Quark-diagram analysis of hyperon radiative decays. General formulation

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We calculate hyperon radiative decays using quark-diagram analysis taking both weak and electromagnetic interactions to be short-distance processes. The second-quantized quark fields are expanded in terms of MIT bag modes, so as to be consistent with quark confinement. In this paper, we present the general formulation of the quark-diagram calculation and use the $\Sigma^+ \to p\gamma$ process to illustrate the procedure. We rederive traditional baryon-pole-model results. The calculation indicates the need of some nonleptonic weak interaction with chiral structure different from $(V-A)\otimes(V-A)$.

I. INTRODUCTION

Weak radiative decays such as $\Sigma^+ \to p\gamma$ and $\Lambda \to n\gamma$ are a good place to study nonleptonic weak interactions. Such radiative decays are presumed to be the combined effects of electromagnetic and nonleptonic weak interactions. Though the experimental information for such radiative decays is more scarce than for pionic decays, the calculation of photon emission is much simpler and more reliable than the calculation of pion emission and we can still learn a lot about the structure of nonleptonic weak interactions from such weak-radiative processes.

Earlier calculations of these processes are based on symmetry principles and the baryon-pole model.[1] These models used tree diagrams as indicated in Fig. 1. Two-body weak transition vertices are also responsible for pionic decays and can also be represented by pole diagrams as in Fig. 2.[2] Later improvements on these simple pole-model calculations were attempted using dispersion-theoretic techniques combined with current algebra.[3]

The Weinberg-Salam model of electroweak interaction is recognized by now as a good theory of leptonic and semileptonic processes. It is to be expected that the weak hadronic currents in this model are also the source for nonleptonic weak processes. In order to test this point, we must have a clear way to separate the strong-interaction effects from the weak-interaction effects. The development of "factorization" theory[4] guarantees that every nonleptonic weak amplitude can be factorized into a product of a coefficient function (to which hard gluons contribute) and a matrix element of some local operator (to which soft gluons contribute).[4] The coefficient function can be calculated by renormalization-group techniques.[5] The matrix element can be either parametrized by some parameters which are fitted by data or calculated from certain quark models [nonrelativistic SU(6) model, MIT bag model, and so on].

Since the only available data are for ($\Sigma^+ \to p\gamma$) process, we calculate it in the standard model. We use quark-diagram analysis and factorization theory so that we can easily see how the observed decay width and angular distribution (or equivalently, the asymmetry parameter) relate to the structure of the four-fermion Hamiltonian.

FIG. 1. Typical pole diagrams for weak-electromagnetic decays of hyperons in the baryon-pole model, also known as Graham-Pakvasa model. In Fig. 1, Y and B denote the initial and final baryons, B' and B'' denote the internal baryons. Open circles represent two-body weak vertex, and the triangles represent the electromagnetic vertex.

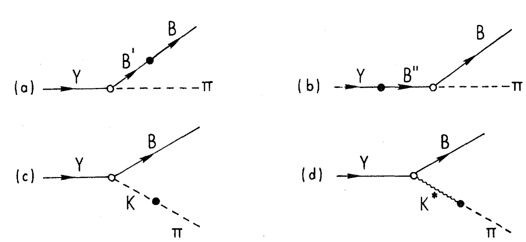

FIG. 2. Particle-pole diagrams for hyperon nonleptonic weak decays, where open circles represent three-body strong-interaction vertices, and shaded circles represent two-body weak-interaction vertices.

Hyperon radiative decay is a higher-order electroweak process, and calculations should be performed in the framework of a full relativistic quantum field theory which is consistent with quark and gluon confinement. The only such theory we have at the present time is the second-quantized MIT bag model.[6] The only undetermined parameter in the dynamical quantities calculated by bag model is the overall normalization constant which relates the matrix element of the operator calculated in hadronic static bag states and the matrix element of the same operator calculated in hadron plane-wave states. Though there are several papers discussing the relations between static bag states and plane-wave states,[7,8] there is still no rigorous calculation of such a normalization constant.

It is believed that one can use one set of data to fix this normalization constant; all other predictions are then unambiguous and are of physical significance. In our analysis of hyperon radiative decays, we use the total decay width of the $\Sigma \to p\gamma$ transition to fix this normalization constant. The asymmetry parameter of this decay is independent of the normalization constant and is a prediction of our model. Decay widths and asymmetry parameters of other hyperon radiative decays, such as $\Lambda \to n\gamma$ and $\Xi^0 \to \Sigma^0\gamma$, can also be predicted, but they will be presented elsewhere.

In this paper, we present the general formulation of quark-model calculation of hyperon radiative decays. We recover the baryon-pole-model result and therefore provide some theoretical justification of that phenomenological model. We also show that the contribution from high radially excited resonant states are ignorable. The standard model predicts too small a value for the asymmetry parameter $\alpha = -0.154$, which may be an indication of the existence of some "intrinsic" nonleptonic current-current weak interactions with chiral structure different from $(V-A)\otimes(V-A)$. This new interaction may help us to understand quantitatively the $\Delta I = \frac{1}{2}$ enhancement in weak interactions and the unequal-lifetime problem in charm mesons $\tau(D^+)/\tau(D^0) = 5$, which the standard model cannot explain unambiguously.

The paper is organized as follows: In Sec. II, we give the kinematics of the radiative decay and define three types of quark transition amplitudes which give the complete third-order electroweak-interaction contribution to this radiative decay. In Sec. III we evaluate the hadronic matrix elements of those quark transition amplitudes. In Sec. IV, we give the numerical results of our calculations and our conclusions. In the Appendix we give the results of two-quark integrals and four-quark integrals which appear in the four-quark transition amplitude.

II. KINEMATICS OF HYPERON RADIATIVE DECAY AND INTERACTION LAGRANGIAN

The most general electromagnetic vertex function involving two-spin-$\frac{1}{2}$ fermions has the following Lorentz-invariant structure[9]:

$$\bar{u}_B(p)\Gamma^\mu u_{B'}(p') = \bar{u}_B(p)\{\gamma^\mu[F_1^V(q^2)+F_1^A(q^2)\gamma_5] + i\sigma^{\mu\nu}q_\nu[F_2^V(q^2)+F_2^A(q^2)\gamma_5] + q^\mu[F_3^V(q^2)+F_3^A(q^2)\gamma_5]\}u_{B'}(p') , \tag{2.1}$$

where $q^\mu = (p-p')^\mu$ and $F_i^V(q^2)$ and $F_i^A(q^2)$, $i = 1,2,3$ are vector and axial-vector form factors, respectively. Conservation of electromagnetic current relates $F_1^V,A(q^2)$ form factors and $F_3^{V,A}(q^2)$ form factors as follows:

$$F_1^V(q^2) = -\frac{q^2}{M_B - M_{B'}} F_3^V(q^2), \quad M_B \neq M_{B'} , \tag{2.2a}$$

$$F_1^A(q^2) = -\frac{q^2}{M_B + M_{B'}} F_3^A(q^2) . \tag{2.2b}$$

$F_3^V(q^2) = 0$ if $M_B = M_{B'}$. The above equations normalize $F_1^{V,A}(q^2=0)$ to zero.

When the photon is a real photon ($q^2 = 0$), the transversality condition $\hat{\epsilon}\cdot\hat{q} = 0$ eliminates the static form factors $F_3^V(0)$ and $F_3^A(0)$; only $F_2^V(0)$ and $F_2^A(0)$ contribute. In this paper, we use C and D to denote $F_2^V(0)$ and $F_2^A(0)$ in $\Sigma^+ \to p\gamma$ process. C is called the transition magnetic dipole moment and D is called the transition electric dipole moment.

The two-body radiative decay amplitude $A(\Sigma^+ \to p\gamma)$, in the rest frame of Σ, has the form[10]

$$A(\Sigma^+ \to p\gamma) = i\left[\frac{m_N}{4\pi q_0 E_N}\right]^{1/2} \bar{u}_N(p)(C + D\gamma_5) \times \sigma^{\mu\nu}\epsilon_\mu q_\nu u_\Sigma(p')\delta^4(p'-p-q) . \tag{2.3}$$

The magnitude of the photon momentum q_ν in the rest frame of Σ is completely determined by baryon masses,

$$|\vec{q}\,| = \frac{(M_\Sigma{}^2 - M_N{}^2)}{2M_\Sigma} . \tag{2.4}$$

From (2.3) the angular distribution of the photon is given by

$$W(\theta) = \frac{1}{16\pi}\left[\frac{M_\Sigma{}^2 - M_N{}^2}{M_\Sigma}\right]^3 \times(|C|^2+|D|^2)[1+\alpha\cdot(\hat{s}\cdot\hat{p})] , \tag{2.5}$$

where $\hat{s}$ is the polarization vector of Σ in its rest frame, $\hat{p}$ is the direction of the proton momentum, and α is the asymmetry parameter given by

$$\alpha = \frac{2\,\mathrm{Re}(C^* D)}{|C|^2+|D|^2} . \tag{2.6}$$

The decay rate is found to be

$$R = \frac{1}{8\pi}\left[\frac{M_\Sigma{}^2 - M_N{}^2}{M_\Sigma}\right]^3(|C|^2+|D|^2) . \tag{2.7}$$

We study this hyperon decay process in the "standard" model. In Feynman—'t Hooft gauge, the electroweak (EW) interaction Lagrangian $\mathscr{L}_{\mathrm{WE}}$ (Ref. 11) consists of four terms

$$\mathscr{L}_{\mathrm{WE}} = \sum_{i=l}^{4} \mathscr{L}_i , \tag{2.8}$$

where $\mathscr{L}_{1,2,3,4}$ represent the quark—gauge-boson, quark—Higgs-boson, gauge-boson— gauge-boson, and gauge-boson—Higgs-boson interactions, respectively. $\mathscr{L}_1$ is given by

$$\mathscr{L}_1(x) = \frac{g}{\sqrt{2}}[J_\mu^{(+)}(x)W^{(-)\mu}(x) + J_\mu^{(-)}(x)W^{(+)\mu}(x)] - (g^2+g'^2)^{1/2}Z_\mu(x)[J_3^\mu(x) - \sin^2\theta_W J_{\mathrm{EM}}^\mu(x)] + eA_\mu(x)J_{\mathrm{EM}}^\mu(x) . \tag{2.9}$$

The charged current $J_\mu^{(+)}(x)$ is defined by

$$J_\mu^{(+)}(x) = \bar{U}_L(x)\gamma_\mu V^C D_L(x) \tag{2.10a}$$

and

$$J_\mu^{(-)}(x) = [J_\mu^{(+)}(x)]^\dagger , \tag{2.10b}$$

where

$$U_L(x) = [u_L(x), c_L(x), t_L(x)] , \tag{2.10c}$$

$$D_L(x) = [d_L(x), s_L(x), b_L(x)] . \tag{2.10d}$$

$U_R(x)$ and $D_R(x)$ are similarly defined. Each flavor quark carries three colors, but we suppress the color index here. V^c is the unitary weak mixing matrix[12]

$$V^c = \begin{bmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1c_3s_2 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{bmatrix} , \quad c_i = \cos\theta_i, s_i = \sin\theta_i . \tag{2.10e}$$

$J_{\mathrm{EM}}^\mu(x)$ is the electromagnetic current and J_3^μ is given by

$$J_3^\mu(x) = \tfrac{1}{2}[\bar{U}_L(x)\gamma^\mu U_L(x) - \bar{D}_L(x)\gamma^\mu D_L(x)] . \tag{2.10f}$$

$\mathscr{L}_2$ is given by

$$\mathscr{L}_2(x) = -\frac{\phi^+(x)}{v}[\bar{U}_L(x)V^c m^{(D)}D_R(x) - \bar{U}_R(x)m^{(U)}V^{c\dagger}D_L(x)] - \frac{\phi^-(x)}{v^*}[\bar{D}_R(x)m^{(D)}V^{c\dagger}U_L(x) - \bar{D}_L(x)V^c m^{(U)}U_R(x)] . \tag{2.11}$$

In the above equation v is the vacuum expectation value of the neutral component of the Higgs doublet and

$$m^{(U)}=\begin{bmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{bmatrix}, \quad m^{(D)}=\begin{bmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{bmatrix} . \tag{2.12}$$

$\mathscr{L}_3$ and $\mathscr{L}_4$ are given by

$$\mathscr{L}_3(x)=ie\{\partial_\mu A_\nu(x)[W^{(-)\mu}(x)W^{(+)\nu}(x)-W^{(+)\mu}(x)W^{(-)\nu}(x)]+[\partial_\mu W_\nu^{(+)}(x)-\partial_\nu W_\mu^{(+)}(x)]W^{(-)\nu}(x)A(x)^\mu$$
$$-[\partial_\mu W_\nu^{(-)}(x)-\partial_\nu W_\mu^{(-)}(x)]W^{(+)\nu}(x)A(x)^\mu\} , \tag{2.13}$$

$$\mathscr{L}_4(x)=\frac{g_e}{\sqrt{2}}[v\phi^{(-)}(x)A^\mu(x)W_\mu^{(-)}(x)+v^*\phi^{(+)}(x)W_\mu^{(+)}(x)A^\mu(x)]$$
$$+(-ie)A^\mu(x)\{[\partial_\mu\phi^{(-)}_{(x)}]\phi^{(+)}(x)-\phi^{(-)}(x)[\partial_\mu\phi^{(+)}_{(x)}]\}$$
$$+\text{terms not relevant to this calculation} . \tag{2.14}$$

To take into account strong interactions, we use the MIT bag model[13] along with QCD. In the bag model the quarks move freely and independently in a cavity, the confinement being produced by an external pressure B. Quark-gluon coupling is considered as weak; hence we can ignore corrections due to the gluon-exchange diagrams which are $O(\alpha/\pi)$ weaker than the zeroth-order result. However, we do make exception for those gluon-exchange diagrams which produce color-octet operators in $\Delta S=1$ weak transitions. These operators which are absent in the $SU_L(2)\times U_Y(1)$ model suppress $\Delta I=\frac{3}{2}$ ones.[14]

We assume that each baryon is made up of three quarks which are combined in such a way as to give the correct quantum numbers of that baryon. The radiative decay amplitude $A(\Sigma\to p\gamma)$ is in the third-order term in the perturbation expansion series of the electroweak Lagrangian $\mathscr{L}_{\rm WE}$. The relevant part is

$$A(\Sigma\to p\gamma)=-i\int d^4x\int d^4y\int d^4z\frac{e^{iq\cdot x}\epsilon_\mu}{4\pi(\pi q_0)^{1/2}}\langle p\,|\,J^\mu_{\rm EM}(x)J^\lambda_{\Delta S=1}(y)J_{\lambda,\Delta S=0}(z)\,|\,\Sigma\rangle(-i)D_W(y-z) , \tag{2.15}$$

where

$$D_W(y-z)=\frac{1}{(2\pi)^4}\int d^4k\frac{e^{-ik\cdot(y-z)}}{k^2-M_W{}^2+i\epsilon} ,$$

and

$$J^\lambda_{\Delta S=1}(y)=\frac{-g}{2\sqrt{2}}\sin\theta_1\cos\theta_3\sum_{\alpha=1}^{3}\bar{u}_\alpha(y)\gamma^\lambda(1-\gamma_5)s_\alpha(y) ,$$
$$J^\lambda_{\Delta S=0}(z)=\frac{g}{2\sqrt{2}}\cos\theta_1\sum_{\alpha=1}^{3}\bar{d}_\alpha(z)\gamma^\lambda(1-\gamma_5)u_\alpha(z) . \tag{2.16}$$

Operator-product-expansion theory can help us to construct an effective local weak Lagrangian from the bilocal one,

$$-i\int d^4y\int d^4z\,J^\lambda_{\Delta S=1}(y)J_{\lambda,\Delta S=0}(z)D_W(y-z)=\frac{G_F}{\sqrt{2}}\sin\theta_1\cos\theta_3\int d^4y\sum_{i=1}C_i{}^i\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}\bar{u}^{(y)}_{\alpha,\sigma}u^{(y)}_{\alpha',\sigma'}\bar{d}^{(y)}_{\beta,\rho}s^{(y)}_{\beta',\rho'} . \tag{2.17}$$

The C_i's are Wilson coefficients which are calculated by QCD renormalization-group equations, ${}^i\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}$ is the color-Lorentz matrix which specifies the color and Lorentz structure of the corresponding operator, $\sigma,\sigma',\ldots$ are spinor indices and $\alpha,\beta,\ldots$ are color indices. Higher-order gluon exchanges make Γ different from the zeroth-order value

$$\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}=\delta_{\alpha\beta'}\delta_{\alpha'\beta}[\gamma^{\lambda}(1-\gamma_5)]^{\sigma\sigma'}[\gamma_{\lambda}(1-\gamma_5)]^{\rho\rho'} .$$

The hyperon-radiative-decay amplitude is therefore the matrix element of the effective electroweak Lagrangian,

$$A(\Sigma\to p\gamma)=-iG_F e\frac{\sin\theta_1\cos\theta_1\cos\theta_3}{4\pi(2\pi q_0)^{1/2}}\sum_{\substack{i,\alpha,\dots,\beta' \\ \sigma\cdots\rho'}}\int d^4x\int d^4y\, e^{iq\cdot x}\epsilon_\mu\langle p\mid J^{\mu}_{\rm EM}(x)\bar{u}_{\alpha,\sigma}(y)u_{\alpha',\sigma'}(y)\bar{d}_{\beta,\rho}(y)s_{\beta',\rho'}(y)\mid\Sigma\rangle$$
$$\times C_i{}^i\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'} . \tag{2.18}$$

Carrying out the Wick reduction of Eq. (2.18), we find three types of terms: (i) terms with "no contractions" between quark fields, (ii) terms with one contraction between quark fields, and (iii) terms with two contractions between quark fields. We shall denote them by A_{6q}, A_{4q}, and A_{2q}, respectively, and represent them diagrammatically in Figs. 3–5. The gauge-invariance condition implies that A_{2q} must be a local magnetic-moment-type transition between s and d quarks:

$$A_{2q}=\frac{i}{2\pi\sqrt{\pi q_0}}\int d^4x\sum_{\alpha=1}^{3}\langle p\mid\bar{d}_\alpha(x)\sigma^{\mu\nu}(F_2^V+F_2^A\gamma_5)s_\alpha(x)\mid\Sigma\rangle\epsilon_\mu q_\nu e^{iq\cdot x} . \tag{2.19}$$

The quark parameters F_2^V and F_2^A are picked up from the one-loop-order s-d-γ vertex[15]:

$$F_2^V=-\frac{G_F e}{4\pi\sqrt{2}\pi^2}(m_d+m_s)\sum_{i=u,c,t}V_{is}V_{id}^*\left[-\frac{11}{36}+\frac{-29t_i{}^2+31t_i-8}{24(1-t_i)^3}+\frac{(2t_i{}^2-3t_i{}^3)}{4(1-t_i)^4}\ln t_i\right] ,$$
$$F_2^A=\left[\frac{m_s-m_d}{m_s+m_d}\right]F_2^V , \tag{2.20}$$

where $t_i=m_i/m_W$ and $m_i=m_u,\ m_c,\ m_t$.

III. HADRONIC MATRIX ELEMENTS

In this section, we shall calculate the hadronic matrix elements of the three transition amplitudes A_{2q}, A_{4q}, and A_{6q} defined in Sec. II, using the MIT bag model, which is a phenomenological model for hadrons in QCD. The bag model has been successfully applied to the nonleptonic weak decays of hyperons and K mesons[16] and K_L-K_S mass-difference estimates.[17] The model has also been used to study the radiative decays of baryon and vector-meson resonances.[18]

In the MIT bag model the baryons are bound states of three quarks. The quark fields satisfy free Dirac equations inside the bag and on the surface satisfy a linear boundary condition,

$$i\not{n}\psi(x)=\psi(x) , \tag{3.1}$$

and a nonlinear boundary condition

$$n_\mu\frac{\partial}{\partial x_\mu}\bar{\psi}(x)\psi(x)=2B , \tag{3.2}$$

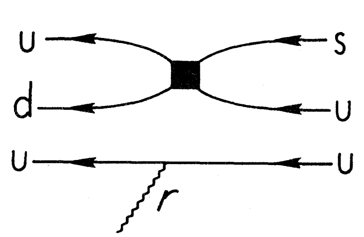

FIG. 3. Quark diagram representing the six-quark transition amplitude A_{6q}, where the shaded box is weak vertex.

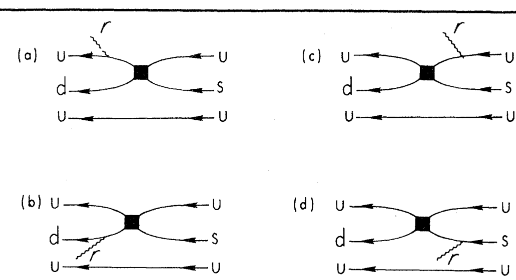

FIG. 4. Quark diagrams representing the four-quark transition amplitude A_{4q}, where the shaded box is the effective local four-fermion weak interaction.

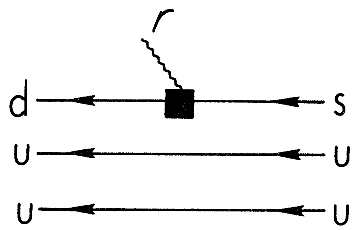

FIG. 5. Quark diagram representing the two-quark transition amplitude A_{2q}.

where n_μ is the unit vector normal to the surface of the bag and B is the "bag volume pressure." The fields vanish outside the bag regions. Assuming a spherical bag, $n=(0,\hat{r})$, and the nonlinear condition implies that only quark modes with total angular momentum $J=\frac{1}{2}$ can exist in the bag. There are two types of solutions with opposite parities: (1) the even-parity solution,

$$\psi_{f,n,(\kappa=-1),M}(\vec{x},t) = \frac{N_n}{\sqrt{4\pi}} \begin{bmatrix} ij_0(k_n r)\chi_M \\ -E_n j_1(k_n r)\vec{\sigma}\cdot\hat{r}\chi_M \end{bmatrix} e^{-i\omega(f,n-1)t}, \tag{3.3a}$$

and (2) the odd-parity solution,

$$\psi_{f,n,(\kappa=+1),M}(\vec{x},t) = \frac{\tilde{N}_n}{\sqrt{4\pi}} \begin{bmatrix} i\tilde{E}_n^{-1} j_1(\tilde{k}_n r)\vec{\sigma}\cdot\hat{r}\chi_M \\ j_0(\tilde{k}_n r)\chi_M \end{bmatrix} e^{-i\omega(f,n,+1)t}, \tag{3.3b}$$

where N_n, $\tilde{N}_n$ are normalization constants, $\omega(f,n,\kappa)$ is the quantized energy, the index f specifies flavor quantum number, index n plays the role of principal quantum number which labels each set of $\kappa=\pm 1$ mode energies in order of increasing values, and κ is the parity index. M is the spin index,

$$\chi_{M=\pm 1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \chi_{M=-1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$j_0(z)$ and $j_1(z)$ are spherical Bessel functions,

$$E_n = \left[\frac{\omega(n,-1)-m_q}{\omega(n,-1)+m_q} \right]^{1/2} \quad \text{with } \omega(n,-1) = \left[k_n{}^2 + m_q{}^2 \right]^{1/2},$$

and (3.4)

$$\tilde{E}_n = \left[\frac{\omega(n,+1)-m_q}{\omega(n,+1)+m_q} \right]^{1/2} \quad \text{with } \omega(n,+1) = \left[\tilde{k}_n{}^2 + m_q{}^2 \right]^{1/2},$$

where the flavor index f is suppressed and m_q is the quark-mass parameter. The momenta k_n and $\tilde{k}_n$ appearing in Eqs. (3.3) and (3.4) are the roots of the quantization equation which result from the linear boundary condition Eq. (3.1),

$$\tan Z_n = \frac{-Z_n}{-1+m_q R_B + \omega(f,n,-1)R_B}, \tag{3.5a}$$

$$\tan \tilde{Z}_n = \frac{\tilde{Z}_n}{1-m_q R_B + \omega(f,n,+1)R_B}, \tag{3.5b}$$

where $Z_n = k_n R_B$, $\tilde{Z}_n = \tilde{k}_n R_B$, and R_B is the bag radius. The rest mass of hadron can be expressed as

$$M_h = \sum_i \omega_i(f,n,\kappa) + BV + (\text{zero-point energy} + \text{gluon energy} + \cdots). \tag{3.6}$$

The charge-conjugate state of $\psi_{f,n,\kappa,M}$, denoted by $\psi^C_{f,n,\kappa,M}$, is its antiparticle state. The symmetry properties

$$Z_n = -\tilde{Z}_{-n} \tag{3.7a}$$

and

$$\omega(f,n,\kappa) = -\omega(f,-n,-\kappa) \tag{3.7b}$$

give the usual "hole" interpretation of antifermion to $\psi^C_{f,n,\kappa,M}$.

Every quark field of a certain flavor f and color i has the expansion

$$q_f^i(\vec{x},t) = \sum_{n,\kappa,M} [a_M(f,n,\kappa,i)\psi_{f,n,\kappa,M}(\vec{x})e^{-i\omega(f,n,\kappa)t} + b_M^\dagger(f,n,\kappa,i)\psi^C_{f,n,\kappa,M}(\vec{x})e^{i\omega(f,n,\kappa)t}]. \tag{3.8}$$

The creation and annihilation operators satisfy the usual anticommutation relations, namely,

$$\{a_M(f,n,\kappa,i), a^\dagger_{M'}(f',n',\kappa',i')\} = \delta_{M,M'}\delta_{f,f'}\delta_{n,n'}\delta_{\kappa,\kappa'}\delta_{i,i'} \tag{3.9a}$$

and

$$\{b_M(f,n,\kappa,i), b^\dagger_{M'}(f',n',\kappa',i')\} = \delta_{M,M'}\delta_{f,f'}\delta_{n,n'}\delta_{\kappa,\kappa'}\delta_{i,i'}. \tag{3.9b}$$

A low-lying baryon (such as $p,n,\Sigma^+,\ldots$) has all its quark constituents in relative s-wave states; each quark model has a parity index $\kappa=-1$ (positive-parity solution) and energy given by the lowest positive solution of Eq. (3.5).

The quark representations of Σ and p are given by

$$|\Sigma^+,\uparrow\rangle=\frac{\epsilon_{ijk}}{\sqrt{18}}[a^\dagger_{M=+1}(u,i)a^\dagger_{M=+1}(u,j)a^\dagger_{M=-1}(s,k)-a^\dagger_{M=+1}(u,i)a^\dagger_{M=-1}(u,j)a^\dagger_{M=+1}(s,k)]\,|\,0\rangle\ , \quad (3.10a)$$

$$|\Sigma^+,\downarrow\rangle=\frac{\epsilon_{ijk}}{\sqrt{18}}[a^\dagger_{M=-1}(u,i)a^\dagger_{M=+1}(u,j)a^\dagger_{M=-1}(s,k)-a^\dagger_{M=-1}(u,i)a^\dagger_{M=-1}(u,j)a^\dagger_{M=+1}(s,k)]\,|\,0\rangle\ , \quad (3.10b)$$

$$|p,\uparrow\rangle=\frac{\epsilon_{ijk}}{\sqrt{18}}[a^\dagger_{M=+1}(u,i)a^\dagger_{M=+1}(u,j)a^\dagger_{M=-1}(d,k)-a^\dagger_{M=+1}(u,i)a^\dagger_{M=-1}(u,j)a^\dagger_{M=+1}(d,k)]\,|\,0\rangle\ , \quad (3.10c)$$

$$|p,\downarrow\rangle=\frac{\epsilon_{ijk}}{\sqrt{18}}[a^\dagger_{M=-1}(u,i)a^\dagger_{M=+1}(u,j)a^\dagger_{M=-1}(d,k)-a^\dagger_{M=-1}(u,i)a^\dagger_{M=-1}(u,j)a^\dagger_{M=+1}(d,k)]\,|\,0\rangle\ . \quad (3.10d)$$

In the above equations, quantum numbers $n=+1$ and $\kappa=-1$ are suppressed.

Two helicity amplitudes are defined as

$$A(\Sigma_\uparrow\to p_\downarrow+\gamma)=i\left[\frac{E_N+M_N}{8\pi q_0 E_N}\right]^{1/2}\left[q+\frac{q^2}{E_N+M_N}\right]$$
$$\times\{[\chi_\downarrow^\dagger(\hat{\epsilon}\times\hat{q})\cdot\vec{\sigma}\chi_\uparrow]C+i[\chi_\downarrow^\dagger(\hat{\epsilon}\cdot\vec{\sigma})\chi_\uparrow]D\}\delta^3(\vec{\mathrm{P}}_N+\vec{\mathrm{q}})\delta(q+E_N-M_\Sigma)\ , \quad (3.11a)$$

$$A(\Sigma_\downarrow\to P_\uparrow+\gamma)=i\left[\frac{E_N+M_N}{8\pi q_0 E_N}\right]^{1/2}\left[q+\frac{q^2}{E_N+M_N}\right]$$
$$\times\{[\chi_\uparrow^+(\hat{\epsilon}\times\hat{q})\cdot\vec{\sigma}\chi_\downarrow]C+i[\chi_\uparrow^+(\hat{\epsilon}\cdot\vec{\sigma})\chi_\downarrow]D\}\delta^3(\vec{\mathrm{P}}_N+\vec{\mathrm{q}})\delta(q+E_N-M_\Sigma)\ , \quad (3.11b)$$

where the parameters C and D are those defined in Eq. (2.3),

$$\chi_\uparrow=\begin{bmatrix}1\\0\end{bmatrix}\ ,\quad \chi_\downarrow=\begin{bmatrix}0\\1\end{bmatrix}\ ,$$

and the two equations are defined in the rest frame of Σ. These two amplitudes can be calculated by the phenomenological Lagrangian

$$\mathscr{L}_{\mathrm{int}}(\vec{\mathrm{x}},t)=i\bar{\psi}_P(x)\sigma^{\mu\nu}(C+D\gamma_5)\psi_\Sigma(x)\frac{F_{\mu\nu}(x)}{2} \quad (3.12)$$

as follows:

$$A(\Sigma_\uparrow\to p_\downarrow+\gamma)=i(2\pi)^3\int dt\langle p_\downarrow,\gamma\,|\,\mathscr{L}_{\mathrm{int}}(0,t)\,|\,\Sigma_\uparrow\rangle\delta^3(\vec{\mathrm{P}}_N+\vec{\mathrm{q}})\ , \quad (3.13a)$$

$$A(\Sigma_\downarrow\to p_\uparrow+\gamma)=i(2\pi)^3\int dt\langle p_\uparrow,\gamma\,|\,\mathscr{L}_{\mathrm{int}}(0,t)\,|\,\Sigma_\downarrow\rangle\delta^3(\vec{\mathrm{P}}_N+\vec{\mathrm{q}})\ . \quad (3.13b)$$

The matrix elements in (3.13) can be calculated by the quark transition amplitudes defined in the last section.

In the following, we give the details of the calculation procedure for $A_{6q}(\Sigma_\uparrow\to p_\downarrow+\gamma)$, $A_{4q}(\Sigma_\uparrow\to p_\downarrow+\gamma)$, and $A_{2q}(\Sigma_\uparrow\to p_\downarrow+\gamma)$ in the MIT bag model. We just state the results for the other helicity amplitudes, since its calculation is essentially similar.

A. Evaluation of six-quark transition amplitude A_{6q}

In calculating $A_{6q}(\Sigma_\uparrow\to p_\downarrow+\gamma)$ the following integral is important:

$$I_1=\sum_{\substack{i,\alpha,\ldots\beta',\gamma\\ \sigma,\sigma',\ldots\rho,\rho'}}\int d^4x\int d^4y\, C_i^{(i)}\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}\langle p_\downarrow\,|\,:\bar{u}_\gamma(x)\not{\epsilon}u_\gamma(x)\bar{u}_{\alpha,\sigma}(y)u_{\alpha',\sigma'}(y)\bar{d}_{\beta,\rho}(y)s_{\beta',\rho'}(y):\,|\,\Sigma_\uparrow\rangle e^{iq\cdot x}\ , \quad (3.14)$$

where the Wilson coefficients C_i's and the color-Lorentz matrices ${}^{(i)}\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}$ are those introduced in Sec. II.

Expanding each quark field in Eq. (3.14) in terms of the bag states as given in Eq. (3.8), and substituting $|\Sigma_\uparrow\rangle$ and $|p_\downarrow\rangle$ by quark representations given in Eq. (3.10), integral I_1 can be reduced to

$$I_1=\sum_{\substack{i,\sigma,\ldots\beta' \\ \sigma,\sigma',\ldots\rho' \\ M_1,\ldots,M_6}} (2\pi)^2 C_i^{(i)}\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'} D^{\alpha,\alpha',\beta,\beta'}_{M_1,M_2,M_3,M_4,M_5,M_6}\,\delta(q_0)\delta(\omega(s,+1,-1)-\omega(l,+1,-1))$$
$$\times\left[\int d^3x\,\bar\psi_{u,M_1}(\vec x)\not\epsilon\psi_{u,M_2}(\vec x)e^{-i\vec q\cdot\vec x}\right]\left[\int d^3y\,\bar\psi_{u,M_3,\sigma}(\vec y)\psi_{u,M_4,\sigma'}(\vec y)\bar\psi_{d,M_5,\rho}(\vec y)\psi_{s,M_6\rho'}(\vec y)\right], \tag{3.15}$$

where all the quark wave functions belong to $n=1$ and $\kappa=-1$ mode, and the coefficients

$$D^{\alpha,\alpha',\beta,\beta'}_{M_1,M_2,M_3,M_4,M_5,M_6}$$

specify the spin and color configuration of quarks in initial and final states. It is given by a sum of products of Kronecker δ's. Notice however that we should have $m_s>m_l$, where l denotes the light quarks u,d, to explain the hyperon-nucleon mass difference. This requires

$$\omega(s,+1,-1)>\omega(l,+1,-1)\,, \tag{3.16}$$

and so the δ function causes the integral to vanish. A_{6q}, being proportional to I_1, vanishes identically. Higher-order gluon exchanges which connect the diagram in Fig. 3 may change this result, but this amplitude is suppressed by higher powers of the weak QCD coupling constant, and it is beyond the scope of our analysis in this paper. Our result is consistent with a recent calculation of hyperon radiative decays,[19] which is done in a nonrelativistic quark model.

B. Evaluation of the four-quark transition amplitude A_{4q}

$A_{4q}(\Sigma_\uparrow\to p_\downarrow+\gamma)$ receives contributions from all the four diagrams in Figs. 4(a), 4(b), 4(c), and 4(d). The amplitudes of the four diagrams are determined by the following integrals:

$$I_2^{(u)}=\sum_{\substack{i,\alpha,\ldots\beta' \\ \sigma,\ldots\rho'}}\int d^4x\int d^4y C_i^{(i,u)}\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}\langle p_\downarrow|:\bar u_\alpha(x)\not\epsilon[iS_F^{(u)}(x-y)]_\sigma u_{\alpha',\sigma'}(y)\bar d_{\beta,\rho}(y)s_{\beta',\rho'}(y):|\Sigma_\uparrow\rangle e^{iq\cdot x}\,, \tag{3.17a}$$

$$I_2^{(d)}=\sum_{\substack{i,\alpha\ldots\beta' \\ \sigma,\ldots\rho'}}\int d^4x\int d^4y C_i^{(i,d)}\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}\langle p_\downarrow|:\bar d_\alpha(x)\not\epsilon[iS_F^{(d)}(x-y)]_\sigma s_{\alpha',\sigma'}(y)\bar u_{\beta,\rho}(y)u_{\beta',\rho'}(y):|\Sigma_\uparrow\rangle e^{iq\cdot x}\,, \tag{3.17b}$$

$$\tilde I_2^{(u)}=\sum_{\substack{i,\alpha\ldots\beta' \\ \sigma,\ldots\rho'}}\int d^4x\int d^4y C_i^{(i,u)}\tilde\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}\langle p_\downarrow|:\bar u_{\alpha,\sigma}(y)[iS_F^{(u)}(y-x)\not\epsilon u_{\alpha'}(x)]_{\sigma'}\bar d_{\beta,\rho}(y)s_{\beta',\rho'}(y):|\Sigma_\uparrow\rangle e^{iq\cdot x}\,, \tag{3.17c}$$

$$\tilde I_2^{(s)}=\sum_{\substack{i,\alpha\ldots\beta' \\ \sigma,\ldots\rho'}}\int d^4x\int d^4y C_i^{(i,s)}\tilde\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}\langle p_\downarrow|:\bar d_{\alpha,\sigma}(y)[iS_F^{(s)}(y-x)\not\epsilon s_{\alpha'}(x)]_{\sigma'}\bar u_{\beta,\rho}(y)u_{\beta',\rho'}(y):|\Sigma_\uparrow\rangle e^{iq\cdot x}\,, \tag{3.17d}$$

where

$$iS_F^{(f)}(x-y)=\sum_{n,\kappa,M}[\psi_{f,n,\kappa,M}(x)\overline{\psi}_{f,n,\kappa,M}(y)\theta(x^0-y^0)-\psi^C_{f,n,\kappa,M}(x)\overline{\psi}^C_{f,n,\kappa,M}(y)\theta(y^0-x^0)]\ . \tag{3.18}$$

Integrating out the two time variables in Eq. (3.17) the four equations can be reduced to the typical form:

$$\begin{aligned}I_2^{(u)}=&\sum_{\substack{i,\alpha,\ldots\beta'\\ \sigma,\ldots,\rho'\\ M_1,M_2,\ldots,M_4}}(2\pi)C_i^{\;(i,u)}\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}G^{\alpha,\alpha',\beta,\beta'}_{M_1,M_2,M_3,M_4}\,\delta(q+E_N-M_\Sigma)\\
&\times\Bigg\{-i\sum_{n,\kappa,M}\frac{1}{\omega(l,n,\kappa)-\omega(s,+1,-1)}\left[\int d^3x\overline{\psi}_{u,M_1}(\vec{x})\not{\epsilon}\psi_{u,n,\kappa,M}(\vec{x})e^{-i\vec{q}\cdot\vec{x}}\right]\\
&\qquad\times\left[\int d^3y\overline{\psi}_{u,n,\kappa,M,\sigma}(\vec{y})\psi_{d,M_2,\sigma'}(\vec{y})\overline{\psi}_{d,M_3,\rho}(\vec{y})\psi_{s,M_4,\rho'}(\vec{y})\right]\\
&\quad+i\sum_{n,\kappa,M}\frac{1}{\omega(l,n,\kappa)+\omega(s,+1,-1)}\left[\int d^3x\overline{\psi}_{u,M_1}(\vec{x})\not{\epsilon}\psi^c_{u,n,\kappa,M}(\vec{x})e^{-i\vec{q}\cdot\vec{x}}\right]\\
&\qquad\times\left[\int d^3y\overline{\psi}^C_{u,n,\kappa,M,\sigma}(\vec{y})\psi_{u,M_2,\sigma'}(\vec{y})\overline{\psi}_{d,M_3,\rho}(\vec{y})\psi_{s,M_4,\rho'}(\vec{y})\right]\Bigg\}\ .\end{aligned}\tag{3.19}$$

We have suppressed the $n=1$, $\kappa=-1$ indices for the ground-state positive-parity wave functions in $I_2^{(u)}$, $I_2^{(d)}$, $\widetilde{I}_2^{(u)}$, and $\widetilde{I}_2^{(s)}$. The coefficient $G^{\alpha,\alpha',\beta,\beta'}_{M_1,M_2,M_3,M_4}$ exhibits all possible color and spin configurations of quarks in the hyperon and proton, and it is given by

$$G^{\alpha,\alpha',\beta,\beta'}_{M_1,M_2,M_3,M_4}=\frac{(\delta_{\alpha\alpha'}\delta_{\beta\beta'}-\delta_{\alpha\beta'}\delta_{\alpha'\beta})}{18}(2\delta^{-1}_{M_1}\delta^{+1}_{M_2}\delta^{-1}_{M_3}\delta^{-1}_{M_4}+2\delta^{-1}_{M_1}\delta^{+1}_{M_2}\delta^{+1}_{M_3}\delta^{+1}_{M_4}-\delta^{-1}_{M_1}\delta^{-1}_{M_2}\delta^{-1}_{M_3}\delta^{+1}_{M_4}-\delta^{+1}_{M_1}\delta^{+1}_{M_2}\delta^{-1}_{M_3}\delta^{+1}_{M_4})\ . \tag{3.20}$$

The two integrations in Eq. (3.17) give only a quark energy conservation δ function, namely,

$$2\pi\delta(q+\omega(l,+1,-1)-\omega(s,+1,-1))\ ;$$

we have to insert the bag energy BV and zero-point energy by hand to give a phenomenologically correct hadronic energy conservation.[20] $I_2^{(d)}$, $\widetilde{I}_2^{(u)}$, and $\widetilde{I}_2^{(s)}$ are similarly calculated:

$$\begin{aligned}I_2^{(d)}=&\sum_{\substack{i,\alpha\ldots,\beta'\\ \sigma,\ldots,\sigma'\\ M_1,\ldots,M_4}}(2\pi)C_i^{\;(i,d)}\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}H^{\alpha,\alpha',\beta,\beta'}_{M_1,M_2,M_3,M_4}\delta(q+E_N-M_\Sigma)\\
&\times\Bigg\{-i\sum_{n,\kappa,M}\frac{1}{\omega(l,n,\kappa)-\omega(s,+1-1)}\left[\int d^3x\overline{\psi}_{d,M_1}(\vec{x})\not{\epsilon}\psi_{d,n,\kappa,M}(\vec{x})e^{-i\vec{q}\cdot\vec{x}}\right]\\
&\qquad\times\left[\int d^3y\overline{\psi}_{d,n,\kappa,M,\sigma}(\vec{y})\psi_{s,M_2,\sigma'}(\vec{y})\overline{\psi}_{u,M_3,\rho}(\vec{y})\psi_{u,M_4,\rho}(\vec{y})\right]\\
&\quad+i\sum_{n,\kappa,M}\frac{1}{\omega(l,n,\kappa)+\omega(s,+1,-1)}\left[\int d^3x\overline{\psi}_{d,M_1}(\vec{x})\not{\epsilon}\psi^C_{d,n,\kappa,M}(\vec{x})e^{-i\vec{q}\cdot\vec{x}}\right]\\
&\qquad\times\left[\int d^3y\overline{\psi}^C_{d,n,\kappa,M,\sigma}(\vec{y})\psi_{s,M_2,\sigma'}(\vec{y})\overline{\psi}_{u,M_3,\rho}(\vec{y})\psi_{u,M_4,\rho'}(\vec{y})\right]\Bigg\}\ ,\end{aligned}\tag{3.21}$$

where

$$H^{\alpha,\alpha',\beta,\beta'}_{M_1,M_2,M_3,M_4}=\frac{(\delta_{\alpha\alpha'}\delta_{\beta\beta'}-\delta_{\alpha\beta'}\delta_{\alpha'\beta})}{18}(2\delta^{-1}_{M_1}\delta^{-1}_{M_2}\delta^{-1}_{M_3}\delta^{+1}_{M_4}+2\delta^{+1}_{M_1}\delta^{+1}_{M_2}\delta^{-1}_{M_3}\delta^{+1}_{M_4}-\delta^{-1}_{M_1}\delta^{+1}_{M_2}\delta^{-1}_{M_3}\delta^{-1}_{M_4}-\delta^{-1}_{M_1}\delta^{+1}_{M_2}\delta^{+1}_{M_3}\delta^{+1}_{M_4})\ ; \tag{3.22}$$

$$\tilde{I}^{(u)}=\sum_{\substack{i,\alpha,\ldots,\beta' \\ \alpha',\ldots,\rho' \\ M_1,\ldots,M_4}}(2\pi)C_i{}^{(i,u)}\tilde{\Gamma}^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}G^{\alpha,\alpha',\beta,\beta'}_{M_1,M_2,M_3,M_4}\delta(q+E_N-M_\Sigma)$$

$$\times\left\{i\sum_{n,\kappa,M}\frac{1}{2\omega(l,+1,-1)-\omega(l,n,\kappa)-\omega(s,+1,-1)}\left[\int d^3x\bar{\psi}_{u,n,\kappa,M}(\vec{x})\epsilon\psi_{u,M_2}(\vec{x})e^{-i\vec{q}\cdot\vec{x}}\right]\right.$$
$$\times\left[\int d^3y\bar{\psi}_{u,M_1,\sigma}(\vec{y})\psi_{u,n,\kappa,M,\sigma'}(\vec{y})\bar{\psi}_{d,M_3,\rho}(\vec{y})\psi_{s,M_4,\rho'}(\vec{y})\right]$$
$$+i\sum_{n,\kappa,M}\frac{1}{2\omega(l,+1,-1)+\omega(l,n,\kappa)-\omega(s,+1,-1)}\left[\int d^3x\bar{\psi}^C_{u,n,\kappa,M}(\vec{x})\epsilon\psi_{u,M_2}(\vec{x})e^{-iq\cdot x}\right]$$
$$\left.\times\left[\int d^3y\bar{\psi}_{u,M_1,\sigma}(\vec{y})\psi^C_{u,n,\kappa,M,\sigma'}(\vec{y})\bar{\psi}_{d,M_3,\rho}(\vec{y})\psi_{s,M_4,\rho'}(\vec{y})\right]\right\}, \tag{3.23}$$

where $G^{\alpha,\alpha',\beta,\beta'}_{M_1,M_2,M_3,M_4}$ is defined in Eq. (3.20);

$$\tilde{I}^{(s)}_2=\sum_{\substack{i,\alpha,\ldots,\beta' \\ \sigma,\ldots,\rho' \\ M_1,\ldots,M_4}}(2\pi)C_i{}^{(i,s)}\tilde{\Gamma}^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}H^{\alpha,\alpha',\beta,\beta'}_{M_1,M_2,M_3,M_4}\delta(q+E_N-M_\Sigma)$$

$$\times\left\{i\sum_{n,\kappa,M}\frac{1}{\omega(l,+1,-1)-\omega(s,n,\kappa)}\left[\int d^3x\bar{\psi}_{s,n,\kappa,M}(\vec{x})\epsilon\psi_{s,M_2}(\vec{x})e^{-i\vec{q}\cdot\vec{x}}\right]\right.$$
$$\times\left[\int d^3y\bar{\psi}_{d,M_1,\sigma}(\vec{y})\psi_{s,n,\kappa,M,\sigma'}(\vec{y})\bar{\psi}_{u,M_3,\rho}(\vec{y})\psi_{u,M_4,\rho'}(\vec{y})\right]$$
$$+i\sum_{n,\kappa,M}\frac{1}{\omega(l,+1,-1)+\omega(s,n,\kappa)}\left[\int d^3x\bar{\psi}^C_{s,n,\kappa,M}(\vec{x})\epsilon\psi_{s,M_2}(\vec{x})e^{-i\vec{q}\cdot\vec{x}}\right]$$
$$\left.\times\left[\int d^3y\bar{\psi}_{d,M_1,\sigma}(\vec{y})\psi^C_{s,n,\kappa,M,\sigma'}(\vec{y})\bar{\psi}_{u,M_3,\rho}(\vec{y})\psi_{u,M_4,\rho'}(\vec{y})\right]\right\}, \tag{3.24}$$

where $H^{\alpha,\alpha',\beta,\beta'}_{M_1,M_2,M_3,M_4}$ is defined in Eq. (3.22).

The helicity amplitude $A_{4q}(\Sigma_\uparrow\to p_\downarrow+\gamma)$ is a linear combination of the four integrals $I_2^{(u)}$, $I_2^{(d)}$, $\tilde{I}_2^{(u)}$, and $\tilde{I}_2^{(s)}$:

$$A_{4q}(\Sigma_\uparrow\to p_\downarrow+\gamma)=-i\pi\left[\frac{2\pi}{q_0}\right]^{1/2}G_Fe\sin\theta_1\cos\theta_1\cos\theta_1\cos\theta_3[\tfrac{2}{3}I_2^{(u)}+\tfrac{2}{3}\tilde{I}_2^{(u)}-\tfrac{1}{3}I_2^{(d)}-\tfrac{1}{3}\tilde{I}_2^{(s)}]\delta^3(\vec{P}_N+\vec{q}) . \tag{3.25}$$

The one-loop-order QCD-corrected weak Lagrangian gives the color-Lorentz matrices

$$\sum_{i=1}^{5}C_i{}^{(i,u)}\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}=\sum_{i=1}^{5}C_i{}^{(i,u)}\tilde{\Gamma}^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}$$
$$=[(C_1+C_3)\delta_{\alpha\alpha'}\delta_{\beta\beta'}+C_2\delta_{\alpha\beta'}\delta_{\alpha'\beta}][\gamma^\lambda(1-\gamma_5)]^{\sigma\sigma'}[\gamma_\lambda(1-\gamma_5)]^{\rho\rho'}$$
$$+[C_4\delta_{\alpha\alpha'}\delta_{\beta\beta'}+C_5\delta_{\alpha\beta'}\delta_{\alpha'\beta}][\gamma^\lambda(1+\gamma_5)]^{\sigma\sigma'}[\gamma_\lambda(1-\gamma_5)]^{\rho\rho'} \tag{3.26}$$

and

$$\sum_{i=1}^{5}C_i{}^{(i,d)}\Gamma^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}=\sum_{i=1}^{5}C_i{}^{(i,s)}\tilde{\Gamma}^{\sigma,\sigma',\rho,\rho'}_{\alpha,\alpha',\beta,\beta'}$$
$$=[(C_1+C_3)\delta_{\alpha\alpha'}\delta_{\beta\beta'}+C_2\delta_{\alpha\beta'}\delta_{\alpha'\beta}][\gamma^\lambda(1-\gamma_5)]^{\sigma\sigma'}[\gamma_\lambda(1-\gamma_5)]^{\rho\rho'}$$
$$+[C_4\delta_{\alpha\alpha'}\delta_{\beta\beta'}+C_5\delta_{\alpha\beta'}\delta_{\alpha'\beta}][\gamma^\lambda(1-\gamma_5)]^{\sigma\sigma'}[\gamma_\lambda(1+\gamma_5)]^{\rho\rho'} . \tag{3.27}$$

The small differences in wave-function normalization constants and momenta between strange and nonstrange quarks produce small $SU_F(3)$-symmetry-breaking effects in the matrix element. Such small "dynamical" symmetry-breaking effects can be ignored, except in the particular case where all the wave functions in two-quark integrals and four-quark integrals are of $n=1$, $\kappa=-1$ mode, since they contribute to the amplitude of the baryon-pole term in our model where $SU_F(3)$-symmetry-breaking effects are significant. We shall discuss this point later. All the two-quark integrals and four-quark integrals in $I_2^{(u)}$, $I_d^{(d)}$, $\widetilde{I}_2^{(u)}$, and $\widetilde{I}_2^{(s)}$ have been evaluated, and they are given in the Appendix.

We find that all positive-parity intermediate states in the propagator $iS_F^{(f)}(x-y)$ contribute to magnetic dipole transition, and all negative-parity intermediate states contribute to electric dipole transition. We give the explicit expression for $A_{4q}(\Sigma_\uparrow \to p_\downarrow + \gamma)$ in the rest frame of Σ:

$$A_{4q}(\Sigma_\uparrow \to p_\downarrow + \gamma) = \sum_{n=1} A_{4q}^n(\Sigma_\uparrow \to p_\downarrow + \gamma) , \tag{3.28}$$

where

$$\begin{aligned} A_{4q}^n(\Sigma_\uparrow \to p_\downarrow + \gamma) = i\pi \left[\frac{2\pi}{q_0}\right]^{1/2} & G_F e \sin\theta_1 \cos\theta_1 \cos\theta_3 \delta^3(\vec{P}_N + \vec{q})\delta(q + E_N - M_\Sigma) \\ & \times \{ [\chi_\downarrow^\dagger(\hat{\epsilon}\times\hat{q})\cdot\vec{\sigma}\chi_\uparrow][(C_1 - C_2 + C_3)(F_{L,L}^{n,\kappa=-1} + G_{L,L}^{n,\kappa=-1}) + (C_4 - C_5)(F_{L,R}^{n,\kappa=-1} + G_{L,R}^{n,\kappa=-1})] \\ & + [i\chi_\downarrow^\dagger(\hat{\epsilon}\cdot\vec{\sigma})\chi_\uparrow][(C_1 - C_2 + C_3)(F_{L,L}^{n,\kappa=+1} + G_{L,L}^{n,\kappa=+1}) + (C_4 - C_5)(F_{L,R}^{n,\kappa=+1} + G_{L,R}^{n,\kappa=+1})] \} . \end{aligned} \tag{3.29}$$

A_{4q}^n is the amplitude, to which the $(n-1)$th radially excited intermediate state contributes. $\chi_\downarrow$ and $\chi_\uparrow$ in Eq. (3.29) are two-component spinors $\binom{0}{1}$ and $\binom{1}{0}$, respectively. $F_{L,L}^{n=1,\kappa=-1}$ is given by

$$F_{L,L}^{n=1,\kappa=-1} = \frac{\frac{2}{3}N_1{}^6 L_{1,1}[(N_1'/N_1)E_1 R^{1(11)}_{(01)} - (N_1'{}^3/N_1{}^3)E_1' R^{1(11)}_{s(01)}]}{\omega(l,+1,-1) - \omega(s,+1,-1)} , \tag{3.30}$$

and for $n \geq 2$

$$\begin{aligned} F_{L,L}^{n,\kappa=-1} = & \left[\frac{1}{\omega(l,n,-1) - \omega(s,+1,-1)} - \frac{2}{3}\frac{1}{2\omega(l,+1,-1) - \omega(l,n,-1) - \omega(s,+1,-1)} \right. \\ & \left. - \frac{1}{3}\frac{1}{\omega(l,+1,-1) - \omega(s,n,-1)} \right] N_1{}^4 N_n{}^2 [E_n R^{1(1n)}_{(01)} + E_1 R^{1(1n)}_{(10)}] L_{1,n} . \end{aligned} \tag{3.31}$$

The primed quantities in Eq. (3.30) are bag-model parameters for the strange quark and the unprimed quantities refer to the nonstrange quarks. The parameters are different since we want to include $SU_F(3)$ breaking effects. $F_{L,L}^{n=1,\kappa=-1}$ vanishes in the exact-$SU_F(3)$ limit, but the smallness of the symmetry breaking is enhanced in this amplitude by the singular denominator $\omega(l,+1,-1) - \omega(s,+1,-1)$. The result is fairly large. $L_{1,1}$ and $L_{1,n}$ come from four-quark integrals and are defined in Eq. (A22) and Eq. (A11). $R^{1(11)}_{(01)}$, $R^{1(11)}_{s(01)}$, and $R^{1(1n)}_{(01)}$. . . in the above two equations come from two-quark integrals, and are defined in Eq. (A8). The subscript s in $R^{1(11)}_{s(01)}$ reminds us that this integral contains strange-quark parameters. $F_{L,L}^{n,\kappa=+1}$ is given by

$$\begin{aligned} F_{L,L}^{n,\kappa=+1} = & \left[\frac{1}{\omega(l,n,+1) - \omega(s,+1,-1)} - \frac{2}{3}\frac{1}{2\omega(l,+1,-1) - \omega(l,n,+1) - \omega(s,+1,-1)} \right. \\ & \left. - \frac{1}{3}\frac{1}{\omega(l,+1,-1) - \omega(s,n,+1)} \right] N_1{}^4 \widetilde{N}_1{}^2 \left[\widetilde{R}^{0(1n)}_{(00)} - \frac{E_1 \widetilde{E}_n{}^{-1}}{3}\widetilde{R}^{0(1n)}_{(11)} + \frac{2E_1\widetilde{E}_n{}^{-1}}{3}\widetilde{R}^{2(1n)}_{(11)} \right] M_{1,n} , \end{aligned} \tag{3.32}$$

where $\widetilde{R}^{0(1n)}_{(00)}$, $\widetilde{R}^{0(1n)}_{(11)}$, and $\widetilde{R}^{2(1n)}_{(11)}$ are defined in Eq. (A9). $M_{1,n}$ comes from four-quark integrals, and it is defined in Eq. (A14). $G_{L,L}^{n,\kappa=-1}$ is given by

$$G_{L,L}^{n,\kappa=-1}=\left[\frac{1}{\omega(l,n,+1)+\omega(s,+1,-1)}+\frac{2}{3}\frac{1}{2\omega(l,+1,-1)+\omega(l,n,+1)-\omega(s,+1,-1)}+\frac{1}{3}\frac{1}{\omega(l,+1,-1)+\omega(s,n,+1)}\right]N_1{}^4\widetilde{N}_n{}^2[\widetilde{E}_n{}^{-1}\widetilde{R}^{1(1n)}_{(01)}-E_1\widetilde{R}^{1(1n)}_{(10)}]\widetilde{L}_{1,n}\ , \tag{3.33}$$

where $\widetilde{R}^{1(1n)}_{(01)}$ and $\widetilde{R}^{1(1n)}_{(10)}$ are defined in Eq. (A9). $L_{1,n}$ comes from four-quark integrals and it is defined in Eq. (A17). $G_{L,L}^{n,\kappa=+1}$ is given by

$$G_{L,L}^{n,\kappa=+1}=\left[-\frac{1}{\omega(l,n,-1)+\omega(s,+1,-1)}-\frac{2}{3}\frac{1}{2\omega(l,+1,-1)+\omega(l,n,-1)-\omega(s,+1,-1)}-\frac{1}{3}\frac{1}{\omega(l,+1,-1)+(s,n,-1)}\right]N_1{}^4N_n{}^2\left[R^{0(1n)}_{(00)}+\frac{E_1E_n}{3}R^{0(1n)}_{(11)}-\frac{2E_1E_n}{3}R^{2(1n)}_{(11)}\right]\widetilde{M}_{1,n}\ , \tag{3.34}$$

where $\widetilde{M}_{1,n}$ is defined in Eq. (A20). $F_{L,R}^{n=1,\kappa=-1}$ is given by

$$F_{L,R}^{n=1,\kappa=-1}=\frac{N_1{}^6[\frac{2}{9}L_{1,1}+\frac{4}{9}L_{2,1}(+1,-1)][(N_1'/N_1)E_1R^{1(11)}_{(01)}-(N_1'/N_1)^3E_1'R_s{}^{1(11)}_{(01)}]}{\omega(l,+1,-1)-\omega(s,+1,-1)}\ , \tag{3.35}$$

where $L_{1,1}$ and $L_{2,1}(+1,-1)$ are defined in Eq. (A22) and Eq. (A23). $SU_F(3)$-symmetry-breaking effect is included in bag integral evaluation. For $n\geq 2$

$$F_{L,R}^{n,\kappa=-1}=\left[\frac{\frac{5}{9}L_{1,n}+\frac{4}{9}L_{2,n}(+1,-1)}{\omega(l,n,-1)-\omega(s,+1,-1)}-\frac{\frac{4}{9}L_{1,n}+\frac{2}{9}L_{2,n}(+1,-1)}{2\omega(l,+1,-1)-\omega(l,n,-1)-\omega(s,+1,-1)}-\frac{\frac{1}{9}L_{1,n}+\frac{2}{9}L_{2,n}(+1,-1)}{\omega(l,+1,-1)-\omega(s,n,-1)}\right]N_1{}^4N_n{}^2[E_nR^{1(1n)}_{(01)}+E_1R^{1(n1)}_{(01)}]\ , \tag{3.36}$$

where $L_{1,n}$ is defined in Eq. (A11) and $L_{2,n}(+1,-1)$ is defined in Eq. (A12) with $a=+1$ and $b=-1$ in that equation. $F_{L,R}^{n,\kappa=+1}$ is given by

$$F_{L,R}^{n,\kappa=+1}=\left[\frac{-\frac{1}{3}M_{1,n}}{\omega(l,n,+1)-\omega(s,+1,-1)}+\frac{\frac{4}{9}M_{1,n}-\frac{2}{9}M_{2,n}(+1,-1)}{2\omega(l,+1,-1)-\omega(l,n,+1)-\omega(s,+1,-1)}+\frac{-\frac{1}{9}M_{1,n}+\frac{2}{9}M_{2,n}(+1,-1)}{\omega(l,+1,-1)-\omega(s,n,+1)}\right]N_1{}^4\widetilde{N}_n{}^2\left[\widetilde{R}^{0(1n)}_{(00)}-\frac{E_1\widetilde{E}_n{}^{-1}}{3}\widetilde{R}^{0(1n)}_{(11)}+\frac{2E_1\widetilde{E}_n{}^{-1}}{3}\widetilde{R}^{2(1n)}_{(11)}\right]\ , \tag{3.37}$$

where $M_{2,n}(+1,-1)$ has its general definition in Eq. (A15) with $a=+1$ and $b=-1$ in that equation. $G_{L,R}^{n,\kappa=-1}$ is given by

$$G_{L,R}^{n,\kappa=-1}=\left[\frac{\frac{5}{9}\widetilde{L}_{1,n}+\frac{4}{9}\widetilde{L}_{2,n}(+1,-1)}{\omega(l,n,+1)+\omega(s,+1,-1)}+\frac{\frac{4}{9}\widetilde{L}_{l,n}+\frac{2}{9}\widetilde{L}_{2,n}(+1,-1)}{2\omega(l,+1,-1)+\omega(l,n,+1)-\omega(s,+1,-1)}+\frac{\frac{1}{9}\widetilde{L}_{1,n}+\frac{2}{9}\widetilde{L}_{2,n}(+1,-1)}{\omega(l,+1,-1)+\omega(s,n,+1)}\right]N_1{}^4\widetilde{N}_n{}^2[\widetilde{E}_n{}^{-1}\widetilde{R}^{1(1n)}_{(01)}-E_1\widetilde{R}^{1(n1)}_{(01)}]\ , \tag{3.38}$$

where $\widetilde{L}_{2,n}(+1,-1)$ has its general definition in Eq. (A18) and $a=+1$, $b=-1$ in that equation. $G_{L,R}^{n,\kappa=-1}$ is given by

$$G_{L,R}^{n,\kappa=+1}=\left[\frac{\frac{1}{3}\widetilde{M}_{1,n}}{\omega(l,n,-1)+\omega(s,+1,-1)}+\frac{\frac{4}{9}\widetilde{M}_{1,n}+\frac{2}{9}\widetilde{M}_{2,n}(+1,-1)}{2\omega(l,+1,-1)+\omega(l,n,-1)-\omega(s,+1,-1)}-\frac{\frac{1}{9}\widetilde{M}_{1,n}+\frac{2}{9}\widetilde{M}_{2,n}(+1,-1)}{\omega(l,+1,-1)+\omega(s,n,-1)}\right]N_1{}^4N_n{}^2\left[R^{0(1n)}_{(00)}+\frac{E_1E_n}{3}R^{0(1n)}_{(11)}-\frac{2E_1E_n}{3}R^{2(1n)}_{(11)}\right]\ , \tag{3.39}$$

where $\widetilde{M}_{2,n}(+1,-1)$ has its general defintion in Eq. (A21) with $a=+1$, $b=-1$ in that equation.

The expression for the other helicity amplitude $A_{4q}(\Sigma_{\downarrow}\to p_{\uparrow}+\gamma)$ can be obtained from Eqs. (3.28) and (3.29) by interchanging the spinors $\chi_{\uparrow}$ and $\chi_{\downarrow}$ in Eq. (3.29).

All the radially excited intermediate states contribute to the amplitude $A_{4q}(\Sigma_{\uparrow}\to p_{\downarrow}+\gamma)$; therefore the upper limit of the summation of n in Eq. (3.28) can even extend to infinity. However, the contributions from higher radially excited intermediate states are small, and it can be shown that the series in Eq. (3.28) converges.

Z_n and $\widetilde{Z}_n$ are both monotonically increasing functions of the principal quantum number n. When n is large, the quark mass effect is ignorable, and the quark energies and wave-function normalization constants are proportional to Z_n or $\widetilde{Z}_n$,

$$\omega(f,n,-1)=\frac{Z_n}{R_B}\approx(n-\tfrac{1}{4})\frac{\pi}{R_B}\ , \qquad (3.40a)$$

$$\omega(f,n,+1)=\frac{\widetilde{Z}_n}{R_B}\simeq(n+\tfrac{1}{4})\frac{\pi}{R_B}\ , \qquad (3.40b)$$

$$N_n\simeq Z_n R_B^{-3/2}\ , \qquad (3.41a)$$

$$\widetilde{N}_n\simeq\widetilde{Z}_n R_B^{-3/2}\ . \qquad (3.41b)$$

The two-quark integrals and four-quark integrals have the general structure

$$J_n=Z_n^{-1}\int_0^1 \begin{Bmatrix}\sin Z_n x\\ \text{or}\\ \cos Z_n x\end{Bmatrix} f(x)x\,dx$$

$$+O(Z_n^{-2})\ . \qquad (3.42)$$

$f(x)$ is a product of two or three spherical Bessel functions of order 0 or 1, and is independent of Z_n. Since $\sin Z_n x$ and $\cos Z_n x$ change phase rapidly for large n, and $f(x)$ is a smooth function, the integral J_n is a decreasing function of n. The amplitude A_{4q}^{n} decreases faster than $1/n$. We expect that the series in Eq. (3.28) converges.

We call the following part of $A_{4q}^{n=1}(\Sigma_{\uparrow}\to p_{\downarrow}+\gamma)$ the baryon-pole term,

$$A_{\text{baryon pole}}(\Sigma_{\uparrow}\to p_{\downarrow}+\gamma)=i\pi\left[\frac{2\pi}{q_0}\right]^{1/2} G_F e\sin\theta_1\cos\theta_1\cos\theta_3[(C_1-C_2-C_3)F_{L,L}^{n=1,\kappa=-1}+(C_4-C_5)F_{L,R}^{n=1,\kappa=-1}]$$
$$\times[\chi_{\downarrow}^{\dagger}(\hat{\epsilon}\times\hat{q})\cdot\vec{\sigma}\chi_{\uparrow}]\delta^3(\vec{P}_N+\vec{q})\delta(q+E_N-M_\Sigma)$$
$$=\frac{ie(\mu_\Sigma-\mu_N)b}{M_\Sigma-M_N}[\chi_{\downarrow}^{\dagger}(\hat{\epsilon}\times\hat{q})\cdot\vec{\sigma}\chi_{\downarrow})]\delta^3(\vec{P}_N+\vec{q})\delta(q+E_N-M_\Sigma)\ , \qquad (3.43)$$

where we approximate $1/\omega(s,+1,-1)-\omega(l,+1,-1)$ in Eq. (3.30) and Eq. (3.35) by $1/(M_\Sigma-M_N)$, and the parameters b, μ_Σ, and μ_N are defined as

$$b=\pi\left[\frac{2\pi}{q_0}\right]^{1/2} G_F N_1^6\sin\theta_1\cos\theta_1\cos\theta_3\{[\tfrac{2}{3}(C_1-C_2+C_3)+\tfrac{2}{9}(C_4-C_5)]L_{1,1}+\tfrac{4}{9}(C_4-C_5)L_{2,1}(+1,-1)\}\ ,$$
$$(3.44)$$

$$\mu_\Sigma=\left[\frac{N_1'}{N_1}\right]^3 E_1' R_s{}^{1(1\,1)}_{\ \ (0\,1)}\ , \qquad (3.45)$$

$$\mu_N=\left[\frac{N_1'}{N_1}\right] E_1 R{}^{1(1\,1)}_{\ \ (0\,1)}\ . \qquad (3.46)$$

Equation (3.43) is the familiar expression for the parity-conserving amplitude in the baryon-pole model[1] for hyperon radiative decays. Our quark-level short-distance analysis can provide some justification of that phenomenological model. The small $SU_F(3)$ symmetry-breaking amplitude $(\mu_\Sigma-\mu_N)b$ is enhanced by the singular denominator $1/(M_\Sigma-M_N)$ and make this baryon-pole term an important contribution to the magnetic transition in $A_{4q}(\Sigma_{\uparrow}\to p_{\downarrow}+\gamma)$.

Our model is also consistent with the phenomenological "$\frac{1}{2}$-resonances model," which has been recently[21] proposed for calculating the parity-violating amplitude of hyperon radiative decay. Notice that $F_{L,L}^{n=1,\kappa=+1}$ and $F_{L,R}^{n=1,\kappa=+1}$ both contain terms

$$\frac{1}{\omega(l,+1,-1)-\omega(s,+1,-1)} \text{ and } \frac{1}{\omega(l,+1,-1)-\omega(s,n,+1)}$$

which can be regarded as being approximately the terms $1/(M_{N^*}-M_\Sigma)$ and $1/(M_N-M_{\Sigma^*})$ of that model, M_{N^*} and M_{Σ^*} being the masses of certain $\frac{1}{2}^-$ nonstrange and strange baryon resonances.

C. Evaluation of the two-quark transition amplitude A_{2q}

Calculation of the two-quark transition amplitude given in Eq. (2.19) is straightforward, and the result is

$$A_{2q}(\Sigma_\uparrow \to p_\downarrow + \gamma) = \left[\frac{-i}{3}\right](4\pi^2)\left[\frac{q_0}{2}\right]^{1/2} N_1{}^2\left[R^{0(11)}_{(00)} + \frac{E_1{}^2}{3}R^{0(11)}_{(11)} - \frac{2E_1{}^2}{3}R^{2(11)}_{(11)}\right]$$

$$\times[\chi_\downarrow^\dagger(\hat{\epsilon}\times\hat{q})\cdot\vec{\sigma}\chi_\uparrow F_2^V + i\chi_\downarrow^\dagger(\hat{\epsilon}\cdot\vec{\sigma})\chi_\uparrow F_2^A]\delta^3(\vec{q}+\vec{P}_N)\delta(q+E_N-M_\Sigma) . \quad (3.47)$$

The coefficients $-\frac{1}{3}$ come from SU(6) Clebsch-Gordan coefficients and color contractions. The other helicity amplitude $A_{2q}(\Sigma_\downarrow \to p_\uparrow + \gamma)$ can be obtained from Eq. (3.47) by interchanging $\chi_\downarrow$ and $\chi_\uparrow$ in that equation. The general structure of the helicity amplitude for such a local two-quark radiative transition has been studied by Gilman and Wise.[22] They write the amplitude as a product of three factors: (1) quark parameters F_2^V and F_2^A, (2) a numerical coefficient which comes from Clebsch-Gordan coefficients and color contractions, (3) a function $F(q)$ which depends on the overlap of the initial and final wave functions of baryons (as well as photon momentum). The amplitude in Eq. (3.47) has exactly that structure, and the function $F(q)$ is the calculable integral

$$F(q) = N_1{}^2\left[R^{0(11)}_{(00)} + \frac{E_1{}^2}{3}R^{0(11)}_{(11)} - \frac{2E_1{}^2}{3}R^{2(11)}_{(11)}\right]$$

in our model.

IV. NUMERICAL RESULTS AND DISCUSSION

We shall give the numerical estimates of quark transition amplitudes in this section. The set of bag model parameters we use is the conventional one which gives good predictions of hadron mass spectrum.[13] The parameters are

$$m_l = 0\ (l, = u,d)\ ,\quad m_s = 0.279 \text{ GeV}\ ,\quad \omega(l,+1,-1)R = 2.04\ ,$$

$$\omega(s,+1,-1)R = 2.909\ ,\quad \omega(l,+1,+1)R = 3.81,\quad \omega(s,+1,+1)R = 4.22\ ,$$

$$\omega(l,2,-1)R = 5.4,\quad \omega(l,2,+1)R = 7\ .$$

R is the integration limit of radial-variable integrations in two-quark and four-quark integrals; we take it as the average of the hyperon and proton radii[21] and $R = 5.285 \text{ GeV}^{-1}$.

The integrals of Bessel functions given in Appendix B are all numerically evaluated. The results for $A_{4q}^{(n)}$, $n = 1,2$ are

$$A_{4q}^{(1)}(\Sigma_\downarrow \to p_\downarrow + \gamma) = i\pi\left[\frac{2\pi}{9}\right]^{1/2} G_F e \sin\theta_1\cos\theta_1\cos\theta_3$$

$$\times\{(\chi_\downarrow^\dagger\hat{\epsilon}\times\hat{q}\cdot\vec{\sigma}\chi_\uparrow)[(C_1-C_2+C_3)-(1.73\times10^{-3})+(C_4-C_5)7.8\times10^{-4}]$$

$$+i(\chi_\downarrow^\dagger\hat{\epsilon}\cdot\vec{\sigma}\chi_\uparrow)[(C_1-C_2+C_3)2.7\times10^{-2}+(C_4-C_5)(-7.1\times10^{-3})]\}\ , \quad (4.1a)$$

$$A^{(2)}_{4q}(\Sigma_\uparrow \to p_\downarrow+\gamma)=i\pi\left[\frac{2\pi}{9}\right]^{1/2} G_F e\,\sin\theta_1\cos\theta_1\cos\theta_3$$
$$\times\{(\chi^\dagger_\downarrow\hat{\epsilon}\times\hat{q}\cdot\vec{\sigma}\chi_\uparrow)[(C_1-C_2+C_3)2.7\times10^{-5}+(C_4-C_5)4.13\times10^{-5}]$$
$$+i(\chi^\dagger_\downarrow\hat{\epsilon}\cdot\vec{\sigma}\chi_\uparrow)[(C_1-C_2+C_3)(-4.68\times10^{-3})+(C_4-C_5)1.67\times10^{-3}]\}\ . \quad (4.1b)$$

$A^{(n)}_{4q}$ is a decreasing sequence, and $A_{4q}(\Sigma_\uparrow\to p_\downarrow+\gamma)$ can be approximated by the sum of $A^{(1)}_{4q}$ and $A^{(2)}_{4q}$. The Wilson coefficients in Eqs. (4.1a) and (4.1b) are $C_1-C_2+C_3=-2.4$ (Ref. 23) and $C_4-C_5=0.06$, which are calculated by Gilman and Wise.

We substitute the expressions in Eq. (2.20) for F^V_2, and F^A_2 in amplitude $A_{2q}(\Sigma_\uparrow\to p_\downarrow+\gamma)$, and use numerical values for the "electromagnetic vertex":

$$N_1^2\left[R^{0(11)}_{(00)}+\frac{E_1^2}{3}R^{0(11)}_{(11)}-\frac{2E_1{}^2}{3}R^{2(11)}_{(11)}\right]=0.726\ .$$

Then

$$A_{2q}(\Sigma^\uparrow\to p_\downarrow+\gamma)=i\left[\frac{2\pi}{9}\right]^{1/2} G_F e\,\sin\theta_1\cos\theta_1\cos\theta_3\{7.6\times10^{-3}[\cos^2\epsilon_2 f(m_c)+\sin^2\theta_2 f(m_t)]\}$$
$$\times[\chi^\dagger_\downarrow(\hat{\epsilon}\times\hat{q})\cdot\vec{\sigma}\chi_\uparrow+i\chi^\dagger_\downarrow(\hat{\epsilon}\cdot\vec{\sigma})\chi_\uparrow]\ , \quad (4.2)$$

where $m_c=1.5$ GeV. The upper bound of $\sin\theta_2$ can be obtained from K_L-K_S mass-difference calculation and charm-meson-decay analysis; we have $\sin\theta_2\le 0.3$. We consider three possible top-quark masses, $m_t=15$ GeV, 30 GeV, and 60 GeV. They represent three different cases where the model has a light, heavy, and superheavy top quark. $f(m_c)$ and $f(m_t)$ are

$$f(m_c)=4.6\times10^{-5}\ ,$$
$$f(m_t=15\text{ GeV})=4.7\times10^{-3}\ ,\quad f(m_t=30\text{ GeV})=0.118\ ,\quad f(m_t=60\text{ GeV})=0.106\ . \quad (4.3)$$

$A_{2q}(\Sigma_\uparrow\to p_\downarrow+\gamma)$ is much smaller than $A_{4q}(\Sigma_\uparrow\to p_\downarrow+\gamma)$ and the latter determines the whole hyperon-decay amplitude,

$$A(\Sigma_\uparrow\to P_\downarrow+\gamma)\simeq N_B A_{4q}(\Sigma_\uparrow\to p_\downarrow+\gamma)\ .$$

N_B is the normalization constant which normalizes the calculated radiative decay width of Σ to the measured value. This normalization constant relates the matrix element evaluated using plane-wave states and the matrix element evaluated using spherical bag states. The asymmetry parameter defined in Eq. (2.6) is independent of N_B, and is predicted by our model as $\alpha=-0.154$, which is much smaller than the measured value $\alpha=-0.7$. Since our calculation is based on first principle, there is no free parameter in the model and we can use this radiative decay to explore the structure of $\Delta S=1$ nonleptonic weak-interaction Lagrangian in the standard model. The wrong prediction of α may indicate that there may be a right-handed charged hadronic current which contributes to $\Delta S=1$ nonleptonic weak transition. Actually the $\Delta S=1$ weak Lagrangian

$$\mathscr{L}_w(x)=\frac{-G_F}{\sqrt{2}}\sin\theta_1\cos\theta_3[(C_1+C_3)\delta_{\alpha\alpha'}\delta_{\beta\beta'}+C_2\delta_{\alpha\beta'}\delta_{\alpha'\beta}]$$
$$\times[L^\lambda+xR^\lambda]^{\sigma\sigma'}[L_\lambda+xR_\lambda]^{\rho\rho'}\bar{u}_{\alpha,\sigma}(x)u_{\alpha',\sigma'}(x)\bar{d}_{\beta,\rho}(x)\delta_{\beta',\rho'}(x)\ . \quad (4.4)$$

$[L^\lambda=\gamma^\lambda(1-\gamma_5),\ R^\lambda=\gamma^\lambda(1+\gamma_5)]$ can give an asymmetry parameter $\alpha=-0.7$ where $x\simeq\frac{2}{3}$.

We summarize the results of this paper as follows.

(1) Our calculation using the second-quantized MIT quark bag model can produce the traditional baryon-pole-model results for the parity-conserving amplitude as indicated in Eq. (3.43), and also give an expression

for the parity-violating amplitude which is somewhat similar to the prediction of the $\frac{1}{2}^-$ baryon-resonance model. Our quark-model calculation gives some justification of these phenomenological models.

(2) High radially excited resonant states give very little contribution to this radiative decay amplitude.

(3) Only the $\Delta I = \frac{1}{2}$ part of the weak-interaction Lagrangian contributes to Σ radiative decay.

(4) There are no ambiguities of relative phase and normalizations between the two-quark transition amplitude and the four-quark transition amplitude in our model. In the standard model, numerical estimates show A_{4q} is much larger than A_{2q}.

(5) There may be a right-handed charged hadronic current which contributes to nonleptonic weak decay. The standard model may not be sufficient for nonleptonic decay.

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APPENDIX: CALCULATION OF TWO-QUARK AND FOUR-QUARK INTEGRALS

The two-quark integrals which appear in $I_2^{(u)}$, $I_2^{(d)}$, $\tilde{I}_2^{(u)}$, and $\tilde{I}_2^{(s)}$ are the photon-emission amplitudes. Since electromagnetic interaction conserves flavor, we omit the flavor indices in the wave functions. The transition amplitudes for emitting a transversely polarized photon are

$$\int \bar{\psi}_{1,-1,M}(\vec{x})\epsilon\psi_{n,-1,M'}(\vec{x})e^{-i\vec{q}\cdot\vec{x}}d^3x = \int \bar{\psi}_{n,-1,M}(\vec{x})\epsilon\psi_{1,-1,M'}(\vec{x})e^{-iq\cdot x}d^3x$$
$$=(-i)N_1N_n[\chi_M^\dagger(\hat{\epsilon}\times\hat{q})\cdot\vec{\sigma}\chi_{M'}][E_nR^{1(1\,n)}_{(0\,1)}+E_1R^{1(1\,n)}_{(1\,0)}] , \tag{A1}$$

$$\int \bar{\psi}_{1,-1,M}(\vec{x})\epsilon\psi_{n,+1,M'}(\vec{x})e^{-i\vec{q}\cdot\vec{x}}d^3x = -\int \bar{\psi}_{n,+1,M}(\vec{x})\vec{\epsilon}\psi_{1,-1,M'}(\vec{x})e^{-i\vec{q}\cdot\vec{x}}d^3x$$
$$=iN_1\tilde{N}_n[\chi_M^\dagger(\hat{\epsilon}\cdot\vec{\sigma})\chi_{M'}]\left[\tilde{R}^{0(1\,n)}_{(0\,0)}-\frac{E_1\tilde{E}_n^{\,-1}}{3}\tilde{R}^{0(1\,n)}_{(1\,1)}+\frac{2E_1\tilde{E}_n^{\,-1}}{3}\tilde{R}^{2(1\,n)}_{(1\,1)}\right] , \tag{A2}$$

$$\int \bar{\psi}_{1,-1,M}(\vec{x})\epsilon\psi^C_{n,+1,M'}(\vec{x})e^{-i\vec{q}\cdot\vec{x}}d^3x = iN_1\tilde{N}_n[\chi_M^\dagger(\hat{\epsilon}\times\hat{q})\cdot\vec{\sigma}\chi^C_{M'}][\tilde{E}_n^{\,-1}\tilde{R}^{1(1\,n)}_{(0\,1)}-E_1\tilde{R}^{1(1\,n)}_{(1\,0)}] , \tag{A3}$$

$$\int \bar{\psi}^C_{n,+1,M}(\vec{x})\epsilon\psi_{1,-1,M'}(\vec{x})e^{-i\vec{q}\cdot\vec{x}}d^3x = iN_1\tilde{N}_n[\chi_M^{C\dagger}(\hat{\epsilon}\times\hat{q})\cdot\vec{\sigma}\chi_{M'}][\tilde{E}_n^{\,-1}\tilde{R}^{1(1\,n)}_{(0\,1)}-E_1\tilde{R}^{1(1\,n)}_{(1\,0)}] , \tag{A4}$$

$$\int \bar{\psi}_{1,-1,M}(\vec{x})\epsilon\psi^C_{n,-1,M'}(\vec{x})e^{-i\vec{q}\cdot\vec{x}}d^3x = -iN_1N_n[\chi_M^\dagger(\hat{\epsilon}\cdot\vec{\sigma})\chi^C_{M'}]\left[R^{0(1\,n)}_{(0\,0)}+\frac{E_1E_n}{3}R^{0(1\,n)}_{(1\,1)}-\frac{2E_1E_n}{3}R^{2(1\,n)}_{(1\,1)}\right] , \tag{A5}$$

$$\int \bar{\psi}^C_{n,-1,M}(\vec{x})\epsilon\psi_{1,-1,M'}(\vec{x})e^{-i\vec{q}\cdot\vec{x}}d^3x = iN_1N_n[\chi_M^{C\dagger}(\hat{\epsilon}\cdot\vec{\sigma})\chi_{M'}]\left[R^{0(1\,1)}_{(0\,0)}+\frac{E_1E_n}{3}R^{0(1\,n)}_{(1\,1)}-\frac{2E_1E_n}{3}R^{2(1\,n)}_{(1\,1)}\right] . \tag{A6}$$

The wave functions in the above equations are those given in Eqs. (3.3a) and (3.3b). The charge-conjugate spinor of χ_M is defined as

$$\chi_M^C = \sigma_2\chi_m . \tag{A7}$$

$R^{l(m\,n)}_{(r\,s)}$ and $\tilde{R}^{l(m\,n)}_{(r\,s)}$ are defined as

$$\tilde{R}^{l(m\,n)}_{(r\,s)} = R^3\int_0^1 dx\, x^2 j_l(\bar{q}x)j_r(Z_mx)j_s(Z_nx) , \tag{A8}$$

$$\tilde{R}^{l(m\,n)}_{(r\,s)} = R^3\int_0^1 dx\, x^2 j_l(\bar{q}x)j_r(Z_mx)j_s(\tilde{Z}_nx) , \tag{A9}$$

where $R=\frac{1}{2}(R_\Sigma+R_p)$ and R_Σ and R_p are the bag radii of Σ and p; $\bar{q}=qR$.

The four-quark integrals are the amplitudes of $\Delta S=1$ weak transitions. The one-loop-order QCD-renormalized $\Delta S=1$ weak-interaction Lagrangian contains operators of $(V-A)\otimes(V-A)$ type and $(V-A)\otimes(V+A)$ type. We therefore calculate the matrix element of a general operator with chiral structure

$$[\gamma^\lambda(1+a\gamma_5)]_{\sigma\sigma'}[\gamma_\lambda(1+b\gamma_5)]_{\rho\rho'}\ .$$

All integrals are evaluated in exact $SU_F(3)$ symmetry by giving all quarks a common mass parameter. Flavor indices of wave functions are suppressed. The results are presented below:

$$\begin{aligned}A_n(a,b)&=\int d^3y[\bar\psi_{n,-1,M_1}(\vec y)\gamma^\lambda(1+a\gamma_5)\psi_{1,-1,M_2}(\vec y)][\bar\psi_{1,-1,M_3}(\vec y)\gamma_\lambda(1+b\gamma_5)\psi_{1,-1,M_4}(\vec y)]\\&=\int d^3y[\bar\psi_{1,-1,M_1}(\vec y)\gamma^\lambda(1+a\gamma_5)\psi_{n,-1,M_2}(\vec y)][\bar\psi_{1,-1,M_3}(\vec y)\gamma_\lambda(1+b\gamma_5)\psi_{1,-1,M_4}(\vec y)]\\&=\frac{N_1{}^3N_n}{4\pi}[(\chi^\dagger_{M_1}\chi_{M_2})(\chi^\dagger_{M_3}\chi_{M_4})L_{1,n}-(\chi^\dagger_{M_1}\sigma^i\chi_{M_2})(\chi^\dagger_{M_3}\sigma^i\chi_{M_4})L_{2,n}(a,b)]\ ,\end{aligned}\tag{A10}$$

where

$$L_{1,n}=R^3\int_0^1dx\,x^2[j_0{}^2(Z_1x)+E_1{}^2j_1{}^2(Z_1x)][j_0(Z_1x)j_0(Z_nx)+E_1E_nj_1(Z_1x)j_1(Z_nx)]\ ,\tag{A11}$$

and

$$\begin{aligned}L_{2,n}(a,b)=R^3\int_0^1dx\,x^2\Bigg\{&ab[j_0{}^2(Z_1x)-E_1{}^2j_1{}^2(Z_1x)][j_0(Z_1x)j_0(Z_nx)-E_1E_nj_1(Z_1x)j_1(Z_nx)]\\&+\left[\frac{2ab+4}{3}\right]E_1j_0(Z_1x)j_1(Z_1x)[E_1j_0(Z_nx)j_1(Z_1x)+E_nj_0(Z_1x)j_1(Z_nx)]\Bigg\}\ ,\end{aligned}\tag{A12}$$

$$\begin{aligned}B_n(a,b)&=\int d^3y[\bar\psi_{n,+1,M_1}(\vec y)\gamma^\lambda(1+a\gamma_5)\psi_{1,-1,M_2}(\vec y)][\bar\psi_{1,-1,M_3}(\vec y)\gamma_\lambda(1+b\gamma_5)\psi_{1,-1,M_4}(\vec y)]\\&=-\int d^3y[\bar\psi_{1,-1,M_1}(\vec y)\gamma^\lambda(1+a\gamma_5)\psi_{n,+1,M_2}(\vec y)][\bar\psi_{1,-1,M_3}(\vec y)\gamma_\lambda(1+b\gamma_5)\psi_{1,-1,M_4}(\vec y)]\\&=i\frac{N_1{}^3\tilde N_n}{4\pi}\{(\chi^\dagger_{M_1}\chi_{M_2})(\chi^\dagger_{M_3}\chi_{M_4})aM_{1,n}-(\chi^\dagger_{M_1}\sigma^i\chi_{M_2})(\chi^\dagger_{M_3}\sigma^i\chi_{M_4})M_{2,n}(a,b)\}\ ,\end{aligned}\tag{A13}$$

where

$$M_{1,n}=R^3\int_0^1dx\,x^2[j_0(Z_1x)j_0(Z_nx)+E_1\tilde E_n{}^{-1}j_1(Z_1x)j_1(Z_nx)][j_0{}^2(Z_1x)+E_1{}^2j_1{}^2(Z,x)]\ ,\tag{A14}$$

and

$$\begin{aligned}M_{2,n}(a,b)=R^3\int_0^1dx\,x^2\Bigg\{&b[j_0{}^2(Z_1,x)-E_1{}^2j_1{}^2(Z_1x)][j_0(Z_1x)j_0(Z_nx)-E_1\tilde E_n{}^{-1}j_1(Z_1x)j_1(Z_nx)]\\&+\left[\frac{4a+2b}{3}\right]E_1j_0(Z_1x)j_1(Z_1x)[E_1j_0(\tilde Z_nx)j_1(Z_1x)+\tilde E_n{}^{-1}j_0(Z_1x)j_1(\tilde Z_nx)]\Bigg\}\ ,\end{aligned}\tag{A15}$$

$$\begin{aligned}C_n(a,b)&=\int d^3y[\bar\psi_{1,-1,M_1}(\vec y)\gamma^\lambda(1+a\gamma_5)\psi^C_{n,+1,M_2}(\vec y)][\bar\psi_{1,-1,M_3}(\vec y)\gamma_\lambda(1+b\gamma_5)\psi_{1,-1,M_4}(\vec y)]\\&=\int d^3y[\bar\psi^C_{n+1,M_1}(\vec y)\gamma^\lambda(1+a\gamma_5)\psi_{1,-1,M_2}(\vec y)][\bar\psi_{1,-1,M_3}(\vec y)\gamma_\lambda(1+b\gamma_5)\psi_{1,-1,M_4}(\vec y)]\\&=\frac{N_1{}^3\tilde N_n}{4\pi}\{(\chi^\dagger_{M_1}\chi_{M_2})(\chi^\dagger_{M_3}\chi_{M_4})\tilde L_{1,n}-(\chi_{M_1}\sigma^i\chi_{M_2})(\chi^\dagger_{M_3}\sigma^i\chi_{M_4})\tilde L_{2,n}(a,b)\}\ ,\end{aligned}\tag{A16}$$

where

$$\widetilde{L}_{1,n}=R^3\int_0^1 dx\,x^2[j_0^2(Z_1x)+E_1^2j_1^2(Z_1x)][j_0(Z_1x)j_0(Z_nx)-E_1\widetilde{E}_n^{-1}j_1(Z_1x)j_1(\widetilde{Z}_nx)]\,, \quad \text{(A17)}$$

and

$$\widetilde{L}_{2,n}(a,b)=R^3\int_0^1 dx\,x^2\left\{ab[j_0^2(Z_1x)-E_1^2j_1^2(Z_1x)][j_0(Z_1x)j_0(\widetilde{Z}_nx)+E_n\widetilde{E}_n^{-1}j_1(Z_1x)j_1(\widetilde{Z}_nx)]\right.$$
$$\left.-\left[\frac{2ab+4}{3}\right]E_1j_0(Z_1x)j_1(Z_1x)[\widetilde{E}_n^{-1}j_0(Z_1x)j_1(\widetilde{Z}_nx)-E_1j_0(\widetilde{Z}_nx)j_1(Z_1x)]\right\}, \quad \text{(A18)}$$

$$D_n(a,b)=\int d^3y[\overline{\psi}_{1,-1,M_1}(\vec{y})\gamma^\lambda(1+a\gamma_5)\psi^C_{n,-1,M_2}(\vec{y})][\overline{\psi}_{1,-1,M_3}(\vec{y})\gamma_\lambda(1+b\gamma_5)\psi_{1,-1,M_4}(\vec{y})]$$
$$=-d^3y[\overline{\psi}^C_{n,-1,M_1}(\vec{y})\gamma^\lambda(1+a\gamma_5)\psi_{1,-1,M_2}(\vec{y})][\overline{\psi}_{1,-1,M_3}(\vec{y})\gamma_\lambda(1+b\gamma_5)\psi_{1,-1,M_4}(\vec{y})]$$
$$=i\frac{N_1^3N_n}{4\pi}\{(\chi^\dagger_{M_1}\chi_{M_2})(\chi^\dagger_{M_3}\chi_{M_4})a\widetilde{M}_{1,n}-(\chi^\dagger_{M_1}\sigma^i\chi_{M_2})(\chi^\dagger_{M_3}\sigma^i\chi_{M_4})\widetilde{M}_{2,n}(a,b)\}\,, \quad \text{(A19)}$$

where

$$\widetilde{M}_{1,n}=R^3\int_0^1 dx\,x^2[j_0(Z_1x)j_0(Z_nx)-E_1E_nj_1(Z_1x)j_1(Z_nx)][j_0^2(Z_1x)+E_1^2j_1^2(Z_1x)] \quad \text{(A20)}$$

and

$$\widetilde{M}_{2,n}(a,b)=R^3\int_0^1 dx\,x^2\left\{b[j_0^2(Z_1x)-E_1^2j_1^2(Z_1x)][j_0(Z_1x)j_0(Z_nx)+E_1E_nj_1(Z_1x)j_1(Z_nx)]\right.$$
$$\left.-\left[\frac{4a+2b}{3}\right]E_1j_0(Z_1x)j_1(Z_1x)[E_nj_0(Z_1x)j_1(Z_nx)-E_1j_1(Z_1x)j_0(Z_nx)]\right\}. \quad \text{(A21)}$$

When the $SU_F(3)$-symmetry-breaking effects are included in $A_1(a,b)$, $L_{1,1}$ and $L_{2,1}(1,-1)$ will have the following expressions:

$$L_{1,1}=\frac{N'}{N}R^2\int_0^1 dx\,x^2[j_0(Z_1x)j_0(Z_1x)+E_1E_1'j_1(Z_1x)j_1(Z',x)][j_0^2(Z_1x)+E_1^2j_1'^2(Z_1x)]\,, \quad \text{(A22)}$$

$$L_{2,1}(+1,-1)=\frac{N'}{N}R^3\int_0^1 dx\,x^2\{\tfrac{2}{3}E_1j_0(Z_1x)j_1(Z_1x)[E_1j_0(Z_1'x)j_1(Z_1x)+E_1'j_0(Z_1x)j_1(Z_1'x)]$$
$$-[j_0^2(Z_1x)-E_1^2j_1^2(Z_1x)][j_0(Z_1x)j_0(Z_1'x)-E_1E_1'j_1(Z_1x)j_1(Z_1'x)]\}\,, \quad \text{(A23)}$$

where symbols with a prime pertain to kinematics of strange quarks.

[1]R. Graham and S. Pakvasa, Phys. Rev. 140, B1144 (1965).
[2]H. Sugawara, Nuovo Cimento 31, 635 (1964); B. W. Lee and A. R. Swift, Phys. Rev. 136, B228 (1964).
[3]G. Farrar, Phys. Rev. D 4, 212 (1971).
[4]H. Galic, Phys. Rev. D 24, 2441 (1981).
[5]C. G. Callan, Phys. Rev. D 2, 1541 (1970); K. Symanzik, Commun. Math. Phys. 18, 227 (1970).
[6]O. V. Maxwell and V. Vento, Report No. DPH-T/81-76 (unpublished).
[7]Carl-Edwin Carlson and Michael Chachkhunashvila, Nordita Report No. 81/18 (unpublished).
[8]J. Donoghue and K. Johnson, Phys. Rev. D 21, 1975 (1980).
[9]B. W. Lee and R. Shrock, Phys. Rev. D 16, 1444 (1977); T. P. Cheng and Ling-Fong Li, *ibid.* 16, 1425 (1977).
[10]R. Marshak, Riazuddin, and C. Ryan, *Theory of*

Weak Interactions in Particle Physics (Wiley-Interscience, New York, 1969).

[11]E. Abers and B. W. Lee, Phys. Rep. 9C, 1 (1973).

[12]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[13]A. Chodos, R. Jaffe, K. Johnson, C. Thorn, and V. Weisskopf, Phys. Rev. D 9, 3397 (1974); A. Chodos, R. Jaffe, K. Johnson, and C. Thorn, *ibid.* 10, 2599 (1974). A good review of the quark bag model can be found in P. Hasenfratz and J. Kuti, Phys. Rep. 40C, 75 (1978); and R. Jaffe, in *Pointlike Structure Inside and Outside Hadrons,* proceedings of the Seventeenth International School of Subnuclear Physics, Erice, 1979, edited by A. Zichichi (Plenum, New York, 1982).

[14]B. W. Lee and M. Gaillard, Phys. Rev. Lett. 33, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. 52B, 351 (1974).

[15]Ernest Ma and A. Pramudita, Phys. Rev. D 24, 1410 (1981).

[16]J. Donoghue, E. Golowich, W. A. Ponce, and B. R. Holstein, Phys. Rev. D 21, 186 (1980).

[17]R. Shrock and S. B. Treiman, Phys. Rev. D 19, 2148 (1979).

[18]A. J. G. Hey, B. R. Holstein, and D. P. Sidhu, Ann. Phys. (N. Y.) 117, 5 (1979); R. H. Hackman, N. G. Deshpande, D. A. Dicus, and V. L. Teplitz, Phys. Rev. D 18, 2537 (1979).

[19]A. Kamal and R. Verma, Alberta Report No. Thy-20-81 (unpublished).

[20]Patrick Hays and Martin V. K. Ulehla, Phys. Rev. D 13, 1339 (1976).

[21]I. Picek, Phys. Rev. D 21, 3169 (1980).

[22]F. Gilman and M. Wise, Phys. Rev. D 19, 976 (1979).

[23]F. Gilman and M. Wise, Phys. Rev. D 20, 2392 (1979).