Radiative weak decays of baryons

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A systematic study of the two-quark transition amplitude, $s+u \to u+d+\gamma$, and its effect on the radiative weak decays of baryons is made. A model involving the single-quark transition amplitude, $s \to d+\gamma$, and the two-quark transition amplitude is shown to be consistent with the existing data.

I. INTRODUCTION

The radiative weak decays of baryons have attracted considerable attention recently. Although these decays occur at the level of a few tenths of a percent in branching ratio one would like to understand these decay rates and the asymmetry parameters within the context of the unified electroweak theory. The data at this stage is very sparse. One has the branching ratios for $\Sigma^+ \to p\gamma$ (Refs. 1–3) and $\Xi^0 \to \Lambda\gamma$ (Ref. 3), the asymmetry parameter for $\Sigma^+ \to p\gamma$ (Refs. 1–3), bounds on the rates for $\Xi^0 \to \Sigma^0\gamma$ and $\Xi^- \to \Sigma^-\gamma$ (Refs. 3 and 4).

Earlier theoretical attempts[5–15] have used a combination of techniques including pole models and internal-symmetry assumptions. Although these attempts produced the rates at the right order of magnitude the asymmetry was more difficult to obtain.

With the advent of the electroweak theory one would like to understand these decays at the quark-lepton level. Most attempts up to this stage, including those that investigate the short-distance behavior of current operators, study the single-quark transition operator [see Fig. 1(a)] for $s \to d\gamma$. The remaining two quarks are assumed to be spectators. The single-quark transition operator is of order $G_F e$ and the transition matrix element can be parametrized as

$$M = G_F e \bar{d}(a + b\gamma_5) k\!\!\!/ \epsilon\!\!/ s , \tag{1.1}$$

where d and s stand for the Dirac spinors with the obvious flavor. a and b are the parity-conserving and parity-violating amplitudes, respectively. If quarks were observed as free particles this amplitude would yield a decay rate

$$\Gamma(s \to d\gamma) = \frac{G_F{}^2 e^2}{\pi}(|a|^2 + |b|^2) k^3 , \tag{1.2}$$

where k is the photon momentum. In the single-quark transition model the rate for the baryon weak radiative decay is given by Eq. (1.2) up to an overall normalization factor.

The angular distribution relative to the s-quark spin is of the form

$$1 + \frac{2\,\mathrm{Re}(ab^*)}{|a|^2 + |b|^2} \hat{s}\cdot\hat{p} , \tag{1.3}$$

where $\hat{s}$ and $\hat{p}$ are unit vectors along the s-quark spin and the momentum of the d quark. The definition of the asymmetry parameter α is

$$\alpha = \frac{2\,\mathrm{Re}(ab^*)}{|a|^2 + |b|^2} . \tag{1.4}$$

The asymmetry parameter is independent of the overall normalization of a and b. It has the same value for $B_i \to B_f\gamma$ as for $s \to d\gamma$ in the single-quark transition model.

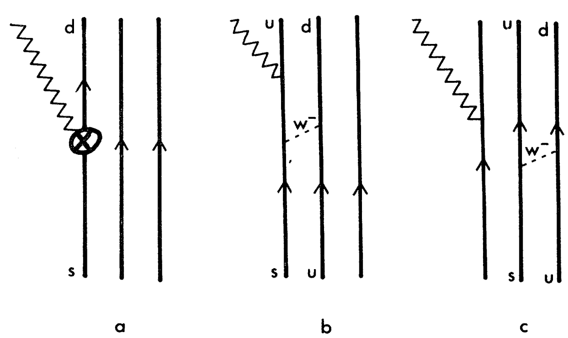

FIG. 1. Single-quark, two-quark, and three-quark transition mechanisms. All permutations of these diagrams have to be considered.

The U-spin properties of the weak and electromagnetic Hamiltonians imply that the parity-violating part of the radiative weak decay $\Sigma^+\to p\gamma$ vanishes in the U-spin-symmetry limit, that is, $\alpha=0$.[16,17] If one assumes that $m_d \neq m_s$, then the ratio of the parity-violating amplitude to the parity-conserving amplitude is $(m_s-m_d)/(m_s+m_d)$, implying a small but positive asymmetry parameter. The old measurement[1] of α_{Σ^+} was $-1.03^{+0.52}_{-0.42}$ while the new CERN measurement[2] yields $\alpha_{\Sigma^+}=-0.53^{+0.38}_{-0.36}$. Although the errors are large the asymmetry appears to be negative. One way to obtain a negative asymmetry parameter would be to invoke right-handed weak currents though this is disputed by some authors.[18]

Gilman and Wise[19] take a different approach and parametrize the single-quark transition operator by two parameters, a and b of Eq. (1.1). These are determined from $\Gamma(\Sigma^+\to p\gamma)$ and α_{Σ^+}. They then generate all the other baryon radiative-weak-decay rates through the quark-model SU(6) wave functions. They find that their predicted rate $\Gamma(\Xi^-\to\Sigma^-\gamma)$ violates the experimental upper bound[3] and unexpectedly large branching ratios are generated for $\Lambda\to n\gamma$ and $\Omega^-\to\Xi^-\gamma$. It was, therefore, concluded that a single-quark transition model is inadequate in describing the baryon radiative weak decays. Note that the parameter b in Eq. (1.1) must be proportional to (m_s-m_d) in order to satisfy the U-spin constraint.[17]

In Secs. II and III of this paper we have investigated a model of two-quark transition amplitude with one spectator [see Fig. 1(b)]. The model we investigate involves a systematic nonrelativistic evaluation of the quark-quark weak bremsstrahlung amplitude $s+u\to u+d+\gamma$. Because of the exchange of $W^\pm$ the quark flavors are as indicated and the matrix element is proportional to $G_F e\sin\theta_C\cos\theta_C$. The spectator quark can have any flavor. This two-quark transition amplitude contributes to the same order in coupling constants as the single-quark transition model of Fig. 1(a). Indeed a three-quark transition amplitude, Fig. 1(c), which has no spectators will also contribute to the same order in coupling constants. There is no compelling reason to assume that any of the processes of Figs. 1(a)–1(c) would dominate the physics. In Sec. IV we demonstrate that a three-parameter model which combines the effects of single-quark and two-quark transition amplitudes, Figs. 1(a) and 1(b), successfully describes the existing data. The paucity of data at this moment does not warrant inclusion of the three-quark transition amplitude of Fig. 1(c). We conclude the paper with a brief discussion in Sec. V.

II. TWO-QUARK TRANSITION AMPLITUDE

The general idea is to evaluate the effective electroweak Hamiltonian for $s+u\to u+d+\gamma$ with one spectator quark and sandwich it between baryon wave functions. This calculation can be done either in configuration space or in momentum space. As the effective Hamiltonian turns out to be quark momentum dependent the calculation is much cleaner in momentum space. In configuration space the quark momenta Fourier transform into space derivatives with the result that the calculation is much more cumbersome.

In the baryon radiative weak decays the energy available to the photon is $\approx 100-200$ MeV. If the photon were soft one would expect the quark-quark weak bremsstrahlung amplitude of Fig. 1(b) to diverge like $\sim 1/k$. In the two-body baryon decays the photon is monochromatic and far from soft. We, therefore, carry out an expansion of the amplitude in k such that each stage of expansion is gauge invariant. A natural expansion parameter which appears from the nonrelativistic reduction of the quark Dirac spinors is $k/2m$, where m is the quark mass. We shall use m for the u- and d-quark mass and m_s for the strange-quark mass.

We work in the Coulomb gauge; $\epsilon_0=0$, $\vec{\epsilon}\cdot\vec{k}=0$. The matrix element for the one process shown in Fig. 1(b) (without the permutations) for $s(p_1)+u(p_2)\to u(p_3)+d(p_4)+\gamma(k)$ is

$$\frac{e_u G_F \sin\theta_C\cos\theta_C}{\sqrt{2}2k\cdot p_3} \times \bar{u}(2\epsilon\cdot p_3+\not{\epsilon}\not{k})\gamma_\mu(1-\gamma_5)s\bar{d}\gamma_\mu(1-\gamma_5)u\bar{q}q\ . \tag{2.1}$$

The factor $\bar{q}q$ comes from the spectator quark. All external quarks have been assumed to be on their mass shells. In the soft-photon limit the terms proportional to $\epsilon\cdot p_3/k\cdot p_3$ diverge like $(1/k)$. We shall see later that these terms do not contribute to the baryon radiative weak decays.

Our procedure is to evaluate the matrix element for the process of Fig. 1(b) and all its permutations, taking care to antisymmetrize the amplitude as appears to be essential to satisfy the U-spin constraint on the parity-violating amplitude for $\Sigma^+\to p\gamma$ (see Fig. 2),

$$s(\vec{p}_1)+u(\vec{p}_2)\to u(\vec{p}_3)+d(\vec{p}_4)+\gamma(k)\ . \tag{2.2}$$

The spectator quark momenta are $\vec{p}_5$ in the initial state and $\vec{p}_6$ in the final state with $\vec{p}_5=\vec{p}_6$. Then we extract the parity-conserving and parity-violating parts of the matrix element. Finally we carry out the nonrelativistic reduction of the Dirac spinors neglecting terms of order $(\vec{p}^2/2m)$. The effective Hamiltonian so obtained is then sandwiched between baryon wave functions in momentum space. The procedure is outlined in the Appendix. Consider first the term in Eq. (2.1) proportional to $\epsilon\cdot p_3/k\cdot p_3$, with similar terms coming from other permutations shown in Fig. 2. In Coulomb gauge this term is proportional to $\vec{\epsilon}\cdot\vec{p}_3/k\cdot p_3$. Once such terms are sandwiched between baryon wave functions in momentum space and the quark momenta $\vec{p}_i$ integrated over, one can only generate terms proportional to $\vec{\epsilon}\cdot\vec{k}$, as $\vec{k}$ is the only available free momentum. By gauge invariance these terms, therefore, vanish.

The remaining term in Eq. (2.1) arising from Fig. 1(b) above is

$$H_{\rm eff}=\frac{G_F\sin\theta_C\cos\theta_C}{\sqrt{2}}\frac{e_u}{2k\cdot p_3}\times[\bar{u}\epsilon\!\!/ k\!\!/\gamma_\mu(1-\gamma_5)s\bar{d}\gamma_\mu(1-\gamma_5)u](\bar{q}q)\ . \tag{2.3}$$

Notice that this term is gauge invariant by itself. All other permutations of Fig. 1(b) shown in Fig. 2 are also, similarly, gauge invariant.

$H_{\rm eff}$ in Eq. (2.3) carries k dependence through two factors: the propagators $1/k\cdot p_3$ and the quark brackets. The propagators expanded in powers of k yield

$$\frac{1}{2p_i\cdot k}\simeq\frac{1}{2m_ik}\left[1+\frac{\vec{p}_i\cdot\vec{k}}{m_ik}\right]. \tag{2.4}$$

On integrating over the quark momenta $\vec{p}_i$ the propagator results in a leading term of $O(1/k)$ and a nonleading term of $O(k^0)$. The quark brackets in Eq. (2.3) on nonrelativistic reduction, result in leading terms of $O(k)$ and nonleading terms of $O(k^2)$. By taking products of appropriate orders we can generate $H_{\rm eff}$ of $O(k^0)$ and $O(k)$, the only two orders we are interested in.

To proceed further we define the following symbols:

$$eG(k)=\frac{e_u}{2p_3\cdot k}+\frac{e_s}{2p_1\cdot k}+\frac{e_d}{2p_4\cdot k}+\frac{e_u}{2p_2\cdot k}$$

and (2.5)

$$eH(k)=\frac{e_u}{2p_3\cdot k}-\frac{e_s}{2p_1\cdot k}+\frac{e_d}{2p_4\cdot k}-\frac{e_u}{2p_2\cdot k}\ .$$

We demonstrate, in some detail, the evaluation of the parity-conserving (PC) effective Hamiltonian. The same detail will not be provided for the parity-violating part of the effective Hamiltonian. We evaluate the contribution from all graphs of Fig. 2 with appropriate antisymmetrization and carry out the nonrelativistic reduction of the quark brackets. The resulting parity-conserving $H_{\rm eff}$ so obtained is

$$\begin{aligned}H_{\rm eff}^{\rm PC}=\frac{eG_F\sin\theta_C\cos\theta_C}{\sqrt{2}}(q^\dagger q)\Big[&(u^\dagger\vec{\sigma}\cdot\vec{\epsilon}sd^\dagger\vec{\sigma}\cdot\vec{k}u-u^\dagger\vec{\sigma}\cdot\vec{k}sd^\dagger\vec{\sigma}\cdot\vec{\epsilon}u)G(k)-i(\vec{\epsilon}\times\vec{k})\cdot(u^\dagger\vec{\sigma}sd^\dagger u-u^\dagger sd^\dagger\vec{\sigma}u)H(k)\\
&+\frac{ik}{2m}\{u^\dagger\vec{\epsilon}\cdot[\vec{\sigma}\times(\vec{p}_1-\vec{p}_3)]sd^\dagger u-u^\dagger sd^\dagger\vec{\epsilon}\cdot[\vec{\sigma}\times(\vec{p}_2-\vec{p}_4)]u\}G(k)\\
&-\frac{ik}{2m}\{u^\dagger\vec{\epsilon}\cdot[\vec{\sigma}\times(\vec{p}_2+\vec{p}_4)]sd^\dagger u-u^\dagger sd^\dagger\vec{\epsilon}\cdot[\vec{\sigma}\times(\vec{p}_1+\vec{p}_3)]u\}H(k)\\
&+\frac{k}{2m}\{u^\dagger\vec{\sigma}\cdot(\vec{p}_1+\vec{p}_3)sd^\dagger\vec{\sigma}\cdot\vec{\epsilon}u-u^\dagger\vec{\sigma}\cdot\vec{\epsilon}sd^\dagger\vec{\sigma}\cdot(\vec{p}_2+\vec{p}_4)u\}G(k)\\
&+\frac{k}{2m}\{u^\dagger\vec{\sigma}\cdot\vec{\epsilon}sd^\dagger\vec{\sigma}\cdot(\vec{p}_1-\vec{p}_3)u-u^\dagger\vec{\sigma}\cdot(\vec{p}_2-\vec{p}_4)sd^\dagger\vec{\sigma}\cdot\vec{\epsilon}u\}H(k)\\
&-i\frac{\xi k}{2m}u^\dagger\vec{\epsilon}\cdot(\vec{\sigma}\times\vec{p}_1)sd^\dagger uG(k)-i\frac{\xi k}{2m}u^\dagger sd^\dagger\vec{\epsilon}\cdot(\vec{\sigma}\times\vec{p}_1)uH(k)\\
&-\frac{\xi k}{2m}u^\dagger\vec{\sigma}\cdot\vec{p}_1sd^\dagger\vec{\sigma}\cdot\vec{\epsilon}uG(k)-\frac{\xi k}{2m}u^\dagger\vec{\sigma}\cdot\vec{\epsilon}sd^\dagger\vec{\sigma}\cdot\vec{p}_1uH(k)\Big]\ ,\end{aligned} \tag{2.6}$$

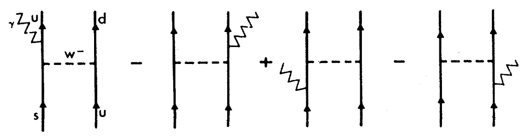

FIG. 2. Permutations of Fig. 1(b) with proper antisymmetrization.

where

$$\xi=(m_s-m)/m_s\ . \tag{2.7}$$

In order to calculate the baryon radiative-weak-decay amplitudes $H_{\text{eff}}^{\text{PC}}$ is sandwiched between the baryon wave functions, in momentum space, and the quark momenta are then integrated over. As demonstrated in the Appendix the evaluation of the space part of the integral leads, up to an overall factor, to the following replacement:

$$(\vec{p}_1,\vec{p}_2)\rightarrow\vec{k}/12,\ \ (\vec{p}_3,\vec{p}_4)\rightarrow-5\vec{k}/12\ . \tag{2.8}$$

The remaining calculation proceeds in the following steps.

(1) Make the replacements shown in (2.4) and (2.8) into (2.6). The replacement (2.8) can, in fact, be made directly into (2.4) since, as stated later, we are not interested in terms higher than $O(k)$. As a consequence we are not required to handle terms in $\vec{p}_i$ of order higher than the first. The resulting $G(k)$ and $H(k)$ are

$$G(k)=\frac{(2+3\xi)}{6mk}-\frac{(2-\xi)}{36m^2}\ ,$$
$$H(k)=-\frac{\xi}{6mk}-\frac{(3+\xi)}{36m^2}\ . \tag{2.9}$$

(2) The first two terms of (2.6) contribute both to $O(k^0)$ and $O(k)$ while the remaining terms contribute to $O(k)$. We ignore terms of order higher than $O(k)$.

(3) Project $H_{\text{eff}}^{\text{PC}}$ into photon helicity states through

$$(\vec{\epsilon}\cdot\vec{\sigma})\underset{\lambda_\gamma=\pm1}{\longrightarrow}\sigma_\mp,\ \ (\vec{\epsilon}\times\vec{k})\cdot\vec{\sigma}\underset{\lambda_\gamma=\pm1}{\longrightarrow}\mp ik\sigma_\mp\ . \tag{2.10}$$

$H_{\text{eff}}^{\text{PC}}$ in $\lambda_\gamma=+1$ helicity state of the photon is then derived to be [$\lambda_\gamma=-1$ projection is not written to avoid a proliferation of indices; $O(\xi^2)$ and $O(k^2)$ terms are also dropped]

$$H_{\text{eff}}^{\text{PC}}(\lambda_\gamma=+1)=\frac{eG_F\sin\theta_C\cos\theta_C}{\sqrt{2}}(q^\dagger q)\left[\frac{(2+3\xi)}{6m}(u^\dagger\sigma_-sd^\dagger\sigma_3u-u^\dagger\sigma_3sd^\dagger\sigma_-u)\right.$$
$$+\frac{\xi}{6m}(u^\dagger\sigma_-sd^\dagger u-u^\dagger sd^\dagger\sigma_-u)$$
$$\left.+\frac{\xi k}{72m^2}u^\dagger\sigma_-sd^\dagger u-\frac{\xi k}{72m^2}u^\dagger\sigma_3sd^\dagger\sigma_-u\right]\ . \tag{2.11}$$

Note that the only surviving $O(k)$ terms are proportional to ξ. In the degenerate-mass limit ($\xi\rightarrow0$) the $O(k)$ terms arising from the first two terms of Eq. (2.6) exactly cancel the $O(k)$ terms arising from the remaining terms of Eq. (2.6). As $\xi\approx\frac{1}{3}$ and one has large denominators in the last two terms of Eq. (2.11) we shall adopt the following as the model $H_{\text{eff}}^{\text{PC}}$:

$$H_{\text{eff}}^{\text{PC}}(\lambda_\gamma=+1)=\frac{eG_F\sin\theta_C\cos\theta_C}{\sqrt{2}}(q^\dagger q)\left[\frac{(2+3\xi)}{6m}(u^\dagger\sigma_-sd^\dagger\sigma_3u-u^\dagger\sigma_3sd^\dagger\sigma_-u)\right.$$
$$\left.+\frac{\xi}{6m}(u^\dagger\sigma_-sd^\dagger u-u^\dagger sd^\dagger\sigma_-u)\right]\ . \tag{2.12}$$

The parity-violating (PV) part of H_{eff} is isolated in the same manner. The final form for $H_{\text{eff}}^{\text{PV}}$ in the photon helicity state $\lambda_\gamma=+1$ is

$$H_{\text{eff}}^{\text{PV}}=\frac{eG_F\sin\theta_C\cos\theta_C}{\sqrt{2}}(q^\dagger q)\left[\frac{(2+3\xi)}{6m}(u^\dagger\sigma_-sd^\dagger u-u^\dagger sd^\dagger\sigma_-u)+\frac{\xi}{6m}(u^\dagger\sigma_-sd^\dagger\sigma_3u-u^\dagger\sigma_3sd^\dagger\sigma_-u)\right]\ . \tag{2.13}$$

Again we have neglected terms of $O(\xi k)$ as they appear with large denominators as in the case of $H_{\text{eff}}^{\text{PC}}$. ξ-independent terms of $O(k)$ cancel out in precisely the same manner as in the case of $H_{\text{eff}}^{\text{PC}}$. In the degenerate-quark-mass limit $H_{\text{eff}}^{\text{PC}}$ is entirely of $O(k^0)$, as is $H_{\text{eff}}^{\text{PC}}$.

III. EXTRACTION OF BARYON-DECAY AMPLITUDES IN TWO-QUARK MODEL

The gauge-invariant form of the radiative-weak-decay amplitude $B_i \to B_f + \gamma$ is

$$M = G_F e \bar{B}_f (A + B\gamma_5) \not{k} \not{\epsilon} B_i \;, \tag{3.1}$$

where B_i and B_f are the Dirac spinors. The decay rate is then given by

$$\Gamma(B_i \to B_f + \gamma) = \frac{G_F{}^2 e^2}{\pi} (\,|A\,|^2 + |\,B\,|^2) k^3 \tag{3.2}$$

and the asymmetry parameter is

$$\alpha = \frac{2\,\mathrm{Re}(\mathrm{AB}^*)}{|A\,|^2 + |\,B\,|^2} \;. \tag{3.3}$$

A nonrelativistic reduction of Eq. (3.1) yields

$$M = G_F e B_f^\dagger [iA\,(\vec{\epsilon} \times \vec{\mathrm{k}}) \cdot \vec{\sigma} - Bk\,\vec{\sigma} \cdot \vec{\epsilon}\,] \;, \tag{3.4}$$

where B_i and B_f are two-component Pauli spinors. Projecting out the two-photon helicity states one gets

$$M(\lambda_\gamma = \pm 1) = G_F e k\,(\pm A - B) B_f^\dagger \sigma_\mp B_i \;. \tag{3.5}$$

Note that the parity-violating part does not change sign while the parity-conserving part does.

The procedure we adopt to calculate the baryon-weak-radiative-decay matrix elements is detailed in the Appendix. In order to extract A and B of Eq. (3.5) we evaluate the $\lambda_\gamma = +1$ amplitudes from H_{eff} and then utilize the proportionality

$$\begin{aligned} \langle B_f \,|\, H_{\text{eff}}^{\text{PC}}(\lambda_\gamma = +1) \,|\, B_i \rangle &\propto kA \;, \\ \langle B_f \,|\, H_{\text{eff}}^{\text{PC}}(\lambda_\gamma = +1) \,|\, B_i \rangle &\propto -kB \;. \end{aligned} \tag{3.6}$$

The effective Hamiltonians for $\lambda_\gamma = +1$ for parity-conserving and parity-violating amplitudes, displayed in (2.12) and (2.13), are to be sandwiched between the baryon wave functions to generate the baryon-radiative-weak-decay amplitudes. Indeed in writing down (2.12) and (2.13) the effect of integrating over the spatial wave functions is already incorporated. What remains now is to evaluate spin and flavor dependence. This is done by using SU(6) wave functions. In Table I we have tabulated A and B up to an overall scale factor for the various processes arising from $H_{\text{eff}}^{\text{PC}}$ and $H_{\text{eff}}^{\text{PC}}$ of (2.12) and (2.13). As is evident from this table, the parity-violating contribution to $\Sigma^+ \to p\gamma$ vanishes in the U-spin limit ($\xi \to 0$) as it should.[16,17]

IV. A THREE-PARAMETER MODEL

The single-quark transition model, with two spectators, is generally parametrized in terms of two parameters.[19] The $s \to d + \gamma$ transition, in the nonrelativistic limit, leads to an effective Hamiltonian

$$H_{\text{eff}}(\text{1-quark}) = eG_F d^\dagger [ia\,(\vec{\epsilon} \times \vec{\mathrm{k}}) \cdot \vec{\sigma} - bk\,\vec{\epsilon} \cdot \vec{\sigma}] s\,(q^\dagger q)(q^\dagger q) \;, \tag{4.1}$$

where s, d, and q are two-component Pauli spinors. a and b are two parameters. $(q^\dagger q)$ arise from the spectator quarks. The photon helicity $\lambda_\gamma = +1$ projections of (4.1) are

$$H_{\text{eff}}^{\text{PC}}(\text{1-quark}) = eG_F k a\,(d^\dagger \sigma_- s)(q^\dagger q)(q^\dagger q) \tag{4.2}$$

and

$$H_{\text{eff}}^{\text{PV}}(\text{1-quark}) = -eG_F k b\,(d^\dagger \sigma_- s)(q^\dagger q)(q^\dagger q) \;. \tag{4.3}$$

Following the method outlined in the Appendix we evaluate the baryon matrix elements by sandwiching the operators of (4.2) and (4.3) between baryon wave functions in momentum space. Spin and flavor dependence are generated by using the SU(6) wave functions. The space part of the matrix element for the single-quark and the two-

TABLE I. Decay amplitudes in two-quark model up to a scale factor.

Decay	Parity conserving	Parity violating
$\Sigma^+ \to p\gamma$	$(2+3\xi)/\sqrt{2}$	$-\xi/\sqrt{2}$
$\Sigma^0 \to n\gamma$	$\xi/2$	$-(2+3\xi)/2$
$\Lambda \to n\gamma$	$-\sqrt{3}(4+7\xi)/6$	$\sqrt{3}(2+5\xi)/6$
$\Xi^0 \to \Sigma^0\gamma$	$-(1+\xi)$	$-(1+\xi)$
$\Xi^0 \to \Lambda\gamma$	$-(1+2\xi)/\sqrt{3}$	$(1+2)\xi)/\sqrt{3}$
$\Xi^- \to \Sigma^-\gamma$	0	0
$\Omega^- \to \Xi^-\gamma$	0	0

TABLE II. Decay amplitudes in three-parameter model.

Decay	Parity conserving	Parity violating
$\Sigma^+ \to p\gamma$	$-a+c(2+3\xi)/\sqrt{2}$	$-b-c\xi/\sqrt{2}$
$\Sigma^0 \to n\gamma$	$-a/\sqrt{2}+c\xi/2$	$-b/\sqrt{2}-c(2+3\xi)/2$
$\Lambda \to n\gamma$	$9a/\sqrt{6}-c(4+7\xi)/\sqrt{12}$	$9b/\sqrt{6}+c(2+5\xi)/\sqrt{12}$
$\Xi^0 \to \Sigma^0\gamma$	$5a/\sqrt{2}-c(1+\xi)$	$5b/\sqrt{2}-c(1+\xi)$
$\Xi^0 \to \Lambda\gamma$	$3a/\sqrt{6}-c(1+2\xi)/\sqrt{3}$	$3b/\sqrt{6}+c(1+2\xi)/\sqrt{3}$
$\Xi^- \to \Sigma^-\gamma$	$5a$	$5b$
$\Omega^-(\frac{3}{2}) \to \Xi^-(\frac{1}{2})\gamma$[a]	$\sqrt{6}a/3$	$\sqrt{6}b/3$
$\Omega^-(\frac{1}{2}) \to \Xi^-(-\frac{1}{2})\gamma$[a]	$\sqrt{2}a$	$\sqrt{2}b$

[a] $\Omega^-(\frac{3}{2}) \to \Xi^-(\frac{1}{2})$, etc. refer to the helicity states.

quark transition operators contains a factor $\exp(-k^2/24\alpha^2)$, where k is the photon momentum. With $\alpha \sim 0.5$ GeV and k, typically, 100–200 MeV we set this factor equal to unity for all decays. The parity-conserving and parity-violating amplitudes are then identified by writing the baryon-decay amplitudes in the forms (3.1) and (3.6). In Table II we have combined the single-quark amplitude with that arising from the two-quark transition model. As the overall normalization of the latter amplitude is not known the amplitudes of Table I are scaled by a factor c before entering them in Table II. The three parameters of the model we discuss in this section are a, b and c. In addition there are processes of the kind shown in Fig. 1(c) which involve all the quarks, with no spectators. The effective Hamiltonian arising from the process shown in Fig. 1(c) is

$$H_{\text{eff}}(\text{3-quark}) = eG_F \frac{\sin\theta_C \cos\theta_C}{\sqrt{2}} d\,(u^\dagger s d^\dagger u - u^\dagger \vec{\sigma} s \cdot d^\dagger \vec{\sigma} u) q^\dagger (\vec{\epsilon} \times \vec{k}) \cdot \vec{\sigma} q \ . \tag{4.4}$$

Notice that H_{eff} of Eq. (4.4) is purely parity conserving. The parity-violating part is of $O(k^2)$ and, hence, neglected. d is an overall constant. There is no reason to assume that any of the three processes shown in Fig. 1 would dominate the physics. The data at this moment is very sparse and a four-parameter model cannot be exploited profitably. In the following we consider a three-parameter model which employs the single-quark and the two-quark transition operators [Figs. 1(a) and 1(b)] only.

The three data points we choose to work with are $\Gamma(\Sigma^+ \to p\gamma)$, α_{Σ^+}, and the bound on $\Gamma(\Xi^- \to \Sigma^-\gamma)$. In terms of a, b, and c of Table II,

$$\Gamma(\Sigma^+ \to p\gamma) = \frac{G_F{}^2 e^2 k^3}{\pi} \{ [a - c(2+3\xi)/\sqrt{2}]^2 + (b + c\xi/\sqrt{2})^2 \} \ , \tag{4.5}$$

$$\Gamma(\Xi^- \to \Sigma^-\gamma) = \frac{G_F{}^2 e^2 k'^3}{\pi} (25)(a^2+b^2) \ , \tag{4.6}$$

$$\alpha_{\Sigma^+} = \frac{2[a - c(2+3\xi)/\sqrt{2}](b + c\xi/\sqrt{2})}{[a - c(2+3\xi)/\sqrt{2}]^2 + (b + c\xi/\sqrt{2})^2} \ . \tag{4.7}$$

Equations (4.5) and (4.7) are used to derive a quadratic in either $(b + c\xi/\sqrt{2})^2$ or $[a - c(2+3\xi)/\sqrt{2}]^2$ in terms of $\Gamma(\Sigma^+ \to p\gamma)$ and α_{Σ^+}. For definiteness we used the branching ratio[3] for $\Sigma^+ \to p\gamma = 1.24 \times 10^{-3}$ and[3] $\alpha_{\Sigma^+} = -0.5$. The four solutions so obtained are listed below. All parameters a, b, and c are expressed in units of 10^{10} $\text{GeV}^{1/2}\,\text{sec}^{-1/2}$. $\xi = 0.36$ is used throughout. Solution I:

$$\begin{aligned} A_\Sigma &\equiv a - c(2+3\xi)/\sqrt{2} = 1.806 \ , \\ B_\Sigma &\equiv b + c\xi/\sqrt{2} = -0.484 \ . \end{aligned} \tag{4.8}$$

Solution II is obtained by reversing the signs of A_Σ and B_Σ. Solution II:

$$a - c(2+3\xi)/\sqrt{2} = -1.806 \, ,$$
$$b + c\xi/\sqrt{2} = +0.484 \, . \tag{4.9}$$

Solutions III and IV are obtained by the interchange ($A_\Sigma \rightleftarrows B_\Sigma$) in solutions I and II, as Eqs. (4.5) and (4.7) are invariant under this interchange.

Equation (4.8) determines a and b in terms of c. Once this replacement is made in (4.6) we get a quadratic inequality for c on using the bound for $\Gamma(\Xi^- \to \Sigma^- \gamma)$ with a branching ratio[3] $< 1.2 \times 10^{-3}$. The domain of c for which this inequality is satisfied for solution I is

$$-1.13 \le c \le -0.56 \, . \tag{4.10}$$

Solution II leads to an allowed domain

$$0.56 \le c \le 1.13 \, . \tag{4.11}$$

Solutions III and IV do not lead to a real solution for c. We do not consider these solutions any further.

The use of Eq. (4.8) with the allowed range of c given by (4.10) yields the allowed ranges for a and b for solution I. A similar use of Eqs. (4.9) and (4.11) generates the allowed ranges for a and b for solution II. Thus the allowed ranges of all the parameters are

Solution I:

$$-0.65 \le a \le +0.58 \, ,$$
$$-0.34 \le b \le -0.20 \, , \tag{4.12}$$
$$-1.13 \le c \le -0.56 \, .$$

Solution II:

$$-0.58 \le a \le +0.65 \, ,$$
$$+0.20 \le b \le +0.34 \, , \tag{4.13}$$
$$+0.56 \le c \le +1.13 \, .$$

If we *saturate* the rate $\Gamma(\Xi^- \to \Sigma^- \gamma)$ then we get the following two solutions from the solution set I.

Solution A:

$$a = 0.58 \, ,$$
$$b = -0.34 \, , \tag{4.14}$$
$$c = -0.56 \, .$$

Solution B:

$$a = -0.65 \, ,$$
$$b = -0.20 \, , \tag{4.15}$$
$$c = -1.13 \, .$$

Similar solutions obtained from set II are obtained by a reversal of sign of all parameters and gives us no new physics. With the set of parameters shown in Eqs. (4.14) and (4.15) we evaluated all the other decay rates and the asymmetry parameters which are displayed in Table III. The differences between the solutions A and B is most striking in the asymmetry parameters. It is clear from Table III that it is possible to find a set of solutions in the three-parameter model consistent with the existing data.

TABLE III. Rates and asymmetries.

Decay	k (MeV)	Calculated branching ratio Sol. A[a]	Sol. B[a]	Calculated asymmetries Sol. A[a]	Sol. B[a]	Experiment Branching ratio	Asymmetries
$\Sigma^+ \to p\gamma$	225	1.24×10^{-3}[b]	1.24×10^{-3}[b]	-0.5[b]	-0.5[b]	$(1.24 \pm 0.18) \times 10^{-3}$	$-1.03^{+0.52}_{-0.42}$
							$-0.53^{+0.38}_{-0.36}$
$\Sigma^0 \to n\gamma$	225	0.42[c]	1.02[c]	$+0.76$	-0.26		
$\Lambda \to n\gamma$	162	5.97×10^{-3}	1.70×10^{-3}	-0.87	$+0.25$		
$\Xi^0 \to \Sigma^0\gamma$	117	1.48×10^{-3}	0.23×10^{-3}	-0.3	-0.99	$< 7 \times 10^{-2}$	
$\Xi^0 \to \Lambda\gamma$	184	1.80×10^{-3}	1.36×10^{-3}	-0.96	-0.45	$(5 \pm 5) \times 10^{-3}$	
$\Xi \to \Sigma^-\gamma$	118	1.20×10^{-3}[b]	1.20×10^{-3}	-0.87	$+0.56$	$< 1.2 \times 10^{-3}$	
$\Omega^- \to \Xi^-\gamma$	314	0.6×10^{-3}	0.6×10^{-3}	-0.87	$+0.56$		

[a]Solutions A and B are shown in Eqs. (4.14) and (4.15), respectively.
[b]Fitted values. We saturate the bound on $\Gamma(\Xi^- \to \Sigma^- \gamma)$.
[c]This is the ratio $\Gamma(\Sigma^0 \to n\gamma)/\Gamma(\Sigma^+ \to p\gamma)$. Data from Refs. 1, 2, and 3.

V. DISCUSSION

Since the single-quark transition model has been demonstrated[19] to be inconsistent with the existing data on radiative weak decays of baryons we have studied the two-quark and three-quark transition models in the nonrelativistic limit. Owing to the paucity of data we have ignored the three-quark transition mechanism and demonstrated that a three-parameter model with the single-quark and the two-quark transition operators is consistent with the existing data. As more data accumulate it will be worthwhile to include the three-quark transition mechanism in the discussion.

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APPENDIX:

DETAILS OF THE CALCULATION

Baryon wave functions

Let the initial quark momenta be $(\vec{p}_1,\vec{p}_2,\vec{p}_5)$ and the final quark momenta be $(\vec{p}_3,\vec{p}_4,\vec{p}_6)$. $\vec{p}_5$ and $\vec{p}_6$ are the spectator quark momenta with $\vec{p}_5=\vec{p}_6$. Define three independent variables with unit Jacobian

$$\begin{aligned}\vec{P}&=\frac{1}{\sqrt{3}}(\vec{p}_1+\vec{p}_2+\vec{p}_5)\,,\\ \vec{p}_\rho&=\frac{1}{\sqrt{2}}(\vec{p}_1-\vec{p}_2)\,,\\ \vec{p}_\lambda&=\frac{1}{\sqrt{6}}(\vec{p}_1+\vec{p}_2-2\vec{p}_5)\,.\end{aligned}\tag{A1}$$

The corresponding contributions for $(\vec{p}_3,\vec{p}_4,\vec{p}_6)$ are $\vec{P}\,'$, $\vec{p}\,'_\rho$, and $\vec{p}\,'_\lambda$.

The initial and final baryon wave functions in the momentum space are (up to an overall normalization)

$$|B_i\rangle=\delta^3(\vec{P})\exp[-(\vec{p}_\rho^{\,2}+\vec{p}_\lambda^{\,2})/2\alpha^2]\tag{A2}$$

$$|B_f\rangle=\delta^3\left[\vec{P}\,'+\frac{\vec{k}}{\sqrt{3}}\right]\exp[-(\vec{p}\,'^{2}_\rho+\vec{p}\,'^{2}_\lambda)/2\alpha^2]\,.$$

The initial baryon is taken to be at rest and the final baryon recoils against the photon with momentum $-\vec{k}$. The wave functions of (A2) correspond to Gaussian wave functions in relative coordinates in configuration space.

Evaluation of baryon-weak-radiative-decay matrix elements

Consider first the terms in $H_{\rm eff}$ which do not depend on the quark momenta $\vec{p}_i$. We shall evaluate here only the space part of the matrix element. One has to remember that the flavor- and spin-dependent parts have to be evaluated using SU(6) wave functions. For such $\vec{p}_i$-independent terms in $H_{\rm eff}$ the baryon-weak-radiative-decay amplitude involves evaluation of an integral of the kind

$$\begin{aligned}I=\int &\delta^3(\vec{p}_5-\vec{p}_6)\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4-\vec{k})\delta^3(\vec{P})\delta^3\left[\vec{p}\,'+\frac{k}{\sqrt{3}}\right]\\ &\times\exp\left[-\frac{(\vec{p}_\rho^{\,2}+\vec{p}_\lambda^{\,2})}{2\alpha^2}\right]\exp\left[-\frac{(\vec{p}\,'^{2}_\rho+\vec{p}\,'^{2}_\lambda)}{2\alpha^2}\right]d^3\vec{P}\,d^3\vec{P}\,'d^3\vec{p}_\rho d^3\vec{p}\,'_\rho d^3\vec{p}_\lambda d^3\vec{p}\,'_\lambda\,.\end{aligned}\tag{A3}$$

Momentum conservation is built in. On performing $\vec{P}$, $\vec{P}\,'$, and $\vec{p}\,'_\lambda$ integrations one is left with an overall-momentum-conserving δ function. The remaining three Gaussian integrals can be evaluated simply, leading to

$$I=\delta\left[\sum\vec{p}_i-\sum\vec{p}_f-\vec{k}\right]\exp\left[-\frac{k^2}{24\alpha^2}\right](2\pi^{3/2}\alpha^3)^3\,,\tag{A4}$$

where $\sum\vec{p}_i\equiv\vec{p}_1+\vec{p}_2+\vec{p}_5$ and $\sum\vec{p}_f=\vec{p}_3+\vec{p}_4+\vec{p}_6$. Equation (A4) is the space part of the matrix element.

The terms linear in $\vec{p}_i$ in $H_{\rm eff}$ require evaluation of integrals of kind

$$\int \vec{p}_1\delta^3(\vec{p}_5-\vec{p}_6)\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4-\vec{k})\delta^3(\vec{P})\delta^3\left[\vec{P}'+\frac{k}{\sqrt{3}}\right]\exp\left[-\frac{(\vec{p}_\rho{}^2+\vec{p}_\lambda{}^2)}{2\alpha^2}\right]\exp[-(\vec{p}\,'_\rho{}^2+\vec{p}\,'_\lambda{}^2)/2\alpha^2]$$

$$\times d^3\vec{P}\,d^3\vec{P}\,'d^3\vec{p}_\rho d^3\vec{p}\,'_\rho d^3\vec{p}_\lambda d^3\vec{p}\,'_\lambda \equiv A\vec{k}\ . \quad \text{(A5)}$$

This integral can only be proportional to $\vec{k}$ which is the only free momentum. The constant of proportionality, A, has to be evaluated. Once $\vec{p}_1$ is expressed in terms of $\vec{P}$, $\vec{p}_\rho$, and $\vec{p}_\lambda$ and the integrations over $\vec{P}$, $\vec{p}\,'_\rho$, and $\vec{p}\,'_\lambda$ carried out one is led to

$$A\vec{k}=\frac{1}{\sqrt{6}}\int \vec{p}_\lambda d^3\vec{p}_\rho d^3\vec{p}\,'_\rho d^3\vec{p}_\lambda \exp\left[-\frac{\vec{p}_\lambda{}^2}{2\alpha^2}\right]\exp\left[-\frac{(\vec{p}_\rho{}^2+\vec{p}\,'_\rho{}^2)}{2\alpha^2}\right]\exp\left[-\frac{\left[\vec{p}_\lambda-\frac{\vec{k}}{\sqrt{6}}\right]^2}{2\alpha^2}\right]. \quad \text{(A6)}$$

On combining the $\vec{p}_\lambda$ type of exponents and shifting the origin, $\vec{p}_\lambda \to \vec{p}_\lambda + k/\sqrt{24}$, one gets

$$A\vec{k}=\frac{I}{12}\vec{k}\ , \quad \text{(A7)}$$

where I is defined in (A4). The result of this type of calculation is that both $\vec{p}_1$ and $\vec{p}_2$ can be replaced by $\vec{k}/12I$ in the evaluation of the matrix elements and that $\vec{p}_3$ and $\vec{p}_4$ can each be replaced by $-5\vec{k}/12I$ so that $\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4=\vec{k}$. Thus, up to an overall normalization, the rule that emerges is

$$(\vec{p}_1,\vec{p}_2)\to\vec{k}/12,\ \ (\vec{p}_3,\vec{p}_4)\to-5\vec{k}/12\ . \quad \text{(A8)}$$

A single overall normalization is fixed by the rate $\Gamma(\Sigma^+\to p\gamma)$. The spin-flavor dependence is worked out by using SU(6) wave functions for the baryons.

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