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Comment on the proliferation of gauge fields in a nonlinear spinor theory

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It is noted that by merely rewriting the Lagrangian in a different form, Dürr and Saller do not realize their aim of inducing an additional hypercharge degree of freedom. It is pointed out that such an increase in the dynamical degrees of freedom would indeed occur if the Weinberg-Salam model could be derived starting from a four-fermion interaction. Existing studies on nonlinear spinor theories indicate that it may be possible to induce such additional symmetries, perhaps only in an approximate sense.

The search for a unified theory of fundamental interactions has been a subject of numerous investigations. One such attempt is the nonlinear spinor theory of Heisenberg[1] within which all the physical fields are considered to be composites of one single nonlinearly self-coupled spinor field. In a series of papers, Dürr and Saller[2–5] have indicated that it is possible, starting with such a self-coupled spinor field, to dynamically rearrange the Lagrangian so that the rearranged Lagrangian appears to have symmetries that were not originally present. This is referred to as the inflation of the dynamical degrees of freedom. They have recently used this approach to induce the hypercharge degree of freedom of the Weinberg-Salam (WS) model[3] and also the color and flavor degrees of freedom associated with the quarks.[5] The purpose of this paper is to examine their inflation scheme more closely and point out some of the difficulties associated with this approach.

For the sake of definiteness, we focus our attention on Ref. 3 wherein it has been claimed that it is possible to dynamically "deflate" the local SU(2) $\times$U(1) symmetry of the WS model down to a local SU(2). This led the authors to suggest that the local hypercharge U(1) group may be redundant. Using the notation of Ref. 3, the Lagrangian for the WS model, to which (following Dürr and Saller) a noninteracting, left-handed, massless fermion singlet L' has been added, is written as

$$\mathscr{L}=\vec{\mathrm{L}}^*\sigma^\mu(\tfrac{1}{2}i\partial_\mu-\tfrac{1}{2}\mathscr{B}_\mu-\tfrac{1}{2}\vec{\tau}\cdot\vec{\mathscr{A}}_\mu)\vec{\mathrm{L}}+R^*\sigma^\mu(\tfrac{1}{2}i\partial_\mu-\mathscr{B}_\mu)R-\frac{1}{4g'^2}\mathscr{B}_{\mu\nu}{}^2-\frac{1}{4g^2}\vec{\mathscr{A}}_{\mu\nu}{}^2$$
$$+\tfrac{1}{2}\,|\,(i\partial_\mu+\tfrac{1}{2}\mathscr{B}_\mu-\tfrac{1}{2}\vec{\tau}\cdot\vec{\mathscr{A}}_\mu)\Phi\,|^2-M_1{}^2\vec{\Phi}^*\cdot\vec{\Phi}+h(\vec{\Phi}^*\cdot\vec{\Phi})^2-G_e[(\vec{\mathrm{L}}^*R)\cdot\vec{\Phi}+\vec{\Phi}^*\cdot(R\vec{\mathrm{L}})]+L'^*\sigma^\mu\frac{i}{2}\partial_\mu L'\ , \tag{1}$$

with the fields transforming under $\mathrm{U}^F(1)\times\mathrm{U}^Y(1,\mathrm{loc})\times\mathrm{SU}^I(2,\,\mathrm{loc})$ as

$$\vec{\mathrm{L}}(x)\to\exp\left[-i\delta-\frac{i}{2}\alpha(x)-\frac{i}{2}\vec{\tau}\cdot\vec{\beta}(x)\right]\vec{\mathrm{L}}(x)\ ,$$
$$R(x)\to\exp[-i\delta-i\alpha(x)]R(x)\ ,$$
$$L'(x)\to\exp(-i\delta)L'(x)\ ,$$
$$\Phi(x)\to\exp\left[-\frac{i}{2}\alpha(x)-\frac{i}{2}\vec{\tau}\cdot\vec{\beta}(x)\right]\Phi(x)\ ,$$
$$\vec{\mathscr{A}}_\mu(x)\underset{\mathrm{inf}}{\to}\vec{\mathscr{A}}_\mu(x)+\vec{\beta}(x)\times\vec{\mathscr{A}}_\mu(x)+\partial_\mu\vec{\beta}(x)\ ,$$
$$\mathscr{B}_\mu(x)\underset{\mathrm{inf}}{\to}\mathscr{B}_\mu(x)+\partial_\mu\alpha(x)\ . \tag{2}$$

The SU(2) doublet can be conveniently parametrized as

$$\vec{\mathrm{L}}=|\mathcal{N}|\begin{bmatrix}\cos\dfrac{\theta}{2}\,e^{i\xi}\\ \sin\dfrac{\theta}{2}\,e^{i\phi'}\end{bmatrix} . \tag{3}$$

SU(2) transformations leave $|\mathcal{N}|$ unaltered. The doublet $\vec{\mathrm{L}}$ can be rewritten as

$$\vec{\mathrm{L}}(x)=|\mathcal{N}|\,e^{i\xi}\vec{c}(x) , \tag{4a}$$

with

$$\vec{c}(x)=\begin{bmatrix}\cos\dfrac{\theta}{2}\\ \sin\dfrac{\theta}{2}\,e^{i\phi}\end{bmatrix} . \tag{4b}$$

Dürr and Saller require by definition that under SU(2) transformations the upper component of $\vec{c}(x)$ remain real. They further require $\vec{c}(x)$ to be inert under hypercharge U(1) transformations. In Ref. 3, the SU(2) transformation of $\vec{c}(x)$ is written as

$$\begin{bmatrix}\cos\dfrac{\theta}{2}\\ \sin\dfrac{\theta}{2}\,e^{i\phi}\end{bmatrix}\underset{\mathrm{SU(2)}}{\longrightarrow} e^{i(\kappa/2)(\theta,\phi,\vec{\beta})-(i/2)\vec{\tau}\cdot\vec{\beta}}\begin{bmatrix}\cos\dfrac{\theta}{2}\\ \sin\dfrac{\theta}{2}\,e^{i\phi}\end{bmatrix} , \tag{5}$$

where κ is determined from the constraint that the upper component remain real. Dürr and Saller, by considering an infinitesimal SU(2) transformation, obtain

$$\kappa=\kappa'(\vec{\beta},\theta,\phi)$$
$$=\beta_3+(\beta_1\cos\phi+\beta_2\sin\phi)\tan\frac{\theta}{2} . \tag{6}$$

At this point, we note that because of the constraint, $\vec{c}(x)$ does not transform as an SU(2) doublet under SU(2) transformations. Moreover, due to its nonlinear character, the infinitesimal transformation cannot be exponentiated to yield the correct finite transformation. Indeed it can be directly checked that the finite transformation (5) with $\kappa=\kappa'$ given in (6) does not maintain the reality of the upper component. We find that for a finite transformation,

$$\kappa(\vec{\beta},\theta,\phi)$$
$$=2\tan^{-1}\left[\tan\frac{\beta}{2}\,\frac{\hat{\beta}_3+\tan\frac{1}{2}\theta(\hat{\beta}_1\cos\phi+\hat{\beta}_2\sin\phi)}{1+\tan\frac{1}{2}\theta\tan\frac{1}{2}\beta(\hat{\beta}_1\sin\phi-\hat{\beta}_2\cos\phi)}\right] \tag{7}$$

with $\hat{\beta}_i\equiv\beta_i/|\beta|$. It is clear that (7) reduces to (6) for $|\beta|<<1$.

Following Ref. 3, we continue the "deflation" process. To this end, we define the deflated fields

$$\vec{c}(x)=\omega(x)\vec{\mathrm{L}}(x) ,$$
$$\vec{\phi}(x)=\omega^*(x)\vec{\Phi}(x) ,$$
$$r(x)=\omega^2(x)R(x) ,$$
$$\mathcal{C}_\mu(x)=\mathcal{B}_\mu(x)-i\omega\partial_\mu\omega^*(x)$$

with

$$\omega(x)=e^{i\xi(x)} . \tag{8}$$

Under SU(2), the deflated fields transform as

$$\vec{c}(x)\to\exp\left[i\frac{\kappa}{2}-\frac{i}{2}\vec{\tau}\cdot\vec{\beta}(x)\right]\vec{c}(x) ,$$
$$r(x)\to\exp\left[\frac{i}{2}\kappa\right]r(x) ,$$
$$\mathcal{C}_\mu(x)\to\mathcal{C}_\mu-\partial_\mu\kappa , \tag{9}$$
$$\vec{\phi}(x)\to\exp\left[-\frac{i}{2}\kappa-\frac{i}{2}\vec{\tau}\cdot\vec{\beta}(x)\right]\vec{\phi}(x) ,$$
$$\omega(x)\to\exp\left[\frac{i}{2}\kappa\right]\omega(x) ,$$

whereas under U(1) transformations

$$\omega(x)\to\exp\left[\frac{i}{2}\alpha(x)\right]\omega(x) , \tag{10}$$

while the fields $\vec{c}(x)$, $r(x)$, $\mathcal{C}_\mu(x)$, and $\vec{\phi}(x)$ are inert. In terms of these deflated fields, the Lagrangian (1) can be written as

$$\mathcal{L}=\vec{c}^{\,*}\sigma^\mu(\tfrac{1}{2}i\partial_\mu-\tfrac{1}{2}\mathcal{C}_\mu-\tfrac{1}{2}\vec{\tau}\cdot\vec{\mathcal{A}}_\mu)\vec{c}+r^*\sigma^\mu(\tfrac{1}{2}i\partial_\mu-\mathcal{C}_\mu)r-\frac{1}{4g'^2}\mathcal{C}_{\mu\nu}^{\ 2}-\frac{1}{4g^2}\vec{\mathcal{A}}_{\mu\nu}^{\ 2}$$
$$+\tfrac{1}{2}|(i\partial_\mu+\tfrac{1}{2}\mathcal{C}_\mu-\tfrac{1}{2}\vec{\tau}\cdot\vec{\mathcal{A}}_\mu)\vec{\phi}\,|^2-M_1^{\,2}\vec{\phi}^{\,*}\cdot\vec{\phi}+h(\vec{\phi}^{\,*}\cdot\vec{\phi})^2-G_e[(\vec{c}^{\,*}r)\cdot\vec{\phi}+\vec{\phi}^{\,*}\cdot(\vec{c}\,r)]+L'^*\sigma^\mu\frac{i}{2}\partial_\mu L' . \tag{11}$$

Since $\omega(x)$ does not appear in (11), the fields that do appear are inert under U(1) transformations.

This led Dürr and Saller to argue that the local SU(2)×U(1) symmetry of the WS model is dynamically deflated to local SU(2). They take this as an indication that the hypercharge degree of freedom may be redundant.

Their modus operandi is to write the fermion fields so that they transform nonlinearly under SU(2) rather than as SU(2) doublets [see Eq. (9)]. The invariance of the Lagrangian under this nonlinear SU(2) transformation of the fermion fields entails the introduction of four (rather than three) gauge fields, $\mathscr{C}_\mu(x)$ and $\vec{\mathscr{A}}_\mu(x)$. $\vec{\mathscr{A}}_\mu(x)$ transforms as a genuine SU(2) gauge field whereas $\mathscr{C}_\mu(x)$ transforms in a rather complicated manner. The fermion fields and the four gauge fields are then dynamically rearranged to yield a local SU(2) $\times$U(1) invariant Lagrangian.

To point out where the additional U(1) enters, we remark that the invariance of the Lagrangian under local, nonlinear SU(2) transformations (9) automatically entails an SU(2)$\times$U(1) invariance. (A simple example of a Lagrangian where an assumed "lower" symmetry automatically entails a "higher" symmetry is that of a Lagrangian that is required to be invariant under rotations about the x and y axes; it is then invariant under rotations about the z axis also.) To see this we note that the Lagrangian (11) is invariant under the transformations (9) even when κ is arbitrary, i.e., when κ does not satisfy (7). But the transformations (9), for arbitrary $\kappa(x)$ are just SU(2)$\times$U(1) transformations. We are thus forced to conclude that *rewriting the Lagrangian (1) in terms of the deflated fields as in (11) does not deflate the dynamical degrees of freedom.*[6] In fact, the nonlinear transformation of Dürr and Saller makes manifest the hypercharge degree of freedom that is latent in Eqs. (9)–(11). We remark in passing that the deflated fields transform nonlinearly under SU(2) transformations and not as ray doublets as SU(2) as suggested in Ref. 3 [SU(2) has no nontrivial ray representations] and hence it is not possible to assign values of I^2 and I_3 to these fields.

Dürr and Saller proceed to conjecture but do not explicitly demonstrate that the Lagrangian (11) can be obtained by a dynamical rearrangement of the fields in the Lagrangian,

$$\mathscr{L} = \vec{\psi}^* \sigma^\mu \left(\tfrac{1}{2} i \partial_\mu - \tfrac{1}{2}\vec{\tau}\cdot\vec{\mathrm{A}}_\mu\right)\vec{\psi} - \frac{1}{4{g_0}^2}{\vec{\mathrm{A}}_{\mu\nu}}^2 - \frac{G}{2M^2}(\vec{\psi}^*\vec{\tau}\psi)^2 \,. \qquad (12)$$

Here, $\vec{\psi}$ is an SU(2) doublet and $\vec{\mathrm{A}}_\mu$ an SU(2) gauge field. The symmetry group of the Lagrangian (12) is U(1,global)$\times$ SU(2,local). The dynamical rearrangement of (12) into the form (11) would thus entail a creation of an additional local U(1) symmetry; it is then that there would be a true inflation of the dynamical degrees of freedom since an additional gauge field would have to be generated. Once again, we state that this has not been demonstrated by the authors.

Although it is not clear how an inflated symmetry may be obtained merely by rewriting the original Lagrangian in a different form, it may be worthwhile to point out that several authors[7] have suggested that a four-fermion theory of the type considered by Dürr and Saller may be cast into the form of a local gauge theory. In a recent paper[8] we have reexamined some of these arguments and concluded that the equivalence of asymptotically free gauge theories and the corresponding four-fermion theory may be possible under certain conditions. We note, however, that the WS model is not asymptotically free and our method of proof does not apply. Furthermore, if the transmutation mechanism for solvable models[9] is also operative for the case of relativistic field theories, the above-mentioned equivalence could at most be in an approximate sense.

In summary, we state that the mere requirement that the fermions transform nonlinearly under SU(2) does not deflate the dynamical degrees of freedom. Admittedly, it may be possible to extract some terms from the four-fermion Lagrangian so that part of the Lagrangian has an inflated symmetry, but it is not clear how such an inflation could be achieved in an (even formally) exact sense. A genuine deflation could occur if it would be possible to dynamically rearrange the Lagrangian (12) to the form of that in (11). Dürr and Saller have conjectured but not demonstrated this possibility. In view of the fact that the methods of Ref. 8 are not applicable here, we are at present not aware of how to argue the equivalence of the Lagrangians of the WS model and the corresponding four-fermion theory.

Note added. After completion of this work we learned that a formula for the finite nonlinear SU(2) transformation [see Eq. (7)] has also been independently derived by Dürr and Saller (H. P. Dürr, private communication).

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[6]The corresponding situation for QED is indicated here. The Lagrangian $\mathscr{L}_1=\bar{\psi}(\not\partial+m-ie\not A)\psi$ can be rewritten in terms of new fields defined by $\psi=S\xi$ with $\xi=(\psi^*\psi)^{1/2}$ as $\mathscr{L}_2=\bar{\xi}(\not\partial+m-ie\not B)\xi$ with $B_\mu=A_\mu-(1/e)S^*\partial_\mu S$. Under the U(1) transformation $\psi\rightarrow e^{i\alpha}\psi$, $A_\mu\rightarrow A_\mu+(1/e)\partial_\mu\alpha$, $S\rightarrow e^{i\alpha}S$ while the deflated field ξ and B_μ are inert simply chosen to be real thereby precluding it from transforming under that particular transformation. The Lagrangian $\mathscr{L}_2$, however, is still invariant under local U(1) transformations, $\xi\rightarrow e^{i\beta}\xi$, $B_\mu\rightarrow B_\mu+(1/e)\partial_\mu\beta$, and so is not U(1) deflated.

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