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Environment-induced superselection rules

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We show how the correlations of a quantum system with other quantum systems may cause one of its observables to behave in a classical manner. In particular, "reduction of the wave packet," postulated by von Neumann to explain definiteness of an outcome of an individual observation, can be explained when a realistic model of an apparatus is adopted. Instead of an isolated quantum apparatus with a number of states equal to the number of possible distinct outcomes of the measurement, discussed by von Neumann, we consider an apparatus interacting with other physical systems, described here summarily as "environment." The interaction of the quantum apparatus with the environment results in correlations. Correlations impose effective *superselection rules* which prevent apparatus from appearing in a superposition of states corresponding to different eigenvalues of the privileged *pointer observable.* It is the propagation of the correlations with the *pointer basis* states which is ultimately responsible for the choice of the pointer observable. It can be thought of as a process of amplification in which the state of many distinct physical systems becomes correlated with the pointer basis state. Whether these environment systems are regarded as a part of the apparatus setup, or as a part of its environment is irrelevant. What is crucial is the redundancy of the record concerning the pointer observable which is imprinted into the correlations. Eigenspaces of the pointer observable provide a natural basis for the pointer of the quantum apparatus and determine the to-be-measured observable of the quantum system. Decay of those elements of the apparatus-system density matrix, which are off-diagonal in the pointer observable, is caused by the natural evolution of the combined system-apparatus-environment object. For a hypothetical finite environment with N distinct eigenvalues of the apparatus-environment interaction Hamiltonian, off-diagonal terms will decay to become of the order of $N^{-1/2}$, and will recur only on a Poincaré time scale. No recurrences will be observed in realistic circumstances. As the correlations spread through the environment on a time scale typically much shorter than the recurrence time scale calculated for the environment already correlated with the pointer observable, the measurement becomes effectively irreversible. Relevance of this model of the measurement process for the understanding of the second law of thermodynamics and its relation to Bohr's "irreversible act of amplification" is briefly discussed. The emergence of the pointer observable can be interpreted as a clue about the resolution of the measurement problem in case of no environment. It points towards the possibility that properties of quantum systems have no absolute meaning. Rather, they must be always characterized with respect to other physical systems.

I. INTRODUCTION

Superselection rules have been often invoked in attempts to solve the problem of measurement in quantum theory.[1-3] Their role was to prevent the apparatus pointer from appearing in the superposition of states corresponding to the distinct outcomes of the measurement. This would allow the apparatus to accomplish what has become known as the second stage of the measurement process: the reduction of the state vector.[4-6]

The aim of this paper is to show that when the quantum apparatus is an open system, interacting with other systems, which will be described below as "an environment," such superselection rules do not need to be postulated: They arise naturally in the course of any Hamiltonian evolution which correlates quantum apparatus with its environment. The state vector of the combined system-apparatus-environment object evolves as follows:

$$|\Phi(t=t_0)\rangle = |\Psi\rangle \otimes |A_0\rangle \otimes |\mathscr{E}(t_0)\rangle \tag{1.1a}$$

$$\rightarrow |\Phi(t=t_1)\rangle = \left\{\sum_n c_n(|n\rangle \otimes |A_n\rangle)\right\} \otimes |\mathscr{E}(t_1)\rangle \tag{1.1b}$$

$$\rightarrow |\Phi(t>t_2)\rangle = \sum_n c_n |n\rangle \otimes |A_n\rangle \otimes |\mathscr{E}_n(t)\rangle \ . \tag{1.1c}$$

Above, the state vector of the system is given at $t=t_0$ by

$$|\Psi\rangle = \sum c_n |n\rangle \ ,$$

while $\{|A_n\rangle\}$ is the basis of the apparatus. $|\mathscr{E}\rangle$ ($|\mathscr{E}_n\rangle$) describe the state of the environment before (after) it has become correlated, as a result of the apparatus-environment interaction, with the different outcomes of the measurement. It is worth stressing that at $t=t_1$ the observable of the system which will be ultimately recorded by the apparatus is not yet defined. Only the establishment of the apparatus-environment correlation removes the ambiguity in the definition of the recorded observable.[7] Part of the apparatus-system correlations, which are present at $t=t_1$ in the wave function $|\Phi(t)\rangle$ are replaced by the apparatus-environment correlations. While the first set of the correlations has provided the apparatus with the information about the quantum system, the second set of the correlations is indispensable to define the measured observable. At the same time the apparent reduction of the state vector is accomplished; the state of the apparatus-system combination must be now described by the density matrix:

$$\begin{aligned}\rho &= \mathrm{Tr}_{\mathscr{E}}[\,|\Phi(t>t_2)\rangle\langle\Phi(t>t_2)|\,] \\ &\cong \sum_n |c_n|^2 |A_n\rangle\langle A_n| \otimes |n\rangle\langle n| \\ &= \rho_{\mathrm{mix}} \ .\end{aligned} \tag{1.2}$$

Above $\mathrm{Tr}_{\mathscr{E}}$ indicates partial trace over the environment degrees of freedom. The size of the off-diagonal terms appearing in ρ_{mix} is determined by the values of the scalar products, $\rho_{nm} \sim |\langle \mathscr{E}_n(t>t_2) | \mathscr{E}_m(t>t_2)\rangle|^2$. When these are small, as is usually the case, the density matrix can be thought of as describing the apparatus in a definite state. The probabilities on the diagonal of the density matrix are there because of our (i.e., outside observer's) ignorance about the outcome of the measurement: It is, as yet, unknown to us, but nevertheless it is already definite.

The manner in which the transition from ρ_{pure} to ρ_{mix} proceeds in the above scenario indicates that the traditional concept of properties which belong to the system is incompatible with quantum mechanics. This follows from the fact that the to-be-measured observable is defined only in the course of the apparatus-environment interaction. Its characterization proceeds in parallel to the reduction of the wave packet. In other words, to maintain that the environment-induced superselection rules arise because the environment removes spurious correlations by perturbing the phase relations between different states of the apparatus (see, e.g., Refs. 8–13) reveals only part of what happens. It is more to the point to note that the environment-apparatus correlations replace some of the apparatus-system correlations. This replacement defines the "classical" pointer observable of the apparatus: Before the off-diagonal terms can be damped out it must be clear what states remain on the diagonal.

It is important to recognize similarities and to stress differences between the meaning attributed to the term "superselection rules" here, and in its earlier uses. Superselection rules were said to operate between the subspaces of a Hilbert space of a given system if the phase factors between the vectors belonging to the two distinct subspaces were unobservable.[22] Such superselection rules imply that an operator which has eigenstates composed of superpositions of vectors contained in two or more subspaces cannot be measured. In the case of environment-induced superselection rules phase coherence between two eigenspaces of the pointer observable is being continuously destroyed by the interaction of the system to which they apply (e.g., apparatus pointer) with the environment: This interaction, equivalent to "monitoring" of the system by the environment, makes the phase between pointer basis states impossible to observe. Thus, the interaction with the environment forces the system to be in one of the eigenstates of the pointer observable, rather than in some arbitrary superposition of such eigenstates. This results in effective, environment-induced superselection rules.

The subject of the following section is to introduce the idea of the environment-induced superselection rules on a simple, exactly soluble example of interacting two-state quantum systems. Section III explores in a more general way conditions under which superselection rules may be induced. In particular, we show that as the size of the environment increases, the superselection rule becomes more effective. Recurrence of the correlation terms (which are shown in Sec. III to be decreased by the factor $N^{1/2}$, where N is the number of the involved states of the environment) is the subject of Sec. IV. Discussion of the environment-induced superselection rules in the context of interpretation of quantum theory is conducted in Sec. V. In this last section we shall also consider connection of our considerations with the problems of irreversibility and amplification.

II. POINTER BASIS IN A BIT-BY-BIT MEASUREMENT

Consider a pair of two-state systems. We shall call one of them "spin" while the other one shall be called "atom." A possible basis of the spin is given by $|\uparrow\rangle, |\downarrow\rangle$—states parallel or antiparallel to the z axis. Alternative orthonormal basis states are then, for example,

$$\begin{aligned} |\odot\rangle &= (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}\,, \\ |\otimes\rangle &= (|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2} \end{aligned} \tag{2.1}$$

or

$$\begin{aligned} |\rightarrow\rangle &= (|\uparrow\rangle + i\,|\downarrow\rangle)/\sqrt{2}\,, \\ |\leftarrow\rangle &= (|\uparrow\rangle - i\,|\downarrow\rangle)/\sqrt{2}\,. \end{aligned} \tag{2.2}$$

The basis of the atom consists of the "ground state" $|\underset{\cdot}{=}\rangle$ and of the "excited state" $|\dot{=}\rangle$. Despite this nomenclature we shall assume that the energy of the atom in each of these states is identical: Neither the atom nor the spin possess self-Hamiltonians. Alternative states of the atom can be, for example,

$$\begin{aligned} |+\rangle &= (|\dot{=}\rangle + |\underset{\cdot}{=}\rangle)/\sqrt{2}\,, \\ |-\rangle &= (|\dot{=}\rangle - |\underset{\cdot}{=}\rangle)/\sqrt{2} \end{aligned} \tag{2.3}$$

or

$$\begin{aligned} |\perp\rangle &= (|\dot{=}\rangle + i\,|\underset{\cdot}{=}\rangle)/\sqrt{2}\,, \\ |\top\rangle &= (|\dot{=}\rangle - i\,|\underset{\cdot}{=}\rangle)/\sqrt{2}\,. \end{aligned} \tag{2.4}$$

The spin shall now be considered, for the purpose of our discussion, as the quantum system which is to be measured, while the atom shall play the role of the apparatus. It is not difficult to show[7,14] that the interaction Hamiltonian

$$\begin{aligned} H^{\mathrm{AS}} = g(&|\perp\rangle\langle\perp| - |\top\rangle\langle\top|) \\ &\otimes(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|) \end{aligned} \tag{2.5}$$

acting over the time interval $\tau_i = \pi\hbar/4g$ transforms the initial, direct product

$$|\varphi_i\rangle = (a\,|\uparrow\rangle + b\,|\downarrow\rangle)\otimes|+\rangle \tag{2.6}$$

into a correlated state vector

$$|\varphi_f\rangle = a\,|\uparrow\rangle\otimes|\dot{=}\rangle + b\,|\downarrow\rangle\otimes|\underset{\cdot}{=}\rangle\,. \tag{2.7}$$

This is still, beyond any doubt, a pure state. Although the correlations between the system and the apparatus have already been established, so far the measurement could not have yielded a definite outcome. First of all, the correlated apparatus-system state vector $|\varphi_f\rangle$, Eq. (2.7), can be brought back to the initial $|\varphi_i\rangle$, if the same interaction, Eq. (2.5), proceeds for the additional time $\tau_r = 3\pi\hbar/4g$. This proves that the apparatus could not have "decided," at the stage described by Eq. (2.7), whether the outcome of the measurement was $|\uparrow\rangle$ or $|\downarrow\rangle$. For, if the initial, direct product $|\varphi_i\rangle$ is to reemerge after $\tau = \tau_i + \tau_r = \pi\hbar/g$, all of the components of $|\varphi_f\rangle$—all of the possible outcomes of the measurement—must have been present at $t = \tau_i$.

More importantly, at the stage described by $|\varphi_f\rangle$ of Eq. (2.7) it is as yet undetermined what the possible states are between which this measurement would distinguish. To show this we rewrite

$$\begin{aligned} |\varphi_f\rangle = [&(a\,|\uparrow\rangle + b\,|\downarrow\rangle)\otimes|+\rangle \\ &+ (a\,|\uparrow\rangle - b\,|\downarrow\rangle)\otimes|-\rangle]/\sqrt{2}\,. \end{aligned} \tag{2.8}$$

Thus, the atom states $|+\rangle$ and $|-\rangle$ are correlated with the definite states of the spin:

$$|S_1\rangle = a\,|\uparrow\rangle + b\,|\downarrow\rangle, \quad |S_2\rangle = a\,|\uparrow\rangle - b\,|\downarrow\rangle\,.$$

The above states are distinct from the states $|\uparrow\rangle, |\downarrow\rangle$ which, we might have thought on the basis of Eq. (2.7), were recorded by the apparatus. In particular, when the state of the spin before the measurement was least certain, i.e., when $a = b = 2^{1/2}$, the final, correlated state vector can be written, for example,

$$|\varphi_f\rangle = (|\odot\rangle\otimes|+\rangle + |\otimes\rangle\otimes|-\rangle)/\sqrt{2}\,, \tag{2.9}$$

as well as in continuously many equivalent ways.

The state of the atom—one is led to conclude—does not, at the stage of $|\varphi_f\rangle$ given by Eqs. (2.7)–(2.9), contain even the information about what observable of the spin it was supposed to record. It would therefore be impossible to claim that the measurement in the sense in which we normally use this word has already occurred. Moreover, even though our arguments following Eqs. (2.7), (2.8), and (2.9) apply directly to the two-state "bit-by-bit" interactions, it is not difficult to modify them so that one is forced to conclude that in a closed apparatus-system object evolving unitarily "reduction of the wave packet" cannot be accomplished. What has, nevertheless, undoubtedly taken place was the transfer of information between the spin and the atom. This information transfer can be quantified.[14] Let us now return to the discussion of the pointer basis of the apparatus, which removes this ambiguity in the choice of the recorded observable.

Consider the environment $\mathscr{E}$ consisting of N two-level atoms, the kth of which has a Hilbert space spanned by the basis $\{\,|\pm)_k, |\mp)_k\}$. (The environment atoms are denoted by parentheses, while Dirac's bra and ket are reserved for the spin and for the apparatus atom.) We assume that the self-Hamiltonians of the atoms taken separately, as well as the interaction Hamiltonian between these atoms, is zero. The only Hamiltonian in the problem is then the apparatus-environment interaction Hamiltonian H^{AE}, which, we assume, separates as follows:

$$H^{\mathrm{AE}}=\sum_k H_k^{\mathrm{AE}}\ . \tag{2.10}$$

Furthermore, we assume that the individual components have a form

$$H_k^{\mathrm{AE}}=g_k(\,|\pm\rangle\langle\pm|-|\mp\rangle\langle\mp|\,)\otimes(\,|\pm)(\pm|-|\mp)(\mp|\,)_k\prod_{j\neq k}\otimes 1_j\ . \tag{2.11}$$

The above interaction Hamiltonian has a very special property; its eigenstates are direct-product states. Components of the direct product belong to Hilbert spaces of the apparatus and environment atoms, respectively. Such Hamiltonians have recently been explored in the context of the nondemolition measurements of quantum observables.[15–18]

When the environment described above interacts with the apparatus atom, superselection rules are induced. They make it impossible for the apparatus to be detected in a superposition of the ground ($|\mp\rangle$) and excited ($|\pm\rangle$) states. To show this, let us assume that the interaction between the apparatus and the environment starts at $t=0$, and that before this instant no correlations with the environment have existed. Thus, the combined system-apparatus-environment state vector has, to begin with, the form

$$|\Phi(0)\rangle=|\varphi_f\rangle\prod_{k=1}^{N}\otimes[\alpha_k|\pm)_k+\beta_k|\mp)_k]\ . \tag{2.12}$$

States $|\pm\rangle, |\mp\rangle$; $|\pm)_k, |\mp)_k$ are the eigenstates of the interaction Hamiltonian, which is the only Hamiltonian acting in the combined system after $t=0$. This allows one to write the state $|\Phi\rangle$ at an arbitrary time t:

$$\begin{aligned}|\Phi(t)\rangle=a\,|\uparrow\rangle\otimes|\pm\rangle\prod_{k=1}^{N}\otimes[\alpha_k\exp(ig_kt)\,|\pm)_k+\beta_k\exp(-ig_kt)\,|\mp)_k]\\ +b\,|\downarrow\rangle\otimes|\mp\rangle\prod_{k=1}^{N}\otimes[\alpha_k\exp(-ig_kt)\,|\pm)_k+\beta_k\exp(ig_kt)\,|\mp)_k]\ .\end{aligned} \tag{2.13}$$

Above, and further on in this paper, we use units in which $\hbar=1$. The geometric representation of this evolution can be given (see Fig. 1). The transition between $|\Phi(0)\rangle$ given by Eq. (2.12) and $|\Phi(t)\rangle$ of Eq. (2.13) establishes the correlation between the state of the apparatus and the state of the environment.[19,20] It is analogous to the transition that occurs between $|\Phi(t=t_1)\rangle$ of Eq. (1.1b) and $|\Phi(t>t_2)\rangle$ of Eq. (1.1c). *The observable $\hat{\Lambda}$ of the apparatus, which is most reliably recorded by the environment, is called the pointer observable.*[7,14] In our case $\hat{\Lambda}$ has a form

$$\hat{\Lambda}=\lambda_1|\pm\rangle\langle\pm|+\lambda_2|\mp\rangle\langle\mp|\ , \tag{2.14}$$

and $\{|\dot{=}\rangle, |\underset{\cdot}{=}\rangle\}$ define the *pointer basis.* Above λ_1 and λ_2 are real and $\lambda_1 \neq \lambda_2$.

We are now ready to calculate, following the prescription of Eq. (1.2), the density matrix describing the spin correlated with the atom apparatus:

$$\rho^{SA} = \mathrm{Tr}_{\mathscr{E}} |\Phi(t)\rangle\langle\Phi(t)| = |a|^2 |\uparrow\rangle\langle\uparrow| \otimes |\dot{=}\rangle\langle\dot{=}| + z(t)ab^* |\uparrow\rangle\langle\downarrow| \otimes |\dot{=}\rangle\langle\underset{\cdot}{=}|$$
$$+ z^*(t)a^*b\, |\downarrow\rangle\langle\uparrow| \otimes |\underset{\cdot}{=}\rangle\langle\dot{=}| + |b|^2 |\downarrow\rangle\langle\downarrow| \otimes |\underset{\cdot}{=}\rangle\langle\underset{\cdot}{=}| \;. \tag{2.15}$$

Above $z(t)$ is the correlation amplitude:

$$z(t) = \prod_{k=1}^{N} [\cos 2g_k t + i(|\alpha_k|^2 - |\beta_k|^2)\sin 2g_k t] \;. \tag{2.16}$$

Note that the correlation amplitude $z(t)$ depends on the initial conditions of the environment only through the probabilities of finding the system in the eigenstates of the interaction Hamiltonain: $p(|\dot{=}\rangle_k) = |\alpha_k|^2$, $p(|\underset{\cdot}{=}\rangle_k) = |\beta_k|^2$. This property is quite important, as it indicates that the ability of $z(t)$ to damp out correlations will be the same for a mixture for which only $p(|\dot{=}\rangle_k)$, $p(|\underset{\cdot}{=}\rangle_k)$ can be given. We shall not give the proof of this fact here. It involves calculations that are as straightforward as they are cumbersome.

Let us moreover note that the correlation amplitude $z(t)$ is equal to the scalar product:

$$z(t) = \langle \mathscr{E}_{|\dot{=}\rangle}(t) \,|\, \mathscr{E}_{|\underset{\cdot}{=}\rangle}(t) \rangle \;. \tag{2.17}$$

States $|\mathscr{E}_{|\dot{=}\rangle}(t)\rangle$, $|\mathscr{E}_{|\underset{\cdot}{=}\rangle}(t)$ are defined by

$$|\mathscr{E}_{|\dot{=}\rangle}(t)\rangle = \prod_{k=1}^{N} \otimes [\alpha_k \exp(ig_k t)\, |\dot{=}\rangle_k + \beta_k \exp(-ig_k t)\, |\underset{\cdot}{=}\rangle_k] \;,$$

$$|\mathscr{E}_{|\underset{\cdot}{=}\rangle}(t)\rangle = \prod_{k=1}^{N} \otimes [\alpha_k \exp(-ig_k t)\, |\dot{=}\rangle_k + \beta_k \exp(ig_k t)\, |\underset{\cdot}{=}\rangle_k] \;.$$

They represent the two distinct records made by the environment of the two alternative outcomes of the measurement.

The time dependence of $z(t)$ is of crucial importance for the successful damping of the off-diagonal correlation terms. It is elementary to see that

(i) $z(t)|_{t=0} = 1$, (2.18)

(ii) $|z(t)|^2 \leq 1$, (2.19)

(iii) $$\langle z(t)\rangle = \lim_{T\to\infty} T^{-1}\int_0^T z(t)dt = 0 \;, \tag{2.20}$$

(iv) $$\langle |z(t)|^2\rangle = 2^{-N}\prod_{k=1}^{N} [1 + (|\alpha_k|^2 - |\beta_k|^2)^2] \;. \tag{2.21}$$

Equation (2.21) implies that unless the initial state of the environment coincides with one of the eigenstates of the interaction Hamiltonian, the expected absolute value of the correlation amplitude, $|z(t)|^2$, is much less than its initial value of unity. Moreover, Fig. 2 suggests that even relatively small environments are quite effective in providing a precise definition of the pointer observable. It is, nevertheless, worth stressing that as long as N is finite the absolute value of $z(t)$ will return arbitrarily closely to $|z(t)|^2 = 1$. This statement is a theorem in the theory of almost-periodic functions.[21] There one defines *"translation numbers"* T_ϵ by requesting that the inequality $1 - |z(t)|^2 < \epsilon$ be satisfied, for a given ϵ, both at $t = 0$ and at $t = T_\epsilon$, but not within the interval $(0, T_\epsilon)$. Translation numbers T_ϵ constitute, for almost-periodic functions, a "moral equivalent" of the period of periodic functions: One can prove that almost-periodic function reaches any value within its range infinitely many times. There is a close analogy between the problem of recurring correlations, exemplified by the existence of T_ϵ for any positive ϵ—and Poincaré recurrences. We shall explore it further in the following two sections.

The aim of this section was to show, on a specific example, how the interaction of the apparatus and the environment can cause an effective reduction of the state vector. Establishment of the correlations between the apparatus and the environment took place at the expense of the previously attained correlations between the apparatus and the system: Rewriting of the final density matrix in the form in which the apparatus contains the information about some arbitrary two states of

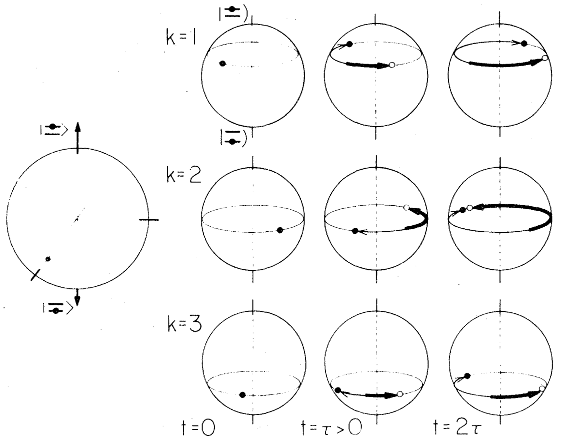

FIG. 1. Pictorial representation of the environment atoms "monitoring" the state of the apparatus atom, as given by Eqs. (2.10) and (2.11). Resulting apparatus-environment correlations replace some of the correlations between the eigenstates of the pointer observable and thus enforce environment-induced superselection rules. The pure state of each of the three ($k=1,2,3$) environment atoms, described by a state vector in a two-dimensional Hilbert space, is represented above by a point on the surface of the appropriate sphere. The initial state of the apparatus atom is represented by a point on the surface of the large sphere on the left. At $t=0$ (first column of the spheres) neither of the environment atoms is, as yet, correlated with the state of the apparatus atom: each of them can be represented by a single point. At $t=\tau>0$, second column, the state of each atom is rotated by an angle; $\varphi_k=\pm g_k t$, around the axis defined by the eigenstates; $|\pm)_k$, $|\mp)_k$, which diagonalize the interaction Hamiltonian;

$$H_k^{\mathrm{AE}}=g_k(|\pm\rangle\langle\pm|-|\mp\rangle\langle\mp|)$$
$$\otimes(|\pm)(\pm|-|\mp)(\mp|)_k\ .$$

Counterclockwise rotations are correlated with the $|\pm\rangle$ state of the apparatus and are marked by the heavy arrow. Clockwise rotations are correlated with the $|\mp\rangle$ state and marked by a lighter arrow. At a still later time, $t=2\tau$, third column, states of the environment atoms rotate still further. This evolution can be thought of as a continuous "measurement," by the environment, of the pointer observable $\hat{\Lambda}=\lambda(|\pm\rangle\langle\pm|-|\mp\rangle\langle\mp|)$ of the apparatus. Therefore, by Bohr's principle of complementarity, only observables which commute with $\hat{\Lambda}$ can be successfully measured or prepared. Indeterminacy is intimately related to the environment-induced superselection rules.

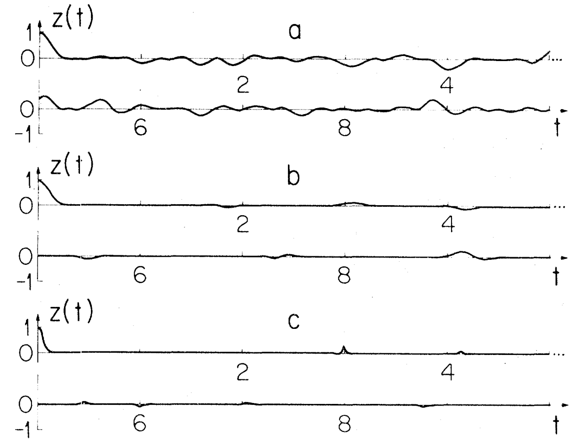

FIG. 2. Evolution of the correlation-damping factor $z(t)$, which multiplies off-diagonal terms of the apparatus-atom density matrix. Diagonal is defined by the pointer basis. Damping results from the process depicted symbolically in Fig. 1. $z(t)$ was calculated from Eq. (2.16) as a function of time for three different sizes of the environment: (a) $N=5$, (b) $N=10$, (c) $N=15$ environment atoms. For simplicity we have assumed that $|\alpha_k|=|\beta_k|$, so that $z(t)=\prod_{k=1}^{N}\cos 2g_k t$. Constants g_k, proportional to the eigenvalues of the interaction Hamiltonians H_k^{AE}, Eq. (2.11), were chosen randomly from the interval (0,1). Note the fluctuations of $z(t)$, and the decrease of their size and frequency with the increase of the number of the environment atoms, N.

the spin is no longer possible when all the off-diagonal terms in the density matrix ρ^{AS}, Eq. (2.15), disappear, even if $a=b=2^{-1/2}$. It is important to stress that through the interaction of the apparatus with the environment we have achieved simultaneously both the apparent reduction of the pure state density matrix into a mixture and the determination of the observable recorded by the apparatus. This dual role of the environment is equivalent to the imposition of the superselection rule. We shall explore this mechanism in more detail, for an arbitrary quantum system, in the following section.

III. ENVIRONMENT-INDUCED SUPERSELECTION RULES

The interaction Hamiltonian coupling the system $\mathscr{S}$ with the environment $\mathscr{E}$ may have the property of commuting with the subspaces $\mathscr{H}_n$ of the Hilbert space $\mathscr{H}$ of the considered system. We shall show that as a result of such interaction the state vector of the system is able to remain pure only if it is completely confined to one of the subspaces $\mathscr{H}_n$. Arbitrary superpositions, with components belonging to two or more subspaces, shall decay into mixtures diagonal in the state vectors belonging to the separate subspaces. This decay is caused by the establishment of the correlation between the quantum system $\mathscr{S}$ and its environment $\mathscr{E}$. Moreover, as long as the environment-system coupling remains stronger than the coupling introduced by the observer attempting to perform the measurement, the set of the observables that can be measured on $\mathscr{S}$ is limited to those that leave subspaces $\mathscr{H}_n$ invariant. Systems which exclude the existence of certain pure states and which restrict the class of possible observables in a manner described above are said to obey superselection rules.[22-25] Below we shall show how the interaction with the environment can induce such rules in $\mathscr{S}$. Once the superselection rules are imposed on $\mathscr{S}$, the system will behave "classically". Thus, conclusions arrived at here are immediately applicable to the problem of measurement. Using $\mathscr{S}$ rather than the "correlated pair of system and apparatus" of the previous section is intended to simplify the notation. Furthermore, it is worth noting that environment-induced superselection rules can be employed to justify not only the classical nature of the apparatus pointer, but apply to a much wider class of "classical observables" of systems which are inherently quantum.

Consider the combined Hilbert space of the system $\mathscr{S}$ and of the environment $\mathscr{E}$:

$$\mathscr{H}_{\mathscr{C}}=\mathscr{H}_{\mathscr{S}}\otimes\mathscr{H}_{\mathscr{E}}\ . \tag{3.1}$$

The Hamiltonian generating the evolution of this combined object is given by some Hermitian operator defined in $\mathscr{H}_{\mathscr{C}}$. We shall assume that this Hamiltonian can be separated into three distinct components. That is, there are self-Hamiltonians of the system, of the environment, and there is the interaction Hamiltonian:

$$H^{\mathrm{S}}=\sum_i \delta_i\,|s_i\rangle\langle s_i|\ , \tag{3.2a}$$

$$H^{\mathrm{E}}=\sum_j \epsilon_j\,|e_j\rangle\langle e_j|\ , \tag{3.2b}$$

$$H^{\mathrm{SE}}=\sum_{ij}\gamma_{ij}\,|s_i\rangle\langle s_i|\otimes|e_j\rangle\langle e_j|+\lambda\sum_i\sum_{i'}\sum_j\sum_{j'}\sigma_{ii'jj'}\,|s_i\rangle\langle s_i'|\otimes|e_j\rangle\langle e_j'|\ . \tag{3.2c}$$

We shall consider only the evolution due to the diagonal part of the interaction Hamiltonian. Hence, from here on, H_0^{SE} will always denote

$$H_0^{\mathrm{SE}}=\sum_{i,j}\gamma_{ij}\,|s_i\rangle\langle s_i|\otimes|e_j\rangle\langle e_j|\ . \tag{3.2d}$$

Setting $\lambda=0$ is an idealization of the situation when the diagonal H_0^{SE} is much greater than the off-diagonal part of the interaction. That is, we go a step beyond saying that $\lambda \ll 1$ when the absolute values of γ_{ij} and $\sigma_{ii'jj'}$ are comparable. Let us also note that we assume that the basis of H_0^{SE} and of H^{S} do coincide. This condition is a little less stringent than it may appear at first sight: Both H^{S} and H_0^{SE} are likely to be highly degenerate, which leaves additional freedom in the choice of the basis.

The assumption of small λ is justified in greater detail by van Hove[26] in his discussion of the approach to equilibrium of quantum systems. Physically, $\lambda \ll 1$ is equivalent to the assertion that the interactions occurring in the real-world physical systems destroy phase coherence between the states of the system on the time scale which is much shorter than the time scale of relaxation to thermal equilibrium. In the original calculations of irreversibility the effect of the diagonal part of the interaction Hamiltonian was taken into account only by adjusting the eigenvalues of the self-Hamiltonian of the system.[26,27] This simplified treatment, justifiable in the derivation of the master equation, would have proven to be a major oversight in the discussion of the measurement process.

Evolution of the combined system-environment state vector, which at $t=0$ was represented by a direct product

$$|\Phi(t=0)\rangle=|\varphi_s\rangle\otimes|\psi_E\rangle =\left[\sum_i \alpha_i\,|s_i\rangle\right]\otimes\left[\sum_j \beta_j\,|e_j\rangle\right]\ , \tag{3.3}$$

can be given immediately as

$$|\Phi(t)\rangle=\sum \alpha_i\beta_j \exp[-it(\delta_i+\epsilon_j+\gamma_{ij})]\,|s_i\rangle\otimes|e_j\rangle\ . \tag{3.4}$$

To show how the superselection rules arise we calculate

$$\rho^S(t)=\mathrm{Tr}_{\mathscr{E}}\,|\Phi(t)\rangle\langle\Phi(t)|\ . \tag{3.5}$$

The elements of $\rho^S(t)$ written in the representation

$$\rho^S(t)=\sum_i \sum_j \rho_{ij}(t)\,|s_i\rangle\langle s_j|$$

are

$$\rho_{ii}(t)=|\alpha_i|^2\sum_k |\beta_k|^2=|\alpha_i|^2\ , \tag{3.6a}$$

$$\rho_{ij}(t)=\alpha_i\alpha_j^*\exp(-it(\delta_i-\delta_j)) \times\sum_k |\beta_k|^2\exp(-it(\gamma_{ik}-\gamma_{jk}))\ . \tag{3.6b}$$

The diagonal elements are time independent when the off-diagonal part of the perturbing potential is absent.

The off-diagonal elements, in the representation in which H_0^{SE} is diagonal, evolve in time in two ways. They "rotate" trivially due to the factor $\exp(-it(\delta_i-\delta_j))$. What is, however, much more important is that they decay as a result of decrease of the *correlation amplitude:*

$$z_{ij}(t)=\sum_k |\beta_k|^2\exp(-it(\gamma_{ik}-\gamma_{jk}))\ . \tag{3.7}$$

The net effect of this second type of time dependence is to bring down the absolute value of the correlation amplitude from $z_{ij}=1$ at the initial $t=0$ to $z_{ij}\ll 1$ for large t. We shall discuss the time dependence of z_{ij} in more detail in the next section. Here let us only note that the average of the correlation amplitude taken over a sufficiently long time interval approaches zero:

$$\langle z_{ij}\rangle_T \xrightarrow[T\to\infty]{} 0\ , \tag{3.8}$$

unless all the differential frequencies

$$\omega_k^{ij}=\gamma_{ik}-\gamma_{jk} \tag{3.9}$$

are equal to zero. (Demanding that all $\omega_k^{ij}=0$ would be equivalent to the statement that the interaction Hamiltonian H_0^{SE} has the diagonal part equal to zero.) It is convenient to rewrite the correlation amplitude as

$$z_{ij}(t)=\sum_k p_k\exp(-i\omega_k^{ij}t)\ , \tag{3.10}$$

where $p_k=|\beta_k|^2$. As in Sec. II, when the environment is in the mixture prior to the interaction with the system, p_k express probabilities of finding the environment in the states corresponding to the distinct eigenvalues of H_0^{SE}. Equation (3.10) remains valid regardless of whether $\mathscr{E}$ is initially in a pure or in a mixed state.

The average absolute value of $z_{ij}(t)$ can be also calculated:

$$\langle |z_{ij}(t)|^2\rangle_T=\frac{1}{T}\int_t^{t+T}|z_{ij}(t)|\,dt \xrightarrow[T\to\infty]{} \sum_{k,k'} p_k p_{k'}\delta(\omega_k^{ij}-\omega_{k'}^{ij})\ . \tag{3.11}$$

Above $\delta(\omega_k^{ij}-\omega_{k'}^{ij})$ is the Kronecker delta. Assuming for simplicity that all ω_k^{ij} are distinct we find that the standard deviation Δ of the correlation amplitude from the average value, $\langle z_{ij}\rangle=0$, is

$$\Delta=(\langle |z_{ij}|^2\rangle-\langle z_{ij}\rangle^2)^{1/2}=\sum_{k=1}^{N} {p_k}^2\ . \tag{3.12}$$

Average fluctuations from zero are then, in the environment of N active states and under the assumption that all p_k are approximately equal, given by

$$\Delta\sim 1/\sqrt{N}\ . \tag{3.13}$$

Large environments can effectively damp out correlations between those states of the system which diagonalize H_0^{SE}.

This last remark brings us back to the discussion of the environment-induced superselection rules. We have proved above that the environment will destroy correlations between the states which correspond to different eigenvalues of H_0^{SE}. Let us stress that there may be many eigenvectors which correspond to the same eigenvalue γ_{nj}. These vectors span a subspace $\mathscr{H}_n$ of the Hilbert space of the system. Although the whole Hilbert space of the system can be reconstructed from such subspaces, it does not, in practice, admit pure states which belong to more than a single $\mathscr{H}_n$ at the same instant. This fact is the ultimate source of the environment-induced superselection rules. As a result, $\mathscr{H}_{\mathscr{S}}$ is a direct sum of the component subspaces:

$$\mathscr{H}_{\mathscr{S}}=\bigoplus_n \mathscr{H}_n \tag{3.14}$$

and all pure states belong to one, and only one of $\mathscr{H}_n$. Moreover, as long as the coupling with some external apparatus does not by far exceed the values of γ_{ij} in H_0^{SE}, the system may not be prepared as or measured in the state which does not remain invariant under the influence of the interaction with the environment. Thus, only these observables which leave every $\mathscr{H}_n$ invariant are admitted: $\hat{A}$ is an observable on a system $\mathscr{S}$ interacting with the environment $\mathscr{E}$ if and only if

$$|\varphi_n\rangle\in\mathscr{H}_n \Longrightarrow \hat{A}\,|\varphi_n\rangle\in\mathscr{H}_n\ . \tag{3.15}$$

The above two conditions, Eqs. (3.14) and (3.15), are equivalent to a more formal definition of the superselection rules.[24,25]

It is tempting to speculate that the well-known charge, and possibly other known superselection rules,[22,24] are in fact induced by the Hamiltonians that are so strong, that the time scale of decay of the pure state not abiding by them into a mixture is beyond our limits of detection, and the measuring Hamiltonian which could prepare a superposition of different "forbidden" states is so difficult to design, that we regard them as rules "in principle" rather than only "in practice."[28]

The pointer observable Λ can now be defined as any observable measurement which allows us to precisely determine the subspace $\mathscr{H}_n$ containing the state of the system. Therefore, if P_n are the projection operators projecting onto respective subspaces $\mathscr{H}_n$, and $\{\lambda_n\}$ are all real and distinct, i.e.,

$$\lambda_n = \lambda_{n'} \Longrightarrow n = n' ,$$

the pointer observable $\hat{\Lambda}$ is given by

$$\hat{\Lambda} = \sum_n \lambda_n P_n . \tag{3.16}$$

The projection operators can be constructed so that they are diagonal in the basis $\{ |s_i\rangle \}$, which diagonalizes H_0^{SE}. Therefore, $\hat{\Lambda}$ commutes with H_0^{SE}:

$$[\hat{\Lambda}, H_0^{\mathrm{SE}}] = 0 . \tag{3.17}$$

Yet, the interaction of the environment with the system establishes a correlation of much the same kind as the correlation between the system and the apparatus. One can therefore conclude that the environment acts as a higher-order apparatus, which performs nondemolition measurement on the state of the system, and consequently destroys coherent superpositions. The eigenbasis of the pointer observable, determined up to the inherent degeneracy of $\hat{\Lambda}$, shall be called *the pointer basis.*[7,14]

The eigenvalues of the self-Hamiltonian H^{S} will be, usually, highly degenerate. The eigenspaces $\mathscr{H}_{\delta_i}$ corresponding to *distinct* eigenvalues δ_i can either (a) contain, (b) be identical with, or (c) be contained by the eigenspaces of the superselection observable $\hat{\Lambda}$:

$$\text{(a)}\quad \mathscr{H}_{\delta_i} \supset \mathscr{H}_n , \tag{3.18a}$$

$$\text{(b)}\quad \mathscr{H}_{\delta_i} \equiv \mathscr{H}_n , \tag{3.18b}$$

$$\text{(c)}\quad \mathscr{H}_{\delta_i} \subset \mathscr{H}_n . \tag{3.18c}$$

The above list is exhaustive as long as $[H^{\mathrm{S}}, H_0^{\mathrm{SE}}] = 0$, as it follows from the commutation relation, Eq. (3.17). In case (a) the interaction with the environment will remove part of the degeneracy in the spectrum of H^{S}. This situation is familiar from the splitting of levels in atomic physics. In case (b) the energy levels of the system will be shifted. In case (c) the state can rotate under the influence of the self-Hamiltonian H^{S} within the subspace $\mathscr{H}_n$ without the loss of coherence.

There are two immediate generalizations of the above discussion. First, let us note that when the environment consists of several independent noninteracting systems, it can be described by the tensor product of the Hilbert spaces of the individual components $\mathscr{H}_{\mathscr{E}} = \prod_{\nu=1}^{N} \otimes \mathscr{H}_{\mathscr{E}}^{\nu}$. In that case the correlation amplitude can be shown to be a product of the correlation amplitudes due to these components:

$$z^{ij}(t) = \prod_{\nu=1}^{N} z_{\nu}^{ij}(t) . \tag{3.19}$$

Above, each of the components is calculated from the equation analogous to Eqs. (3.7)–(3.10). The previous section has afforded a simple example of such situations. The second generalization concerns Hamiltonians (either H^{S} or H_0^{SE}) which have a continuous spectrum: This generalization is straightforward in principle, but cumbersome in practice. We shall comment on it briefly in the next section.[29]

One may imagine situations in which the assumptions stated at the beginning of this section are violated. What observable $\hat{\Lambda}$ of $\mathscr{S}$, if any, can be then regarded as a pointer observable? There are several ways in which our initial assumptions may break down. There are also several, essentially equivalent approaches one might take in an attempt to define $\hat{\Lambda}$. One may search for an operator which, evolving under the full Hamiltonian $H^{\mathrm{S}} + H_0^{\mathrm{SE}}$, satisfied best the criterion for the nondemolition observable $[\hat{\Lambda}(t), \hat{\Lambda}(t')] = 0$.[17] Alternatively, one may search for an observable which has the longest relaxation time in the eventual approach of the system to equilibrium. Finally, one may inquire which observable $\hat{\Lambda}$ will best preserve the information about some other system $\tilde{\mathscr{S}}$, with which $\mathscr{S}$ was nonseparably correlated in the past. The above list is almost certainly not exhaustive. We shall return to these different ways of looking at the pointer observable in the last section of this paper.

Bohr's complementarity and its relation with the above discussion is the final subject we want to bring up in this section. The interaction Hamiltonian H_0^{SE} is clearly capable of inducing correlations between the system $\mathscr{S}$ and some other system $\mathscr{E}$, which we call here environment, and which, we have argued, acts much as an apparatus. The same structure of the Hamiltonian would be used in coupling the real apparatus to the measured system in an effort to measure $\hat{\Lambda}$, the pointer observable. Each Hamiltonian of this kind will then induce, as we have shown above, certain superselection rules, which may prevent other Hamiltonians, measuring other observable $\hat{\Gamma}$, from yielding a definite result. It is not difficult to show that the measurements of $\hat{\Lambda}$ and $\hat{\Gamma}$ will interfere with each other if

$$[\hat{\Lambda},\hat{\Gamma}]\neq 0 \; . \tag{3.20}$$

For, it is clearly impossible to have the same quantum system obey two distinct and mutually exclusive superselection rules. This mutual exclusion is responsible for complementarity, and related to the principle of indeterminacy.[30,31]

IV. DECAY, FLUCTUATIONS, AND THE RECURRENCE OF CORRELATIONS

The physical content of Eq. (3.10) for the correlation amplitude can be best understood in terms of the following geometrical analogy: Each of the individual contributions, $p_k \exp(-i\omega_k^{ij} t)$, is a vector rotated by the phase

$$\varphi_k = \omega_k^{ij} t$$

on a two-dimensional complex plane (see Fig. 3). At an arbitrarily chosen time instant individual vectors point in random directions. The problem is then to evaluate their sum. Clearly, this is a two-dimensional random-walk problem with the restriction on the total length of the path:

$$\sum_{k=1}^{N} p_k = 1 \; . \tag{4.1}$$

This random-walk analogy is the intuitive reason for the standard deviation formula, $\Delta \sim N^{-1/2}$, of the previous section. We shall return to the discussion of the correlation amplitude later on in this section. Before, let us explain why $z^{ij}(t)$ is so closely related to the environment-induced superselection rules.

The strictness of the environment-induced superselection rules can be, at least in principle, verified in the experiment in which one tries to determine the degree of superposition between the two states $|i\rangle$ and $|j\rangle$, which belong to two distinct subspaces $\mathscr{H}_i$ and $\mathscr{H}_j$ of $\mathscr{H}_{\mathscr{S}}$. For example, one could prepare two ensembles of system $\mathscr{S}$ in a pure state:

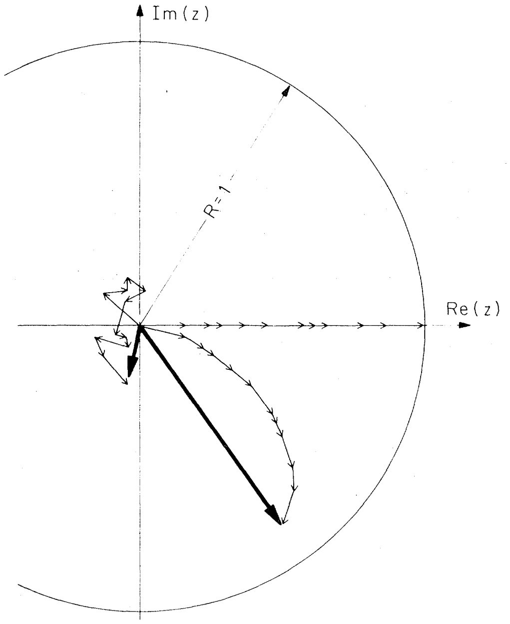

FIG. 3. Various contributions to the correlation-damping factor

$$z(t) = \sum_{k=1}^{N} p_k \exp(-i\omega_k t)$$

and the loss of coherence between them which is responsible for the decay of correlations between the distinct eigenstates of the pointer observable is shown at three different time instants. Individual contributions, $p_k \exp(-i\varphi_k)$, where $\varphi_k = \omega_k t$, correspond to the individual vectors on the two-dimensional plane shown above. The maximal length of z is limited by the normalization condition, $\sum_{k=1}^{N} p_k = 1$. The size of their sum depends on the distribution p_k, as well as on whether they add in phase or out of phase. At $t=0$ correlation amplitude is, of course, $z=1$. At an arbitrarily chosen $t \gg 0$ the phases between the contributions can be thought of as random, unless $\{\omega_k\}$ are related in a particular manner. Individual vectors point in random directions, and the problem reduces to a two-dimensional random walk with the probability of the size of the individual steps given by the distribution of probabilities p_k and with the restriction that the total path should equal one. Small fluctuations can be therefore treated using the theory of random walk. For a large class of $\{p_k\}$ the distribution of small fluctuations is approximately Gaussian with the standard deviation Δ given by $\Delta^2 = \sum_{k=1}^{N} p_k{}^2$, as for any random-walk problem. Large fluctuations, where $1 - |z|^2 = \epsilon \ll 1$ are less likely than for a Gaussian distribution because of the restriction on the total length of the path of the random walk.

$$| + \rangle = (| i \rangle + | j \rangle) / \sqrt{2} \; . \tag{4.2a}$$

Members of one ensemble shall be kept completely isolated, to provide a comparison standard. Systems in the other ensemble shall be allowed to interact, starting at $t = 0$, with the environment $\mathscr{E}$. For our purpose it suffices to characterize $\mathscr{E}$ by the distribution of p_k's and by the interaction Hamiltonian H_0^{SE}. Let us moreover suppose, for simplicity, that we are dealing with the case when both $| i \rangle$ and $| j \rangle$ belong to the same eigenvalue of the self-Hamiltonian of the system, $\langle i | H^{\text{S}} | i \rangle = \langle j | H^{\text{S}} | j \rangle$ [the case classified as (3.18a) in the previous section]. Now at the time t we measure, employing a measurement Hamiltonian much stronger than the environment-system coupling Hamiltonian, whether the system is in the state $| + \rangle$, or in the alternative state

$$| - \rangle = (| i \rangle - | j \rangle) / \sqrt{2} \; . \tag{4.2b}$$

The isolated ensemble will yield, at any time, in 100% of the cases result $| + \rangle$. What shall be the result for the systems coupled with the environment? Presence of the system-environment interaction will cause the density matrix of the system to be, in general, in a mixed state,

$$\begin{aligned}\rho^S(t) &= \tfrac{1}{2} [\, | i \rangle \langle i | + z_{ij}(t) | i \rangle \langle j | + | j \rangle \langle j | + z_{ji}(t) | j \rangle \langle i | \,] \\ &= \tfrac{1}{2} \{ [1 + \text{Re} z_{ij}(t)] \, | + \rangle \langle + | + \text{Im} z_{ij}(t) \, | + \rangle \langle - | + [1 - \text{Re} z_{ij}(t)] \, | - \rangle \langle - | + \text{Im} z_{ij}(t) \, | - \rangle \langle + | \, \} \; .\end{aligned} \tag{4.3}$$

Above we have recognized that $z_{ij} = z_{ji}^*$.

Deviation from the completely random situation when there is a 50% probability of the system being in the state $| + \rangle$, and a 50% probability of a system being in the state $| - \rangle$, is the evidence of superposition between states $| i \rangle$ and $| j \rangle$. It is of course directly related to the real part of the correlation amplitude, as Eq. (4.3) indicates:

$$r_{ij} = \text{Re} z_{ij} = \sum p_k \cos \omega_k^{ij} t \; . \tag{4.4}$$

Let us also remark that the conclusion of the above discussion does not change significantly when

$$\delta_i = \langle i | H^{\text{S}} | i \rangle \neq \delta_j = \langle j | H^{\text{S}} | j \rangle \; .$$

It is then not difficult to calculate that the important quantity is

$$\begin{aligned}&\text{Re}[\exp(-it(\delta_i - \delta_j)) z_{ij}(t)] \\ &\qquad = \sum p_k \cos(\delta_i - \delta_j + \omega_k^{ij}) t \; .\end{aligned} \tag{4.4a}$$

Additional frequency $\bar{\omega}^{ij} = \delta_i - \delta_j$ can be incorporated into the differential frequency ω_k^{ij}, and so in the further discussion of fluctuations and recurrences we do not need to go beyond the situation represented by Eq. (4.4).

Two related, but distinct questions can now be posed. First, one may inquire how often will the correlation amplitude assume certain value b ($b \in [-1,1]$). Second, one may ask what is the distribution of values assumed by $r_{ij}(t)$, if $r_{ij}(t)$ is sampled at arbitrarily chosen time intervals. We shall give now the answers to both of the above questions.

The first question leads us to consider the roots of the equation

$$\sum p_k \cos \omega_k t - b = f(t) - b = 0 \; . \tag{4.5}$$

(Above, and in the calculations below we omit cumbersome and always the same indices i,j.) The question: "How often, on the average, do fluctuations of size b occur?" is really the question about the average number $E(b)$ of roots of Eq. (4.4) for given b. When $N_T(b)$ gives the number of such roots in the interval $(0,T)$, $E(b)$ can be defined as

$$E(b) = \lim_{T \to \infty} N_T(b)/T \; . \tag{4.6}$$

Kac[32] has considered the distribution of roots of an almost-periodic function $f(t)$ defined by Eq. (4.5), under the assumption that all the frequencies ω_n are linearly independent. He was able to show that

$$E(b) = (2\pi^2)^{-1} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \eta^{-2} \cos bz \left\{ \prod_{k=1}^{N} J_0(p_k z) - \prod_{k=1}^{N} J_0[p_k (z^2 + \omega_k{}^2 \eta^2)^{1/2}] \right\} d\eta \, dz \; , \tag{4.7}$$

where J_0 is the Bessel function. This result can be further simplified in the limit of large N, providing that $p_1 = p_2 = \cdots = p_N = 1/N$, and that

$$\lim_{N \to \infty} N^{-1} \sum_{n=1}^{N} \omega_n{}^2 = \omega^2 \; , \tag{4.8}$$

$$\lim_{N\to\infty} N^{-2}\sum_{n=1}^{N}\omega_n{}^4=0\ . \qquad (4.9)$$

Then Kac shows

$$E(b)\sim\omega\pi^{-2}\exp(-b^2N/2)\ . \qquad (4.10)$$

This formula breaks down, of course, before b achieves values close to 1 because of the normalization condition Eq. (4.1). Moreover, the assumptions of equal p_k's as well as additional conditions, Eqs. (4.8) and (4.9), make direct application of Eq. (3.10) to more realistic and physically relevant situations all but impossible. It is, nevertheless, worth stressing that the qualitative features encountered under this very special set of assumptions are likely to reemerge in real-world situations. In particular, in the limit of large N, fluctuations that depart significantly from $\Delta\sim N^{-1/2}$ are going to be exceedingly rare.

This conclusion is confirmed by the answer to the second question concerning the distribution of the values of $f=\sum p_k\cos\varphi_k$, where $\varphi_k=\omega_k t$. We shall show that under many plausible conditions this distribution is normal:

$$p(f)=\sigma^{-1}(2\pi)^{-1/2}\exp(-f^2/2\sigma^2) \qquad (4.11)$$

and that the standard deviation is given by the equation

$$\sigma=\lim_{N\to\infty}\left[\tfrac{1}{2}\sum_{k=1}^{N}p_k{}^2\right]^{1/2}$$
$$=\Delta/\sqrt{2}\ . \qquad (4.12)$$

To argue this we employ the classical limit theorem of the theory of probability.[33] Behavior of random variable X which itself is a sum of random variables x_k: $X=\sum_{k=1}^{N}x_k$ is the limit of large N given by the normal distribution providing that the so-called Lindeberg condition is fulfilled. In our case it suffices to demand that the largest p_k will uniformly approach zero as N increases to infinity:

$$\lim_{N\to\infty}\max(p_k)=0\ . \qquad (4.13)$$

If that is true, then the probability

$$\text{Prob}\left[B_N{}^{-1}\sum_{k=1}^{N}p_k\cos\varphi_k<X\right]$$
$$\to(2\pi)^{-1/2}\int_{-\infty}^{X}e^{-z^2/2}dz\ . \qquad (4.14)$$

Above B_N is the standard deviation of the sum and is defined by

$$B_N{}^2=\sum_{k=1}^{N}\frac{1}{2\pi}\int_0^{2\pi}(p_k\cos\varphi_k)^2d\varphi_k$$
$$=\tfrac{1}{2}\sum p_k{}^2\ . \qquad (4.15)$$

Therefore, $p(f)$ can be expected, for large N, to be reliably approximated by the normal distribution, Eq. (4.11).

The standard deviation of σ of Eq. (4.11) is given by

$$\sigma^2=\lim_{N\to\infty}B_N^2\ . \qquad (4.16)$$

When Eq. (4.13) holds one can expect that σ^2 can be written, for large N, as

$$\sigma^2=\tfrac{1}{2}\sum_{k=1}^{N}p_k{}^2=\lambda^2N\ , \qquad (4.17)$$

where λ^2 characterizes distribution of p_k's. It is possible to show that $\lambda^2\geq\frac{1}{2}$, where the equality applies only if all p_k are the same. Using Eq. (4.17) one may now rewrite Eq. (4.11) as

$$p_N(f)\simeq\lambda(2\pi N)^{1/2}\exp(-f^2N/2\lambda^2)\ . \qquad (4.18)$$

For large N, one can show $p(f)$ approaches Dirac's δ function[34]:

$$\lim_{N\to\infty}p_N(f)=\delta(f)\ . \qquad (4.19)$$

We conclude that as the size of the environment correlated with the quantum system in question increases, the superselection rules are less likely to be violated, and the correlation terms in the density matrix approach zero with the increase of N. It is perhaps worth stressing that this decay of correlations does not result in the decay of information; all the evolutions are generated by Hermitian Hamiltonians, and are therefore strictly reversible. The information is never destroyed; it is only displaced.[7]

Recurrences of correlations and the rate of decay of $z_{ij}(t)$ are the next two subjects of this section. We have already discovered that as the time increases $|z_{ij}|$ decreases to zero on what one may call a correlation decay time scale. This time of decay of correlations is followed by a presumably much longer period during which z_{ij} fluctuates with the average standard deviation $\sigma\sim N^{-1/2}$. How often, if ever, will one encounter a fluctuation substantially greater than σ? Let us answer this question in stages. First, we note that both $z_{ij}(t)$ and $r_{ij}(t)$ are almost periodic.[21] Therefore, one can rigorously show that they return, in the course of their evolution, arbitrarily close to any value within their range infinitely many times. In particular, one can define recurrence time,[35] or, as mathematicians call it, translation number T_ϵ,[21] which for a given almost-periodic correlation amplitude $z_{ij}(t)$ and for a given value of ϵ, $0<\epsilon\ll 1$, shall satisfy the inequality

$$1-|z_{ij}(T_\epsilon)|^2<\epsilon\ . \tag{4.20}$$

Existence of recurrence cycles proves that in a finite system information can never disappear. For macroscopic environments this time may actually prove longer than the lifetime of the Universe, i.e., than the time between big bang and big crunch.[36] More accurate predictions about the recurrences of the large fluctuations may be obtained from Eq. (4.7). These recurrences depend, nevertheless, on the precise distribution of p_k and on the values of ω_k's, and we shall not discuss them here.

The case when the spectrum of the interaction Hamiltonian is continuous rather than discrete can be readily included in our considerations. In a way, we have already implicitly approached that case by taking the $N\rightarrow\infty$ limit in the calculations above. It is nevertheless important to discuss briefly decay of correlations induced by the interaction Hamiltonian with the eigenbasis labeled within the Hilbert space of the environment, $\mathscr{H}_{\mathscr{E}}$, by a continuous parameter. The eigenbasis of both the self-Hamiltonian and of the interaction Hamiltonian within the Hilbert space of the system, $\mathscr{H}_{\mathscr{S}}$, may be either also continuous or it may remain discrete as before; the difference between the two cases is almost entirely notational. For definiteness, we shall then consider elements of the density matrix of the system labeled by continuous indices u and v:

$$\rho^S(u,v,t)=\alpha(u)\alpha^*(v)\exp(-it(\delta_u-\delta_v))f(u,v,t)\ . \tag{4.21}$$

The analogy with the discrete case, Eq. (3.6), is self-evident. As before, the size of the correlation is determined by the correlation amplitude

$$f(u,v,t)=\int p(\nu)\exp\{-it[\gamma(u,\nu)-\gamma(v,\nu)]\}d\nu\ . \tag{4.22}$$

The continuous index ν may express, for instance, the energy of the environment, while $\gamma(u,\nu)$ is the eigenvalue of the interaction Hamiltonian:

$$V|u\rangle\otimes|\nu\rangle=\gamma(u,\nu)|u\rangle\otimes|\nu\rangle\ . \tag{4.23}$$

The notation can be somewhat simplified if we fix our attention of a given pair (u,v) and, with the help of

$$\omega^{uv}(\nu)=\gamma(u,\nu)-\gamma(v,\nu)\ , \tag{4.24}$$

rewrite

$$f(t)=\int p(\omega(\nu))e^{-i\omega(\nu)t}d\nu\ . \tag{4.25}$$

As for decay of unstable states, $f(t)$ is a Fourier transform of a certain probability amplitude.[37–40] Qualitative behavior observed in the decay of unstable states can therefore guide us in the present discussion. First of all, one may note that when the spectrum is continuous the correlation amplitude may decay to zero:

$$\lim_{t\rightarrow\infty} f(u,v,t)=0 \quad \text{for } u\neq v\ . \tag{4.26}$$

Moreover, the correlation amplitude is no longer almost periodic; the recurrence time becomes infinitely long, and in this sense the decay of the off-diagonal elements may be considered irreversible. These properties of infinite systems—the only quantum systems endowed with continuous spectra—have prompted some to claim that a rigorous collapse of the state vector can be achieved[41] in the limit described by Eq. (4.26). However, Bell[42] has pointed out that this cannot be a resolution of the measurement problem: In all physically relevant situations both the time t and the size of the involved systems are finite, and therefore exact equality $f(t)=0$ should not be of crucial importance. We may add to Bell's comment that even though $f(t)$ may approach zero the information residing in correlations is always conserved. In course of the Hamiltonian evolution the information cannot ever be destroyed; it can only be displaced. In a case of a large environment it becomes "dissolved" in all the available degrees of freedom, so that its "density," represented by $f(t)$, decreases. Nevertheless, the total amount of information remains always the same.

Qualitative differences between the discrete and continuous cases can also be discussed in terms of the geometrical analogy of Fig. 3. Now instead of a sum of a countable number of discrete steps the correlation amplitude is represented by a section of a continuous line parametrized by the continuous index ν. At the initial $t=0$ the local curvature is equal to zero everywhere, for all ν. The line is perfectly straight. As the evolution progresses, the line becomes increasingly curved, with the local radius of curvature proportional to $t^{-1}(dp(\omega)/$

$d\omega)|_{\nu}$. At any finite time this curved line may be always approximated by a piecewise "random-walk" trajectory. However, because there is no limit to how small individual steps can become, the absolute value of their sum may be arbitrarily small.

Decay of the correlation amplitude is the subject closing our considerations. As we have noticed before, there is a formal similarity between the description of the unstable states and the decay of the correlation amplitude. In particular, properties of $p(\omega)$ play a decisive role in determining the rate of decay $\lambda(t)$ defined by the equation

$$df(t)/dt = -\lambda(t)f(t) \; . \tag{4.27}$$

The natural question that arises at this point is whether $\lambda = \text{const}$ and the decay is exponential. Let us first note that in the sufficiently small neighborhood of $t=0$ this may never be the case. For, the decay rate $\lambda(t)$ is given by

$$\lambda(t) = \frac{i \int \omega(\nu) p(\omega(\nu)) e^{-i\omega(\nu)t} d\nu}{\int p(\omega(\nu)) e^{-i\omega(\nu)t} d\nu} \; , \tag{4.28}$$

which, in the limit $t \rightarrow 0$, is purely imaginary;

$$\lambda(0) = i\langle \omega \rangle \; . \tag{4.29}$$

The decay does not start exponentially; it starts cosinusoidally, as the equation

$$d^2 f(t)/dt^2 |_{t=0} = -\left[\int \omega^2(\nu) p(\omega(\nu)) d\nu \right] f(0) \tag{4.30}$$

implies. This "slow start" is, in the decay of unstable states, responsible for the so-called "Zeno paradox"[43-45] also known as "watchdog effect"[46]: The unstable state, observed sufficiently frequently by the outside observer, is by each observation brought back to the $f(0)=1$ initial condition. In the vicinity of $t=0$ the decay is quadratic in time:

$$f(t) \simeq 1 - \tfrac{1}{2} \langle \omega(\nu)^2 \rangle t^2 \; . \tag{4.31}$$

Therefore, it appears that one can prevent the decay of the unstable state by watching it sufficiently closely.[43-46]

Finally, let us add that the requirement of an exponential decay would put rather stringent conditions on $p(\omega)$. It may nevertheless prove that the distributions $p(\omega)$ and functions $\omega(\nu)$ conspire as to make $f(t)$ exponential for a large range of physically important values of t.

V. DISCUSSION

Minimizing the loss of information is the key concern in the design of experiments and apparatuses. There is, therefore, something deeply paradoxical in the generally accepted—see, e.g., Refs. 4, 5, 8–11, and 29–31—opinion according to which the destruction of information must be invoked in order to explain the second stage of the measurement process. Von Neumann, in his classic analysis of the reduction of the state vector, was first to formulate this view.[4] It has since become customary to identify correlation terms as a source of all problems of measurement theory, and the unitary, information-conserving evolutions as the greatest obstacle on the way to a satisfactory interpretation of the act of observation.

Von Neumann gave his analysis of the quantum measurement a few years before Einstein, Podolsky, and Rosen showed, in a celebrated paper, consequences of the fact that in quantum mechanics the wave function representing a combination of two correlated systems can be reexpressed in many different bases. Therefore, they concluded that none of the systems can be said to be in a definite state.[47] The observable that one will decide to measure on one of the two systems will determine the set of states the second system can assume. The outcome of the first measurement will unambiguously fix the state of the second, yet-unmeasured system. This phenomenon, known as the "nonseparability paradox" is of direct significance for the correlated pair of quantum apparatus and measured system, considered by von Neumann in his analysis. For, as we have argued in Sec. II of this paper, quantum correlations allow one to determine the to-be-measured observable already after the interaction of the apparatus with the system has occurred. In particular, one could use this phenomenon to measure spin in the direction of the x axis with the help of the reversible Stern-Gerlach apparatus with all the field gradients directed along the z axis.[7] Moreover, one could modify Einstein's version of the double-slit experiment so that the state of the photon approaching the final photographic plate will depend on whether it was the momentum or the position of the double-slitted screen which was registered already after the photon has passed through it.[48] Thus, the EPR "paradox" proves that even after its interaction with the measured system has been completed, the isolated quantum apparatus cannot "know" between which states of the system the measurement is going to distinguish.

Throughout this paper we have argued that the two problems of measurement—the apparent decay of information in the course of the reduction of the state vector and the choice of the measured observable—can be solved in a single stroke, by recognizing that the apparatus is an open system, interacting with the environment. Formally, the resolution is accomplished by correlating the state of the apparatus with the state of the environment, and then by tracing out the environment. The resulting density matrix describing the correlated apparatus-system pair will be, to an excellent accuracy, diagonal in the pointer observable of the apparatus. In the real-world apparatuses the role of the environment will be usually assumed by a part of the apparatus setup itself.[7]

The fundamental question that can be raised at this point concerns the "legality" of invoking open systems as apparatuses. One might have hoped that the reduction of the wave packet occurs already in closed systems. Why should contact with other physical systems be necessary? Destruction of the information contained in the off-diagonal terms of the density matrix, even though important in the second state of the measurement process, provides no clue to the answer to the above question. The choice of the preferred pointer observable does: One may argue that *quantum observable remains undefined until it is defined operationally, with respect to other physical systems.*[49] If the observable is defined with respect to the environment, the system (or the apparatus endowed with this observable) must establish a correlation, must interact, with the environment. It is difficult to overlook the analogy between the operational definition of "observables" in Einstein's theory of relativity and the approach which must be adopted in the operational interpretation of quantum-mechanical observables: There, the Michelson-Morley experiment forced one to abandon the idea of the absolute space filled with an ether, in which light waves were to propagate, and use light signals as basic entities, propagation of which defines rather than is defined by the spacetime. Now the Einstein-Podolsky-Rosen experiment challenges us to abandon the idea that the systems are endowed with immutable, absolute properties between which correlations may exist and forces us to consider correlations as basic entities, existence of which defines rather than is defined by the properties.

Relativity of the properties of quantum systems is the key new concept arising in the context of environment-induced superselection rules. Let us also briefly consider other issues which are important for the understanding of the role of the environment in the process of measurement. The first of our concerns is the application of environment-induced superselection rules in practice. Irreversibility and its place in the measurement scenario based on the environment-induced superselection rules is the subject of the second comment. Amplification and the sense in which it enters into our considerations will be the last, but possibly most important of our concerns: We shall argue that the environment provides multiple records of the outcome of the measurement, and this process "fixes" the to-be-recorded observable.

A simple model in which decay of the off-diagonal elements of the density matrix is afforded by the "dephasing" relaxation phenomena, which are encountered in nuclear magnetic resonance as well as in some quantum optics experiments.[50,51] There one can show that the off-diagonal elements of the density matrix decay on a time scale T_2

$$\rho_{ij}(t)=\rho_{ij}(0)\exp(-t/T_2)\ , \tag{5.1}$$

which can be much shorter than the time scale T_1 on which diagonal elements approach equilibrium distribution:

$$\rho_{ii}(t)=\rho_{ii}(\infty)+[\rho_{ii}(0)-\rho_{ii}(\infty)]\exp(-t/T_1)\ . \tag{5.2}$$

When $T_2 \ll T_1$, projection operators appearing on the diagonal of density matrix ρ_{ij} define the pointer basis: The record contained in this basis will be preserved for $t<T_1$. The physical mechanism responsible for this behavior—as shown by Harris and Stodolsky[20] on an example where ρ describes a chemical molecule with two possible chiralities—can be based on the interaction between the system and the environment. Such interaction results in the environment-induced superselection rules, and is reminiscent of the model discussed in Sec. II of this paper.

The position of the macroscopic pointer can be also regarded as an approximate pointer observable: It is clearly capable of leaving imprints on a typical environment. This can be traced back to the fact that typical interaction potentials depend on the distance and hence force the environment to monitor the position of the pointer. One should remember, however, that position is never an exact pointer basis—at least not in terms of our idealized criterion that the pointer observable should be an exact nondemolition observable of the system: The self-Hamiltonian of any system will always contain $H^S=\vec{p}^2/2m$, which does not commute with $H_0^{SE}=V(\vec{r})$. Moreover, one can imagine situations (realized, for example, in solid-state physics) where $V(\vec{r})$ is periodic and eigenstates with definite wave vector, and therefore, completely undetermined position diagonalize total $H^S+H_0^{SE}$. Nevertheless, for a macroscopic (i.e., sufficiently large) quantum system H_0^{SE} will be usually aperiodic and sufficiently strong to force the position $\vec{r}$ to be an effective pointer observable. An interesting way of looking at this problem was already suggested some time ago, but published only recently by Wigner,[13] who argues that the density matrix representing the state of an object should be first expressed in terms of the moment expansion:

$$\rho=\rho(X,Y,Z,\mathscr{L}_0,\mathscr{L}_{21},\mathscr{L}_{22},\ldots,\mathscr{L}_{25},\mathscr{L}_{31},\ldots;X',Y',Z',\mathscr{L}'_0,\mathscr{L}'_{21},\mathscr{L}'_{22},\ldots,\mathscr{L}'_{25},\mathscr{L}'_{31},\ldots)\ . \tag{5.3}$$

Above X,Y,Z are the coordinates of the center of mass, while $\mathscr{L}_0$, $\mathscr{L}_{21},\ldots$ are the usual multipole expansion moments calculated with respect to the mass center. Next, Wigner suggests that "It is reasonable. . .to add to the [usual von Neumann equation] other terms which decrease the off-diagonal elements of ρ and to write"

$$i\hbar\partial\rho/\partial t=[H^S,\rho]-i\hbar\sum_{l,m}\epsilon_l(\mathscr{L}_{lm}-\mathscr{L}'_{lm})^2\rho\ . \tag{5.4}$$

H^S is the self-Hamiltonian of the considered system, while ϵ_l depend on the environment but " . . . their effectiveness should decrease with increasing l . . . as l^{-2}/a^l, where a is a characteristic length."

Little is known about the properties of the solutions of Eq. (5.4). Clearly, it would be difficult to expect it to be universally valid—after all, there is no universal environment: Moreover, arguments that lead to Eq. (5.4) seem to imply that the environment should actually alter the structure of the energy levels or perturb the system in question in some sense.[13] We have seen that this is not necessary—the state of the system may decay into a mixture not because it is perturbed, but because it is recorded by the environment. The interest in Eq. (5.4) stems rather from the fact that it has certain symmetries—it is a so-called "Galilean invariant"—which lead one to believe that it may be approximately valid under a great many circumstances. If this hope could be substantiated by the experiment, Eq. (5.4) could prove truly useful.

Irreversibility is sometimes invoked as a justification for the reduction of the wave packet. While the connection between the indelibility of the records resulting from the measurement process and the second law of thermodynamics is undeniable, there is also little doubt that the explanation of one in terms of the other can be circular,[52] and usually avoids the really deep question: How can one reconcile either of them with the unitarity of quantum-mechanical evolutions? We have invoked environment, and the transfer of information from the apparatus-system object to the environment-apparatus correlations as the ultimate cause of the apparent wave packet collapse. Can one understand microscopic origins of the second law of thermodynamics in this manner? In a many-body system of a finite, if large size relaxation of the system into equilibrium on the relatively short time scale can be understood as a consequence of the correlations that form between its various components. From the viewpoint of the observer such many-body system can be perceived, prepared, or utilized only a "piece at a time," one subsystem after the other. Therefore, the information relevant for him resides in the separate "bodies" of the many-body system: He cannot effectively use the information hidden in the interparticle correlations. This can be thought of as a consequence of a superselection rule which forces us to regard the Hilbert space of the many-body system as a direct sum of the Hilbert spaces of its components:

$$\tilde{\mathscr{H}}_{\mathscr{S}}=\bigoplus_n\mathscr{H}_n$$

while the evolution Hamiltonian induces rotations which occur in the much larger product Hilbert space

$$\mathscr{H}_{\mathscr{S}}=\prod_n\otimes\mathscr{H}_n\ .$$

The total information I—a constant of motion under the unitary evolution—is given by

$$I=\ln(\dim(\mathscr{H}_{\mathscr{S}}))+\mathrm{Tr}\rho\ln\rho\ . \tag{5.5}$$

The information residing in the nth subsystem is given by

$$I_n=\ln(\dim(\mathscr{H}_{\mathscr{S}}))+\mathrm{Tr}\rho_n\ln\rho_n\ , \tag{5.6}$$

where ρ_n is the density matrix of the nth subsystem, and can be obtained from ρ, the total density matrix, by a partial trace. The total information residing in the subsystems is equal to

$$\mathscr{I} = \sum I_n \ , \tag{5.7}$$

where $\mathscr{I}$ can be regarded as *negentropy.* On time scales much smaller than the Poincaré time scale, $\mathscr{I}$ will decrease whenever at $t=0$ the many-body system was prepared or measured: it has to be measured subsystem after subsystem, and hence $\mathscr{I}$ has increased in the course of the measurement. In a sense, one can maintain that the many-body structure of the large system provides a natural coarse graining, and can be utilized much in the same fashion as the Gibb's original concept. Moreover, its application in quantum mechanics is more natural than in classical mechanics: First, in quantum mechanics the many-body structure of the system provides a natural definition of coarse graining. Second, the state of the individual bodies—individual "grains" in the phase space—becomes indeed indefinite in a quantum system. Thus, the increase of entropy in quantum systems can be regarded not only as not in contradiction with, but almost as a consequence of the conservation of the total information I.[14]

In quantum mechanics recurrences of the state of low entropy appear even more inevitable in a closed finite system than in classical physics.[35] Following Zermelo one may inquire why all the systems which are large but finite appear nevertheless to comply with the second law of thermodynamics. Boltzmann's remark pointing to the duration of the Poincaré cycle provides certainly a partial answer to this question. We would like to argue that it is strengthened by the observation that the idealization of an isolated system is much more difficult to realize in the context of quantum mechanics. For, the entropy of a quantum system increases whenever a system-environment correlation is established. It is therefore reasonable to contend that a system is "isolated" only when it is not able to establish any correlations with the environment over the Poincaré cycle time scale. Thus, the upper limit on the strength of the environment-system decreases very rapidly with the increase in size or in complexity which in turn cause the increase of the recurrence time of the system which is to be isolated.

Bohr has often emphasized that the quantum phenomenon is "brought to a close" only by an irreversible act of amplification. In what sense, if at all, does the measurement model based on the environment-induced superselection rules realize Bohr's postulate of amplification? To answer this question we note that already establishment of the apparatus-environment correlation, the transition $(\sum_n c_n \,|\, n\rangle \otimes |\, A_n\rangle) \otimes |\, \mathscr{E}\rangle \to \sum_n c_n \,|\, n\rangle \otimes |\, A_n\rangle \otimes |\, \mathscr{E}_n\rangle$, Eq. (1.1b) and Eq. (1.1c), can be interpreted as an elementary act of amplification, provided that $|\langle \mathscr{E}_n \,|\, \mathscr{E}_m\rangle|^2 \simeq \delta_{nm}$. For now both the apparatus and the environment are correlated with a state of the quantum system: The information about a pointer observable $\hat{\Pi} = \sum \pi_n \,|\, n\rangle\langle n \,|$ can be extracted from any of them. In this simple sense the amplification has already occurred. Two interrelated aspects of this elementary act of amplification are worthy of special attention. First, we can claim that the amplification has occurred only because we treat apparatus and environment as two distinct systems, which can be "read off" separately. This remark ties in with the discussion of irreversibility caused by the fact that we perceive many-body systems as consisting of many subsystems. Second, there is just one basis set $\{\,|\, n\rangle\}$ for which records of the apparatus and the environment will always agree: One can therefore argue that only this observable of the quantum system can be regarded as truly measured which is recorded *redundantly,* that is, correlated with the state of many distinct quantum systems.

This last remark brings us back to the key idea emerging from the discussion of the environment-induced superselection rules: Correlations between the properties of quantum systems are more basic than the properties themselves. This order of importance, in which a correlation—a record of a property—comes before the property, reverses the ordinary hierarchy to which one is accustomed within the realm of everyday experience. It may, nevertheless, prove to be essential in the deeper understanding of the seemingly paradoxical features of the nonseparably correlated quantum systems. It may also be the only way to further reconcile the key lesson of Bohr's interpretation of quantum theory—"No phenomenon is a phenomenon until it is a recorded phenomenon"[53]—with the unitarity of quantum-mechanical evolutions.

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