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Theory of two Z bosons

Yasunari Tosa
Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061
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The general theory of two-Z-boson electroweak models is developed, without using gauge theories. All low-energy neutral-current couplings are expressed in terms of the generalized Weinberg angle and other parameters. The sufficient condition for giving the same neutrino-induced neutral-current reactions as the Glashow-Weinberg-Salam (GWS) model does is obtained. The best place to see the deviation from the GWS model is the forward-backward asymmetry in $e^+e^-\rightarrow\mu^+\mu^-$. We find a model based on $SU(2)_L\otimes U(1)_K\otimes U(1)_{B-L}$, which yields exactly the same predictions for charged-current reactions and neutrino-induced neutral-current reactions as the GWS model does and still gives the larger forward-backward asymmetry.

I. INTRODUCTION

The successes of the predictions of the Glashow-Weinberg-Salam (GWS) electroweak model[1] in neutrino-induced neutral-current reactions and the qualitative agreements in electroweak interferences in $e^+e^-\rightarrow\mu^+\mu^-$ and eH reactions may lead to the question why such a simple theory holds in Nature. The group is *not* semisimple because of the ugly U(1) factor. The parity is broken by hand explicitly, i.e., SU(2) doublets are all left-handed, while singlets are all right-handed. The symmetry is broken by just using the simplest doublet Higgs scalar(s). The naive extension of the GWS model to grand unification[2] gives us the embarrassing desert where nothing new is expected from 10^2 to 10^{15} GeV.

Perhaps, Nature conceals herself in such a way that at low energies only a simple but prejudiced look can be seen by us. We could find, after some illusions, oases where flowers are blooming in the desert. Thus, we can consider the possibilities of having two or more Z bosons in the electroweak interactions. Our first try would be two-Z-boson theories. We have only a few possibilities, because the rank is just three: $SU(2)\otimes U(1)\otimes U(1)$, $SU(2)\otimes SU(2)\otimes U(1)$, and $SU(3)\otimes U(1)$, if we consider only doublet or triplet quarks and leptons. (Theorems proved by Okubo[3] help us enumerate these possibilities quite a lot.) All of them actually have been proposed.[4] However, analyses have always been plagued by the arbitrary choice of Higgs scalars and are therefore model-dependent. Some progress has been done toward the general neutral-current structures in gauge theories, without going into details of Higgs scalars. Georgi and Weinberg[5] have proved a theorem which shows the existence of expanded gauge theories, without changing the predictions for neutrino-induced neutral currents. In these cases, at least one of the Z bosons must be lighter than the Z of the GWS model. In the framework of $SU(2)_L\otimes SU(2)_R\otimes U(1)_{B-L}$, Li and Marshak[6] have analyzed the most general neutral-current structures and the present author[7] has obtained the general mass relation among W's and Z's.

The desert may be blooming because of the possible existence of the inner structures of quarks and leptons. Some people[8] argue that weak interactions are just the residual parts of binding forces of quarks and leptons. Attempts have been made in this direction of thinking, initially by Bjorken[9], and by Hung and Sakurai.[10]

Considering these facts, we are led to investigate the general theory of two Z bosons, using the information from gauge theories as little as possible. This is the purpose of this paper.

The most general Z-boson theories can be said to be the theories where neutral vector bosons Z_j^0 mix together and produce a photon field A and Z-boson fields Z_j. There are two possible types of mixing scheme: (1) mass mixing and (2) kinetic mixing. The mass mixing occurs because of the nondiagonal terms of the mass matrix, while the kinetic mixing is caused by the nondiagonal terms in the kinetic terms. In the text, *we only consider the mass-mixing theories, since gauge theories yield only these.* Two-Z-boson theories with kinetic mixing are discussed in Appendix A.

In Sec. II we discuss the possible types of boson mass matrix. We introduce the parametrization of the diagonalization matrix of the mass matrix in

Sec. III. In Sec. IV, we express the low-energy neutral-current interactions in terms of the parameters of the theory. Section V is devoted to the discussion of how we obtain the same predictions for neutrino-induced reactions as the GWS model does. We find the conspiracy condition and discuss its implication for gauge theories.

In Appendix A we show that the mass and the kinetic mixings are not compatible in a single-Z-boson theory, if we insist that the predictions of this theory yield exactly the same as the GWS model does. Thus, we have either one of the two types, but no mixture of the two: The mass-mixing case is the GWS model, although it is not necessary to gauge the theory, while the kinetic-mixing case corresponds to that of Hung and Sakurai.[10] However, for theories with two or more Z bosons, we have in general the mixture of two types. Therefore, the general analyses become complex, although specific cases can be done. (For example, Barbieri and Mohapatra[11] have discussed the left-right-symmetric two-Z-boson theory.)

In Appendix B we discuss a model based on $SU(2)_L \otimes U(1)_K \otimes U(1)_{B-L}$ which satisfies the conspiracy condition and gives the larger forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$.

II. CLASSIFICATION OF TWO-Z-BOSON THEORIES

Consider a general mass matrix for photon and two Z bosons, which is real and symmetric. The non-negative eigenvalue property yields[12]

$$M^2 = P^t P\,, \tag{2.1}$$

where P is a real upper-triangle matrix and t denotes the transpose operation. Since one of the eigenvalues of M^2 gives the mass of the photon, we must have

$$\det M^2 = \prod_j p_{jj}{}^2 = 0\,, \tag{2.2}$$

where p_{jj} is the diagonal element of P. Therefore, we have only three possible types of mass matrix M^2:

Case 1, $p_{11}=0$,

$$M^2 = \begin{bmatrix} 0 & 0 & 0 \\ A & C & 0 \\ B & D & E \end{bmatrix} \begin{bmatrix} 0 & A & B \\ 0 & C & D \\ 0 & 0 & E \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & A^2+C^2 & AB+CD \\ 0 & AB+CD & B^2+D^2+E^2 \end{bmatrix}. \tag{2.3}$$

Case 2, $p_{22}=0$,

$$M^2 = \begin{bmatrix} A & 0 & 0 \\ B & 0 & 0 \\ C & D & E \end{bmatrix} \begin{bmatrix} A & B & C \\ 0 & 0 & D \\ 0 & 0 & E \end{bmatrix} = \begin{bmatrix} A^2 & AB & AC \\ AB & B^2 & BC \\ AC & BC & C^2+D^2+E^2 \end{bmatrix}. \tag{2.4}$$

Case 3, $p_{33}=0$,

$$M^2 = \begin{bmatrix} A & 0 & 0 \\ B & D & 0 \\ C & E & 0 \end{bmatrix} \begin{bmatrix} A & B & C \\ 0 & D & E \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} A^2 & AB & AC \\ AB & B^2+D^2 & BC+DE \\ AC & BC+DE & C^2+E^2 \end{bmatrix}. \tag{2.5}$$

The use of gauge theories automatically ensures the type of mass matrices to be one of the three cases above. However, since it is not obvious that similar relations hold in *composite-boson* cases, one must be careful in constructing such models.

It is easy to see that case 1 corresponds to the case where the photon does not mix with other bosons. We show that case 2 corresponds to the case where the photon is made of two neutral bosons. Case 3 is the most general one where the photon is a mixture of three bosons. In terms of the gauge-theory language, the electric-charge operator is given by $Q=f_1(T_1{}^0)$, $Q=f_2(T_1{}^0,T_2{}^0)$, and $Q=f_3(T_1{}^0,T_2{}^0,T_3{}^0)$ for cases 1, 2, and 3, respectively, where T_j^0 denote the neutral generator of some groups corresponding to the Z_j^0 bosons.

Let us denote the diagonalization matrix U, which is real and orthogonal, as

$$U = \begin{bmatrix} x & p & s \\ y & q & t \\ z & r & u \end{bmatrix}, \tag{2.6}$$

by which we obtain the physical photon and two Z bosons as

$$\begin{bmatrix} A \\ Z_1 \\ Z_2 \end{bmatrix} = U^t \begin{bmatrix} Z_1^0 \\ Z_2^0 \\ Z_3^0 \end{bmatrix}, \quad \begin{bmatrix} Z_1^0 \\ Z_2^0 \\ Z_3^0 \end{bmatrix} = U \begin{bmatrix} A \\ Z_1 \\ Z_2 \end{bmatrix}, \tag{2.7}$$

where t denotes the transpose.

Now, we discuss case 2. The 11 component of the mass matrix in terms of the physical basis is given by

$$\widetilde{M}^2{}_{11}=(Ax+By+Cz)^2+(D^2+E^2)z^2$$
$$=0 \qquad (2.8)$$

since we take $\widetilde{M}^2{}_{11}$ as the photon mass. Hence, $z=0$, which yields

$$A=xZ_1^0+yZ_2^0 \qquad (2.9)$$

which shows that the photon is a mixture of two Z's.

III. PARAMETRIZATION

We discuss case 3 only without loss of generality, since we can parametrize the diagonalization matrix U such that it can produce all three cases. The parametrization needs some physics such that we can have the generality of producing all possible two-Z-boson gauge theories, such as SU(2) $\otimes$ U(1) $\otimes$ U(1), or $\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R\otimes\mathrm{U}(1)_{B-L}$, etc. We use three parameters θ, ϕ, ϵ, and express U as

$$\begin{bmatrix} \sin\theta & \cos\theta\cos\phi & \cos\theta\sin\phi \\ \epsilon\sin\theta & -\tan\theta\left[\epsilon\sin\theta\cos\phi+\dfrac{\sin\phi}{\sin\theta}z\right] & \tan\theta\left[-\epsilon\sin\theta\sin\phi+\dfrac{\cos\phi}{\sin\theta}z\right] \\ z & \tan\theta(\epsilon\sin\phi-z\cos\phi) & -\tan\theta(\epsilon\cos\phi+z\sin\phi) \end{bmatrix}, \qquad (3.1)$$

where $z=(\cos^2\theta-\epsilon^2\sin^2\theta)^{1/2}$. The angle θ is the generalized Weinberg angle. The case where $\epsilon=0$ corresponds to case 2, while the case where $\theta=0$ corresponds to case 1. Because the mass matrix has five parameters. A, B, C, D, E, we can express everything in terms of A, B, θ, ϕ, ϵ. For example, two physical Z-boson masses are

$$M_{Z_1}{}^2=\frac{M^2{}_{11}}{\cos^2\theta}\left[1-\frac{1}{z}\left[\epsilon\sin^2\theta+\cos^2\theta\frac{M^2{}_{12}}{M^2{}_{11}}\right]\tan\phi\right],$$
$$M_{Z_2}{}^2=\frac{M^2{}_{11}}{\cos^2\theta}\left[1+\frac{1}{z}\left[\epsilon\sin^2\theta+\cos^2\theta\frac{M^2{}_{12}}{M^2{}_{11}}\right]\cos\phi\right], \qquad (3.2)$$

where

$$M^2{}_{11}=A^2,\ M^2{}_{12}=AB, \qquad (3.3)$$

$$\frac{\sin\theta}{z}=-\frac{C}{A+\epsilon B}=-\frac{E}{\epsilon D}, \qquad (3.4)$$

$$\tan 2\phi=-\frac{(2A/z)(A\epsilon\sin^2\theta+B\cos^2\theta)}{A^2-(1/z^2)(A\epsilon\sin^2\theta+B\cos^2\theta)^2-D^2\cos^4\theta/z^2}. \qquad (3.5)$$

One notices that the limit $\phi\to+0$ corresponds to the standard GWS model, if gauged, because in that case, we have $M^2{}_{11}=g^2\Phi^\dagger T_3{}^2\Phi$ where Φ denotes the (reducible) Higgs scalar(s).

IV. PHYSICAL NEUTRAL-CURRENT COUPLINGS

We assume that the Z_j^0 interact with Dirac fermions as

$$\mathscr{L}=\sum_j Z_{j\mu}^0\bar\psi\gamma^\mu(G_V^j-G_A^j\gamma_5)\psi. \qquad (4.1)$$

Using Eq. (2.7), we can express this in terms of physical photon A and two Z_j. One *strong constraint* is that the photon must couple to fermions with a vector coupling. This constraint yields

$$eQ=xG_V^1+yG_V^2+zG_V^3, \qquad (4.2)$$

$$0=xG_A'+yG_A^2+zG_A^3. \qquad (4.3)$$

Now, we can have the physical neutral-current coupling constants in terms of $G_{V,A}^j$ $(j=1,2)$ and Q:

$$\mathscr{L}=A^\mu\bar\psi\gamma_\mu eQ\psi+\sum Z_j^\mu\bar\psi\gamma_\mu(g_V^j-g_A^j\gamma_s)\psi, \qquad (4.4)$$

where

$$g_V^1=\frac{1}{\cos\theta}\left[\left[\cos\phi-\frac{\sin\phi}{z}\epsilon\sin^2\theta\right](G_V^1-eQ\sin\theta)-\frac{\sin\phi}{z}\cos^2\theta(G_V^2-eQ\epsilon\sin\theta)\right], \tag{4.5}$$

$$g_A^1=\frac{1}{\cos\theta}\left[\left[\cos\phi-\frac{\sin\theta}{z}\epsilon\sin^2\theta\right]G_A^1-\frac{\sin\phi}{z}\cos^2\theta G_A^2\right], \tag{4.6}$$

$$g_V^2=\frac{1}{\cos\theta}\left[\left[\sin\phi+\frac{\cos\phi}{z}\epsilon\sin^2\theta\right](G_V^1-eQ\sin\theta)+\frac{\cos\phi}{z}\cos^2\theta(G_V^2-eQ\epsilon\sin\theta)\right], \tag{4.7}$$

$$g_A^2=\frac{1}{\cos\theta}\left[\left[\sin\phi+\frac{\cos\phi}{z}\epsilon\sin^2\theta\right]G_A^1+\frac{\cos\phi}{z}\cos^2\theta G_A^2\right]. \tag{4.8}$$

The factor $G_V^1-eQ\sin\theta$ is actually equal to the familiar form

$$\tfrac{1}{2}g(T_3-2Q\sin^2\theta),$$

if $G_{V,A}^1$ is gauged as $SU(2)_L$.

Unfortunately, the parameters which can be found by experiments are not g_V^j, g_A^j, but those which characterize the low-energy neutral-current interactions.[13] The low-energy neutral-current interactions are defined as follows:

νH and νe:

$$\mathscr{L}=\frac{G_F}{\sqrt{2}}[\bar{\nu}\gamma^\mu(1-\gamma_5)\nu]J_\mu^H, \tag{4.9}$$

where

$$J_\mu^H=\bar{\psi}\gamma_\mu(g_V-g_A\gamma_5)\psi \tag{4.10}$$

and g_V, g_A are matrices and ψ denotes fermions. We assume the absence of flavor-changing neutral current.

e^- H:

$$\mathscr{L}=\frac{G_F}{\sqrt{2}}[C_1(\bar{e}\gamma^\mu\gamma_5 e)(\bar{\psi}\gamma_\mu\psi)+C_2(\bar{e}\gamma^\mu e)(\bar{\psi}\gamma_\mu\gamma_5\psi)], \tag{4.11}$$

where ψ denotes quarks.

$e^+e^-\to\mu^+\mu^-$:

$$\mathscr{L}=\frac{G_F}{\sqrt{2}}h_{AA}(\bar{e}\gamma^\mu\gamma_5 e+\bar{\mu}\gamma^\mu\gamma_5\mu)(\bar{e}\gamma_\mu\gamma_5 e+\bar{\mu}\gamma_\mu\gamma_5\mu)+\cdots. \tag{4.12}$$

We will express g_V, g_A, C_1, C_2, h_{AA} in terms of our parameters, θ, ϕ, ϵ, A, B. In our theory, the effective neutral-current interactions can be obtained from

$$\mathscr{L}(NC)=\tfrac{1}{2}\sum\frac{1}{M_{Z_j}{}^2}J_\mu^j J^{j\mu}, \tag{4.13}$$

where

$$J_\mu^j=\sum_k\bar{\psi}_k\gamma_\mu(g_V^j-g_A^j\gamma_5)\psi_k \tag{4.14}$$

and k goes over quarks and leptons. For neutrino-induced reactions, we use only the left-handed part of neutrinos. Hence, we get

$$\frac{G_F}{\sqrt{2}}g_V=\sum_j\left[\frac{g_V^j+g_A^j}{2}\right]_\nu\frac{g_V^j}{M_{Z_j}{}^2}, \tag{4.15}$$

$$\frac{G_F}{\sqrt{2}}g_A=\sum_j\left[\frac{g_V^j+g_A^j}{2}\right]_\nu\frac{g_A^j}{M_{Z_j}{}^2}, \tag{4.16}$$

since

$$\bar{\nu}\gamma_\mu(g_V^j-g_A^j\gamma_s)\nu=\bar{\nu}\gamma_\mu\left[\frac{g_V^j+g_A^j}{2}\right](1-\gamma_5)\nu+\bar{\nu}\gamma_\mu\left[\frac{g_V^j-g_A^j}{2}\right](1+\gamma_5)\nu. \tag{4.17}$$

For electron-hadron reactions, we have

$$\frac{G_z}{\sqrt{2}}C_1=\sum_j\frac{1}{M_{Z_j}{}^2}(g_A^j)_e g_V^j, \tag{4.18}$$

$$\frac{G_F}{\sqrt{2}}C_2=\sum_j\frac{1}{M_{Z_j}{}^2}(g_V^j)_e g_A^j. \tag{4.19}$$

For $e^+e^-\to\mu^+\mu^-$, we have

$$\frac{G_7}{\sqrt{2}}h_{AA}=\sum_j\frac{1}{M_{Z_j}{}^2}(g_{Ae}^j)^2. \tag{4.20}$$

The following are the explicit forms of g_V, g_A, C_1, C_2, and h_{AA}:

$$\frac{G_F}{\sqrt{2}}g_V=\frac{1}{\cos^2\theta}(G_V^1-eQ\sin\theta)\left[a_1\left[\alpha-\frac{\gamma^2}{\beta}+\frac{1}{\beta}\psi^2\right]+a_2\frac{\cos^2\theta}{z}\psi\right]+\frac{1}{z}(G_V^2-eQ\epsilon\sin\theta)\left[a_1\psi+a_2\frac{\cos^2\theta}{z}\beta\right], \tag{4.21}$$

$$\frac{G_F}{\sqrt{2}}g_A=\frac{1}{\cos^2\theta}G_A^1\left[a_1\left[\alpha-\frac{\gamma^2}{\beta}+\frac{1}{\beta}\psi^2\right]+a_2\frac{\cos^2\theta}{z}\psi\right]+\frac{1}{z}G_A^2\left[a_1\psi+a_2\frac{\cos^2\theta}{z}\beta\right], \tag{4.22}$$

$$\frac{G_F}{\sqrt{2}}C_1=\frac{1}{\cos^2\theta}(G_V^1-eQ\sin\theta)\left[G_{Ae}^1\left[\alpha-\frac{\gamma^2}{\beta}+\frac{1}{\beta}\psi^2\right]+G_{Ae}^2\frac{\cos^2\theta}{z}\psi\right]$$
$$+\frac{1}{z}(G_V^2-eQ\epsilon\sin\theta)\left[G_{Ae}^1\psi+G_{Ae}^2\frac{\cos^2\theta}{z}\beta\right], \tag{4.23}$$

$$\frac{G_F}{\sqrt{2}}C_2=\frac{1}{\cos^2\theta}G_A^1\left[(G_{Ve}^1-eQ_e\sin\theta)\left[\alpha-\frac{\gamma^2}{\beta}+\frac{1}{\beta}\psi^2\right]+(G_{Ve}^2-eQ_e\epsilon\sin\theta)\frac{\cos^2\theta}{z}\psi\right]$$
$$+\frac{1}{z}G_A^2\left[(G_{Ve}^1-eQ_e\sin\theta)\psi+(G_{Ve}^2-eQ_e\epsilon\sin\theta)\frac{\cos^2\theta}{z}\beta\right], \tag{4.24}$$

$$\frac{G_F}{\sqrt{2}}h_{AA}=\frac{1}{\cos^2\theta}\left[(G_{Ae}^1)^2\left[\alpha-\frac{\gamma^2}{\beta}+\frac{1}{\beta}\psi^2\right]+2G_{Ae}^1G_{Ae}^2\frac{\cos^2\theta}{z}\psi+(G_{Ae}^2)^2\left[\frac{\cos^2\theta}{z}\right]^2\beta\right], \tag{4.25}$$

where

$$a_j=\tfrac{1}{2}(G_V^j+G_A^j)_\nu\,, \tag{4.26}$$

$$\psi=-\gamma+\frac{\epsilon}{z}\sin^2\theta\beta\,, \tag{4.27}$$

$$\alpha=\frac{\cos^2\phi}{{M_{Z_1}}^2}+\frac{\sin^2\phi}{{M_{Z_2}}^2}\,,\quad \beta=\frac{\sin^2\phi}{{M_{Z_1}}^2}+\frac{\cos^2\phi}{{M_{Z_2}}^2}\,,$$

$$\gamma=\cos\phi\sin\phi\left[\frac{1}{{M_{Z_1}}^2}-\frac{1}{{M_{Z_2}}^2}\right]. \tag{4.28}$$

So far, we have not used any information from gauge theories. We list the conversion table for gauge theories as follows:

(1) The Glashow-Weinberg-Salam model:

$$G_V^2=G_A^2=0,\quad G_V^1=G_A^1=\tfrac{1}{2}gT_3\,,$$

$$a_1=g/2,\quad a_2=0$$

$$\alpha=1/M_Z{}^2,\quad \beta=\gamma=0\,.$$

(2) The left-right-symmetric model, $SU(2)_L\otimes SU(2)_R\otimes U(1)_{B-L}$:

$$G_V^1=G_A^1=\tfrac{1}{2}gT_3\,,$$

$$G_V^2=-G_A^2=\tfrac{1}{2}gT_3\,,$$

$$a_1=g/2,\quad a_2=0\,,$$

where

$$\bar{q}T_3q=\tfrac{1}{2}(\bar{u}u-\bar{d}d)+\tfrac{1}{2}(\bar{c}c-\bar{s}s)+\cdots\,,$$

$$\bar{l}T_3l=\tfrac{1}{2}(\bar{\nu}_e\nu_e-\bar{e}e)+\tfrac{1}{2}(\bar{\nu}_\mu\nu_\mu-\bar{\mu}\mu)+\cdots$$

and q and l denote quarks and leptons.

It is easy to establish such relations for other models.

V. CONSPIRACY MODEL

Our formulas for g_V, g_A, C_1, C_2, h_{AA} are so complicated that they appear to be of no use. However, one can draw rather general conclusions, *provided* that two-Z-boson theories have exactly the same neutral-current (NC) structures in neutrino-induced reactions as the GWS model does. This requirement can be satisfied if

$$G_0\frac{M_0}{\cos^2\theta_W}=G_V^1\frac{M}{\cos^2\theta}+G_V^2\frac{N}{z}\,, \tag{5.1}$$

$$G_0\frac{M_0}{\cos^2\theta_W}=G_A^1\frac{M}{\cos^2\theta}+G_A^2\frac{N}{z}\,, \tag{5.2}$$

$$\frac{\sin\theta_W}{\cos^2\theta_W}M_0=\frac{\sin\theta}{\cos^2\theta}M+\frac{\epsilon\sin\theta}{z}N\,, \tag{5.3}$$

where

$$G_0=\tfrac{1}{2}g_{\rm GWS}T_3\ ,$$
$$M_0=\tfrac{1}{2}g_{\rm GWS}\frac{1}{M_Z{}^2}\ ,$$
$$M=a_1\left[\alpha-\frac{\gamma^2}{\beta}+\frac{1}{\beta}\psi^2\right]+a_2\frac{\cos^2\theta}{z}\psi\ , \tag{5.4}$$
$$N=a_1\psi+a_2\frac{\cos^2\theta}{z}\beta\ ,$$

and M_Z is the Z mass of the GWS model. From Eqs. (5.1) and (5.2), we have *either*

$$G_V^2-G_A^2=0,\ \text{i.e., } (V-A) \text{ or vanishing } G_V^2, G_A^2 \tag{5.5}$$

or

$$N=a_1\psi+a_2\frac{\cos^2\theta}{z}\beta=0 \tag{5.6}$$

if $G_V^1-G_A^1=0$, i.e., $(V-A)$ type. We assume hereafter that G_V^1 and G_A^1 are exactly the same as in the GWS model. This assumption is trivial if the theory is gauged as $G=\mathrm{SU}(2)_L\otimes\cdots$ with its electric-charge operator Q as $Q=T_{3L}+\cdots$. We assume furthermore that Eq. (5.5) is not satisfied. Then, Eq. (5.6) leads to

$$\alpha-\frac{\gamma^2}{\beta}=\frac{1}{M_Z{}^2}\ . \tag{5.7}$$

The case where $G_V^2=G_A^2=0$ for ordinary quarks and leptons [i.e., Eq. (5.5) is satisfied] has been discussed by Barger *et al.*[14] The condition Eq. (5.6) is sufficient for having exactly the same NC reactions for neutrino-induced reactions, but not necessary, as can be seen from the argument above. The condition Eq. (5.6) can be simplified, using Eqs. (3.2) and (4.28):

$$a_1B=a_2A \tag{5.8}$$

since

$$\beta=\frac{1}{M_{Z_1}{}^2M_{Z_2}{}^2}\frac{A^2}{\cos^2\theta}\ ,$$
$$\gamma=\frac{1}{M_{Z_1}{}^2M_{Z_2}{}^2}\frac{A^2}{\cos^2\theta}\frac{1}{z}\left[\epsilon\sin^2\theta+\cos^2\theta\frac{B}{A}\right]\ .$$

The implication of Eq. (5.8) in gauge theories is the following: Using

$$A^2=g_1^2\Phi^\dagger(T^{(1)})^2\Phi\ ,$$
$$AB=g_1g_2\Phi^\dagger T^{(1)}T^{(2)}\Phi\ ,$$
$$a_1=g_1p_1,\ \ a_2=g_2p_2\ ,$$

we get

$$p_1\Phi^\dagger T^{(1)}T^{(2)}\Phi=p_2\Phi^\dagger(T^{(1)})^2\Phi\ , \tag{5.9}$$

where g_j and $T^{(j)}$ are the corresponding coupling constant and the generator for the Z_j^0. The p_j are the strength of the left-handed neutrino coupled to the Z_j^0 boson. [For example, if $G=G_1\otimes G_2\otimes\mathrm{U}(1)$ and $G_1=\mathrm{SU}(2)_L$, then $p_1=\frac{1}{2}$.] We call Eq. (5.9) *the conspiracy condition.* The simplest way to satisfy the conspiracy condition is

$$\Phi^\dagger T^{(1)}T^{(2)}\Phi=0 \text{ and } p_2=0\ . \tag{5.10}$$

Georgi and Weinberg[5] achieve this by doing the following: The gauge symmetry $G=G_1\otimes G_2\otimes\mathrm{U}(1)$ is broken by two Higgs scalars ϕ_1 and ϕ_2. The ϕ_1 transforms nontrivially under G_1 and U(1) but is neutral under G_2, while the ϕ_2 transforms nontrivially under G_2 and U(1) but is neutral under G_1. Thus, we have $\Phi^\dagger T^{(1)}T^{(2)}\Phi=0$ for $\Phi=\langle\phi_1\rangle+\langle\phi_2\rangle$. Furthermore, if neutrinos are neutral under G_2, then we get $p_2=0$.

Thus, our condition for obtaining the same neutrino-induced neutral-current reactions as the GWS does is more general than that of Georgi and Weinberg.[5]

Now, neutral-current coupling constants at low energies become very much simplified as follows:

$$\frac{G_F}{\sqrt{2}}g_V=\frac{1}{\cos^2\theta}(G_V^1-eQ\sin\theta)a_1\tilde{\alpha}\ , \tag{5.11}$$

$$\frac{G_F}{\sqrt{2}}g_A=\frac{1}{\cos^2\theta}G_A^1a_1\tilde{\alpha}\ , \tag{5.12}$$

$$\frac{G_F}{\sqrt{2}}C_1=\frac{1}{\cos^2\theta}(G_V^1-eQ\sin\theta)\left[G_{Ae}^1\tilde{\alpha}+\beta\left[\frac{\cos^2\theta}{z}\right]^2\frac{a_2}{a_1}Y\right]-\frac{1}{z}(G_V^2-eQ\epsilon\sin\theta)\left[\frac{\cos^2\theta}{z}\beta\right]Y\ , \tag{5.13}$$

$$\frac{G_F}{\sqrt{2}}C_2=\frac{1}{\cos^2\theta}G_A^1\left\{(G_{Ve}^1-eQ_e\sin\theta)\tilde{\alpha}+\beta\left[\frac{\cos^2\theta}{z}\right]^2\frac{a_2}{a_1}\left[(G_{Ve}^1-eQ_e\sin\theta)\frac{a_2}{a_1}-(G_{Ve}^2-eQ_e\epsilon\sin\theta)\right]\right\}$$

$$-\frac{1}{z}G_A^2\frac{\cos^2\theta}{z}\beta\left[(G_{Ve}^1-eQ_e\sin\theta)\frac{a_2}{a_1}-(G_{Ve}^2-eQ_e\epsilon\sin\theta)\right], \tag{5.14}$$

$$\frac{G_F}{\sqrt{2}}h_{AA}=\frac{1}{\cos^2\theta}\left[(G_{Ae}^1)^2\tilde{\alpha}+\left[\frac{\cos^2\theta}{z}\right]^2\beta Y^2\right], \tag{5.15}$$

where

$$\tilde{\alpha}=\frac{1}{M_Z{}^2}=\alpha-\frac{\gamma^2}{\beta},\quad \beta=\frac{M_Z{}^2}{M_{Z_1}{}^2M_{Z_2}{}^2},$$

$$Y=G_{Ae}^1\frac{a_2}{a_1}-G_{Ae}^2,$$

$$e=g\sin\theta,\quad G_V^1=G_A^1=\tfrac{1}{2}gT3,$$

using Eqs. (4.28), (5.6), and (5.8).

The Z-boson masses have relations

$$M_{Z_1}\le M_Z,\qquad M_{Z_2}\ge M_Z, \tag{5.16}$$

which are derived from Eqs. (4.28) and (5.8), i.e.,

$$\frac{1}{M_{Z_1}{}^2}\frac{1}{M_{Z_2}{}^2}=\frac{1}{M_Z{}^2}\left[\frac{1}{M_{Z_2}{}^2}+\left[\frac{1}{M_{Z_1}{}^2}-\frac{1}{M_{Z_2}{}^2}\right]\sin^2\phi\right]. \tag{5.17}$$

Using Eq. (5.17), we can express the heavier-Z-boson mass in terms of the lighter-Z-boson mass and the mixing angle ϕ:

$$M_{Z_2}{}^2=M_{Z_1}{}^2\left[1+\frac{1}{\sin^2\phi}\left[\frac{1}{M_{Z_2}{}^2}-1\right]\right]. \tag{5.18}$$

Note that *relations Eqs. (5.11)–(5.18) hold for any two-Z-boson theory, if the conspiracy condition is satisfied.* For another type of two-Z theory which predicts $M_{Z_1}, M_{Z_2}\ge M_Z$, see Barger *et al.*[14]

As can be seen from Eq. (5.15), the best way to distinguish these two-Z-boson theories from the GWS model before producing Z bosons is to measure the precise value of h_{AA} in $e^+e^-\to\mu^+\mu^-$ (Ref. 15), as long as $Y\neq 0$, i.e.,

$$2G_{Ae}^2\neq-(G_V^2+G_A^2)_\nu,$$

which implies that ν and/or e must couple with Z_2^0. Since the GWS term $(G_{Ae}^1)^2\tilde{\alpha}$ and the deviation term $(\cos^2\theta/z)^2\beta Y^2$ are of the same order (if $Y\sim G_{Ae}^1$), we can expect a forward-backward asymmetry for two-Z-boson theories twice as large as that of the GWS model. Furthermore, this effect is enhanced by the propagator of the lighter Z boson $M_{Z_1}{}^2/(s-M_{Z_1}{}^2)$ (>1). In Appendix B, we give one gauge-theory model which illustrates the point.

The other possible way of getting the larger asymmetry would be the case where neutrinos are Majorana particles and ν_L and ν_R mix together with an angle δ. The reason is that neutrino-induced reactions now have the factor $\cos^2\delta$, while e^+e^- and eH reactions do not have this factor. Since we fix the parameters in neutrino-induced reactions, e^+e^- and eH reactions are renormalized by the amount $1/\cos^2\delta$. The trouble with this idea is that the Cabibbo universality requires a small mixing angle δ.[16]

Here, we examined the question of whether it is possible to obtain different NC predictions for e^+e^- and e^-H reactions, even though neutrino-induced reactions are exactly the same as in the GWS model. In the gauge-theory framework, Barger, Ma, and Whisnant[17] have examined the same question under the assumption that quarks and leptons transform only under $SU(2)_L\otimes U(1)$ out of $SU(2)_L\otimes U(1)\otimes G$. (We have not used this assumption.) However, because of this assumption, the NC structure is modified by only $J_{EM}{}^2$. Hence, their theory *cannot* predict larger asymmetries for $e^+e^-\to\mu^+\mu^-$, contrary to our predictions.

It is amusing to note that if ν and e do not couple with Z_2^0, then we have exactly the same low-energy neutral-current reactions as in the GWS model, and still have two Z bosons.

In the text, we have not concerned ourselves about the charged-current reactions. The reason is that the connection between charged- and neutral-current reactions is established by the content of Higgs scalars in gauge theories and we do not want to go into the details of Higgs scalars. (As shown in Appendix A, it is possible to find the relation in the case of the kinetic mixing.)

We have found the general neutral-current structures for two-Z-boson theories, *without* using gauge principles. They are given in Eqs. (4.21)–(4.25). The masses of Z bosons are given in Eq. (3.2). We have found the generalized conspiracy condition for obtaining the same neutrino-induced reactions as the GWS model does in Eq. (5.5).

Notes added

The model discussed in Appendix B needs one more Higgs scalar whose quantum numbers are $(\frac{1}{2}, \frac{1}{2}, 0)$ for $(\mathrm{SU}(2)_L, \mathrm{U}(1)_K, \mathrm{U}(1)_{B-L})$, in order to make fermions massive at the tree level. The vacuum expectation value of this Higgs scalar is assumed to be very small. Thus, our conclusion is essentially unchanged.

After the completion of this paper, we noticed papers which use the gauge group $\mathrm{SU}(2)_L \otimes \mathrm{U}(1) \otimes \mathrm{U}(1)$. They are N.G. Deshpande and D. Iskandar, Phys. Lett. 87B, 393 (1979); Nucl. Phys. B167, 223 (1980); S. Rajpoot, Phys. Lett. 108B, 303 (1982); R. W. Robinett and J. L. Rosner, Phys. Rev. D 25, 3036 (1982); M. K. Parida and A. Raychaudhuri, *ibid.* (to be published); G. Fogleman and T. G. Rizzo, Phys. Lett. 113B, 240 (1982); N. G. Deshpande and R. J. Johnson, University of Oregon Report No. OITS-188, 1982 (unpublished).

The bounds on boson masses in the general $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L}$ have been obtained by V. Barger, E. Ma, and K. Whisnant, University of Hawaii Report No. UH-511-462-82, 1982 (unpublished).

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APPENDIX A

Here, we develop single- and double-Z-boson theories with both mass and kinetic mixings. The method is due to Hung and Sakurai.[10] The starting Lagrangian is

$$\mathscr{L} = -\tfrac{1}{4}\Phi_{\mu\nu}^T K \Phi_{\mu\nu} - \tfrac{1}{2}\Phi_\mu^T M^2 \Phi_r + \Phi_\mu^T \Psi_\mu , \quad \text{(A1)}$$

where K and (non-negative) M^2 are real and symmetric matrices and

$$\begin{aligned} \Phi_{\mu\nu}^T &= (F_{\mu\nu}^0, F_{\mu\nu}^1, F_{\mu\nu}^2, \dots) , \\ \Phi_\mu^T &= (Z_\mu^0, Z_\mu^1, Z_\mu^2, \dots) , \\ \Psi_\mu^T &= (g_0 J_\mu^0, g_1 J_\mu^1, g_2 J_\mu^2, \dots) . \end{aligned} \quad \text{(A2)}$$

The inverse propagator $\Delta^{-1}(q^2)$ can be written as

$$\Delta^{-1}(q^2) = q^2 K + M^2 . \quad \text{(A3)}$$

The physical vector-boson masses can be found from

$$\det \Delta^{-1}(q^2)\,|_{q^2=-M_i{}^2} = 0 . \quad \text{(A4)}$$

Since we must have the photon, i.e., the pole at $q^2=0$, we have

$$\det M^2 = 0 . \quad \text{(A5)}$$

Hence, the form of M^2 is fixed as discussed in Sec. II.

1. Single-Z-boson theories

In this case, we have

$$K = \begin{bmatrix} 1 & \lambda \\ \lambda & 1 \end{bmatrix}, \quad M^2 = \begin{bmatrix} A^2 & AB \\ AB & B^2 \end{bmatrix} . \quad \text{(A6)}$$

The Z mass is given by

$$M_Z{}^2 = \frac{1}{1-\lambda^2}(A^2 + B^2 - 2\lambda AB) . \quad \text{(A7)}$$

The effective current-current interactions are given by

$$\tfrac{1}{2}\Psi_\mu^T \Delta(q^2) \Psi_\mu . \quad \text{(A8)}$$

Using

$$\Delta(q^2) = \frac{1}{1-\lambda^2}\frac{1}{M_Z^2} \begin{bmatrix} \dfrac{B^2}{q^2} + \dfrac{M_Z{}^2 - B^2}{q^2 + M_Z{}^2} & -\left[\dfrac{AB}{q^2} + \dfrac{\lambda M_Z{}^2 - AB}{q^2 + M_Z{}^2}\right] \\ -\left[\dfrac{AB}{q^2} + \dfrac{\lambda M_Z{}^2 - AB}{q^2 + M_Z{}^2}\right] & \dfrac{A^2}{q^2} + \dfrac{M_Z{}^2 - A^2}{q^2 + M_Z{}^2} \end{bmatrix} . \quad \text{(A9)}$$

Therefore, for the case where

$$(\lambda A - B)(A - \lambda B) > 0 \text{ and } 1-\lambda^2 > 0 ,$$

we have the interactions as

$$\mathscr{L}(\text{int}) = \frac{1}{2}\frac{1}{1-\lambda^2}\frac{1}{M_Z{}^2}\left[\frac{1}{q^2}(g_0 \,|B|\, J_0 - g_1 \,|A|\, J_1)^2 + \frac{1}{q^2+M_Z{}^2}[g_0(M_Z{}^2-B^2)^{1/2}J_0 - g_1(M_Z{}^2-A^2)^{1/2}J_1]^2\right] , \tag{A10}$$

since

$$(M_Z{}^2-A^2)^{1/2}(M_Z-B^2)^{1/2} = |\,\lambda M_Z{}^2 - AB\,| ,$$
$$\lambda M_Z{}^2 - AB = (\lambda A - B)(A - \lambda B)/(1-\lambda^2) ,$$
$$M_Z{}^2 - A^2 = (\lambda A - B)^2/(1-\lambda^2) \geq 0 ,$$
$$M_Z{}^2 - B^2 = (A - \lambda B)^2/(1-\lambda^2) \geq 0 . \tag{A11}$$

Since the interaction terms must be of the form

$$\mathscr{L}(\text{int}) = \frac{1}{2}\frac{e^2}{q^2}J_{\text{EM}}{}^2 + \frac{1}{2}\frac{g_{\text{NC}}{}^2}{q^2+M_Z{}^2}J_{\text{NC}}{}^2 , \tag{A12}$$

we identify

$$\pm e J_{\text{EM}} = \left[\frac{1}{(1-\lambda^2)M_Z{}^2}\right]^{1/2}(g_0 \,|B|\, J_0 - g_1 \,|A|\, J_1) , \tag{A13}$$

$$\pm g_{\text{NC}} J_{NC} = \left[\frac{1}{(1-\lambda^2)M_Z{}^2}\right]^{1/2} \times (g_0 \,|A - \lambda B|\, J_0 - g_1 \,|\lambda A - B|\, J_1) .$$

Obviously, there are two ways of getting exactly the same predictions as the GWS model does:

Case 1: $J_1 = J_{3L}$, $J_0 = J_Y$,

$$e = -g_0\left[\frac{B^2}{(1-\lambda^2)M_Z{}^2}\right]^{1/2} = g_1\left[\frac{A^2}{(1-\lambda^2)M_Z{}^2}\right]^{1/2} .$$

Case 2: $J_0 = J_{\text{EM}}$, and $A = 0$.

In case 1, in order to have

$$1/e^2 = 1/g_0{}^2 + 1/g_1{}^2 , \tag{A14}$$

we must have

$$\lambda = 0 . \tag{A15}$$

One must note, then, that the resultant neutral-current interactions agree with those of the GWS.

In case 2, we automatically have

$$g_0 = e \tag{A16}$$

since $A = 0$. The low-energy neutral-current interactions are given by

$$\mathscr{L}(\text{NC}) = \frac{1}{2}\frac{g_1{}^2}{B^2}\left[J_1 - \frac{e}{g_1}\lambda J_{\text{EM}}\right]^2 .$$

Thus, we have the celebrated relation $\rho = 1$, if $B^2 = M_W{}^2$, which is natural.

We have shown that in order to have the same predictions as the GWS model does, we have two possible choices: (1) $\lambda = 0$; the mass mixing or (2) $A = 0$; the kinetic mixing. The two choices are incompatible.

2. Two-Z-boson theories

Now, we discuss two-Z-boson theories where

$$K = \begin{bmatrix} 1 & \lambda & \delta \\ \lambda & 1 & \omega \\ \delta & \omega & 1 \end{bmatrix} , \qquad M^2 = \begin{bmatrix} A^2 & AB & AC \\ AB & B^2+D^2 & BC+DE \\ AC & BC+DE & C^2+E^2 \end{bmatrix} . \tag{A17}$$

As usual, the masses of Z bosons can be found from $\det\Delta^{-1}(q^2)$:

$$\begin{aligned} \det\Delta^{-1}(q^2) = {} & (1+2\lambda\delta\omega-\lambda^2-\omega^2-\delta^2)q^6 \\ & + [(1-\omega^2)A^2 + (1-\delta^2)(B^2+D^2) + (1-\lambda^2)(C^2+E^2) \\ & \quad -2(\delta\omega-\lambda)AB + 2(\omega\lambda-\delta)AC + 2(\lambda\delta-\omega)(BC+DE)]q^4 \\ & + [A^2(D^2+E^2-2\omega DE) + (BE-CD)^2 + (BE-CD)(-2\lambda AE + 2\delta AD)]q^2 \\ = {} & (1+2\lambda\delta\omega-\lambda^2-\delta^2-\omega^2)q^2(q^2+M_{Z_1}{}^2)(q^2+M_{Z_2}{}^2) . \end{aligned} \tag{A18}$$

The effective current-current interactions are given by

$$\frac{1}{2}\Psi_\mu^T \Delta(q^2)\Psi_\mu \ .$$

We prove that *the mass and kinetic mixings are compatible* by showing that $J_0 = J_{\rm EM}$ does not lead to the vanishing of the off-diagonal terms of the mass matrix. Note that in a single-Z-boson theory $J_0 = J_{\rm EM}$ yields the vanishing of the off-diagonal terms plus the vanishing photon mass term.

Using the identity

$$\frac{\alpha q^4+\beta q^2+\gamma}{q^2(q^2+M_{Z_1}{}^2)(q^2+M_{Z_2}{}^2)} = \frac{\gamma}{M_{Z_1}{}^2 M_{Z_2}{}^2}\frac{1}{q^2} + \frac{\beta-\alpha M_{Z_1}{}^2-\dfrac{\gamma}{M_{Z_1}{}^2}}{M_{Z_2}{}^2-M_{Z_1}{}^2}\frac{1}{q^2+M_{Z_1}{}^2} - \frac{\beta-\alpha M_{Z_2}{}^2-\dfrac{\gamma}{M_{Z_2}{}^2}}{M_{Z_2}{}^2-M_{Z_1}{}^2}\frac{1}{q^2+M_{Z_2}{}^2} \ , \qquad (A19)$$

one can see that it is enough to find the q^2-independent part of the cofactor of $\Delta^{-1}(q^2)$, in order to get the electromagnetic current-current interactions. After some calculation, we find the electromagnetic interactions are given by

$$\frac{1}{2}\frac{1}{\kappa M_{Z_1}{}^2 M_{Z_2}{}^2}\frac{1}{q^2}[g_0{}^2(BE-CD)^2 J_0{}^2 + g_1{}^2 A^2 E^2 J_1{}^2 + g_2{}^2 A^2 D^2 J_2{}^2 + \cdots] \ , \qquad (A20)$$

where

$$\kappa = 1+2\lambda\delta\omega-\lambda^2-\delta^2-\omega^2 \ .$$

Equation (A20) should factorize into

$$\frac{1}{2}\frac{1}{\kappa M_{Z_1}{}^2 M_{Z_2}{}^2}\frac{1}{q^2}(g_0\,|\,BE-CD\,|\,J_0 - g_1\,|\,AE\,|\,J_1 + g_2\,|\,AD\,|\,J_2)^2 \ . \qquad (A21)$$

As mentioned before, we identify $g_0 J_0$ as $eJ_{\rm EM}$.

Then, we must have

$$AE = AD = 0 \qquad (A22)$$

which yields

$$\kappa M_{Z_1}{}^2 M_{\mu_{Z_2}}{}^2 = (BE-CD)^2$$

and the correct electromagnetic interactions.

Note that the condition Eq. (A22) does not necessarily lead to the vanishing of the nondiagonal terms of the mass matrix. Barbieri and Mohapatra[11] have failed to observe this, since they have worked under the condition that $\omega=0$, $B=0$, $AC=DE$, $C\neq 0$, $D\neq 0$.

Having seen that the mass and kinetic mixings are compatible, we are forced to analyze two-Z-boson theories with eight parameters λ,δ,ω, A,B,C,D,E. Because of the complexity, we will be satisfied here to discuss the kinetic-mixing case, since the mass-mixing case is dealt in the text.

3. Kinetic-mixing two-Z-boson theories

In this case, things simplify quite a lot, compared with the mass-mixing case. We have

$$K = \begin{bmatrix} 1 & \lambda & \delta \\ \lambda & 1 & \omega \\ \delta & \omega & 1 \end{bmatrix}, \quad M^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & m_1{}^2 & 0 \\ 0 & 0 & m_2{}^2 \end{bmatrix} \qquad (A23)$$

and

$$\det\Delta^{-1}(q^2) = \kappa q^2 \left\{ q^4 + \frac{1}{\kappa}[(1-\delta^2)m_1{}^2 + (1-\lambda^2)m_2{}^2]q^2 + \frac{1}{\kappa}m_1{}^2 m_2{}^2 \right\}$$

$$= \kappa q^2 (q^2+M_{Z_1}{}^2)(q^2+M_{Z_2}{}^2) \ , \qquad (A24)$$

where

$$\kappa M_{Z_1}{}^2 M_{Z_2}{}^2 = m_1{}^2 m_2{}^2, \quad \kappa(M_{Z_1}{}^2+M_{Z_2}{}^2) = (1-\delta^2)m_1{}^2 + (1-\lambda^2)m_2{}^2 \ . \qquad (A25)$$

The effective current-current interactions are given by $\Delta(q^2)$:

$$\frac{1}{\det\Delta^{-1}(q^2)}\begin{bmatrix}(1-\omega^2)q^4+(m_1{}^2+m_2{}^2)q^2+m_1{}^2m_2{}^2 & (\delta\omega-\lambda)q^4-\lambda m_2{}^2q^2 & (\lambda\omega-\delta)q^4-\delta m_1{}^2q^2\\ (\delta\omega-\lambda)q^4-\lambda m_2{}^2q^2 & (1-\delta^2)q^4+m_2{}^2q^2 & (\lambda\delta-\omega)q^4\\ (\lambda\omega-\delta)q^4-\delta m_1{}^2q^2 & (\lambda\delta-\omega)q^4 & (1-\lambda^2)q^4+m_1{}^2q^2\end{bmatrix}. \tag{A26}$$

Although the exact interactions can be given, we give the low-energy limit of two-Z-boson interactions. That is, we separate $\Delta(q^2)$ into

$$\Delta^{\text{photon}}(q^2)+\Delta^Z(q^2)$$

and take the limit $q^2\to 0$ for $\Delta^Z(q^2)$. Using the formula Eq. (A19), we have

$$\frac{\alpha q^4+\beta q^2+\gamma}{\kappa q^2(q^2+M_{Z_1}{}^2)(q^2+M_{Z_2}^2)}\to\frac{\gamma}{\kappa M_{Z_1}{}^2M_{Z_2}{}^2}\frac{1}{q^2}+\frac{1}{\kappa M_{Z_1}{}^2M_{Z_2}{}^2}\left[\beta-\gamma\frac{M_{Z_1}{}^2+M_{Z_2}{}^2}{M_{Z_1}{}^2M_{Z_2}{}^2}\right]. \tag{A27}$$

Hence,

$$\Delta^{\text{photon}}(q^2)=\frac{1}{q^2}\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix},$$

which implies $J_0=J_{\text{EM}}$ and

$$\Delta^Z(0)=\frac{1}{m_1{}^2m_2{}^2}\begin{bmatrix}\delta^2m_1{}^2+\lambda^2m_2{}^2 & -\lambda m_2{}^2 & -\delta m_1{}^2\\ -\lambda m_2{}^2 & m_2{}^2 & 0\\ -\delta m_1{}^2 & 0 & m_1{}^2\end{bmatrix}, \tag{A28}$$

which is ω independent. The low-energy neutral-current interactions are

$$\mathscr{L}(\text{NC})=\frac{1}{2}\frac{g_2{}^2}{m_1{}^2}\left[J_1-\frac{e}{g_1}\lambda J_{\text{EM}}\right]^2+\frac{1}{2}\frac{g_2{}^2}{m_2{}^2}\left[J_2-\frac{e}{g_2}\delta J_{\text{EM}}\right]^2. \tag{A29}$$

Note that even with the mixing terms proportional to ω, the low-energy interactions are factorized and we have the generalized relation $\rho_i=1$ ($i=1,2$), if the corresponding charged-current interactions are characterized by $m_i{}^2$ ($i=1,2$). Also, note that we *cannot* have the same conspiracy as in the case of the mass mixing, since in this case

$$\frac{G_F}{\sqrt{2}}g_V=\frac{1}{m_1{}^2}a_1(G_V^1-\lambda eQ)+\frac{1}{m_2{}^2}a_2(G_V^2-\delta eQ). \tag{A30}$$

Compare Eq. (A30) with (4.21).

APPENDIX B

We discuss one gauge-theory model which yields exactly the same charged-current reactions and neutrino-induced reactions as the GWS model does and still gives the larger forward-backward asymmetry.

The mobel is based on the group[18]

$$SU(2)_L\otimes U(1)_K\otimes U(1)_{B-L}, \tag{B1}$$

where the electric-charge operator is given by

$$Q=T_{3L}+K+\tfrac{1}{2}(B-L). \tag{B2}$$

The form of the charge operator yields

$$e=g_1\sin\theta=g_2\epsilon\sin\theta=g_3z. \tag{B3}$$

Hereafter, we assume

$$g_1=g_2\equiv g,\ \text{i.e.,}\ \epsilon=1. \tag{B4}$$

By examining the charges of quarks and leptons, we see

$$K_L=0\ \text{and}\ K_R=T_3. \tag{B5}$$

We have the *effective* T_3 *symmetry* in the right-handed sector. We assume that ν is *left-handed*, i.e.,

$$K_\nu=0,\ \text{i.e.,}\ p_2=0. \tag{B6}$$

We take two Higgs scalars ϕ_1 and ϕ_2, whose quantum numbers are

$$\phi_1=(2,0,\tfrac{1}{2})\ \text{and}\ \phi_2=\left[0,\frac{\omega}{2\alpha},-\frac{\omega}{2\alpha}\right]. \tag{B7}$$

The numbers in parentheses are the $SU(2)_L$, K, $\frac{1}{2}(B-L)$ quantum numbers, respectively. Therefore, the conspiracy condition is satisfied.

The vacuum expectation values for ϕ_1 and ϕ_2 are denoted by

$$\langle\phi_1\rangle=\begin{bmatrix}0\\ v\end{bmatrix},\quad \langle\phi_2\rangle=\alpha\ .$$

Hence, the ϕ_1 gives the mass of the W in the same way as in the GWS model. The mass matrix for the neutral bosons is given by

$$M^2=\frac{1}{4}\begin{bmatrix} g^2v^2 & 0 & -gg_3v^2 \\ 0 & g^2w^2 & -gg_3w^2 \\ -gg_3v^2 & -gg_3w^2 & g_3{}^2(v^2+w^2)\end{bmatrix} \tag{B8}$$

which gives

$$\begin{aligned} &A=gv,\quad B=0,\quad C=-g_3v\ ,\\ &D=gw,\quad E=-g_3w \end{aligned} \tag{B9}$$

in the notation of Eq. (2.5). Thus, Eq. (B3) is consistent with Eq. (3.4).

The effective low-energy neutral-current parameters are

$$\begin{aligned} \frac{G_F}{\sqrt{2}}C_1&=\left[\frac{G_F}{\sqrt{2}}C_1\right]_{\text{GWS}}\left[1-\frac{\cos^4\theta}{z^2}\frac{M_Z{}^4}{M_{Z_1}{}^2M_{Z_2}{}^2}\right],\\ \frac{G_F}{\sqrt{2}}C_2&=\left[\frac{G_F}{\sqrt{2}}C_2\right]_{\text{GWS}}\left[1-\frac{\cos^4\theta}{z^2}\frac{M_Z{}^4}{M_{Z_1}{}^2M_{Z_2}{}^2}\right],\\ \frac{G_F}{\sqrt{2}}h_{AA}&=\left[\frac{G_F}{\sqrt{2}}h_{AA}\right]_{\text{GWS}}\left[1+\frac{\cos^4\theta}{z^2}\frac{M_Z{}^4}{M_{Z_1}{}^2M_{Z_2}{}^2}\right], \end{aligned} \tag{B10}$$

using Eqs. (5.9)–(5.11), since for Dirac fermions,

$$\begin{aligned} &G_V^1=\tfrac{1}{2}gT_3,\quad G_A^1=\tfrac{1}{2}gT_3\ ,\\ &G_V^2=\tfrac{1}{2}gT_3,\quad G_A^2=-\tfrac{1}{2}gT_3\ . \end{aligned} \tag{B11}$$

Note that only ν is the exception for Eq. (B11).

We have the forward-backward asymmetry larger than the GWS predicts. Note that the measurement of the deviations from the predictions of the GWS model can give the masses of Z_1 and Z_2,

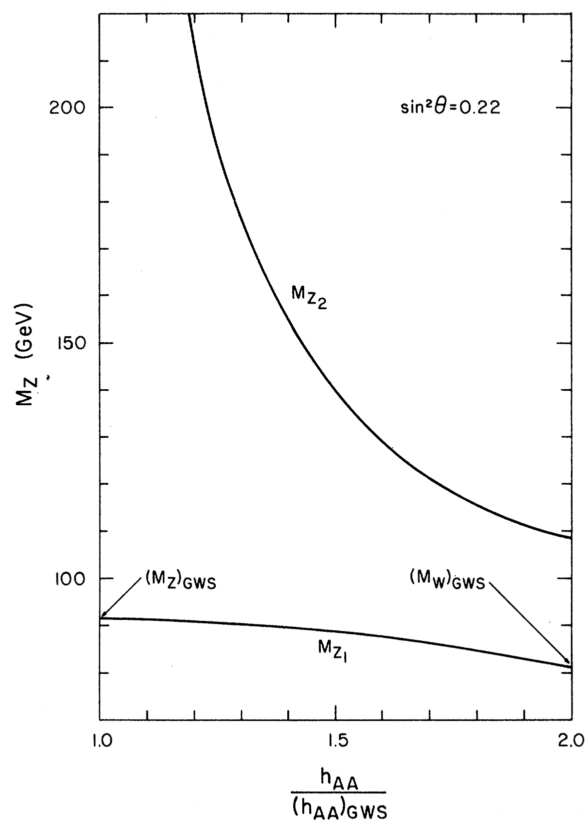

FIG. 1. Masses of two Z bosons.

using Eqs. (3.2), (5.7), and $A^2/\cos^2\theta=M_Z{}^2$. The explicit formulas are

$$\begin{aligned} M_{Z_1}{}^2&=M_Z{}^2\left[1-\frac{\sin\theta}{z}\tan\phi\right],\\ M_{Z_2}{}^2&=M_Z{}^2\left[1+\frac{\sin\theta}{z}\cot\phi\right], \end{aligned} \tag{B12}$$

where

$$\tan\phi=\frac{1}{2\Delta z\sin^2\theta}[F+(F^2+4\Delta^2z^2\sin^4\theta)^{1/2}]\ , \tag{B13}$$

$$F=(\Delta-1)z^2-(\Delta+1)\sin^4\theta\ , \tag{B14}$$

$$z=(\cos^2\theta-\sin^2\theta)^{1/2}\ , \tag{B15}$$

$$\Delta=\frac{h_{AA}-(h_{AA})_{\text{GWS}}}{(h_{AA})_{\text{GWS}}}\ . \tag{B16}$$

We show the dependence on Δ in Fig. 1. Note that if $\Delta=1$, $M_{Z_1}=M_W$.

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