Properties of Yang-Mills theories with gauge-fixing conditions on the field strength

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(Received 1 July 1981)

Within the Hamiltonian formulation and by means of Dirac's constraint formalism we investigate the properties of Yang-Mills theories when the gauge-fixing conditions involve the field strength E_a^i. We show, e.g., that $C_{abi}E_b^i=0$, where C_{abc} are the structure constants of the gauge group, are satisfactory gauge conditions for any semisimple gauge group in a basis where the first three indices represent a minimal SU(2) embedding. We also give an unconstrained formulation of the theory. This generalizes a previous similar treatment of the SU(2) case by Goldstone and Jackiw. We argue that this formalism could be used to study the quantum excitations around, for instance, the Julia-Zee dyon solution.

I. INTRODUCTION

We shall consider, to start with, a pure Yang-Mills theory with a compact semisimple gauge group given by the Lagrangian density

$$\mathscr{L}(x)=-\tfrac{1}{4}F_{a\mu\nu}(x)F_a^{\mu\nu}(x)\ , \tag{1.1}$$

where

$$F_{a\mu\nu}(x)\equiv\partial_\mu A_{a\nu}-\partial_\nu A_{a\mu}+gC_{abc}A_{b\mu}A_{c\nu}\ . \tag{1.2}$$

C_{abc} are the totally antisymmetric structure constants of the gauge group which are assumed to satisfy $C_{acd}C_{bcd}\propto\delta_{ab}$. (Our general results are valid even if the representation does not satisfy this relation, but a more careful analysis is then required.) The group index a runs from 1 to m (the order of the group). The spacetime metric is timelike. The transition to the Hamiltonian formulation is obtained by means of the canonical conjugate momenta to $A_{a\mu}$ defined by

$$E_a^\mu(x)\equiv\frac{\partial\mathscr{L}(x)}{\partial\dot{A}_{a\mu}}=F_a^{\mu 0}(x)\ , \tag{1.3}$$

which satisfy the basic equal-time Poisson-bracket (PB) relations

$$\{A_a^\mu(x),\ E_{b\nu}(y)\}=\delta_\nu^\mu\delta_{ab}\delta^3(x-y)\ . \tag{1.4}$$

The canonical Hamiltonian density is in turn given by

$$\mathscr{H}_c(x)=-\tfrac{1}{2}E_a^i(x)E_{ai}(x)+\tfrac{1}{4}F_a^{ij}(x)F_{aij}(x)-gA_a^0(x)G_a(x)\ , \tag{1.5}$$

where

$$G_a(x)\equiv\frac{1}{g}D_iE_a^i\equiv\frac{1}{g}\partial_iE_a^i(x)-C_{abc}E_b^iA_{ci}\ . \tag{1.6}$$

Now this is a Hamiltonian system with constraints[1] since (1.3) contains the primary constraints

$$E_a^0(x)\approx 0\ . \tag{1.7}$$

Consistency $[\dot{E}_a^0(x)\approx 0]$ requires furthermore the secondary constraints

$$G_a(x)\approx 0 \tag{1.8}$$

for which $\dot{G}_a(x)\approx 0$ is satisfied. All these constraints are first class,[1] since

$$\begin{aligned}&\{G_a(x),\ G_b(y)\}=C_{abc}G_c(x)\delta^3(x-y)\ ,\\&\{E_a^0(x),\ G_b(y)\}=0\ .\end{aligned} \tag{1.9}$$

According to Dirac's constraint formalism such constraints may be eliminated by means of gauge-fixing conditions. E_a^0 is eliminated by fixing the variables A_a^0. This is a trivial elimination leaving the phase space spanned by E_a^i and A_a^i. The physical phase space is then reached by means of m gauge conditions

$$\chi_a(x)\approx 0 \tag{1.10}$$

to $G_a(x)\approx 0$. In order for χ_a to be satisfactory the matrix

$$M_{ab}(x,y)\equiv\{G_a(x),\ \chi_b(y)\} \tag{1.11}$$

must have an inverse in the following sense:

$$\int d^3z\, M_{ab}(x,z)(M^{-1})_{bc}(z,y)=\int d^3z (M^{-1})_{ab}(x,z) M_{bc}(z,y)=\delta_{ac}\delta^3(x-y) . \tag{1.12}$$

When this is the case, the Poisson brackets for the physical phase space may be expressed in terms of the original ones by means of the Dirac brackets

$$\begin{aligned}\{A(x),B(y)\}^*=\{A(x),B(y)\}+\int\int d^3z\,d^3z'\,[\,\{B(y),\chi_a(z)\}(M^{-1})_{ab}(z,z')\{G_b(z'),A(x)\}\\ -\{A(x),\chi_a(z)\}(M^{-1})_{ab}(z,z')\{G_b(z'),B(y)\}\,]\\ -\int\int\int\int d^3z\,d^3z'\,d^3z''\,d^3z'''\,[\,\{A(x),G_a(z)\}(M^{-1})_{ba}(z,z')\{\chi_b(z'),\chi_c(z'')\}\\ \times(M^{-1})_{cd}(z'',z''')\{G_d(z'''),B(y)\}\,] .\end{aligned} \tag{1.13}$$

Usually the transformation properties of the field under global symmetry transformations look differently in the physical space than in the original one. Let J be a generator of a global symmetry transformation. Then J is gauge invariant, i.e.,

$$\{G_a(x),J\}\approx 0 \tag{1.14}$$

and by means of the Dirac brackets (1.13) we find

$$\{A_{ai}(x),J\}^*=\{A_{ai}(x),J\}+D_i W_{Ja} , \tag{1.15}$$

where

$$W_{Ja}(x)=\frac{1}{g}\int d^3z\{J,\chi_b(z)\}(M^{-1})_{ba}(z,x) . \tag{1.16}$$

Hence it is the symmetry properties of the gauge conditions that determine the transformation properties in the physical space. Take, e.g., $J=H$ where H is the Hamiltonian, i.e.,

$$\begin{aligned}&H=H_0+H_G ,\\ &H_0=\int d^3x(-\tfrac{1}{2}E_a^iE_{ai}+\tfrac{1}{4}F_a^{ij}F_{aij}) ,\\ &H_G=-g\int d^3x A_a^0(x)G_a(x) ,\end{aligned} \tag{1.17}$$

where H_G is just a gauge transformation. Inserted into (1.15) we get

$$\begin{aligned}\dot{A}_{ai}(x)&=\{A_{ai}(x),H\}^*\\ &=\{A_{ai}(x),H_0\}^*\\ &=-E_{ai}(x)+D_i\widetilde{A}_a^0(x) ,\end{aligned} \tag{1.18}$$

where

$$\widetilde{A}_a^0=\frac{1}{g}\int d^3z\{H_0,\chi_b(z)\}(M^{-1})_{ba}(z,x) . \tag{1.19}$$

Hence if we have made the gauge choice $A_a^0=\widetilde{A}_a^0$, then we would have $\dot{\chi}_a=0$ and

$$\{A_{ai}(x),H\}^*=\{A_{ai}(x),H\} . \tag{1.20}$$

We shall come back to global transformation properties later.

There is only a limited set of gauge conditions considered so far in the literature. There are the standard Coulomb $[\partial_i A_a^i(x)\approx 0]$ and axial $[A_a^3(x)\approx 0]$ gauge choices for which $(M^{-1})_{ab}(x,y)$ exists at least in perturbation theory. (See, e.g., Ref. 2.) Other gauge choices are, e.g., the radial gauge $x^iA_{ai}(x)\approx 0$, (see, e.g., Ref. 3) and $\epsilon_{abi}E_b^i\approx 0$ in the SU(2) case treated by Goldstone and Jackiw[4,5] and the more complicated gauge choices considered in Ref. 6.

In the present paper we shall investigate some properties of linear gauge conditions on the field strength E_a^i including the Goldstone-Jackiw one for SU(2). General conditions on coefficients as well as on the field strengths are derived.

II. GAUGE FIXING OF E_{ai}

First let us for simplicity eliminate A_a^0 from the theory and then introduce the Euclidean metric for the space indices. We have then the basic PB relation

$$\{E_{ai}(x),A_{bj}(y)\}=\delta_{ij}\delta_{ab}\delta^3(x-y) . \tag{2.1}$$

We shall consider gauge conditions of the type

$$\chi_a\equiv B_{abi}E_{bi}(x)\approx 0 , \tag{2.2}$$

where B_{abi} are constants. A necessary condition for this to be a satisfactory gauge condition is that χ_a represents m algebraically independent variables different from zero. Thus we have the following.

Condition I (necessary): The constants B_{abi} when viewed as an $m\times(m\times 3)$ matrix must have rank m. A both sufficient and necessary condition is that the matrix

$$M_{ab}(x,y)=\{G_a(x),\chi_b(y)\}$$
$$=-C_{acd}E_{ci}(x)B_{bdi}\delta^3(x-y) \qquad (2.3)$$

is invertible in the sense of (1.12), which here leads to the following.

Condition II (necessary and sufficient): The matrix

$$M_{ab}(x)\equiv C_{acd}E_{ci}(x)B_{bdi} \qquad (2.4)$$

must be nonsingular. Far from all B's which satisfy condition I, also satisfy condition II. Take, e.g., $B_{cdk}=\delta_{k3}\delta_{cd}$ which yields the gauge condition $E_{a3}=0$, but which implies $M_{ab}=0$. Thus the constants B_{bdi} have at least to mix group and space indices in order for (2.4) to be invertible. A natural choice of B_{bdi} which satisfies condition I for any semisimple gauge group is

$$B_{bdi}=C_{bdi}\ , \qquad (2.5)$$

where C_{abc} are the structure constants in a basis where the first three indices correspond to the so-called minimal embedding of the SU(2) algebra. This embedding (which always exists) has the property that no (nonzero) element of the gauge group algebra commutes with all elements of the SU(2) subalgebra (see Ref. 7). This property implies that if $\alpha_a C_{abk}=0$ for all b and k, then $\alpha_a\equiv 0$, which is equivalent to the statement that C_{abk} considered as an $m\times(m\times 3)$ matrix has rank m. That minimal embedding exists for the special case SU(N) can easily be shown by considering the irreducible, Hermitian N-dimensional representation of SU(2). The matrices of such a representation obviously form a subalgebra. However, since the representation is irreducible, then according to Schur's lemma there is no matrix other than a multiple of the unit matrix which commutes with the SU(2) matrices. Now the unit matrix does not belong to the Lie algebra of SU(N) and therefore there is no Lie-algebra element which commutes with the SU(2) algebra.

The minimal embedding has some further noteworthy properties. When the adjoint representation of the gauge group algebra is decomposed with respect to the SU(2) subalgebra, only integer spins j occur and furthermore $j\geq 1$ (see Ref. 7). Therefore the gauge group algebra is decomposed into orthogonal subspaces j_l ($l=1,2,\ldots$) of dimension $(2j_l+1)$ under the action of the SU(2) subalgebra. Also the field strength E_{di} can be similarly decomposed into subspaces when d labels vectors in the different subspaces j, but they are of course not irreducible under SU(2) since E_{di} also carries the space index i. Under the joint action of SU(2) in the gauge group and SO(3) on the spatial indices, the linear combination $C_{bdi}E_{di}$ can be shown to transform as a spin-j_l vector if d is restricted to the subspace j_l. (Use the fact that C_{aci} are representation matrices in the adjoint representation.) Therefore our gauge condition can be stated as the elimination of the spin-j_l states occurring in the coupling of the spin j_l from the gauge group and the spin 1 from the spatial degrees of freedom. The joint action of SU(2) in the gauge group and SO(3) is then also the natural realization of rotational symmetry since it respects the gauge condition. The generators of rotational symmetry are thus J_k+T_k, where J_k are the generators of SO(3) and T_k those of SU(2) in the gauge group.

We would like to mention that there exist other similar expressions for B_{bdi} which satisfy the necessary condition [e.g. $B_{bdi}=C_{bdi+3}$ in the representation (2.14) of SO(4)]. However they are not of such a general character as (2.5) and we shall therefore consider only the minimal-embedding case for B_{bdi} in what follows.

We turn now to the analysis of the matrix (2.4). When (2.5) is inserted we obtain

$$M_{ab}(x)=C_{acd}E_{ci}(x)C_{dib}\ . \qquad (2.6)$$

Using the Jacobi identity of the structure constants we find

$$M_{ab}(x)=M_{ba}(x)-C_{abc}\chi_c(x)\ , \qquad (2.7)$$

where $\chi_a(x)=C_{abi}E_{bi}(x)$. Hence, when the gauge condition $\chi_a=0$ is imposed the matrix $M_{ab}(x)$ becomes symmetric. Another property of M_{ab} is

$$\mathrm{Tr}M_{ab}(x)=\eta E_{ii}(x)\ ,$$

where η is a constant related to the normalization of the structure constants. The necessary and sufficient condition for $\chi_a=0$ to be a gauge condition is Det $M_{ab}(x)\neq 0$ which divides $E_{ci}(x)$ in at least two sectors; those characterized by Det $M_{ab}(x)>0$ and those for which Det $M_{ab}(x)<0$. The values of $E_{ci}(x)$ for which Det $M_{ab}(x)=0$ must be excluded to start with, but one could try to add them in a consistent fashion to the final physical theory. What we should prove is that the set of values of $E_{ci}(x)$ for which Det $M_{ab}(x)=0$ is a null set. This we shall not do in its full generality, but we shall give strong arguments that this is the case. First we show that there exists an open set of E_{ci} for which Det $M_{ab}\neq 0$. Consider $E_{ci}=\lambda(x)\delta_{ci}+\epsilon_{ci}$, where ϵ_{ci} are small, then $M_{ab}=\lambda(x)m_{ab}+\eta_{ab}(x)$ where $m_{ab}=C_{aid}C_{dib}$ and $\eta_{ab}(x)$ small. m_{ab} is nonsingular, since condition I is satisfied. Hence, when λ is large compared to ϵ_{ci} we have Det $M_{ab} \approx \lambda^m(x)$ Det $\rho_{ab}+\mathrm{Tr}\eta_{ab}(x)\neq 0$. Next we consider some explicit examples.

In an SU(2) Yang-Mills theory the minimal embedding is trivial. We have the gauge condition (cf. Ref. 5)

$$\chi_a(x)=\epsilon_{abi}E_{bi}(x)\approx 0 \ . \tag{2.8}$$

Hence, $E_{bi}(x)$ is a symmetric 3×3 matrix. The matrix M_{ab} becomes here

$$M_{ab}(x)=\epsilon_{acd}E_{ci}(x)\epsilon_{dib}=\delta_{ab}\mathrm{Tr}E-E_{ab} \ , \tag{2.9}$$

which leads to

$$\mathrm{Det}M_{ab}(x)=\tfrac{1}{3}[(\mathrm{Tr}E)^3-\mathrm{Tr}E^3] \ . \tag{2.10}$$

Since $E_{ai}(x)$ is symmetric, there exists an orthogonal matrix R_{ab} which diagonalizes $E_{ai}(x)$. We have

$$E_{ab}(x)=R^T_{ac}D_{cd}R_{db} \ , \tag{2.11}$$

where $D_{ab}(x)=\delta_{ab}\lambda_b(x)$. When this expression is inserted into (2.10) we get

$$\mathrm{Det}M_{ab}(x)=(\lambda_1+\lambda_2)(\lambda_1+\lambda_3)(\lambda_2+\lambda_3) \ . \tag{2.12}$$

Those values of E_{ai} for which Det $M_{ab}=0$ are thus characterized by the surfaces $\lambda_1=-\lambda_2$ or $\lambda_1=-\lambda_3$ or $\lambda_2=-\lambda_3$. These values have to be excluded, which then divide E_{ai} into eight nonsingular sectors. (This analysis should be equivalent to the apparently different analysis in Ref. 4.) For a general Yang-Mills theory we shall first consider a very particular case, namely when $E_{\bar{c}k}=0$ for $\bar{c}\neq 1,2,3$. The matrix M_{ab} becomes then

$$M_{kl}(x)=\epsilon_{kjm}E_{ji}(x)\epsilon_{mil} \ ,$$
$$M_{\bar{a}k}(x)=0, \quad \bar{a}\neq 1,2,3 \ , \tag{2.13}$$
$$M_{\bar{a}\bar{b}}(x)=C_{\bar{a}j\bar{d}}E_{ji}(x)C_{\bar{d}i\bar{b}}, \quad \bar{a},\bar{b}\neq 1,2,3$$

which implies Det $M_{ab}=\mathrm{Det}M_{kl}\mathrm{Det}M_{\bar{a}\bar{b}}$. The requirement Det $M_{ab}\neq 0$ is thus equivalent to the SU(2) conditions above together with Det $M_{\bar{a}\bar{b}}\neq 0$. We expect therefore that the number of nonsingular sectors of E_{ji} increases with the order of the gauge group. For arbitrary $E_{\bar{c}k}$ the positions of these sectors depend on $E_{\bar{c}k}$ or we may decrease the number of nonsingular sectors of E_{ji} by dividing also $E_{\bar{c}k}$ into such sectors. We illustrate these properties by an explicit example. In the SO(4) case with the nonzero structure constants C_{abc} given by

$$C_{ijk}=\epsilon_{ijk}, \ C_{i+3,j+3,k}=\epsilon_{ijk} \tag{2.14}$$

we have $M_{i+3,j+3}=M_{ij}$ and Det $M_{ab}=(\mathrm{Det}M_{ij})^2-(\mathrm{Det}\tilde{M}_{ij})^2$ where $\tilde{M}_{ij}=M_{i+3,j}=\epsilon_{ilm}E_{l+3,k}\epsilon_{mkj}$. Hence, when $E_{l+3,k}=0$ the nonsingular sectors of $E_{ij}(x)$ are exactly those of SU(2), and when $E_{l+3,k}\neq 0$ the positions of these sectors have changed and a further sector has entered. We see also the alternative possibility of letting $E_{l+3,k}$ be divided into these nonsingular sectors leaving E_{ij} free. In the SU(3) case which is more complex we have checked that the general feature as described above still holds. [In this connection we would like to mention that the SU(3) treatment given in Ref. 8 is different from ours although the minimal embedding is important there as well.] We conclude that apart from a null set of field configurations the gauge condition

$$\chi_a(x)\equiv C_{abi}E_{bi}(x)\approx 0 \tag{2.15}$$

is fully satisfactory.

When (2.15) is imposed we may construct a completely unconstrained formulation of the Yang-Mills theory. The expression for $A_{ai}(x)$ in the physical phase space is obtained by solving Gauss's law $G_a(x)=0$, Eq. (1.6). Obviously the following expression is a solution (cf. Ref. 9):

$$A_{ai}(x)=S_{ai}(x)-C_{aib}(M^{-1})_{bc} \times\left[-\frac{1}{g}\partial_k E_{ck}(x)+C_{cde}E_{dk}S_{ek}\right] , \tag{2.16}$$

where $S_{ai}(x)$ may be chosen to satisfy

$$C_{abi}S_{bi}(x)=0 \ . \tag{2.17}$$

This last equation makes it possible to invert (2.16) yielding

$$S_{ai}(x)=A_{ai}(x)-C_{aib}\rho_{bc}C_{cdk}A_{dk} \ , \tag{2.18}$$

where ρ_{bc} is the inverse of the matrix $C_{aib}C_{aic}$, which is nonsingular since condition I is satisfied. By means of the Dirac brackets (1.13) one obtains

$$\{S_{ai}(x),\ S_{bj}(y)\}^*=0 \ ,$$
$$\{E_{ai}(x),\ S_{bj}(y)\}^*=K_{ai,bj}\delta^3(x-y) \ , \tag{2.19}$$
$$K_{ai,bj}\equiv\delta_{ab}\delta_{ij}-C_{aic}\rho_{cd}C_{bjd} \ .$$

The constants $K_{ai,bj}$ satisfy $C_{cai}K_{ai,bj}=C_{cbj}K_{ai,bj}=0$ and $K_{ai,ai}=2m$. Hence $S_{ai}(x)$ and $E_{ai}(x)$ are $(4m)$ canonical variables that span the physical phase space. When (2.16) is inserted into the Hamiltonian (1.16) we obtain a completely unconstrained formulation of the Yang-Mills theory which may be quantized by standard techniques (cf. Ref. 6).

III. SOME PROPERTIES OF THE YANG-MILLS THEORY UNDER GAUGE CONDITIONS (2.15)

What are the properties of the Yang-Mills theory when the gauge condition (2.15) is imposed? First we notice that the conventional perturbation expansion is not valid. This may, e.g., be understood from the fact that Gauss's law (1.16) becomes $\partial_i E_{ai}(x)\approx 0$ in the $g\to 0$ limit which then has zero PB with χ_a. Hence, the gauge condition $\chi_a=0$ is not effective in the $g\to 0$ limit (see also Refs. 4 and 5). The solutions are thus nonperturbative. Maybe the theory describes, e.g., a monopole sector. Indeed in this gauge spherically symmetric monopole solutions are easy to obtain. To see this we first notice that since the gauge condition is invariant under global transformations generated by J_k+T_k, $k=1,2,3$, where J_k is the generator of rotations and T_a the generator of global gauge transformations, we have, due to (1.15)

$$\begin{aligned}\{A_{ai}(x),\, J_k+T_k\}^* &= \{A_{ai}(x),J_k+T_k\}\ ,\\ \{E_{ai}(x),J_k+T_k\}^* &= \{E_{ai}(x),J_k+T_k\}\ .\end{aligned} \tag{3.1}$$

Hence, the equations

$$\{A_{ai}(x),J_k+T_k\}=\{E_{ai}(x),J_k+T_k\}=0 \tag{3.2}$$

may be consistently imposed even in the physical phase space. (For the general connection between these equations and monopole solutions, see, e.g., Ref. 10.) The solutions are

$$\begin{aligned}E_{ai}(x)&=\alpha(r,t)\delta_{ai}+\beta(r,t)x^a x^i\ ,\\ A_{ai}(x)&=\gamma(r,t)\delta_{ai}+\rho(r,t)x^a x^i\\ &\quad+\epsilon_{aik}x^k H(r,t)\end{aligned} \tag{3.3}$$

for $a=1,2,3$. Equation (2.18) implies

$$S_{ai}(x)=\gamma(r,t)\delta_{ai}+\rho(r,t)x^a x^i\ . \tag{3.4}$$

Comparison with the expression (2.16) for $A_{ai}(x)$ leads to the relation

$$x^k H(r_1 t)=\frac{1}{g}(M^{-1})_{kl}\partial_i E_{li} \tag{3.5}$$

which by means of (3.3) becomes

$$H(r,t)=\frac{1}{g}\frac{\alpha'(r,t)+\beta'(r,t)r^2+4\beta r}{2\alpha r}\ . \tag{3.6}$$

[Notice that $(M^{-1})_{kl}$ is the inverse of M_{kl} when $E_{ak}=0$ for $a\neq 1,2,3$.] Now choosing $S_{ai}=0$ and $\alpha(r,t)=c_1/r^2$, $\beta(r,t)=c_2/r^4$, we get $H(r,t)=-1/gr^2$, which when inserted into (3.3) yields the Wu-Wang monopole solution.[11] However, this choice is only a solution to the equations of motion in the limit $E_{ai}\to 0(c_1,c_2\to 0)$, i.e., when we scale E_{ai} to zero in (3.5). This is a general property. When we introduce Higgs fields, we have to add

$$\{\varphi_a(x),J_k+T_k\}=0,\quad a=1,2,3 \tag{3.7}$$

to Eq. (3.2), whose solution is

$$\varphi_a(x)=x^a h(r,t)\ . \tag{3.8}$$

There always exists an expression for E_{ai} which yields the standard monopoles solutions.[12] [Since $P_a=D_0\phi_a=0$ for monopole solutions, the expression (3.5) still holds.] However, they are only solutions to the equations of motion when we scale E_{ai} to zero in (3.5). (In addition, we have to set $A_{ai}=E_{ai}=\phi_a=0$ for $a\neq 1,2,3$.) The problem is now that $E_{ai}=0$ is the most singular value in the theory (leading to $M_{ab}=0$) and we do not know if this value may be included in a consistent fashion. What we need is to determine the restrictions of the $E_{ai}\to 0$ limit of the theory which seems very difficult. However, there is a spherical symmetric solution to Yang-Mills for which $E_{ai}\neq 0$, namely, the Julia-Zee dyon solution[13] (for vanishing Higgs potential[12]):

$$\begin{aligned}A_{ai}&=-\epsilon_{aij}\frac{x^j}{gr^2}[1-K(\xi)]\ ,\\ \varphi_a&=\frac{x^a}{gr^2\cos\theta}H(\xi)\ ,\\ E_{ak}&=(D_k\varphi)_a\sin\theta\ ,\end{aligned} \tag{3.9}$$

where

$$\begin{aligned}&K(\xi)=\frac{\xi}{\sinh\xi},\quad H(\xi)=\xi\coth\xi-1\ ,\\ &\xi=\beta r\cos\theta,\quad \tan\theta=\frac{q_E}{q_M}\ ,\end{aligned} \tag{3.10}$$

where in turn q_E and q_M are the electric and magnetic charge of the dyon, respectively. One may easily check that E_{ai} ($a=1,2,3$) is of the form (3.3) and that Eq. (3.6) is satisfied [(3.5) is valid since $D_0\phi_a=0$]. The determinant of the matrix M_{ij} is given by

$$\mathrm{Det}M_{ij}=\left[\frac{1}{g}\tan\theta\right]^3\frac{2}{r^6}H(\xi)K(\xi)\times\{1+K(\xi)[H(\xi)-K(\xi)]\}^2\,, \quad (3.11)$$

which is positive definite and finite for finite r and at $r\to0$, but goes like $e^{-\beta\cos\theta r}$ at $r\to\infty$. Hence, M_{ij} is nonsingular for all r except in the limit $r\to\infty$.

IV. CONCLUSIONS

We have given an unconstrained formulation of the Yang-Mills theory for an arbitrary gauge group which generalizes the SU(2) construction by Goldstone and Jackiw. The minimal embedding of SU(2) in the given gauge group played an important role in our construction. The resulting theory may not be used to study quantum fluctuations either around the conventional vacuum or around the spherically symmetric monopole solutions. It may, however, be used to study fluctuations around the Julia-Zee dyon solution since in this case the gauge condition is nonsingular.

ACKNOWLEDGMENT

R.M. was supported by the Swedish Natural Science Research Council under Contract No. F-UR 7310-108.

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