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Tachyons and the radiation of an accelerated charge

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The motion of an accelerated charge in vacuum is analyzed, via the superposition principle and Fourier analysis, into uniform-motion components, which include bradyonic as well as tachyonic contributions. It is shown that the former contribute only to the induction fields whereas the latter are the source of the radiation emitted by the charge, via the Sommerfeld-Čerenkov mechanism. This result calls for a reexamination of some recently formulated theories of superluminal particles.

We want to show that the origin of the radiation emitted by an accelerated charge *in vacuum* can be traced to the Sommerfeld-Čerenkov mechanism, i.e., to the fact that a charge moving in a given medium with a velocity larger than the velocity of light in that medium does radiate.[1]

As will be shown below, this comes about because the motion of an accelerated charge can be analyzed into uniform-motion components, via Fourier analysis. These components can be classified as tachyonic and bradyonic, and it is the former, that is, the tachyonic components, which give rise to the radiation fields of an accelerated charge.[2]

To illustrate this contention we shall discuss in detail the case of a point charge q in vacuum performing a general one-dimensional motion along the z axis.[3] The electromagnetic fields can be determined from the potentials $\Phi(\vec{r},t)$ and $A_z(\vec{r},t)$. We shall consider in detail only the scalar potential $\Phi(\vec{r},t)$; the vector potential $A_z(\vec{r},t)$ obeys similar equations and can likewise be determined. Within the Lorentz gauge the potential Φ satisfies

$$\nabla^2\Phi-\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}=-4\pi\rho(\vec{r},t)$$
$$=-4\pi q\delta(x)\delta(y)\delta(z-z(t)) , \quad (1)$$

where $z(t)$ represents the one-dimensional law of motion of the charge.

By invoking the superposition principle we can solve Eq. (1) performing a Fourier transform of the factor $\delta(z-z(t))$ in the form

$$\delta(z-z(t))=\int_{-\infty}^{\infty}dk\int_{-\infty}^{\infty}d\omega\, g(k,\omega)\times e^{i(kz-\omega t)} , \quad (2)$$

where

$$g(k,\omega)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}e^{-ikz(t)}e^{i\omega t}dt . \quad (3)$$

With this, we can write the source $\rho(\vec{r},t)$ as a sum of components $\rho(k,\omega,\vec{r},t)$ which represent line charge densities moving along the z axis with uniform velocities $v_{k\omega}=\omega/k$, i.e.,

$$\rho(\vec{r},t)=\int_{-\infty}^{\infty}dk\int_{-\infty}^{\infty}d\omega\,\rho(k,\omega,\vec{r},t)$$
$$=\int_{-\infty}^{\infty}dk\int_{-\infty}^{\infty}d\omega\, q\delta(x)\delta(y)g(k,\omega)\times e^{i(kz-\omega t)} ,$$
$$J_z(\vec{r},t)=\int_{-\infty}^{\infty}dk\int_{-\infty}^{\infty}d\omega\, j_z(k,\omega,\vec{r},t)$$
$$=\int_{-\infty}^{\infty}dk\int_{-\infty}^{\infty}d\omega\,\rho(k,\omega,\vec{r},t)v_{k\omega} . \quad (4)$$

The solution of Eq. (1) can thus be written

$$\Phi(\vec{r},t)=\int_{-\infty}^{\infty}dk\int_{-\infty}^{\infty}d\omega\,\varphi(k,\omega,\vec{r},t) . \quad (5)$$

The component $\varphi(k,\omega,\vec{r},t)$ satisfies an equation analogous to Eq. (1) whose source is $\rho(k,\omega,\vec{r},t)$. Taking into account the symmetry of the problem this equation reads

$$\frac{1}{\rho}\frac{d}{d\rho}\left[\rho\frac{d}{d\rho}\varphi(k,\omega,\vec{r},t)\right]+\Omega^2\varphi(k,\omega,\vec{r},t)=-4qg(k,\omega)\frac{\delta(\rho)}{\rho}e^{i(kz-\omega t)} , \quad (6)$$

where $\rho^2=x^2+y^2$ and $\Omega^2=\omega^2/c^2-k^2$.

For $\Omega^2<0$, which implies $|v_{k\omega}|<c$, the solution of Eq. (6) is[4]

$$\varphi(k,\omega,\vec{r},t)=2qg(k,\omega)K_0(\xi)e^{i(kz-\omega t)}\,, \tag{7}$$

where $\xi=(-\Omega^2)^{1/2}\rho$ and $K_0(\xi)$ is the modified Bessel function which behaves as $-\ln\xi$ for $\xi<<1$ and as $(\pi/2\xi)^{1/2}e^{-\xi}$ for $\xi>>1$.

For $\Omega^2>0$, which implies $|v_{k\omega}|>c$, there are two different solutions, depending on the sign of ω, given by[5]

$$\begin{aligned}\omega<0:\ \ &\varphi(k,\omega,\vec{r},t)=-i\pi qg(k,\omega)H_0^{(2)}(\eta)e^{i(kz-\omega t)}\,,\\ \omega>0:\ \ &\varphi(k,\omega,\vec{r},t)=i\pi qg(k,\omega)H_0^{(1)}(\eta)e^{i(kz-\omega t)}\,,\end{aligned} \tag{8}$$

where $\eta=(\Omega^2)^{1/2}\rho$ and $H_0^{(1)}(\eta)$, $H_0^{(2)}(\eta)$ are the Hankel functions which behave as $\pm i(2/\pi)\ln\eta$ for $\eta<<1$ and as $(2/\pi\eta)^{1/2}e^{\pm i(\eta-\pi/4)}$ for $\eta>>1$.

It should be noted here that in the equations above all values of k and ω are present. Therefore, all values of the velocities $v_{k\omega}$ of the line charge densities are possible. This means that the field of a charged particle performing an arbitrary one-dimensional motion will contain in general bradyonic and tachyonic parts. We shall call the "bradyonic" part of the field that field associated with line charge densities moving with constant velocities $v_{k\omega}$ with $|v_{k\omega}|<c$. We use the word "tachyonic" for those fields associated with line charge densities moving with constant velocities $v_{k\omega}$ with $|v_{k\omega}|>c$. According to this, in building up the total solution, we can split $\Phi(\vec{r},t)$ into its bradyonic and tachyonic parts $\Phi(\vec{r},t)=\Phi^B(\vec{r},t)+\Phi^T(\vec{r},t)$. The bradyon-part is given by

$$\Phi^B(\vec{r},t)=\int_{-\infty}^{\infty}dk\int_{-|k|c}^{|k|c}d\omega\,2qg(k,\omega)K_0(\xi)e^{i(kz-\omega t)} \tag{9}$$

and the tachyonic part by

$$\begin{aligned}\Phi^T(\vec{r},t)=&\int_{-\infty}^{\infty}dk\int_{-\infty}^{-|k|c}d\omega(-i\pi q)g(k,\omega)H_0^{(2)}(\eta)e^{i(kz-\omega t)}\\ &+\int_{-\infty}^{\infty}dk\int_{|k|c}^{\infty}d\omega(i\pi q)g(k,\omega)H_0^{(1)}(\eta)e^{i(kz-\omega t)}\,.\end{aligned} \tag{10}$$

By carrying out a similar analysis for the vector potential $A_z(\vec{r},t)$ we obtain $A_z=A_z^B+A_z^T$, where A_z^B $[A_z^T]$ is obtained from Eq. (9) [Eq. (10)] by replacing $g(k,\omega)$ with $(\omega/ck)g(k,\omega)$. Finally, from $\vec{E}=-\vec{\nabla}\Phi-(1/c)\partial\vec{A}/\partial t$ and $\vec{H}=\vec{\nabla}\times\vec{A}$, we see that the electromagnetic fields of the charged particle split in a natural way into bradyonic and tachyonic components, i.e., $\vec{E}=\vec{E}^B+\vec{E}^T$ and $\vec{H}=\vec{H}^B+\vec{H}^T$. The bradyonic fields are given by

$$\begin{aligned}E_\rho^B(\vec{r},t)&=\int\int_{\Omega^2<0}\mathscr{E}_\rho^B(k,\omega,\vec{r},t)dk\,d\omega=\int_{-\infty}^{\infty}dk\int_{-|k|c}^{|k|c}d\omega\,2qg(k,\omega)(-\Omega^2)^{1/2}K_1(\xi)e^{i(kz-\omega t)}\,,\\ E_z^B(\vec{r},t)&=\int\int_{\Omega^2<0}\mathscr{E}_z^B(k,\omega,\vec{r},t)dk\,d\omega=\int_{-\infty}^{\infty}dk\int_{-|k|c}^{|k|c}d\omega\,2iqg(k,\omega)\frac{\Omega^2}{k}K_0(\xi)e^{i(kz-\omega t)}\,,\\ H_\theta^B(\vec{r},t)&=\int\int_{\Omega^2<0}\mathscr{H}_\theta^B(k,\omega,\vec{r},t)dk\,d\omega=\int_{-\infty}^{\infty}dk\int_{-|k|c}^{|k|c}d\omega\,2qg(k,\omega)\frac{\omega}{ck}(-\Omega^2)^{1/2}K_1(\xi)e^{i(kz-\omega t)}\,,\end{aligned} \tag{11}$$

and $E_\theta^B=H_\rho^B=H_z^B=0$. Here $K_1(\xi)$ is the modified Bessel function which behaves as ξ^{-1} for $\xi<<1$ and as $(\pi/2\xi)^{1/2}e^{-\xi}$ for $\xi>>1$.

The tachyonic fields are given by

$$\begin{aligned}E_\rho^T(\vec{r},t)=\int\int_{\Omega^2>0}\mathscr{E}_\rho^T(k,\omega,\vec{r},t)dk\,d\omega=&\int_{-\infty}^{\infty}dk\int_{-\infty}^{-|k|c}d\omega(-i\pi q)g(k,\omega)(\Omega^2)^{1/2}H_1^{(2)}(\eta)e^{i(kz-\omega t)}\\ &+\int_{-\infty}^{\infty}dk\int_{|k|c}^{\infty}d\omega(i\pi q)g(k,\omega)(\Omega^2)^{1/2}H_1^{(1)}(\eta)e^{i(kz-\omega t)}\,,\\ E_z^T(\vec{r},t)=\int\int_{\Omega^2>0}\mathscr{E}_z^T(k,\omega,\vec{r},t)dk\,d\omega=&\int_{-\infty}^{\infty}dk\int_{-\infty}^{-|k|c}d\omega\,\pi qg(k,\omega)\frac{\Omega^2}{k}H_0^{(2)}(\eta)e^{i(kz-\omega t)}\\ &+\int_{-\infty}^{\infty}dk\int_{|k|c}^{\infty}d\omega(-\pi q)g(k,\omega)\frac{\Omega^2}{k}H_0^{(1)}(\eta)e^{i(kz-\omega t)}\,,\\ H_\theta^T(\vec{r},t)=\int\int_{\Omega^2>0}\mathscr{H}_\theta^T(k,\omega,\vec{r},t)dk\,d\omega=&\int_{-\infty}^{\infty}dk\int_{-\infty}^{-|k|c}d\omega(-i\pi q)g(k,\omega)\frac{\omega}{ck}(\Omega^2)^{1/2}H_1^{(2)}(\eta)e^{i(kz-\omega t)}\\ &+\int_{-\infty}^{\infty}dk\int_{|k|c}^{\infty}d\omega(i\pi q)g(k,\omega)\frac{\omega}{ck}(\Omega^2)^{1/2}H_1^{(1)}(\eta)e^{i(kz-\omega t)}\,,\end{aligned} \tag{12}$$

and $E_\theta^T = H_\rho^T = H_z^T = 0$. Here $H_1^{(1)}(\eta)$, $H_1^{(2)}(\eta)$ are the Hankel functions which behave as $\mp(2i/\pi)\eta^{-1}$ for $\eta << 1$ and as $\mp i(2/\pi\eta)^{1/2} e^{\pm i(\eta - \pi/4)}$ for $\eta >> 1$.

As described by Eq. (11), the *bradyonic fields* appear as a superposition of transverse magnetic (TM), cylindrical *evanescent waves*. These are solutions of the wave equation in vacuum except at $\rho = 0$. For $\xi = (-\Omega^2)^{1/2}\rho = K\rho >> 1$, they behave as $(1/\sqrt{K\rho})e^{-K\rho} e^{i(kz - \omega t)}$, giving no contribution to the energy flux across a cylindrical surface of arbitrary radius coaxial with the particle's track. These evanescent waves propagate along the z axis, bound to their sources, with the phase velocities $v_{k\omega}^{\text{ph}} = v_{k\omega} = (\omega/k)$.

As given by Eq. (12), the *tachyonic fields* appear as a superposition of TM cylindrical *outgoing waves*. For $\eta = (\Omega^2)^{1/2}\rho = k_\rho \rho >> 1$, they can be written as ($\Omega^2 > 0$; $\omega \gtrless 0$; $k_\rho \rho >> 1$):

$$\begin{aligned}
\mathscr{E}_\rho^T(k,\omega,\vec{r},t) &\cong \sqrt{2\pi} q g(k,\omega) k_\rho \frac{1}{(k_\rho \rho)^{1/2}} e^{\pm i(k_\rho \rho \pm kz - |\omega| t - \pi/4)} , \\
\mathscr{E}_z^T(k,\omega,\vec{r},t) &\cong \mp\sqrt{2\pi} q g(k,\omega) \frac{k_\rho^{\,2}}{k} \frac{1}{(k_\rho \rho)^{1/2}} e^{\pm i(k_\rho \rho \pm kz - |\omega| t - \pi/4)} , \\
\mathscr{H}_\theta^T(k,\omega,\vec{r},t) &\cong \sqrt{2\pi} q g(k,\omega) \frac{\omega k_\rho}{ck} \frac{1}{(k_\rho \rho)^{1/2}} e^{\pm i(k_\rho \rho \pm kz - |\omega| t - \pi/4)} ,
\end{aligned} \tag{13}$$

which shows that, far from their sources, they behave as outgoing cylindrical waves traveling with the phase velocity $|\omega|/(k_\rho^{\,2} + k^2)^{1/2} = c$ in the direction of the "asymptotic wave vector" $\vec{k}_a = k_\rho \hat{\rho} \pm k\hat{z}$, while their sources travel with velocities $v_{k\omega}$ with $|v_{k\omega} > c$ in the z direction. The angle φ between $\vec{k}_a$ and the z axis satisfies

$$\cos\varphi = \frac{\pm k}{(k_\rho^{\,2} + k^2)^{1/2}} = \frac{k}{\omega/c} = \frac{c}{v_{k\omega}} , \tag{14}$$

which is just the *Sommerfeld-Čerenkov radiation condition in vacuum.* The fields given by Eq. (13), which are transverse to the direction of propagation, decay as $\rho^{-1/2}$ and therefore do represent radiation fields in the sense that they contribute to the energy flux across a cylindrical surface of arbitrary radius coaxial with the z axis.

Let us first consider the case of a particle moving with constant velocity, for which $z(t) = v_0 t$ ($v_0 \geq 0$) and $g(k,\omega) = (1/2\pi)\delta(\omega - kv_0)$. For $v_0 < c$ only the bradyonic part of the field contributes [cf. Eqs. (11) and (12)], giving the fields of the *bradyonic* particle resolved into cylindrical *evanescent* waves. For a "real" tachyonic particle $v_0 > c$ and only the tachyonic part of the fields is nonvanishing [cf. Eqs. (11) and (12)]. From $S_\rho = -(c/4\pi)E_z^T H_\theta^T$ the total power radiated by the "real" tachyon results

$$\begin{aligned}
P &= \int_0^{2\pi} \rho\, d\theta \int_{-\infty}^{\infty} S_\rho(\rho,\theta,z) dz \\
&= q^2 \frac{v_0}{c^2} \left[1 - \frac{c^2}{v_0^{\,2}} \right] \int_0^\infty \omega\, d\omega .
\end{aligned} \tag{15}$$

This expression clearly diverges. But if we consider a charge of finite size and assume that the radiation it emits is limited to the waves which are larger than its diameter, we obtain a finite expression which results almost identical with that obtained by Sommerfeld.[6]

For general physical motions ($|\dot{z}(t)| < c$), the properties of the fields can be summarized in the following three theorems, the proof of which will be published elsewhere.[7]

Theorem I. For any physical motion there are always nonvanishing bradyonic fields, i.e.,

$$|\dot{z}(t)| < c \Longrightarrow \exists k,\omega$$

with $\omega^2/c^2 < k^2$ such that $g(k,\omega) \neq 0$.

Theorem II. For any charged particle in physical motion in which the acceleration is not identically zero there are always nonvanishing tachyonic fields. And conversely, if in any physical motion there are nonvanishing tachyonic fields, the charge's acceleration is not identically zero, i.e., $\ddot{z}(t)$ not identically zero $\Longleftrightarrow \exists k,\omega$ with $\omega^2/c^2 > k^2$ such that $g(k,\omega) \neq 0$.

Theorem III. For any physical motion the tachyonic fields are the only ones which contribute to the total energy radiated by the charge, which results in

$$\begin{aligned}
I &= \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} 2\pi R S_\rho dz \\
&= 8\pi^3 q^2 \int_{-\infty}^{\infty} dk \int_{|k|c}^{\infty} d\omega\, |g(k,\omega)|^2 \frac{\omega}{k^2} \\
&\quad \times \left[\frac{\omega^2}{c^2} - k^2 \right] .
\end{aligned} \tag{16}$$

Using the above given description of the fields of an accelerated charge, we have worked out the well-known example of the harmonic oscillator. We found that the tachyonic fields in the dipolar approximation give rise to the Larmor formula. This example and some other applications will be reported elsewhere.[8]

Our present analysis shows, via the superposition principle, that the radiation of an accelerated charge in vacuum can be traced to the Sommerfeld-Čerenkov mechanism of radiation by faster-than-light particles. This result is at variance with the conclusion of several authors who, either based on what they call "extended special relativity"[9] or using *ad hoc* procedures,[10] claim to have shown that uniformly moving charged tachyons do not emit Sommerfeld-Čerenkov radiation in vacuum. If this were true, according to our results, any accelerated charge would not emit radiation at all. In particular the assertion due to R. Mignani and E. Recami that "it is necessary *not to confuse* electromagnetic Čerenkov radiation (EČR) with usual electromagnetic radiation (UER) that a charged particle emits (even in vacuum) when it is *accelerated*"[11] should be reviewed in light of the current analysis.

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[1]As early as 1904, Sommerfeld arrived at the conclusion that radiation could be emitted in free space by a charged particle moving with constant velocity exceeding that of light [see A. Sommerfeld, Proceedings of the Amsterdam Academy 8, 346 (1904)]. After the advent in 1905 of the Einstein theory of special relativity, the Sommerfeld tachyonic solution was discarded. Radiation by faster-than-light particles found its application in the explanation of the Čerenkov effect given by Frank and Tamm in 1937 [P.Čerenkov, C. R. Acad. Sci. URSS 8, 451 (1934); I. Frank and Ig. Tamm, C. R. Acad. Sci. URSS, 14, 109 (1937)]. In this case the physical charged particles move faster than the velocity of light in the material medium without violating the principles of relativity. In 1962 E.C.G. Sudarshan *et al.* [Am. J. Phys. 30, 718 (1962)] showed that the special theory of relativity does not in fact forbid particles from moving faster than c, but rather that it inhibits subluminal particles from breaking the light barrier.

[2]Superluminal particles were named tachyons by G. Feinberg [Phys. Rev. 159, 1089 (1967)]. The name bradyons for subluminal particles was proposed by R. G. Cawley [Ann. Phys. (N.Y.) 54, 132 (1969)].

[3]In this paper we consider only one-dimensional motions, but the conclusions can be generalized to three-dimensional ones.

[4]See J. L. Agudín and A. M. Platzeck, J. Opt. Soc. Am. 70, 1329 (1980).

[5]This can be inferred from Ref. 4 and from the fact that $H_0^{(1)}(\eta)$, $H_0^{(2)}(\eta)$ are equal to $\mp(2/\pi)iK_0(\mp i\eta)$. The choice between $H_0^{(1)}$ and $H_0^{(2)}$ is made in order to obtain only outgoing solutions.

[6]See, for example, Ig. Tamm, J. Phys. 1, 439 (1939), principally the discussion following Eq. (4.10), where Sommerfeld's result is compared with the Frank and Tamm formula for the Čerenkov radiation. It is worth noting here that we have reobtained Sommerfeld's result by summing up the fields produced by all the tachyonic densities with $v_{k\omega}=v_0$ which are the only ones excited by the tachyonic particle.

[7]The proof of Theorem III appeared in Phys. Lett. 83A, 423 (1981).

[8]J. L. Agudín and A. M. Platzeck, Lett. Nuovo Cimento 31, 421 (1981).

[9]R. Mignani and E. Recami, Lett. Nuovo Cimento 7, 388 (1973); E. Recami and R. Mignani, Riv. Nuovo Cimento 4, 209 (1974); H. Lemke, Lett. Nuovo Cimento 12, 342 (1975).

[10]C. M. Ey and C. A. Hurst, Nuovo Cimento 39B, 76 (1977); C. C. Chiang, Center for Particle Theory, University of Texas Report No. CPT 117, 1971 (unpublished).

[11]R. Mignani and E. Recami, Lett. Nuovo Cimento 9, 362 (1974).