Effective Weinberg angle for weak-electromagnetic gauge theories

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We define the effective Weinberg parameter x_W which plays the role of $\sin^2\theta_W$ when the standard SU(2)$\times$U(1) weak-electromagnetic theory is embedded in a larger group. The experimental value of $\sin^2\theta_W$ places strict limits on the charges of the fundamental fermions in these theories, and thus eliminates a large class of models that may seem to be valid. We also investigate weak-electromagnetic models based on SU(N) where the Weinberg angle is $\theta_W = 30°$ naturally.

The success of the now standard $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ weak-electromagnetic theory[1] lies in its predictions for neutral-current interactions.[2] For all the phenomenological success of the standard model, it still may be worthwhile to consider the standard model as the effective "low-energy" (100 GeV) limit of a larger weak-electromagnetic gauge theory. For example, the multigeneration structure of the weak interaction might be explained as a result of triangle-anomaly cancellation in an SU(N)$\times$U(1) weak-electromagnetic gauge theory.[3]

In the context of grand unified theories, if we assume that the strong interaction is color SU(3), and the standard model is the weak-electromagnetic interaction, then the one-generation grand unified Georgi-Glashow model[4] is most likely unique.[5] Assuming that the unbroken weak theory is larger than SU(2)$\times$U(1) can lead to multigeneration models,[6,7] and possibly realistic dynamically broken grand unified theories.[7] In addition, semisimple grand unified theories, such as the Pati-Salam model[8] and others based on SU(N)$\times$SU(N) (Ref. 9) have been postulated with an unbroken weak gauge group larger than SU(2)$\times$U(1).

Assuming that the SU(2)$\times$U(1) model is the correct low-energy theory, we wish to study how it may be embedded in larger gauge theories. Specifically we will show how the experimental value of the Weinberg angle θ_W places strict limits on how one can embed the standard model in larger groups. In addition, we also consider one-coupling-constant weak-electromagnetic theories based on SU(N) which naturally lead to a Weinberg angle $\theta_W = 30°$.

For simplicity let us consider embedding the $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ model in an $\mathrm{SU}(N)_L \times \mathrm{U}(1)$ theory. We then place the leptons in an $\underline{N}$ representation:

$$(\chi_a)_L = \begin{bmatrix} \nu_e(0) \\ e(-1) \\ l_1(n_1) \\ l_2(n_2) \\ \cdots \\ l_{N-2}(n_{N-2}) \end{bmatrix}_L , \tag{1}$$

where the charges of each lepton have been placed in parentheses. We have $N-2$ new leptons in the model, labeled by l_i, each with a charge n_i. Let us consider these charges to be arbitrary, each n_i assumed to be an integer. Since our model will break down to SU(2)$\times$U(1), these new particles will not appear at low energies, and will not affect low-energy phenomenology. Quarks may also be placed in an $\underline{N}$ representation as follows:

$$(\psi_a)_L = \begin{bmatrix} u(\frac{2}{3}) \\ d(-\frac{1}{3}) \\ q_1(\frac{2}{3}+n_1) \\ q_2(\frac{2}{3}+n_2) \\ \cdots \\ q_{N-2}(\frac{2}{3}+n_{N-2}) \end{bmatrix}_L . \tag{2}$$

Again we have $n-2$ new quarks with their charges in parentheses. The n_i's in Eqs. (1) and (2) are the same.

Since our model is SU(N)$\times$U(1) we have (N^2-1) gauge fields W_μ^j $(j=1,2,\ldots,N^2-1)$ associated with SU(N), and a singlet field B_μ. We can then write the covariant derivatives for this model as follows:

$$\mathscr{D}_\mu(\psi_a)_L=\{\delta_{ab}\partial_\mu-ig\lambda^j_{ab}W^j_\mu + ig'\delta_{ab}[\tfrac{2}{3}-\sigma(N)]B_\mu\}(\psi_b)_L\ , \tag{3a}$$

$$\mathscr{D}_\mu(\chi_a)_L=\{\delta_{ab}\partial_\mu-ig\lambda^j_{ab}W^j_\mu + ig'\delta_{ab}[-\sigma(N)]B_\mu\}(\chi_b)_L\ , \tag{3b}$$

where g and g' are the appropriate coupling constants. The λ^j_{ab}'s are the generators for SU(N) and are normalized so that the Fermi constant is

$$\frac{G_F}{\sqrt{2}}=\frac{g^2}{8m^2(W^\pm)}\ , \tag{4a}$$

where

$$W^\pm_\mu=\frac{1}{\sqrt{2}}(W^1_\mu\mp iW^2_\mu) \tag{4b}$$

just as in the standard model. The function $\sigma(N)$ in Eqs. (3a) and (3b) is defined as

$$\sigma(K)=\frac{1-\sum_{j=1}^{K-2}n_j}{K}\ ,\quad K=3,4,\ldots,N\ . \tag{5}$$

All the right-handed fields are singlets. The covariant derivative of a right-handed singlet s_R of charge Q_s is then

$$\mathscr{D}_\mu s_R=(\partial_\mu+ig'Q_sB_\mu)s_R\ . \tag{6}$$

The photon A_μ is found by mixing the singlet field B_μ with a linear combination of the "diagonal fields" of SU(N) which reflects the charge structure in Eqs. (1) or (2). Specifically, if we call this linear combination of diagonal fields F_μ,

$$F_\mu\equiv\frac{1}{\eta}[C_3W^3_\mu+C_8W^8_\mu+\cdots C_{N^2-1}W^{N^2-1}_\mu]\ , \tag{7}$$

then we have

$$\begin{bmatrix}F_\mu\\ B_\mu\end{bmatrix}=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}X^0_\mu\\ A_\mu\end{bmatrix}\ . \tag{8}$$

The angle θ is the SU(N)$\times$U(1) analog of the SU(2)$\times$U(1) Weinberg angle θ_W. The field X^0_μ is a neutral gauge field much like the Z_μ of the standard model. Finally, from Eqs. (1)–(3) and (5), the coefficients C_{K^2-1} in Eq. (7) are calculated to be

$$\begin{aligned}&C_3\equiv 1\ ,\\ &C_8=-(1+2n_1)/\sqrt{3}\ ,\\ &\ldots\\ &C_{K^2-1}=[2K(K-1)]^{1/2}[\sigma(K)-\sigma(K-1)]\ .\end{aligned} \tag{9}$$

The normalization constant η in Eq. (7) is just seen to be

$$\begin{aligned}\eta^2&\equiv\sum_{j=2}^{N}(C_{j^2-1})^2\\ &=2\left[1+\sum_{j=1}^{N-2}(n_j)^2-\frac{1}{N}\left[1-\sum_{j=1}^{N-2}n_j\right]^2\right]\ .\end{aligned} \tag{10}$$

We reproduce electromagnetism by setting

$$g'\cos\theta=g\frac{\sin\theta}{\eta}=e\ . \tag{11}$$

Let us compare this to the standard model where

$$g'\cos\theta_W=g\sin\theta_W=e\ . \tag{12}$$

Both g and e are the same in both models. The electric charge e is the same, and the g is set by the weak interaction, namely, the Fermi constant in Eq. (4). Equation (11) is always true, independent of how the gauge bosons acquire mass. If we assume that our SU(N)$\times$U(1) model breaks down to the low-energy SU(2)$\times$U(1) model, then comparing Eqs. (11) and (12) tells us that the parameter x_W,

$$x_W\equiv\frac{\sin^2\theta}{\eta^2}\ , \tag{13}$$

plays the role of $\sin^2\theta_W$, the Weinberg angle. A complete description of gauge fields and their mixing is found in Appendix A.

From Eqs. (9) and (10) we see that η^2, a normalization constant, must be greater than or equal to one. This means that

$$x_W\leq\frac{1}{\eta^2}\ . \tag{14}$$

From the experimental value[2] of $\sin^2\theta_W=0.231\pm0.010$, a "safe" limt on $1/\eta^2$ is

$$\frac{1}{\eta^2}\geq\frac{1}{4}\ . \tag{15}$$

Since η^2 depends on the parameters n_i in Eq. (10), the experimental value of x_W sets limits on the parameters n_i, and thus on the charges of the fermions in Eqs. (1) and (2).

From the form of η^2 we can make some general comments on the structure of allowed gauge theories.

(i) η^2 is invariant on the substitution of all $n_j \to -(n_j+1)$. Formally this means replacing the $\underline{N}$ representations by $\underline{N}^*$ representations. We would thus have

$$(\chi_a)_L \to (\chi^a)_L = [e(-1), \nu_e(0), l_1(-1-n_1), \ldots, l_{N-1}(-1-n_{N-2})] \tag{16a}$$

or

$$(\psi_a)_L \to (\psi^a)_L = [d(-\tfrac{1}{3}), u(\tfrac{2}{3}), q_1(-\tfrac{1}{3}-n_1), \ldots, q_{N-2}(-\tfrac{1}{3}-n_{N-2})] \,. \tag{16b}$$

Since the $\underline{2}$ and $\underline{2}^*$ representations of SU(2) are equivalent, this substitution will make no difference on the low-energy SU(2)$\times$U(1) limit of any model.

(ii) For all $n_i=0$, or all $n_i=-1$ we find

$$\frac{1}{\eta^2}=\frac{1}{2}\left[\frac{1}{1-1/N}\right] . \tag{17}$$

Thus all models of this type are acceptable.

(iii) For $n_1=-1$, $n_2=n_3=\cdots n_{N-2}=0$ or $n_1=0$, $n_2=n_3=\cdots=n_{N-2}=-1$,

$$\frac{1}{\eta^2}=\frac{1}{4}\left[\frac{1}{1-2/N}\right] . \tag{18}$$

These too are safe models.

(iv) For an arbitrary $n_i=-1$ or 0,

$$\frac{1}{\eta^2}\geq\frac{1}{4} \quad \text{if } N\leq 9 \,. \tag{19}$$

All models with an arbitrary selection of "normal" quark and lepton charges $(\frac{2}{3},-\frac{1}{3},0,-1)$ are allowed only for $N\leq 9$.

(v) For $n_1=1$, $n_2=n_3=\cdots=n_{N-2}=0$ or $n_1=-2$, $n_2=n_3=\cdots=n_{N-2}=-1$,

$$\frac{1}{\eta^2}=\frac{1}{4} \quad \text{for every } N \,. \tag{20}$$

These are the only allowed set of values for the n_i's which permit "exotic charged" fermions, i.e., fermions with charges other than $(\frac{2}{3},-\frac{1}{3},0,-1)$. Thus the experimental values of $\sin^2\theta_W$ places strict limits on the type and number of new "exotic" fermions in models of this form.

For example, consider models based on SU(3) $\times$U(1). Those which place quarks (and/or leptons) in triplets with charges[10] $[u(\frac{2}{3}),\ d(-\frac{1}{3})b(-\frac{1}{3})]$ or $[u(\frac{2}{3}),\ d(-\frac{1}{3})t(\frac{2}{3})]$ are "safe" since $1/\eta^2=\frac{3}{4}$. If there is an "exotic" charged quark,[11] it can only have a charge of $\frac{5}{3}$ or $-\frac{4}{3}$; then $1/\eta^2=\frac{1}{4}$.

For an SU(4)$\times$U(1) model, an example is a Pati-Salam-type model with a multiplet[12] $[u(\frac{2}{3}),\ d(-\frac{1}{3}), s(-\frac{1}{3})\ c(\frac{2}{3})]$. Again we can see by inspection it is a "safe " model since $1/\eta^2=\frac{1}{2}$. Safe models up through $N=6$ are given in Table I. The value of η^2 is unchanged by a permutation of the n_i's. The SU(2)$\times$U(1) limit is not changed by rearranging the order of the electric charges on the last $N-2$ elements.

This type of analysis can be extended to models of the form $SU(N)_L\times SU(M)_R\times U(1)$ which break down to $SU(2)_L\times U(1)$. The F_μ field remains the same. The singlet field B_μ, though, is now a linear combination of the "diagonal fields" of $SU(M)_R$ and the new singlet field associated with the new U(1). The result in Eq. (15) is still valid when we place no restrictions on the three coupling constants (see Appendix B).

We can also extend our analysis to cover truly unified, one-coupling-constant models of weak-electromagnetic interactions based on the group SU(N). For these cases, the B_μ field is absent so that we must identify the photon A_μ with the field F_μ in Eq. (7). From Eq. (8) we see that $\sin^2\theta\equiv 1$ so that

$$x_W=\frac{1}{\eta^2} \,. \tag{21}$$

Thus the effective Weinberg angle is uniquely determined. To within 2σ experimentally $x_W=\frac{1}{4}$. Using Eqs. (1), (2), and (10) we can, by just looking at the charge structure of the fermions, find models which can reproduce the known phenomenology.

For examples, consider a model based on SU(3) where the quarks are placed in triplets $[u(\frac{2}{3}),\ d(-\frac{1}{3}),\ b(-\frac{1}{3})]$ and the leptons in octets.[13] Just from the charge structure we can see that $x_W=\frac{3}{4}$ and the model is unacceptable. (In addition, the u-quark and electron neutral currents are incon-

TABLE I. A list of safe $SU(N)\times U(1)$ models for $N\leq 6$.

$SU(N)\times U(1)$	n_i	$1/\eta^2$
$N=3$	$n_i=n_1$	
	0	$\frac{3}{4}$
	-1	$\frac{3}{4}$
	1	$\frac{1}{4}$
	-2	$\frac{1}{4}$
$N=4$	$n_i=n_1,n_2$	
	0,0	$\frac{2}{3}$
	$-1,-1$	$\frac{2}{3}$
	$-1,0$	$\frac{1}{2}$
	0,1	$\frac{1}{4}$
	$-2,-1$	$\frac{1}{4}$
$N=5$	$n_i=n_1,n_2,n_3$	
	0,0,0	$\frac{5}{8}$
	$-1,-1,-1$	$\frac{5}{8}$
	$-1,0,0$	$\frac{5}{12}$
	$-1,-1,0$	$\frac{5}{12}$
	0,0,1	$\frac{1}{4}$
	$-2,-1,-1$	$\frac{1}{4}$
$N=6$	$n_i=n_1,n_2,n_3,n_4$	
	0,0,0,0	$\frac{3}{5}$
	$-1,-1,-1,-1,$	$\frac{3}{5}$
	$-1,0,0,0,$	$\frac{3}{8}$
	$-1,-1,-1,$ 0	$\frac{3}{8}$
	$-1,-1,0,0$	$\frac{1}{3}$
	0,0,0,1	$\frac{1}{4}$
	$-2,-1,-1,-1$	$\frac{1}{4}$

sistent with experiment.) By referring to Table I we see that if $n_1=1$, $x_W=\frac{1}{4}$. This is the SU(3) model of Pugh.[14] Here the leptons are placed in triplets $[\nu_e, e(-1), e^c(+1)]$. The quarks are integer-charge Han-Nambu quarks,[15] and are placed in triplets and antitriplets so as to cancel anomalies. This model has also been shown to reproduce the standard model phenomenology after a spontaneous breakdown of symmetry.

We are now in a position to see if this model can be extended from SU(3) to $SU(N)$. Since all the leptons would be placed in $\underline{N}$ representations, both lepton and antilepton, and the sum of the charges in any representation must be zero, we see that, since $n_1=1$,

$$\sum_{j=2}^{N-2} n_j=0 . \tag{22}$$

The condition that $x_W=\frac{1}{4}$ with $n_1=1$ requires

$$\sum_{j=2}^{N-2} (n_j)^2=0 . \tag{23}$$

Thus we can trivially extend Pugh's model to arbitrary $SU(N)$ provided that all the new leptons have zero charge.[16]

We can also investigate other types of $SU(N)$ models where $x_W=\frac{1}{4}$. For example, there is a viable extension of the models in Ref. 13. If we look at SU(9) and place the quarks in $\underline{9}$ representations,

$$\begin{bmatrix} u(\frac{2}{3}) \\ d(-\frac{1}{3}) \\ b(-\frac{1}{3}) \\ u'(\frac{2}{3}) \\ d'(-\frac{1}{3}) \\ b'(-\frac{1}{3}) \\ u''(\frac{2}{3}) \\ d''(-\frac{1}{3}) \\ b''(-\frac{1}{3}) \end{bmatrix}_L \quad \begin{bmatrix} u''(\frac{2}{3}) \\ d''(-\frac{1}{3}) \\ b''(-\frac{1}{3}) \\ u'(\frac{2}{3}) \\ d'(-\frac{1}{3}) \\ b'(-\frac{1}{3}) \\ u(\frac{2}{3}) \\ d(-\frac{1}{3}) \\ b(-\frac{1}{3}) \end{bmatrix}_R ,$$

and place the leptons in the adjoint (80-dimensional) representation, then $x_W=\frac{1}{4}$. This model, when broken down to $SU(2)\times U(1)$, can reproduce a standard phenomenology.

Finally, we can look at a "bizarre" model based **on SU(6) where quarks and antiquarks are both placed in the same $\underline{6}$ representations:**

$$\begin{bmatrix} u(+\frac{2}{3}) \\ d(-\frac{1}{3}) \\ t(\frac{2}{3}) \\ u^c(-\frac{2}{3}) \\ d^c(+\frac{1}{3}) \\ t^c(-\frac{2}{3}) \end{bmatrix}_L .$$

Here we have $n_1=0$, $n_2=-\frac{4}{3}$, $n_3=-\frac{1}{3}$, $n_4=-\frac{4}{3}$, $N=6$, and by (10), $x_W=\frac{1}{4}$. The leptons in such a scheme could be placed in a representation which has some integer-charge elements, such as the totally symmetric $\underline{21}$. Since the n_i are not all integers in this model, the $\underline{21}$ representation will have fractionally charged members in it, as well as integer-charge ones. Again, if we break the symmetry to SU(2)×U(1), we can reproduce standard neutral currents.

APPENDIX A

Let us consider the $(N-1)$ "diagonal" gauge fields associated with SU(N). These fields are

$$W^3, W^8, W^{15}, \ldots, W^{N^2-1} .$$

Here the Lorentz indices have been dropped for ease of notation. If we define a generalized normalization constant $\eta(K)$,

$$[\eta^2(K)] \equiv \sum_{j=2}^{K} (C_{j^2-1})^2 = 2\left[1+\sum_{j=1}^{K-2}(n_j)^2 - \frac{1}{K}\left[1-\sum_{j=1}^{K-2} n_j\right]^2\right] , \qquad \text{(A1)}$$

we can then write the $(N-1)$ diagonal gauge fields in the orthonormal basis

$$F=\frac{1}{\eta(N)}[W^3+C_8W^8+C_{15}W^{15}+\cdots+C_{N^2-1}W^{N^2-1}] \equiv \frac{1}{\eta(N)}\{W^3+[\eta^2(N)-1]^{1/2}V^0\} ,$$

$$H \equiv \frac{1}{\eta(N)}\{-[\eta^2(N)-1]^{1/2}W^3+V^0\} ,$$

$$X^2 \equiv \frac{1}{[C_8{}^2+C_{15}{}^2]^{1/2}}[C_{15}W^8-C_8W^{15}] , \qquad \text{(A2)}$$

$$\cdots$$

$$X^j \equiv \left[\frac{1}{\eta^2(j+2)-1}\right]^{1/2}\left[\frac{C_{(j+2)^2-1}}{[\eta^2(j+1)-1]^{1/2}}\left[\sum_{K=3}^{j+1} c_{K^2-1}W^{K^2-1}\right] - [\eta^2(j+1)-1]^{1/2}W^{(j+2)^2-1}\right] , \quad j=2,3,\ldots,N-2 . \qquad \text{(A2)}$$

The advantage of this parametrization lies in the fact that we can first reproduce electromagnetism by mixing the F field and the singlet B field by

$$\begin{bmatrix} F \\ B \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} X^0 \\ A \end{bmatrix} . \qquad \text{(A3)}$$

Second, we can reproduce the low-energy SU(2) ×U(1) limit by mixing the X^0 and H fields by an angle ϵ,

$$\tan\epsilon = \frac{(\eta^2-1)^{1/2}}{\cos\theta} , \qquad \text{(A4)}$$

where

$$\begin{bmatrix} X^0 \\ H \end{bmatrix} = \begin{bmatrix} \cos\epsilon & -\sin\epsilon \\ \sin\epsilon & \cos\epsilon \end{bmatrix}\begin{bmatrix} Z \\ X^1 \end{bmatrix} . \qquad \text{(A5)}$$

Thus using (A2)–(A5), we have

$$A = -\sqrt{x_W}\,W^3 + (1-x_W)^{1/2}\left[\frac{B\cos\theta - V^0\sqrt{x_W}(\eta^2-1)^{1/2}}{(1-x_W)^{1/2}}\right] ,$$

$$Z = (1-x_W)^{1/2}W^3 + \sqrt{x_W}\left[\frac{B\cos\theta - V^0\sqrt{x_W}(\eta^2-1)^{1/2}}{(1-x_W)^{1/2}}\right] , \qquad \text{(A6)}$$

$$X^1 = \frac{B\sqrt{x_W}(\eta^2-1)^{1/2}+V^0\cos\theta}{(1-x_W)^{1/2}} .$$

Let us now compare the fields A and Z in (A6) with the SU(2)×U(1) A and Z:

$$A = -\sin\theta_W W^3 + \cos\theta_W D ,$$

$$Z = \cos\theta_W W^3 - \sin\theta_W D , \qquad \text{(A7)}$$

where D here is the $SU(2)\times U(1)$-singlet field. As can be seen, the parameter x_W plays the role of $\sin^2\theta_W$, while the linear combination of fields

$$\frac{B\cos\theta - V^0\sqrt{x_W}(\eta^2-1)^{1/2}}{(1-x_W)^{1/2}}$$

plays the role of the $SU(2)\times U(1)$-singlet D.

After a spontaneous breakdown to the $SU(2)\times U(1)$ level, all the fields $X^1, X^2,\ldots,X^{N-2}$ acquire very large masses, and are effectively frozen out of the theory. This can be accomplished in an $SU(N)\times U(1)$ theory by considering a set of N Higgs scalars $\phi_a^{(l)}$, $l=1,2,\ldots,N$, each transferring as an $\underline{N}$ under $SU(N)$.[17] Their covariant derivatives are

$$D_\mu\phi_a^{(l)}=[\delta_{ab}\partial_\mu - ig\lambda_{ab}^j W_\mu^j + ig'\delta_{ab}\beta(l)B_\mu]\phi_b^{(l)} , \tag{A8}$$

where

$$\begin{aligned}
\beta(1)&=-\sigma(N) ,\\
\beta(2)&=1-\sigma(N) ,\\
\beta(3)&=-n_1-\sigma(N) ,\\
&\ldots\\
\beta(l)&=-n_{l-2}-\sigma(N) .
\end{aligned} \tag{A9}$$

The vacuum expectation values are then taken to be

$$\langle\phi_a^{(l)}\rangle_0 = v_a\delta_{al} \quad (\text{no sum}) . \tag{A10}$$

To break $SU(N)\times U(1)$ to $SU(2)\times U(1)$ requires that

$$v_{1,2} << v_b \quad (b=3,4,5,\ldots,N) . \tag{A11}$$

We then find that

$$m^2(A)=0 ,$$

and neglecting terms of the order

$$\left[\frac{v_{1,2}}{v_b}\right]^2 , \tag{A12}$$

$$\frac{g^2}{2}(v_1{}^2+v_2{}^2)=m^2(W^\pm)=m^2(Z)(1-x_W)$$

$$=\frac{\pi\alpha}{\sqrt{2}G_F x_W} \tag{A13}$$

and *all* the other diagonal and off-diagonal gauge bosons acquire masses proportional to $(v_b)^2$. The mass relationship (A13) is exactly the one in the $SU(2)\times U(1)$ theory when only Higgs doublets are allowed.

APPENDIX B

Consider weak-electromagnetic theories of the form $SU(N)_L\times SU(M)_R\times U(1)$. The quark and lepton multiplets $(\psi_a)_L$ and $(\chi_a)_L$ given in Eqs. (1) and (2) still couple only to $SU(N)_L\times U(1)$. We now postulate two new quark and lepton multiplets, $(\tilde{\psi}_a)_R$ and $(\tilde{\chi}_a)_R$ which transform as $\underline{M}$ representations under $SU(M)$. In analogy to Eqs. (1) and (2), we have

$$(\tilde{\chi}_a)_R=\begin{bmatrix} \nu'(m') \\ E(-1+m'') \\ L_1(m_1) \\ L_2(m_2) \\ \cdots \\ L_{M-2}(m_{M-2}) \end{bmatrix}_R , \tag{B1}$$

$$(\tilde{\psi}_a)_R=\begin{bmatrix} U(\frac{2}{3}+m') \\ D(-\frac{1}{3}+m'') \\ Q_1(\frac{2}{3}+m_1) \\ Q_2(\frac{2}{3}+m_2) \\ \cdots \\ Q_{M-2}(\frac{2}{3}+m_{M-2}) \end{bmatrix}_R .$$

The charges are in parentheses. The parameters $m', m'', m_1,\ldots,m_{M-2}$ are all arbitrary constants.

In analogy to Eq. (3a), the covariant derivative for $(\tilde{\psi}_a)_R$ is given as

$$\mathscr{D}_\mu(\tilde{\psi}_a)_R=\{\delta_{ab}\partial_\mu - i\tilde{g}\tilde{\lambda}_{ab}^j\tilde{W}_\mu^j + ig'\delta_{ab}[\tfrac{2}{3}+m'-\tilde{\sigma}(M)]B_\mu\}(\tilde{\psi}_b)_R , \tag{B2}$$

where $\tilde{g}$, $\tilde{\lambda}_{ab}^j$, and $\tilde{W}_\mu^j$ are, respectively, the coupling constant, generators, and gauge fields associated with $SU(M)$. The function $\tilde{\sigma}(M)$ in (B2) is just

$$\tilde{\sigma}(K)=\frac{1-m'-m''-\sum_{j=1}^{K-2}m_j}{K} . \tag{B3}$$

The U(1) coupling constant in Eqs. (3) and (B2) is, of course, the same. Thus a left-handed singlet $\tilde{s}_L$ under $SU(M)$ with charge $\tilde{Q}_s$ has covariant derivative

$$\mathscr{D}_\mu\tilde{s}_L=(\partial_\mu+ig'\tilde{Q}_sB_\mu)\tilde{s}_L . \tag{B4}$$

The photon A_μ is now found by mixing the singlet field B_μ, F_μ of Eq. (7), *and* $\widetilde{F}_\mu$. $\widetilde{F}_\mu$ is

$$\widetilde{F}_\mu=\frac{1}{\widetilde{\eta}}(\widetilde{C}_3\widetilde{W}^3_\mu+\widetilde{C}_8\widetilde{W}^8_\mu+\dots+\widetilde{C}_{M^2-1}W^{M^2-1}_\mu)\ , \tag{B5}$$

where

$$\widetilde{C}_3=1+m'-m''$$
$$\widetilde{C}_{15}=(1-m'-m''+2m_1)/\sqrt{3}\ , \tag{B6}$$
$$\widetilde{C}_{K^2-1}=[2K(K-1)]^{1/2}[\widetilde{\sigma}(K)-\widetilde{\sigma}(K-1)]\ ,$$

and

$$\widetilde{\eta}^2=2\left\{(m')^2+(-1+m'')^2+\sum_{j=1}^{M-2}(m_j)^2-M[\widetilde{\sigma}(M)]^2\right\}. \tag{B7}$$

There are now three mixing angles θ_1, θ_2, and θ_3. Formally the F_μ, $\widetilde{F}_\mu$, and B_μ mixing can be given by

$$\begin{bmatrix}F_\mu\\ \widetilde{F}_\mu\\ B_\mu\end{bmatrix}=\begin{bmatrix}C_1C_3-S_1C_2S_3 & C_1S_3+S_1C_2C_3 & S_1S_2\\ -S_1C_3-C_1C_2S_3 & -S_1S_3+C_1C_2C_3 & C_1S_2\\ S_2S_3 & -S_2C_3 & C_2\end{bmatrix}\begin{bmatrix}X^0_\mu\\ \widetilde{X}^0_\mu\\ A_\mu\end{bmatrix}, \tag{B8}$$

where $S_i=\sin\theta_i$, $C_i=\cos\theta_i$. Electromagnetism is reproduced when

$$\frac{g\sin\theta_1\sin\theta_2}{\eta}=\frac{\widetilde{g}\cos\theta_1\sin\theta_2}{\widetilde{\eta}}=g'\cos\theta_2=|e|\ . \tag{B9}$$

After a spontaneous breakdown of symmetry to the $SU(2)_L\times U(1)$ limit, we find

$$x_W=\frac{\sin^2\theta_1\sin^2\theta_2}{\eta^2} \tag{B10}$$

in analogy to Eq. (13). If we place no restrictions on g, $\widetilde{g}$, η, and $\widetilde{\eta}$, then a "safe" model has $1/\eta^2\geq\frac{1}{4}$ as before.

Let us assume that

$$\frac{\widetilde{g}}{\widetilde{\eta}}=k\frac{g}{\eta}\ , \tag{B11}$$

where k is a proportionality constant. Equation (B9) tells us that

$$\sin^2\theta_1=\frac{k^2}{k^2+1} \tag{B12}$$

and (B10) gives us

$$x_W=\frac{k^2}{k^2+1}\frac{\sin^2\theta_2}{\eta^2}\ . \tag{B13}$$

An interesting special case is the symmetric model. By this we mean $N=M$, $g=\widetilde{g}$, $m'=m''=0$, and $n_i=m_i$. Equation (B11) gives us $k=1$ and (B13) tells us

$$x_W=\frac{\sin^2\theta_2}{2\eta^2} \tag{B14}$$

so that safe models in this case have $1/\eta^2\geq\frac{1}{2}$. This parametrization is also applicable to an $SU(2)_L\times SU(2)_R\times U(1)$-symmetric model.[18] Here $\eta\equiv1$ and $x_W=\frac{1}{2}\sin^2\theta_2$.

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