Constraints on supersymmetric-particle masses from $(g-2)_\mu$

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From the experimental value of $(g-2)_\mu$ we derive absolute lower bounds on the masses of the supersymmetric fermionic partners of the W and Z bosons and the scalar partners of the muon and the neutrino.

Supersymmetry constitutes an extremely elegant theory which provides a high degree of symmetry in the description of nature.[1] As a by-product, it might help to solve one of the outstanding problems of present grand unified theories. In fact, the so-called no-renormalization theorems of supersymmetric theories suggest a possible solution of the hierarchy problem of symmetry-breaking scales.[2] However, an obvious fact about supersymmetry is that it is not manifestly realized in the particle spectrum of the real world. That is, Bose-Fermi symmetry is clearly broken at low energies. Therefore, the relevant issue is to what extent is supersymmetry broken. Several scenarios entailing different symmetry-breaking patterns have been entertained.[3] However, the theoretical possibilities still remain widely open. Also, from the phenomenological standpoint, few constraints on the parameters of these theories are known.

In theories with spontaneous supersymmetry breaking through a new U(1) gauge symmetry, mass relations of the type $m_{sf}{}^2 - m_{sf'}{}^2 = m_f{}^2 - m_{f'}{}^2$ emerge, and in the simplest versions of these models, an upper limit for the mass of the supersymmetric scalar partners (sf) of the fermions (f) of about $M_W/2$ is obtained.[4] A recent survey concerning flavor-changing neutral-current interactions puts stringent bounds on the relative mass differences of supersymmetric leptons and quarks belonging to different family generations.[5] However, for the absolute value of the masses themselves, very little is known from experiment, apart from the obvious requirement that the charged supersymmetric particles should be heavier than about 20 GeV since none of them have been seen at PETRA or PEP. Consequently, any experimental evidence that narrows down the range of masses in supersymmetry is welcome. In this respect it has been noted that the muon anomaly forbids masses below ~100 GeV for the supersymmetric scalar partner $(\tilde{\mu})$ of the muon in theories where there is a diagonal Yukawa coupling of the type μ-μ-$\tilde{\mu}$.[6] In more constrained models, however, where a conserved R quantum number exists, the above-mentioned coupling is absent and the lower bound that one finds in the literature for the $\tilde{\mu}$ mass is ~15 GeV, which, of course, if of no practical use.[4]

In this Rapid Communication we reexamine more closely the constraints which follow from the muon anomalous magnetic moment in the framework of global supersymmetry for models with a conserved R quantum number. The contribution to $a_\mu \equiv (g-2)_\mu/2$ due to supersymmetry comes about from the diagrams shown in Fig. 1.

The supersymmetric particles involved in these amplitudes are: the spin-$\frac{1}{2}$ partners of the W boson, the Z boson, and the photon (denoted by $\tilde{W}$, $\tilde{Z}$, and $\tilde{\gamma}$, respectively), along with the scalar partners of the muon $(\tilde{\mu}_1, \tilde{\mu}_2)$ and the scalar partner of the neutrino $(\tilde{\nu})$. They all carry the quantum numbers of their conventional partners. We denote the masses of these supersymmetric particles by their names, whereas we use m for the muon mass.

We start with the contribution to a_μ from diagram 1(a). We find

$$a_\mu^{(\tilde{W})} = \frac{\alpha}{4\pi \sin^2\theta_W} \int_0^1 dx \, \frac{m^2x^2(1-x)}{m^2x^2 + (\tilde{W}^2 - \tilde{\nu}^2 - m^2)x + \tilde{\nu}^2} ,$$

where α is the fine-structure constant and θ_W the Weinberg angle. The above expression is positive for any values of the masses $\tilde{W}$ and $\tilde{\nu}$. Introducing $\lambda \equiv \tilde{\nu}^2/(\tilde{W}^2 - \tilde{\nu}^2 - m^2)$ and considering $m^2 << \tilde{W}^2$, which is obviously true since $\tilde{W} \gtrsim 20$ GeV (or $\tilde{W} = 1.8$ GeV if it were the τ lepton), we get

$$a_\mu^{(\tilde{W})} = \frac{\alpha}{4\pi \sin^2\theta_W} \frac{m^2}{\tilde{\nu}^2} [\lambda(1-3\lambda-6\lambda^2)/6 + \lambda^3(\lambda+1)\ln(1+1/\lambda)] . \quad (1)$$

Let us now turn to the $\tilde{G}$ $(\equiv \tilde{\gamma}$ or $\tilde{Z})$ exchange contributions of Figs. 1(b) and 1(c). Denoting, in general, the $\mu\tilde{\mu}_i\tilde{G}$ vertex by

$$[\bar{\mu}(g_V - g_A\gamma_5)_{\tilde{G},i}\tilde{G}]\tilde{\mu}_i ,$$

the contribution of $\tilde{G}$ exchange to a_μ is then

$$a_\mu^{(\tilde{G})} = -\sum_{i=1,2} \frac{1}{8\pi^2} m \int_0^1 dx \, \frac{[\tilde{G}(g_V^2 - g_A^2)_{\tilde{G},i} + m(g_V^2 + g_A^2)_{\tilde{G},i} x(1-x)]}{m^2x^2 + (\tilde{\mu}_i^2 - \tilde{G}^2 - m^2)x + \tilde{G}^2} .$$

The main difference with Eq. (1) is the presence of the term proportional to $g_V^2 - g_A^2$ which is linear in m and can, in principle, be the most important contribution. This term is automatically absent in the $\tilde{\gamma}$ contribution since in this case $\tilde{G}$ is essentially massless ($\tilde{\gamma} < 50$ eV in theories with R symmetry)[6] and only the quadratic term survives, leading to

$$a_\mu^{(\tilde{\gamma})} = -\frac{\alpha}{12\pi} m^2 \left(\frac{1}{\tilde{\mu}_1^2} + \frac{1}{\tilde{\mu}_2^2} \right) . \qquad (2)$$

In the case of the $\tilde{Z}$ exchange the linear term is present since $\tilde{G} = \tilde{Z} \neq 0$. However, if $\tilde{\mu}_L$ and $\tilde{\mu}_R$ are mass eigenstates we have $|g_V| = |g_A|$ and the linear term again vanishes. The reason is that there is a flip in the muon helicity in the configuration contributing to $(g-2)_\mu$. The only way in which the linear term can contribute is through a $\tilde{\mu}_L, \tilde{\mu}_R$ mixing (this would imply a nondegeneracy of the $\tilde{\mu}_{1,2}$ masses). If we denote the mass eigenstates by

$$\tilde{\mu}_1 = \cos\phi \, \tilde{\mu}_L + \sin\phi \, \tilde{\mu}_R ,$$

$$\tilde{\mu}_2 = -\sin\phi \, \tilde{\mu}_L + \cos\phi \, \tilde{\mu}_R ,$$

it can be easily seen that $g_V^2 - g_A^2 \propto \sin\phi \cos\phi$.

Assuming maximal mixing $\phi = \pi/4$, the contribution of the linear term is

$$a_\mu^{(\tilde{Z})}(\text{lin}) = -\sum_{i=1,2} (-1)^i \frac{\alpha(1-2\sin^2\theta_W)}{4\pi \cos^2\theta_W} \frac{m\tilde{Z}}{(\tilde{\mu}_i^2 - \tilde{Z}^2)^3} \left[\tfrac{1}{2}(\tilde{\mu}_i^4 - \tilde{Z}^4) - \tilde{\mu}_i^2 \tilde{Z}^2 \ln\frac{\tilde{\mu}_i^2}{\tilde{Z}^2} \right] . \qquad (3)$$

In the limit $\tilde{\mu}_1 = \tilde{\mu}_2$, both contributions cancel, as expected. Conversely, the maximal contribution is obtained for $\mu_{1,2}^2 \ll \mu_{2,1}^2$. In that case the linear piece, due to $\tilde{Z}$ exchange, is by far the most important contribution to $(g-2)_\mu$.

Since the limits on Δa_μ allowed by experiment are[7]

$$-20 \times 10^{-9} < \Delta a_\mu < +26 \times 10^{-9} \quad (95\% \text{ C.L.}) , \qquad (4)$$

we may saturate this inequality with the contribution of $\tilde{W}$ exchange. For the case where $\mu_1^2 \ll \mu_2^2$, $a_\mu^{\tilde{Z}}(\text{lin}) > 0$, and we obtain the allowed region in the $(\tilde{W}, \tilde{\nu})$ plane (there are no cancellations, since $a_\mu^{(\tilde{W})} > 0$ also), displayed in Fig. 2. In the opposite case, however, since $a_\mu^{(\tilde{Z})}(\text{lin})$ is negative, one cannot reach any conclusion at all since both contributions $(\tilde{Z} + \tilde{W})$ potentially cancel. We may now saturate inequality (4) with the $a_\mu^{(\tilde{Z})}(\text{lin})$ piece, which may be positive ($\mu_1^2 \ll \mu_2^2$) or negative ($\mu_2^2 \ll \mu_1^2$). In the first case, shown in Fig. 3, the forbidden region in the $(\tilde{\mu}, \tilde{Z})$ plane is absolutely forbidden because the additional (positive) contribution from $a_\mu^{(\tilde{W})}$ would enlarge it. If any sort of theory or experiment gives information on one of the two masses we can immediately obtain from Fig. 3 interesting bounds on the other mass. In the second case $[a_\mu^{(\tilde{Z})}(\text{lin}) < 0]$, no conclusion can be drawn from $(g-2)_\mu$ alone. However, knowing that $\tilde{W} \gtrsim 17$ GeV, the maximal contribution to a_μ from $\tilde{W}$ exchange is $+8 \times 10^{-9}$. Therefore the lower bound in Eq. (4) is effectively extended to -28×10^{-9}, thus giving similar results to those shown in Fig. 3 (with $\tilde{\mu} \equiv \tilde{\mu}_2$).

As already stated, these results rely heavily on the assumption of left-right mixing and large mass splittings among the spin-zero partners of the muon. In

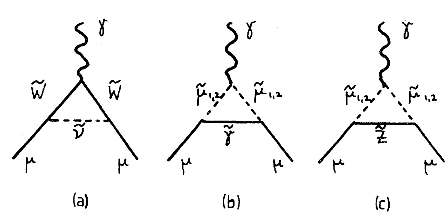

FIG. 1. Diagrams corresponding to the contribution to $(g-2)_\mu$ arising from supersymmetric particles.

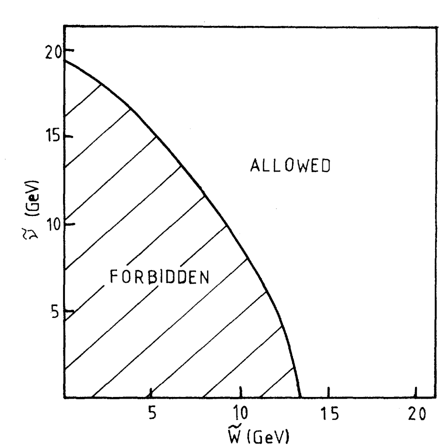

FIG. 2. Allowed region for the masses of the $\tilde{W}$ and the $\tilde{\nu}$ in the case described in the text.

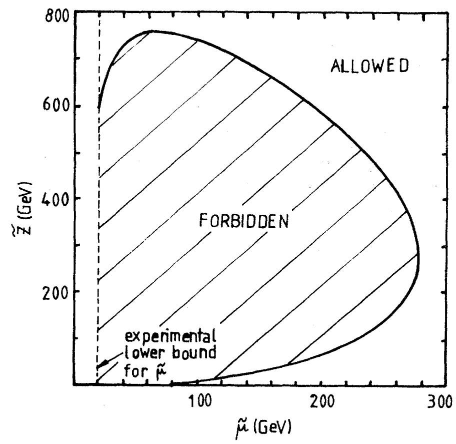

FIG. 3. Allowed region for the masses of the $\tilde{\mu}$ and the $\tilde{Z}$ in the model described in the text. Here, $\tilde{\mu}_1 \equiv \tilde{\mu}$ and $\tilde{\mu}_2 \equiv \infty$.

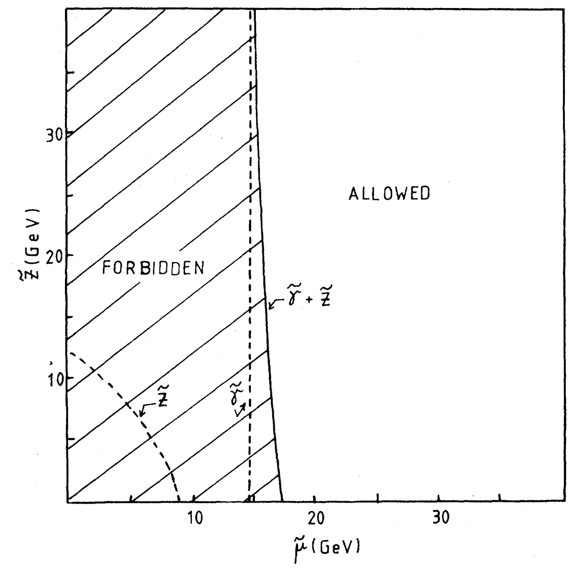

FIG. 4. Allowed region for the masses of the scalar muon and the $\tilde{Z}$ in the situation described in the text. Here, $\tilde{\mu}_1 = \tilde{\mu}_2 \equiv \tilde{\mu}$.

Ref. 8, two classes of models existing in the literature have been analyzed in this context. They all render a too small contribution to the linear piece in $(g-2)_\mu$, even lower than the quadratic term we have been neglecting so far. Let us therefore consider the situation where $a_\mu^{(\tilde{Z})}(\text{lin})$ is absent. In this case, we have on one hand the contribution from Fig. 1(a) and, on the other hand, two contributions involving one common parameter ($\tilde{\mu}$): the $\tilde{\gamma}$ term [Fig. 1(b)] and the $\tilde{Z}$ term [Fig. 1(c)]. They are both negative, whereas the $\tilde{W}$-exchange piece is positive. Again cancellations may occur which prevent us to reach conclusions. Nevertheless, if we assume, guided by the recent analysis of Ellis and Nanopoulos[5] on flavor-changing neutral interactions, $\tilde{W}$ to be 100 GeV or heavier (a very reasonable assumption), then its contribution to $(g-2)_\mu$ is negligible (2.3×10^{-10}). In this circumstance we may evaluate the bounds derived from Eq. (4) for the $\tilde{Z}$ and $\tilde{\mu}$ masses. Figure 4 shows the allowed region in the $(\tilde{\mu}, \tilde{Z})$ plane where the individual $\tilde{Z}$ and $\tilde{\gamma}$ boundaries are separately shown. Unfortunately, the bounds obtained are of no practical use since we already know that $\tilde{\mu} > 17$ GeV and, as far as $\tilde{Z}$ is concerned, no bound at all is obtained.

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