Possibility of SU(N) antisymmetric-tensor Higgs fields

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The possibility of introducing SU(N) antisymmetric Higgs fields is raised, and a pattern of spontaneous symmetry breaking by an antisymmetric Higgs field is presented in terms of quartic coupling constants. A detailed application is given for breaking an SU(7) gauge symmetry to a realistic $SU(3)_c \times SU(2) \times U(1)$ by two antisymmetric fields $H^{\alpha\beta}$ and $H^{\alpha\beta\gamma}$.

Recently, Georgi[1] has discussed a large gauge theory based on SU(N) groups. The principal motivation for extending the SU(5) gauge theory[2] to a still bigger theory is to look for a solution of the so-called "flavor question." In addition to this flavor question, the "gauge hierarchy problem" has been a theoretical obstacle to the grand unified theories for many years. To understand the problem of gauge hierarchy Susskind[3] considered the hypercolor gauge group in addition to the chromoelectroweak gauge group $SU(3)_c \times SU(2) \times U(1)$. Weinberg,[4] on the other hand, advocated a scheme toward a gauge hierarchy by assuming a massless Higgs doublet at the grand unification mass scale. So far no explicit example of the Weinberg mechanism is known. The ideas of hypercolor gauge group or the massless Higss field of Weinberg may be achievable in a larger gauge group which contains an SU(5) gauge theory. Therefore, the study of SU(N) gauge theory has been a focus of recent attention.[1,5,6] With a larger symmetry, we hope that a realistic example of the Weinberg mechanism can be found.

In this spirit, we study the first step (grand unification mass scale) of spontaneous symmetry breaking by a completely antisymmetric-tensor complex Higgs field of order $2 \leq n \leq N/2$ in a grand unification gauge group SU(N) (this representation has dimension ${}_N C_n$),

$$H^{\alpha_1\alpha_2 \cdots \alpha_n} \text{ or } H_{\alpha_1\alpha_2 \cdots \alpha_n} . \quad (1)$$

The completely antisymmetric Higgs fields being only SU(2) doublets or singlets will lead automatically to a relation $M_Z{}^2 = M_W{}^2 \sec^2\theta_W$. In case some Higgs fields survive the breaking to the chromoelectroweak gauge group, these antisymmetric Higgs fields are *natural choices* for the above weak isospin relation. Previously, only adjoint-representation breaking has been suggested.[2,7]

For simplicity, let us introduce one Higgs field. Although I assume that this field is elementary, the following theorem will be applicable when the effective potential for composite Higgs fields takes the same form as in our case. In this paper I restrict my attention to potentials symmetric under reflection: $H \to -H$.

A completely antisymmetric tensor of order n of U(N) can be formed by n arbitrary vectors and the generalized Kronecker δ's[8]

$$A^{\beta_1\beta_2 \cdots \beta_n} \equiv \delta^{\beta_1\beta_2 \cdots \beta_n}_{\alpha_1\alpha_2 \cdots \alpha_n} X^{\alpha_1}_{(1)} X^{\alpha_2}_{(2)} \cdots X^{\alpha_n}_{(n)} . \quad (2)$$

If the sets of numbers $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ and $\{\beta_1, \beta_2, \ldots, \beta_n\}$ are equal and $\beta_1, \beta_2, \ldots, \beta_n$ are all different, $\delta^{\beta_1 \cdots \beta_n}_{\alpha_1 \cdots \alpha_n} = 1$ or -1. We can choose gauge axes such that the n independent vectors are

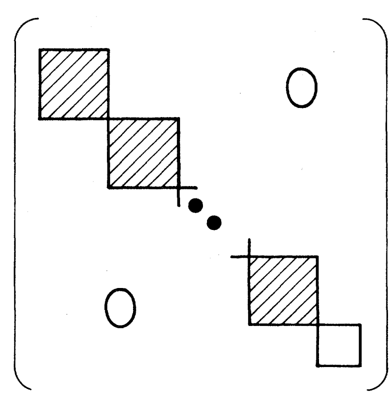

FIG. 1. A standard form of $H^{\alpha_1\alpha_2 \cdots \alpha_n}$. The shaded box is $n \times n$ dimensional.

$X_{(1)}=(10\ldots0,0\ldots0),\ldots,X_{(n)}=(00\ldots1,0\ldots0)$ up to normalization. Therefore, $A^{\beta_1\beta_2\cdots\beta_n}=+1$ for $\beta_i=i$ (and appropriate cyclic permutation of this), and all the other elements are zeros. Since the sum of tensors of the same order, type, and weight can be added to give another tensor of the same order, type, and weight, and we can choose gauge axes such that N independent vectors are $X^{\gamma_i}_{(i)}=\delta_{i,\gamma_i}$ ($i=1,2,\ldots,N$), an arbitrary antisymmetric tensor of order n can be reducible to a direct sum of (2), as shown in Fig. 1.

In Fig. 1, the n-dimensional shaded boxes (when $n=2$, these are 2×2 matrices) are irreducible and all the other elements are zeros. The small open box, whose dimension is smaller than the shaded boxes, is absent when $[N/n]n=N$,[9] otherwise zeros. Hence vacuum expectation values can be made block diagonal in the sense of Fig. 1 with $[N/n]$ different vacuum expectation values: $H^{12\ldots n}=v_1,\ H^{n+1\ldots2n}=v_2,\ldots,\ H^{nl-n+1\ldots nl}=v_l$ and their cyclic permutations with zeros for all the other elements. The most general Higgs potential is

$$V=-\frac{\mu^2}{n!}J+\frac{\Lambda}{(n!)^2}J^2+\sum_{i=1}^{[n/2]}c_i^{[N,n]}K_i\ , \tag{3}$$

where

$$J=H_{\alpha_1\alpha_2\cdots\alpha_n}H^{\alpha_1\alpha_2\cdots\alpha_n}\ , \tag{3a}$$

$$K_i=H_{\alpha_1\cdots\alpha_i\alpha_{i+1}\cdots\alpha_n}H^{\beta_1\cdots\beta_i\alpha_{i+1}\cdots\alpha_n}H^{\alpha_1\cdots\alpha_i\beta_{i+1}\cdots\beta_n}H_{\beta_1\cdots\beta_i\beta_{i+1}\cdots\beta_n}\ , \tag{3b}$$

with the coefficients

$$c_i^{[N,n]}=\frac{c_i}{i!(n-i)!n!},\quad C\equiv\sum_{i=1}^{[n/2]}c_i\ . \tag{3c}$$

The c_i are chosen such that the vacuum expectation value of V takes a simple form,

$$\begin{aligned}\langle V\rangle=&-\mu^2(v_1^2+v_2^2+\cdots+v_l^2)\\&+\Lambda(v_1^2+v_2^2+\cdots+v_l^2)^2\\&+C(v_1^4+v_2^4+\cdots+v_l^4)\ .\end{aligned} \tag{4}$$

The extremum conditions of $\langle V\rangle$ are

$$v_i[\Lambda(v_1^2+v_2^2+\cdots+v_l^2)+Cv_i^2]=\frac{\mu^2}{2}v_i\ . \tag{5}$$

For $\mu^2>0$, not all v_i can be zero. I choose v_1 as a nonvanishing entry,

$$Cv_1^2+\Lambda(v_1^2+v_2^2+\cdots+v_{[N/n]}^2)=\frac{\mu^2}{2}\ . \tag{6}$$

If another entry, say v_k is nonvanishing, I have

$$v_1^2=v_k^2 \tag{7}$$

for the case $C\neq0$, by considering (6) and (5) for $i=k$.

Therefore, I obtain $[N/n]$ different possibilities for the symmetry-breaking pattern:

$$\begin{aligned}&\text{case (1)}\quad v_1^2\neq0,\quad v_2^2=\cdots v_{[N/n]}^2=0\ ,\\&\text{case }(k)\quad v_1^2=v_2^2=\cdots=v_k^2\neq0\ ,\\&\qquad\qquad v_{k+1}^2=\cdots=v_{[N/n]}^2=0\ ,\end{aligned} \tag{8}$$

for $1\le k\le[N/n]$. Certainly, the minimum of the $[N/n]$ cases lead to the absolute minimum of the classical potential V, when the bounded energy condition is satisfied. For case (k), we have

$$F^k(\Lambda,C)\equiv\frac{\langle V\rangle}{\mu^4}=\frac{-1}{4(\Lambda+C/k)}\ . \tag{9}$$

If case (k) is the minimum of $\langle V\rangle$, $\langle V\rangle$ for case (k) should be less than those of other cases. Therefore, it is appropriate to calculate a testing function T,

$$\begin{aligned}T^{k,m}&\equiv F^{k+m}-F^k\\&=-\frac{m}{4k(k+m)}\frac{C}{(\Lambda+C/k)[\Lambda+C/(k+m)]}\end{aligned} \tag{10}$$

with $1\le(k\text{ and }k+m)\le[N/n]$, where m can be negative or positive.

On the other hand, the condition for the bounded energy

$$\Lambda+\frac{C}{k'}>0 \tag{11}$$

should be satisfied for all $1\le k'\le[N/n]$. Therefore, $\Lambda+C/[N/n]>0$ satisfies (11) for $C>0$, and $\Lambda+C>0$ satisfies (11) for $C<0$. For the symmetry breaking to realize the value k for $\mu^2>0$, and nonzero Λ and C, we need

$$C\left[\Lambda+\frac{C}{k}\right]\left[\Lambda+\frac{C}{k+m}\right]\begin{cases}<0\,,\\>0\end{cases} \tag{12}$$

for all the integers

$$m\begin{cases}>0\,,\\<0\,,\end{cases}$$

respectively, satisfying $1\le k+m\le[N/n]$. From (11) and (12), we obtain for $n\le[N/2]$

$$\begin{aligned}&k=[N/n]\quad\text{for } C>0\,,\\&k=1\quad\text{for } C<0\,.\end{aligned} \tag{13}$$

Therefore, the SU(N) gauge symmetry-breaking pattern is

(A) Sp($2k$) where $k=[N/2]$, when $n=2$, $C>0$, and $\Lambda+C/k>0$,

(B) $[\mathrm{SU}(n)]^k$ with $k=[N/n]$ and $N-n[N/n]=0$ or 1, when $3\le n\le[N/2]$, $C>0$, and $\Lambda+C/k>0$,

(C) SU(n)$\times$SU($N-n$) when $2\le n<[N/2]$, $C<0$, and $\Lambda+C>0$.

The cases (A) and (C) for $n=2$ were considered by Li.[10] For the overlapping case of (C), our pattern is slightly different from his. The unbroken Sp($2k$) or SU(n) groups are obtained by calculating the gauge-boson masses.

In the following, we discuss an SU(7) gauge theory as an application of the above result. We consider two antisymmetric Higgs fields $H^{\alpha\beta}$ and $H^{\alpha\beta\gamma}$. The potential consistent with the individual reflection symmetries is

$$V=-\frac{\mu_1{}^2}{2}H_{\alpha\beta}H^{\alpha\beta}-\frac{\mu_2{}^2}{6}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma}+\frac{\lambda_1}{4}\left[H_{\alpha\beta}H^{\alpha\beta}\right]^2+\frac{\lambda_2}{36}\left[H_{\alpha\beta\gamma}H^{\alpha\beta\gamma}\right]^2+\frac{h_1}{2}H^{\alpha\rho}H_{\alpha\beta}H^{\gamma\beta}H_{\gamma\rho}$$
$$+\frac{h_2}{12}H^{\mu\nu\rho}H_{\mu\nu\sigma}H^{\alpha\beta\sigma}H_{\alpha\beta\rho}+\frac{\lambda_3}{6}H_{\alpha\beta}H^{\alpha\beta}H_{\mu\nu\rho}H^{\mu\nu\rho}+\frac{\lambda_4}{2}H^{\alpha\beta}H_{\alpha\mu}H^{\mu\rho\sigma}H_{\beta\rho\sigma}+\frac{\lambda_5}{2}H^{\alpha\beta}H_{\alpha\beta\mu}H_{\rho\sigma}H^{\rho\sigma\mu}\,. \tag{14}$$

To apply the above result, let us assume

$$|\lambda_3|,|\lambda_4|,|\lambda_5|\ll|\lambda_1|,|\lambda_2|,|h_1|,|h_2|\,. \tag{15}$$

Now we can use the symmetry-breaking pattern separately for two different Higgs fields. For the $H^{\alpha\beta}$ case $\Lambda=\lambda_1$ and $C=h_1$, and for the $H^{\alpha\beta\gamma}$ case $\Lambda=\lambda_2$ and $C=h_2$. We obtain

$$\mathrm{SU}(7)\to\mathrm{SU}(5)\times\mathrm{SU}(2)\quad\text{for } h_1<0 \text{ and } \lambda_1>|h_1|\,, \tag{16}$$

$$\mathrm{SU}(7)\to\mathrm{Sp}(6)\quad\text{for } h_1>0 \text{ and } \lambda_1>-h_1/3\,, \tag{17}$$

$$\mathrm{SU}(7)\to\mathrm{SU}(4)\times\mathrm{SU}(3)\quad\text{for } h_2<0 \text{ and } \lambda_2>|h_2|\,, \tag{18}$$

$$\mathrm{SU}(7)\to\mathrm{SU}(3)\times\mathrm{SU}(3)\quad\text{for } h_2>0 \text{ and } \lambda_2>-h_2/2\,. \tag{19}$$

The locking of $H^{\alpha\beta}$ and $H^{\alpha\beta\gamma}$ is determined by mixed terms λ_3, λ_4, and λ_5. I consider only case (16). In this case I choose nonvanishing elements of $\langle H^{\alpha\beta}\rangle$ as

$$\langle H^{67}\rangle=-\langle H^{76}\rangle=v_2 \tag{20}$$

and all the other elements are zeros. For Eq. (18), there are three possibilities for nonvanishing $\langle H^{\alpha\beta\gamma}\rangle=v_3$: (a) $\langle H^{567}\rangle$, (b) $\langle H^{457}\rangle$, and (c) $\langle H^{345}\rangle$ without loss of generality. The three cases lead to the classical vacuum energies due to mixed terms, $\Delta\langle V\rangle/v_2{}^2v_3{}^2=2(\lambda_3+\lambda_4+\lambda_5)$, $2(\lambda_3+\lambda_4/2)$, and $2\lambda_3$, for cases (a), (b), and (c), respectively (Fig. 2). A physically realistic situation results when case (b) is the minimum of the three cases: *SU(7) breaks to SU(3)$\times$SU(2)$\times$U(1).* This happens when

$$\lambda_4<0,\quad \lambda_4+2\lambda_5>0\,. \tag{21}$$

A similar consideration for Eq. (19) leads to unphysical situations.

For Eqs. (16), (18), and (21), the unbroken gauge symmetry axes are 1,2,3 for $\mathrm{SU}(3)_c$ and 4,5 for SU(2). Defining the charge of the charged weak gauge boson $W_5{}^4$ as $+1$, the charges of the fundamental representation are uniquely determined for $e_5=0$ since the sum of the charges adds to zero:

$$(-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},1,0,1,-1)\,. \tag{22}$$

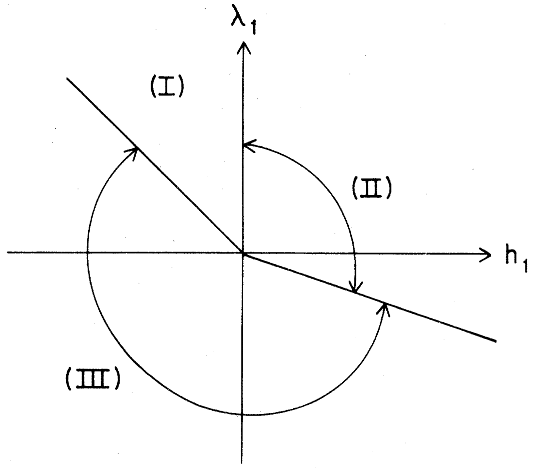

FIG. 2. An example of $H^{\alpha\beta}$ in SU(7). The lines are $\lambda_1 = -h_1$ and $-h_1/3$. The SU(7) gauge symmetry breaks down to (I) SU(5)×SU(2) and (II) Sp(6). The region (III) is unphysical.

The choice of charge assignment is determined by the choice of the quartic coupling constants.

Note added. An erroneous statement regarding a massless Higgs doublet has been removed from the original version of this paper, Report No. UPR-144T, 1980 (unpublished). The symmetry-breaking pattern (22) has been used by J. E. Kim [Phys. Rev. Lett. 45, 1916 (1980) and Phys. Rev. D 23, 2706 (1981)], and by S. Dimopoulos and F. Wilczek, ITP report, 1981 (unpublished).

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[9] $[x]$ means the largest integer not greater than x. In this paper $l = [N/n]$.

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