coulomb action. Even though the massive Schwinger model is not soluble, much is known about it and it is a convenient starting point for a discussion of the physics of our system.

We would now like to explain a peculiar feature of the $J=0$ fermion world which plays an important role in what follows. In Ref. 1 we gave expressions for fermion bilinears in terms of the equivalent one-dimensional Dirac fields. In this paper we will need the reduction of the following current density operators to their equivalent $J=0$, one-dimensional forms:

$$J_0=\bar{\psi}\hat{x}\cdot\vec{T}\gamma_0\psi \cong -\frac{1}{8\pi r^2}(\bar{\chi}_+\bar{\gamma}_1\chi_+ - \bar{\chi}_-\bar{\gamma}_1\chi_-)\ ,$$
$$J_r=\bar{\psi}\hat{x}\cdot\vec{T}\hat{x}\cdot\vec{\gamma}\psi \cong -\frac{1}{8\pi r^2}(\bar{\chi}_+\bar{\gamma}_0\chi_+ - \bar{\chi}_-\bar{\gamma}_0\chi_-), \quad (2.10)$$
$$J_0^5=\bar{\psi}\gamma_0\gamma_5\psi \cong -\frac{1}{8\pi r^2}(\bar{\chi}_+\bar{\gamma}_0\chi_+ - \bar{\chi}_-\bar{\gamma}_0\chi_-)\ .$$

We shall also need the reduction of two operators which appear as mass terms in our Lagrangians:

$$\bar{\psi}\psi = +\frac{1}{8\pi r^2}(\bar{\chi}_+\chi_- + \bar{\chi}_-\chi_+)\ ,$$
$$\bar{\psi}\hat{x}\cdot\vec{T}\psi = \frac{1}{8\pi r^2}(\bar{\chi}_+\gamma_5\chi_- + \bar{\chi}_-\gamma_5\chi_+)\ . \quad (2.11)$$

The expression for the radial electric-charge current density $J_r$ contains a remarkable result. Since $\bar{\chi}\bar{\gamma}_0\chi=\chi^*\chi$ is positive definite, the *flow* of charge for each kind of particle (+ or − helicity; particle or antiparticle) will have a definite sign. This in turn means that the *charge* of each such particle type will depend on whether the particle is moving toward or away from the monopole core,[12,13] despite the fact that the fermions have $I=\frac{1}{2}$ and therefore would be expected to have *two* charge states ($\pm e/2$) for any kinematical configuration. The available charge states are summarized in Table I.

TABLE I. A list of the charge helicity and motion states available to $J=0$ massless fermions.

| Helicity | Fermion No. | Motion | Charge |
|---|---|---|---|
| + | + | out | $-e/2$ |
| + | + | in | $+e/2$ |
| − | + | out | $+e/2$ |
| − | + | in | $-e/2$ |
| + | − | out | $+e/2$ |
| + | − | in | $-e/2$ |
| − | − | out | $-e/2$ |
| − | − | in | $+e/2$ |

According to this table, there is no way to scatter a fermion from the monopole and conserve both charge and helicity. If helicity is to be conserved, the charge carried by the fermion must change by one unit; if the charge carried by the fermion is to be conserved, then helicity must change. Neither alternative is impossible. In the first case, we make use of the dyon degree of freedom of the monopole to absorb the charge lost by the fermion. In the second case we recall that the existence of the dyon degree of freedom means that $\vec{E}\cdot\vec{B}$ need not be zero and that chirality may therefore fail to be conserved because of the anomaly. (We are implicitly talking about the case that the fermion mass is so small that we can neglect explicit chirality violation through the mass term.) Which of the two scenarios is realized can only be decided on the basis of a real dynamical argument such as will be given in the following sections. The point to bear in mind now is that the peculiar charge structure of the $J=0$ fermions is intimately bound up with the chiral anomaly on the one hand and the dynamics of the dyon on the other.

## III. BOSONIZATION

We now want to "solve" the system described by $L_{\mathrm{MF}}$. We will use a variant of the "bosonization" trick which has proven so useful in studies of one-dimensional fermion theories. The first step is to establish a connection between fermion and boson fields at the level of free massless field theory (that this is possible at all is a peculiar feature of the one-dimension case). Because our fermions live on the half line and satisfy a boundary condition at $r=0$, the method of Mandelstam,[5] which applies to fermions living on the whole line, does not work. By dint of some experimentation, however, one can establish that the following connection does work:

$$\chi_u(r,t)=\exp\left[i\sqrt{\pi}\left[\phi(r,t)-\int_0^r ds\,\dot{\phi}(s,t)\right]\right]\ ,$$
$$\chi_l(r,t)=i\exp\left[i\sqrt{\pi}\left[\phi(r,t)+\int_0^r ds\,\dot{\phi}(s,t)\right]\right]\ , \quad (3.1)$$

where

$$\chi=\begin{bmatrix}\chi_u\\ \chi_l\end{bmatrix}$$

is our one-dimensional Fermi field in a $\bar{\gamma}_5=\bar{\gamma}_0\bar{\gamma}_1$

diagonal representation and $\phi$ is a free massless scalar field, living on the half line and satisfying the boundary condition $\phi'(r=0)=0$. [Warning: the one-dimensional $\gamma$ matrices defined after Eq. (2.9) are not in the $\gamma_5$-diagonal representation, so a transformation has to be carried out to make use of the above correspondence.] The correspondence "works" in the following sense. The free scalar field with the specified boundary condition has the propagator

$$\Delta(rt;r't')=-\frac{1}{4\pi}\{\ln[(r-r')^2-(t-t')^2]+\ln[(r+r')^2-(t-t')^2]\} . \tag{3.2}$$

The correspondence of Eq. (3.1) then implies that $\chi$ has the propagator (the propagator of the exponential of a Gaussian, or free, field is always calculable)

$$\begin{aligned} S(r,t;r't')&=S_0(rt;r't')+S_0(rt;-r't')\bar{\gamma}_0 , \\ S_0(rt;r't')&=\frac{1}{\pi}\frac{[\bar{\gamma}_0(t-t')+\bar{\gamma}_1(r-r')]}{(t-t')^2-(r-r')^2} . \end{aligned} \tag{3.3}$$

But this *is* the propagator for a free massless Fermi field satisfying the "bag" boundary condition at $r=0$ described in Sec. II. Certain purely algebraic factors, needed to guarantee the proper anticommutation of field components,[14] but which play no role in our problem, have been dropped from Eq. (3.1). Mandelstam's correspondence is similar to ours, but there is no boundary condition on either the scalar or the Fermi field. It is, we think, remarkable that imposing a boundary condition on the Fermi field preserves the correspondence at the price of a simple boundary condition on the scalar field.

The next step is to write the various bilinear Fermi operators appearing in the fermion Hamiltonian as functions of the corresponding boson fields and so to construct an equivalent boson Hamiltonian. Since we have two one-dimensional Fermi fields $\chi_+$ and $\chi_-$ we will need two boson fields $\phi_+$ and $\phi_-$. The essential operator correspondences for the currents are

$$\bar{\chi}_\pm\bar{\gamma}_\mu\chi_\pm=\frac{1}{\sqrt{\pi}}\partial_\mu\phi_\pm . \tag{3.4}$$

The correspondences relevant to mass terms are

$$\begin{aligned} \bar{\chi}_+\chi_-&=\mu\colon \exp[i\sqrt{\pi}(\phi_--\phi_+)]\cos\sqrt{\pi}\int_0^r ds[\dot{\phi}_+(s,t)+\dot{\phi}_-(s,t)] , \\ \bar{\chi}_+\gamma_5\chi_-&=i\mu\colon \exp[i\sqrt{\pi}(\phi_--\phi_+)]\sin\sqrt{\pi}\int_0^r ds[\dot{\phi}_+(s,t)+\dot{\phi}_-(s,t)] . \end{aligned} \tag{3.5}$$

The various mass terms follow from Eq. (3.2) by straightforward algebra. (This sort of structure was first studied by Halpern.[14]) The normal-ordering instruction cures an ultraviolet divergence and $\mu$ is the mass with respect to which normal ordering is done. Although $\mu$ is needed to give the right dimensionality to the mass term, it is arbitrary and cancels out of any physical answer. The line of argument that leads to the much simpler vector-current correspondence is the conventional one given by Mandelstam [although in his case the right-hand side $\partial_\mu\phi$ is replaced by $\epsilon_{\mu\nu}\partial^\nu\phi$ because the underlying Fermi-boson correspondence is not quite the same as in Eq. (3.1)].

The dynamical system $L_{\rm MF}$ describes the variables $\chi_+$, $\chi_-$, and $\lambda'$. We now know how to replace $\chi_\pm$ by equivalent boson variables and to complete our program we have only to eliminate the gauge variables $\lambda'$. The terms in $L_{\rm MF}$ involving $\lambda$ are

$$\begin{aligned} L_{\rm MF}^\lambda=&\frac{1}{2e^2}\int dt4\pi r^2dr(\dot{\lambda}')^2+\frac{\theta}{2\pi}\int dt\,dr\dot{\lambda}' \\ &+\int dt\int dr\frac{\lambda'}{2}(\bar{\chi}_+\gamma_0\chi_+-\bar{\chi}_-\gamma_0\chi_-) . \end{aligned} \tag{3.6}$$

Using the replacement

$$\bar{\chi}_+\gamma_0\chi_+-\bar{\chi}_-\gamma_0\chi_-=\frac{1}{\sqrt{\pi}}(\dot{\phi}_+-\dot{\phi}_-)$$

and integrating the third term in $L_{\rm MF}^\lambda$ by parts in time we get

$$\begin{aligned} L_{\rm MF}^\lambda=&\frac{1}{2e^2}\int dt\int 4\pi r^2dr(\dot{\lambda}')^2 \\ &+\frac{\theta}{2\pi}\int dt\,dr\dot{\lambda}' \\ &+\int dt\,dr\frac{\dot{\lambda}'}{2\sqrt{\pi}}(\phi_+-\phi_-) . \end{aligned} \tag{3.7}$$

Since $\lambda$ now appears only as a quadratic in $\dot{\lambda}'$, we may eliminate it from the system by solving the classical equation for $\dot{\lambda}'$. We obtain an expression for the radial electric field in terms of the boson fields:

$$E_r = \dot{\lambda}' = \frac{e^2}{8\sqrt[3]{\pi} r^2}\left[\phi_+ - \phi_- - \frac{\theta}{\sqrt{\pi}}\right], \tag{3.8}$$

which, substituted back in $L_{\rm MF}^{\lambda}$ gives the "bosonized" interaction energy

$$H_{\rm int} = \int dr \frac{e}{32\pi^2 r^2}\left[\phi_+ - \phi_- - \frac{\theta}{\sqrt{\pi}}\right]^2 . \tag{3.9}$$

To get the full boson effective Hamiltonian we just append the free massless boson term and the boson equivalent of the fermion mass term:

$$\begin{aligned} H_{\rm eff} = \int dr \Bigg[ & \tfrac{1}{2}{\pi_+}^2 + \tfrac{1}{2}(\phi'_+)^2 + \tfrac{1}{2}(\pi_-)^2 + \tfrac{1}{2}(\phi'_-)^2 \\ & + \frac{e^2}{32\pi^2 r^2}\left[\phi_+ - \phi_- - \frac{\theta}{\sqrt{\pi}}\right]^2 \\ & + m\mu {:}\cos\sqrt{\pi}(\phi_+ - \phi_-) \\ & \times \cos\sqrt{\pi}\int_0^r ds[\pi_+(s) + \pi_-(s)]{:} \Bigg] . \end{aligned} \tag{3.10}$$

We have rewritten $\dot{\phi}$ everywhere it occurs as $\pi$ since we are constructing the Hamiltonian. As an aside, we remark that if we had used the Higgs-mechanism mass term the cosines in $H_{\rm eff}$ would simply have been replaced by sines.

It is clear why the zero-fermion-mass case is so simple: if $m=0$, $H_{\rm eff}$ is a quadratic, soluble Hamiltonian. It is easy to verify that all of our previous results on the zero-fermion-mass case can be reproduced in this way. In particular, it is obvious that all dependence on $\theta$ can be shifted away and that the expectation of $E_r$ must therefore vanish (since $\langle E_r\rangle$ is the $\theta$ derivative of the ground-state energy). If $m\neq 0$, $H_{\rm eff}$ looks rather a mess, but it is clear that physics will now depend on $\theta$ with periodicity $2\pi$.

To make some sense of the finite-mass case, we carry out the canonical transformation

$$\begin{aligned} \Phi &= \tfrac{1}{2}(\phi_+ - \phi_-) + \tfrac{1}{2}\int_0^r ds[\pi_+(s) + \pi_-(s)] , \\ Q &= \tfrac{1}{2}(\phi_+ - \phi_-) - \tfrac{1}{2}\int_0^r ds(\pi_+(s) + \pi_-(s)) , \\ \Pi &= \tfrac{1}{2}(\pi_+ - \pi_-) + \tfrac{1}{2}(\phi'_+ + \phi'_-) , \\ P &= \tfrac{1}{2}(\pi_+ - \pi_-) - \tfrac{1}{2}(\phi'_+ + \phi'_-) \end{aligned} \tag{3.11}$$

obtaining the much more transparent system

$$\begin{aligned} H_{\rm eff} = \int_0^\infty dr \Bigg[ & \tfrac{1}{2}\Pi^2 + \tfrac{1}{2}(\Phi')^2 + \tfrac{1}{2}P^2 + \tfrac{1}{2}(Q')^2 \\ & + \frac{e^2}{32\pi^2 r^2}\left[\Phi + Q - \frac{\theta}{\sqrt{\pi}}\right]^2 \\ & + \frac{m\mu}{2}\cos(2\sqrt{\pi}\Phi) \\ & + \frac{m\mu}{2}\cos 2\sqrt{\pi} Q \Bigg] . \end{aligned} \tag{3.12}$$

So, despite an initial appearance of extreme complexity, the system boils down to two sine-Gordon systems coupled by the Coulomb interaction term and, secretly, by the boundary conditions at $r=0$ which have been scrambled by the canonical transformation. The boundary conditions which guarantee that Eq. (3.12) describes the same physics as the original fermion theory are

$$\Phi(r=0) = Q(r=0) ,$$

$$\Phi'(r=0) = -Q'(r=0) .$$

As a first-order check that the new boson theory is equivalent to the original fermion theory we will verify that the right number and kind of particle states are present. Far from the monopole the theory reduces to two decoupled sine-Gordon models whose excitations are the usual sine-Gordon solitons. There are eight possible soliton "states" (field $\Phi$ or $Q$, soliton or antisoliton, moving in or moving out) and they should correspond to the eight possible $J=0$ fermion states (helicity $+$ or $-$, particle or antiparticle, moving in or out) described in Sec. II. To establish the correspondence, one has only to calculate the electric charge and axial charge (or helicity) of each soliton via the relations

$$\begin{aligned} Q &= \frac{1}{2\sqrt{\pi}}\int_0^\infty dr(\phi'_+ - \phi'_-) \\ &= \frac{1}{2\sqrt{\pi}}\int_0^\infty dr(\Phi' + Q') , \\ Q_5 &= \frac{1}{2\sqrt{\pi}}\int_0^\infty dr(\pi_+ - \pi_-) \\ &= \frac{1}{2\sqrt{\pi}}\int_0^\infty dr(\dot{\Phi} + \dot{Q}) , \end{aligned} \tag{3.13}$$

which in turn follow from Eqs. (2.10) and (3.4). The results are summarized in Table II, and they of course tell us that the soliton states are exactly of the right number and kind to match the $J=0$ states of the original fermion theory. Since asymp-

TABLE II. A list of the possible asymptotic soliton states of the equivalent boson theory and their correspondence with the possible $J=0$ fermion states.

| Field | Boson Type | Motion | Charge | Fermion Helicity | Fermion No. |
|---|---|---|---|---|---|
| $\Phi$ | soliton | in | $+e/2$ | + | + |
| $\Phi$ | soliton | out | $+e/2$ | − | + |
| $\Phi$ | antisoliton | in | $-e/2$ | + | − |
| $\Phi$ | antisoliton | out | $-e/2$ | − | − |
| $Q$ | soliton | in | $+e/2$ | − | − |
| $Q$ | soliton | out | $+e/2$ | + | − |
| $Q$ | antisoliton | in | $-e/2$ | − | + |
| $Q$ | antisoliton | out | $-e/2$ | + | + |

totic fermion states are just sine-Gordon solitons, it will be possible to study scattering of fermions from the monopole by rather straightforward soliton methods, once the monopole ground state is properly understood.

## IV. THE MONOPOLE GROUND STATE

We would now like to establish the properties (charge, charge distribution, mass,. . .) of the monopole ground state and their dependence on the free parameters of the theory, $e$, $m$, and $\theta$. We will simply do a classical analysis of the bosonized Hamiltonian, derived in the previous section. This is potentially dangerous since the coupling constant of the sine-Gordon part of the theory is not particularly weak. General one-dimensional experience suggests, however, that this procedure should not lead us astray as far as qualitative properties are concerned.

The electric charge of the system is just $4\pi r^2 E_r/e$, evaluated at large $r$. According to Eqs. (3.8) and (3.11) this may be expressed in terms of the boson fields as

$$q=\frac{e}{2\sqrt{\pi}}\left[\phi_+-\phi_--\frac{\theta}{\sqrt{\pi}}\right]\Bigg|_{r=\infty}$$
$$=\frac{e}{2\sqrt{\pi}}\left[\Phi+Q-\frac{\theta}{\sqrt{\pi}}\right]\Bigg|_{r=\infty} . \tag{4.1}$$

But a finite-energy state of the effective Hamiltonian, Eq. (3.12), is obviously characterized by asymptotic values of $\Phi$ and $Q$ lying at minima of the mass term: $\Phi_N=\sqrt{\pi}N$, $Q_M=\sqrt{\pi}M$ with $N$, $M$ being positive or negative integers. The allowed values of monopole charge are therefore

$$q_{N,M}=e\left[\frac{N+M}{2}-\frac{\theta}{2\pi}\right] . \tag{4.2}$$

This is perfectly consistent with the presence of a monopole obeying the "standard" monopole charge formula of Witten

$$q_N=e\left[N-\frac{\theta}{2\pi}\right] \tag{4.3}$$

plus some number of charge $\pm e/2$ fermion excitations. In short, the charge quantization rules for the monopole are mass independent: as long as the system lives in an infinite volume, any mass, no matter how small, imposes the same quantization rules. The $\theta$ dependence of this total charge does *not* vanish smoothly as the fermion mass goes to zero.

To understand more clearly what is going on, we must find the energy and charge distribution associated with these states. The classical ground state of the effective Hamiltonian is characterized by $\Pi=P=0$ and $\Phi$, $Q$ equal to some functions of $r$ which minimize the total energy. We have already argued that we must have $\Phi(\infty)=\sqrt{\pi}N$, and $Q(\infty)=\sqrt{\pi}M$ in order for the total energy not to blow up at large $r$. At the same time, it would seem that we must have

$$\left[\Phi(0)+Q(0)-\frac{\theta}{\sqrt{\pi}}\right]=0 \tag{4.4}$$

in order for the Coulomb energy not to blow up at $r=0$. (There is a physical subtlety here: we have taken the monopole core radius to be zero while in fact it is of order $m_W^{-1}$, where $m_W$ is the vector boson mass; our conclusions therefore apply to the limit $m/m_W \ll 1$.) This means that any electric charge must be located outside the core, in the Fermi vacuum.

If $\theta=0$, a consistent solution for the ground state is $\Phi(r)=Q(r)=0$: no charge, charge density or energy density. If $\theta\neq 0$ this solution is no longer possible—the total charge, as we have just pointed out, must be $\theta/2\pi$ and this charge must be distributed through the vacuum in some fashion. An examination of the energy density suggests that a plausible solution is $\Phi(r)=Q(r)=F(r)$ with $F(\infty)=0$ and $F(0)=\theta/2\sqrt{\pi}$, and with the spatial scale of variation of $F$ being set by $m$, the only mass parameter in the theory (the renormalization scale $\mu$ is, as we have explained, a fake parameter). The energy of such a state, as compared to the $\theta=0$ ground-state energy, will inevitably be of order $m$. We can always add charge in units of $\pm e/2$ at energy cost $m$ by adding asymptotic solitons, representing the basic fermions, to our system. Whether or not there will exist higher charge states stable against decay to fermions is a detailed energetic question which we will not try to answer here since our analysis is only approximate.

The key point is that when $\theta\neq 0$, the fractional charge $e\theta/2\pi$ of the monopole ground state is present as a "vacuum polarization" cloud of size $1/m$ in the fermion vacuum. [This follows from the fact that radial charge density is $\Phi'(r)+Q'(r)$ and from the proposed $r$ dependence of $\Phi$ and $Q$.] In the limit $m\to 0$, this polarization cloud maintains its net charge, but spreads out over larger and larger spatial regions. The electric field and charge density at a *fixed distance* from the core of course go to zero as $m$ vanishes. It is this spatial nonuniformity of the zero-mass limit which reconciles the apparent contradiction between the results on dyon charge obtained here and those of our work on the strictly massless theory. The basic physical point is that the monopole becomes a truly extended object when coupled to light fermions.

In the conventional picture of the monopole, all the charge is on the core. How do we reach that limit? We alluded earlier to the fact that the core radius is small, but not actually zero. This means that the Coulomb energy is approximately

$$E_{\rm Coul}\sim e^2[\Phi(0)+Q(0)-\theta/\sqrt{\pi}]^2 m_W .$$

If the fermion mass is large compared to $e^2 m_W$, it will be energetically unfavorable to move $\Phi$ and $Q$ away from the minima of the fermion mass terms in order to reduce the Coulomb energy, and we will have $\Phi(r)=Q(r)=0$. The total charge will again be $e\theta/2\pi$, but all of that fractional charge will be concentrated on the monopole core and the energy will be approximately $e^2 m_W(\theta/2\pi)^2$. This is the conventional picture, and it is reassuring to recover it in the limit of large $m$.

Finally we would like to say a few words about the peculiar ability of the Fermi vacuum to carry fractional charge. This is perhaps even less intuitive than the original discovery that the charge rotator degree of freedom of the monopole can be quantized to give fractional charge.[6] After all, the charge rotator is a dynamical object which could reasonably have eigenvalues depending continuously on a quantization phase, while one is used to the notion that the charge carried by the Dirac sea must come in units of the basic fermion charge, which is definitely not variable. However, one can use a slightly unfamiliar version of the chiral anomaly to show that in the presence of a magnetic monopole the charge of the Fermi vacuum itself is continuously variable. The point is that in a static magnetic monopole background field the chiral charge and electric charge densities for a Fermi field do not commute: although their canonical commutators vanish, there is an anomalous $c$-number Schwinger term in the presence of a background magnetic field.[15] In the language we have been using, the $J=0$ pieces of the radial chiral and electric charge densities may be expressed in terms of our "one-dimensional" Fermi fields as

$$J_0=\bar{\chi}_+\bar{\gamma}_1\chi_+-\bar{\chi}_-\bar{\gamma}_1\chi_- ,$$
$$J_0^5=\bar{\chi}_+\bar{\gamma}_0\chi_+-\bar{\chi}_-\bar{\gamma}_0\chi_- .$$

These densities may in turn be expressed in terms of the equivalent scalar fields as

$$J_0=\frac{1}{2\sqrt{\pi}}(\phi'_+-\phi'_-) ,$$
$$J_0^5=\frac{1}{2\sqrt{\pi}}(\pi_+-\pi_-) . \tag{4.5}$$

Consequently the densities have the anomalous equal-time commutator

$$[J_0(r),J_0^5(r')]=\frac{i}{2\pi}\delta'(r-r') . \tag{4.6}$$

That this commutator fails to vanish is a direct reflection of the presence of the monopole magnetic field. The total charge then fails to commute with a *local* chiral rotation[16] parametrized by a function $\lambda(r)$:

$$[Q_5^\lambda,Q]=[\lambda(\infty)-\lambda(0)]/2\pi ,$$

where

$$Q=\int_0^\infty dr\,J_0(r) ,$$
$$Q_5^\lambda=\int_0^\infty dr\,\lambda(r)J_0^5(r) . \tag{4.7}$$

If we carry out the transformation $\exp(iQ_5^\lambda)$ on the conventional charge-zero vacuum, we will get a "vacuum" state with a total charge, $[\lambda(\infty)-\lambda(0)]/2\pi$, which can be anything we want. The *energy* of this state will depend on the functional form of $\lambda(r)$. To get the lowest energy state in a given charge sector one must minimize the energy with respect to $\lambda(r)$, keeping $[\lambda(\infty)-\lambda(0)]$ fixed. The analysis done earlier in this section amounts to carrying out this minimization in a simple approximation.

## V. MISCELLANEOUS APPLICATIONS

We would now like to use these methods to discuss some of the physics questions raised in the introductory sections. First of all, let us study what happens when the fermion gets its mass from a Yukawa coupling to the Higgs field:

$$L_m = g\int d^4x\, \bar{\psi}\vec{\phi}\cdot\vec{\mathrm{T}}\psi\ . \tag{5.1}$$

Jackiw and Rebbi showed that in this theory the fermion moving in the monopole field has a strictly zero-energy eigenvalue[8] and that the monopole ground state is two-fold degenerate since this state can be filled or unfilled. Further, because a zero-energy state is neither particle nor antiparticle, it turns out to be necessary to assign fermionic charge $\pm\frac{1}{2}$ to the ground states. We are naturally led to ask what charge is carried by these half-fermion monopoles: given that fermions carry charge $\pm e/2$, should "half" fermions carry charge $\pm e/4$?

The Higgs-mass term, when reexpressed in terms of the $J=0$ Fermi variables, takes the form

$$L_m = m\int dr\, dt(\bar{\chi}_+\gamma_5\chi_- + \bar{\chi}_-\gamma_5\chi_+)\ . \tag{5.2}$$

This in turn can be reexpressed in terms of the equivalent scalar fields, $\Phi$ and $Q$, as

$$L_m = \frac{\mu m}{2}\int dr\, dt[\cos 2\sqrt{\pi}\Phi - \cos 2\sqrt{\pi}Q]\ . \tag{5.3}$$

To compare with the Jackiw and Rebbi treatment we must set $\theta=0$. The various conditions at the origin then imply that $\Phi(r=0)=Q(r=0)=0$. At $r=\infty$ the system must sit at a minimum of the mass term: $\Phi(\infty)=(N+\frac{1}{2})\sqrt{\pi}, Q(\infty)=M\sqrt{\pi}$. A simple energy argument shows that the lowest-energy option for satisfying the boundary conditions at $r=0$ and $r=\infty$ corresponds to $Q(r)=0$, $\Phi(\infty)=\pm 1/2\sqrt{\pi}$ (by an obvious symmetry, these two possibilities are degenerate in energy). According to Eq. (4.1) the allowed charges of the two degenerate states are $\pm e/4$. So our method *does* recover the degeneracy structure of the ground state and, furthermore, informs us that these states actually carry the charges that one would expect of "half fermions." Since we have learned that the monopole charge is a complex collective effect of the Fermi vacuum, we should not be surprised that the original treatment of Jackiw and Rebbi,[8] which works in the "naive" Fermi vacuum, does not show it.

Next we will study what happens when the fermion mass is produced by a Peccei-Quinn—type mechanism.[7] This mechanism was invented to exorcise "strong *CP* violation" ($\theta$ dependence) from gauge theories and we want to understand whether and how it eliminates $\theta$-dependent monopole charge. We adopt a cartoon version of this mechanism:

$$L_m = g\int d^4x[\phi\bar{\psi}(1+\gamma_5)\psi + \text{H.c.}]\ , \tag{5.4}$$

where $\phi$ is a complex $I=0$ scalar having a potential $V(\phi^*\phi)$ which has a symmetry-breaking minimum at $|\phi|=\phi_0$. If we write $\phi=\phi_0 e^{i\alpha}$, the field $\alpha$ is a Goldstone boson (the axion[17]) which couples to ordinary matter through the fermion mass term and must be included in the minimization procedure to find the ground state. The effective Hamiltonian for the scalar fields $Q$, $\Phi$ may be computed as before. If we define $g\phi_0=m$, the pieces of the fermionic Hamiltonian which depend on $\alpha$ and $\theta$ can be written as

$$\frac{e^2}{32\pi^2 r^2}\left[\Phi+Q-\frac{\theta}{\sqrt{\pi}}\right]^2 + \frac{m\mu}{2}\cos(2\sqrt{\pi}\Phi+\alpha) + \frac{m\mu}{2}\cos(2\sqrt{\pi}Q+\alpha)\ . \tag{5.5}$$

Because of anomalous chiral-symmetry breaking associated with instantons, $\alpha$ is only a would-be Goldstone boson: instanton effects generate an effective potential for $\alpha$ of the form

$$V_{\text{eff}} \cong M^4(\alpha+\theta)^2\ , \tag{5.6}$$

where $\theta$ is the same vacuum angle as always and $M$ is a mass scale which will depend on the precise nature of the axion couplings. Taking Eqs. (5.5) and (5.6) together, we see that it is possible to eliminate $\theta$ from the problem altogether by making the shifts

$$\begin{aligned} &\alpha \to \alpha - \theta\ , \\ &\Phi \to \Phi + \theta/2\sqrt{\pi}\ , \\ &Q \to Q + \theta/2\sqrt{\pi}\ . \end{aligned} \tag{5.7}$$

This means that the allowed charges of the monopole are integral multiples of $e/2$, as required to satisfy $CP$ invariance.

Things are slightly different if the weak interactions carry explicit $CP$ violation. Then the instanton-generated effective potential for $\alpha$ will be

$$V_{\rm eff}=M^4(\alpha-\theta-\theta_w)^2\ , \tag{5.8}$$

where $\theta_w$ is a small calculable phase which reflects the effect of explicit $CP$ violation. Now when we carry out the shifts of Eq. (5.7) we will be left with an effective potential for $\Phi$ and $Q$ which has the same physics as Eq. (3.12) with $\theta$ replaced by $\theta_w$. In other words, explicit weak-interaction $CP$ violation shows up in the physics of the monopole system as a small determinate effective $\theta$ parameter having all the effects discussed in previous sections.

Finally, we wish to discuss the scattering of fermions from the monopole. In our model asymptotic fermions are described by solitons of two decoupled sine-Gordon theories for fields $\Phi$ and $Q$. At finite distances from the monopole, these fields are coupled by a Coulomb interaction term [Eq. (3.12)]. Consider a "$\Phi$ soliton" heading in toward the uncharged core of a $\theta=0$ monopole. If this soliton has low energy, it will simply reflect from the Coulomb barrier of its own Coulomb self-energy at a large distance from the monopole core and head back out to spatial infinity as an outgoing $\Phi$ soliton. In this scattering event the charge of the scattering soliton does not change, and neither does the charge of the monopole. According to Table II the only thing that changes is the helicity of the scattering particle and the scattering event can be summarized as

$$q_R+M\rightarrow q_L+M \tag{5.9}$$

(where $q_R$ and $q_L$ stand for the helicity $\pm\frac{1}{2}$, $I=\frac{1}{2}$ fermions of the basic model). In other words, the system chooses to scatter by conserving charge (of both the fermion and the monopole) and *not* conserving chirality. The amplitude for this process is essentially independent of the explicit fermion mass and the helicity nonconservation must be a reflection of the chiral anomaly. In fact, in our previous discussion of the massless fermion case, we pointed out that there is a large vacuum expectation value of the chiral noninvariant quantity $\bar{\psi}\psi=\bar{\psi}_R\psi_L+\bar{\psi}_L\psi_R$ in the neighborhood of the monopole and the scattering event just described can be thought of as a scattering of the fermion from this chiral condensate. Note that although the anomalous nonconservation of chirality is a nonperturbative effect, it occurs here with none of the usual $e^{-8\pi^2/e^2}$ nonperturbative suppression factors. Also, since $m_w$ plays no role in this analysis, the cross section for the process of Eq. (5.9) must have no weak-interaction suppression factors of inverse powers of $m_w$. In other words, the cross section must be roughly $\sigma=c\times E^{-2}$ where $E$ is the fermion energy and $c$ is constant "of order unity." Needless to say, this cross section can be large.

It should be pointed out that this sort of "dimensionless" cross section is characteristic of the scattering of spin-$\frac{1}{2}$ particles on a monopole. In the $J=0$ partial wave certain charge or helicity states exist *only* as ingoing or outgoing waves, so that conservation of probability requires certain $J=0$ quantum-number-flip amplitudes not to vanish even in the limit of zero monopole size. The basic phenomenon has been observed before[18] although it seems not to have been explicitly realized that it implies a large cross section for quantum-number exchange between monopoles and external fermions.

Rather than attempting to obtain a quantitative estimate of this cross section we would like to discuss how the same scattering mechanism would work in a theory with a more phenomenologically realistic structure. Suppose we have not one, but two $I=\frac{1}{2}$ Dirac Fermi fields ($\psi_1$ and $\psi_2$). Then the chiral anomaly should lead to a vacuum expectation value of the *four*-Fermi operator $\bar{\psi}_{1R}\psi_{1L}\bar{\psi}_{2R}\psi_{2L}$ in the vicinity of the monopole, and the scattering process analogous to Eq. (5.9) is

$$q_{1R}+q_{2R}+M\rightarrow q_{1L}+q_{2L}+M\ . \tag{5.10}$$

The previous charge-flow analysis for the $J=0$ partial wave works for $\psi_1$ and $\psi_2$ independently and we again conclude that there is no change in the charge carried by the fermions or by the monopole.

The $SU_5$ theory with one fermion $5+\overline{10}$ generation has in fact precisely this structure: the $SU_5$ monopole lives in an SU(2) subgroup[10] which mixes color and weak isospin and the fermions fill out two Dirac SU(2) doublets plus a number of $SU_2$ singlets (which play no role in the vacuum dynamics of this monopole). We can display the particle identification of the components of the SU(2) doublets, using the notation of the previous paragraph, as follows:

$$\psi_{1R}=\begin{bmatrix} e^- \\ \bar{d}_3 \end{bmatrix}\ ,\quad \psi_{1L}=\begin{bmatrix} \bar{u}_2 \\ u_1 \end{bmatrix}\ ,$$
$$\psi_{2R}=\begin{bmatrix} d^3 \\ e^+ \end{bmatrix}\ ,\quad \psi_{2L}=\begin{bmatrix} \bar{u}_1 \\ u_2 \end{bmatrix}\ , \tag{5.11}$$

where $u^i$, $d^i$ stand for the color components of the nonstrange quarks. Our discussion of charge flow in the $J=0$ partial wave translates into the statement that only the lower components of the $L$ fields and only the upper components of the $R$ fields can appear as ingoing waves and *vice versa* for outgoing waves. The reaction of Eq. (5.10), with $SU_5$ particle identification supplied therefore reads

$$u_{1L}+u_{2L}+M\to\bar{d}_{3R}+e_R^{+}+M \ . \tag{5.12}$$

This conserves $SU_3$ and ordinary charge but violates baryon number. It is reminiscent of the baryon decay found by 't Hooft[19] to be caused by $SU_2\times U_1$ instantons and the associated chiral anomaly. It is also very similar to the process $u_L^1+u_R^2\to\bar{d}_R^3+e_L^+$ which occurs via $X$-boson exchange. The difference is that, for the reasons discussed in the previous paragraphs, the monopole-catalyzed version of baryon decay has a large cross section: its amplitude contains no inverse powers of $m_X$ and no factors of $e^{-8\pi^2/g^2}$. Exactly how this process manifests itself phenomenologically is not completely clear to us. Since the monopole can carry color charge, the confining effects of the ordinary QCD vacuum will presumably play an important role in determining rates and selection rules. We hope, by means of the methods discussed in this paper, to soon be able to give a quantitative calculation of the rate of monopole-catalyzed baryon decay. The potential cosmological importance of this phenomenon is quite obvious.

We would also like to remark that these and similar nonelectromagnetic processes ought to be major contributors to the energy loss of monopoles passing through matter. This, too, could be of cosmological importance and deserves further study.

## VI. CONCLUSIONS

The holes in this treatment of the dynamics of the monopole-fermion system are numerous. Our aim has been to show that by making use of the available arsenal of field-theory tricks, it is possible to make progress in a direct attack on the apparently very complicated dynamics of this system. We believe that we have added weight to the claims of Rubakov, Wilczek, and ourselves that some rather remarkable baryon-decay catalysis effects would occur in the presence of a monopole. An urgent problem, which we hope to be able to attack with an extension of these methods, is the semiquantitative computation of the corresponding rates.

*Note added.* I have been informed by V. Rubakov that the reference to his original suggestion of monopole catalysis of baryon decay is Pis'ma Zh. Eksp. Teor. Fiz. 33, 658 (1981) [JETP Lett. 33, 644 (1981)], and that he has also solved the $J=0$ massless fermion-monopole system by bosonization methods essentially identical to those of our Ref. 2.

## ACKNOWLEDGMENTS

I am pleased to acknowledge stimulating discussions with E. Witten and also with C. Besson whose thesis work gave me the initial impulse to study this class of problems. This work was supported in part by the National Science Foundation under Grant No. PHY80-19754.

[1]C. G. Callan, Jr., Phys. Rev. D 25, 2141 (1982).
[2]B. Julia and A. Zee, Phys. Rev. D 11, 2227 (1975).
[3]G. 't Hooft, Nucl. Phys. B79, 276 (1974); A. M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. 20, 430 (1974) [JETP Lett. 20, 194 (1974)].
[4]F. Wilczek, Phys. Rev. Lett. 48, 1146 (1982).
[5]S. Mandelstam, Phys. Rev. D 11, 3026 (1975); S. Coleman, *ibid.* 11, 2088 (1975).
[6]E. Witten, Phys. Lett. 86B, 283 (1979).
[7]R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977).
[8]R. Jackiw and C. Rebbi, Phys. Rev. D 13, 3398 (1976).
[9]D. Boulware, L. Brown, R. Cahn, S. Ellis, and C. Lee, Phys. Rev. D 14, 2708 (1976).
[10]C. Dokos and T. Tomaras, Phys. Rev. D 21, 2940 (1980).
[11]Private communication from P. Langacker.
[12]A. S. Goldhaber, Phys. Rev. D 16, 1815 (1977).
[13]A. Blaer, N. Christ and J. Tang, Phys. Rev. Lett. 47, 1364 (1981); Phys. Rev. D 25, 2128 (1982).
[14]M. Halpern, Phys. Rev. D 12, 1684 (1975).
[15]R. Jackiw, in *Lectures on Current Algebra and its Applications,* edited by S. B. Treiman, R. Jackiw, and D. J. Gross (Princeton University, Princeton, N.J., 1972).
[16]C. Besson, Princeton University Ph.D. thesis, 1981 (unpublished).
[17]S. Weinberg, Phys. Rev. Lett. 40, 223 (1978); F. Wilczek, *ibid.* 40, 1146 (1978).
[18]Y. Kazama, Int. J. Theor. Phys. 17, 249 (1978); P. Rossi, Nucl. Phys. B127, 518 (1977).
[19]G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976).