Associated heavy-vector-meson production in e^+e^- annihilation

L. Clavelli
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439
(Received 27 May 1982)

As a contribution to the inclusive process $e^+e^- \to V+X$ where V is a ${}^3S_1$ bound state of heavy quarks q_1 and $\bar{q}_2$, we consider the QCD process $e^+e^- \to V+\bar{q}_1+q_2$. The rate is found to dominate asymptotically over other, previously calculated, perturbative mechanisms. An apparently accidental partial cancellation in the case of the like-quark bound states (ϕ,ψ,Υ) enhances the relative rate of flavor-nonsinglet vector mesons.

I. INTRODUCTION

One of the highest priorities of present and near-future experimental physics is the observation of prompt leptons in various interactions as a signal of heavy quarks, leptons, intermediate bosons, or other new particles. As a background to these searches as well as a test of our understanding of heavy-particle production processes it is important to study inclusive production of the known narrow resonances such as ϕ, ψ, and Υ. The simplest production mechanism from a theoretical point of view is e^+e^- annihilation since here there are no ambiguities arising from parton distributions in the target and strong collinear radiation from the initial state. The parton-level processes which have been given consideration are

$$e^+e^- \to V+g \ , \tag{1.1}$$

$$e^+e^- \to V+g+g \ , \tag{1.2}$$

$$e^+e^- \to V+\bar{q}_1+q_2 \ . \tag{1.3}$$

In the above g represents a gluon and V a vector bound state of quarks q_1 and $\bar{q}_2$. It is convenient to normalize the rates and angular distributions to the total μ-pair-production cross section:

$$R_V=\sigma(e^+e^- \to V+X)/\sigma(e^+e^- \to \mu^+\mu^-) \ . \tag{1.4}$$

Process (1.1) is referred to as the "color evaporation model" (CEM), because soft-gluon radiation is postulated to occur from the $\bar{q}q$ state "with probability one" to yield a color-singlet[1] vector meson V. In practice color and binding effects are ignored in model (1.1) and all of the events with $\bar{q}q$ masses near the V mass are counted as V production since one is below the $\bar{q}q$ continuum threshold. The narrow charmonium states are predicted to occur with roughly equal rate and no prediction is made for the charmonium states above threshold. One predicts a falling R contribution[1]:

$$R_\psi \cong \alpha_s \frac{(4m_D{}^2-m_\psi{}^2)^{3/2}}{s m_D} \ . \tag{1.5}$$

The corresponding CEM model for inelastic hadroproduction or photoproduction of ψ does not well reproduce empirical results,[2] whereas a color-singlet model[3] matches the data in shape if not in overall normalization.

It seems likely, therefore, that a more theoretically well founded approach, taking into account color and charmonium wave functions at the origin is given, in lowest order, by the processes of Eqs. (1.2) and (1.3). The first of these, production of ψ together with two gluons, has been recently treated in detail using the technique of Ref. 3.[4] The asymptotic result for the total rate can be written

$$R_V=\tfrac{16}{3}\alpha_s{}^2Q^2\frac{A^2}{s}\ln^2 s/m_V{}^2 \ , \tag{1.6}$$

where A is proportional to the V wave function at the origin. A, in the normalization of Berger and Jones, is related to the leptonic decay rate by

$$\Gamma(V\to\mu^+\mu^-)=8\pi\alpha^2Q^2A^2/M_V \ , \tag{1.7}$$

where Q is the fractional quark charge, $\frac{2}{3}$ in the case of ψ.

The production of a V meson in association with continuum $q\bar{q}$ [Eq. (1.3)] has been estimated previously[5,6] without attention to color and charmonium binding. In these treatments the associated q and $\bar{q}$ are not necessarily of the same flavor as the constituents of the V meson and one again relies on a quantitatively uncertain "color evaporation" to restore color conservation. Symptomatic of the theoretical limitations of this approach, the inclusive ψ production is found, then, to rise asymp-

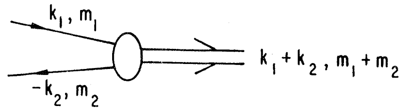

FIG. 1. Vector bound state of quark-antiquark in the zero-binding approximation. Lines are labeled by particle momentum and mass.

totically above the total hadronic cross section.[6]

In the present paper we treat the V production of (1.3) normalizing to the charmonium wave function and imposing explicit color conservation in the manner of Ref. 3. In the present case, of course, the matrix element is not obtainable by crossing from a ψ decay matrix element since the ψ is below threshold for decay into free charm. The associated quarks in (1.3) carry the same flavors as the constituents of V. This associated V production is found to dominate (in the PEP-PETRA energy range and above) over the contribution of (1.2) although the predicted rate is small. In Sec. II we discuss the total rate and differential cross sections in angle and energy for various $^3S_1$ states. Our conclusions are presented in Sec. III together with a discussion of possible nonperturbative effects. The hadronic tensor for process (1.3) is given in the Appendix for a vector bound state of heavy quarks of arbitrary mass and charge.

II. ASSOCIATED HEAVY-VECTOR-MESON PRODUCTION

Following the standard charmonium-type model with quark binding energy neglected[7] we adopt the simplified wave function for quarks of momenta, mass, and spinor indices k_1, m_1, i and k_2, m_2, j in a $^3S_1$ state of momentum, mass, and polarization $2P$, $2m$, ϵ as shown in Fig. 1:

$$\phi_{ij}=\frac{\epsilon_\alpha A}{\sqrt{2N_c}}\int d^4k_1 d^4k_2[\lambda^0(-\not{k}_2-m_2)\gamma_\alpha(\not{P}+m)(\not{k}_1-m_1)]_{ij}\delta^4(k_1+k_2-2P)\delta^4\left[\frac{k_1-k_2}{2}-m_r P\right]. \tag{2.1}$$

The constant A is proportional to the charmonium wave function at the origin. N_c is the number of colors (taken to be 3) and λ^0 is the unit matrix in the color space of the quarks. The integral in (2.1) is understood to be done after insertion of ϕ_{ij} in a Feynman graph and the factors $\pm\not{k}_i-m_i$ serve to amputate the incoming-quark propagators. We have defined

$$m_r\equiv\frac{m_1-m_2}{2m} \tag{2.2}$$

and the zero-binding approximation consists, in part, in our replacing an arbitrary function of (k_1-k_2) by the second δ function in (2.1).

The muonic decay rate of a flavor-neutral vector meson, calculated by using (2.1) in the Feynman graph of Fig. 2, is given by (1.7). ρ, ω, ϕ, ψ, and Υ leptonic rates are consistent empirically with A^2/M_V being constant,[8]

$$\frac{A^2}{M_V}\cong 7.4\times10^{-3}\text{ GeV}, \tag{2.3}$$

and we extend this flavor-independent constant to flavor-nonsinglet bound states.

The associated vector-meson production in e^+e^- annihilation is then given by the four Feynman graphs of Fig. 3.[9] The corresponding matrix element is

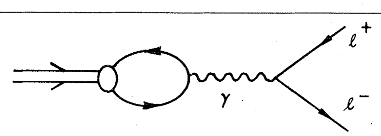

FIG. 2. Feynman graph for the leptonic decay of a flavor-neutral vector bound state.

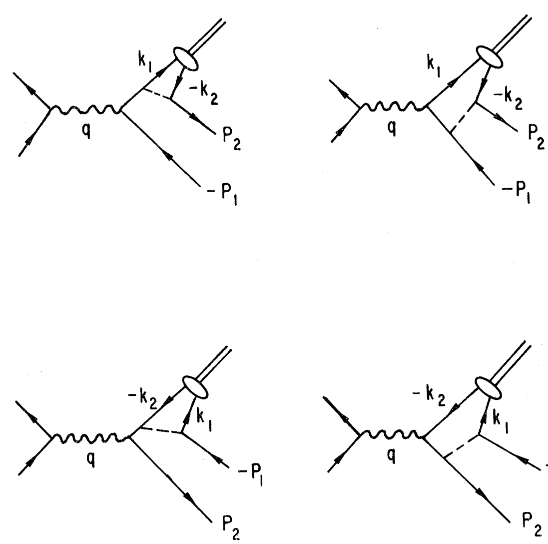

FIG. 3. Feynman graphs for e^+e^- annihilation into a $^3S_1(q_1\bar{q}_2)$ state in association with free quarks of like flavors.

$$M_\mu=\epsilon_\alpha\sqrt{4\pi\alpha}4\pi\alpha_s\frac{A}{\sqrt{2N_c}}\bar{u}(P_2)\sum_\gamma(\lambda^\gamma)^2\tilde{M}_{\mu\alpha}v(P_1)\,, \tag{2.4}$$

$$\tilde{M}_{\mu\alpha}=\gamma_\lambda\frac{\gamma_\alpha(\not P+m)}{(P_2+k_2)^2}\left[\gamma_\lambda\frac{\not q-\not p_1+m_1}{2H_1}\gamma_\mu Q_1+\gamma_\mu Q_1\frac{\not k_1-\not q+m_1}{2H_b}\gamma_\lambda\right]$$
$$+\left[\gamma_\mu Q_2\frac{\not P_2-\not q+m_2}{2H_2}\gamma_\lambda+\gamma_\lambda\frac{\not q-\not k_2+m_2}{2H_c}\gamma_\mu Q_2\right]\frac{\gamma_\alpha(\not P+m)}{(P_1+k_1)^2}\gamma_\lambda\,. \tag{2.5}$$

The notation is such that the virtual photon, with momentum q_μ, couples to γ_μ and the produced vector meson V has polarization ϵ_α. Q_1 and Q_2 are quark fractional charges. In addition

$$m_V=2m=m_1+m_2\,, \tag{2.6a}$$

$$Q_V=Q_1-Q_2\,, \tag{2.6b}$$

$$P_V=2P=k_1+k_2\,, \tag{2.6c}$$

$$q=P_1+P_2+2P\,, \tag{2.6d}$$

$$H_1=\frac{q^2}{2}-q\cdot P_1\,, \tag{2.6e}$$

$$H_2=\frac{q^2}{2}-q\cdot P_2\,, \tag{2.6f}$$

$$H_b=\frac{q^2}{2}-q\cdot k_1\,, \tag{2.6g}$$

$$H_c=\frac{q^2}{2}-q\cdot k_2\,, \tag{2.6h}$$

$$H_p=\frac{q^2}{2}-q\cdot P\,. \tag{2.6i}$$

The zero-binding approximation constrains the momenta to satisfy

$$k_{i\mu}=\frac{m_i}{m}P_\mu\,, \tag{2.6j}$$

$$(P_1+k_1)^2=\frac{m_1}{m}H_2\,, \tag{2.6k}$$

$$(P_2+k_2)^2=\frac{m_2}{m}H_1\,. \tag{2.6l}$$

The differential cross section summed over the produced-vector-meson polarization and quark spins is

$$d\sigma(e^+e^-\to V+q+\bar{q})=\frac{d\Omega}{2s}\frac{4\pi\alpha}{s^2}L_{\mu\nu}H_{\mu\nu} \tag{2.7}$$

with lepton tensor

$$L_{\mu\nu}+P_{e_\mu}P_{\bar{e}_\nu}+P_{\bar{e}_\nu}P_{e_\mu}-P_e\cdot P_{\bar{e}}g_{\mu\nu} \tag{2.8}$$

and hadronic tensor

$$H_{\mu\nu}=4\pi\alpha(4\pi\alpha_s)^2\frac{A^2}{2}\left[\frac{N_c{}^2-1}{2N_c}\right]^2\tilde{H}_{\mu\nu}\,, \tag{2.9}$$

$$\tilde{H}_{\mu\nu}=\mathrm{Tr}(\not P_2+m_2)\tilde{M}_{\mu\alpha}(\not P_1-m_1)$$
$$\times\tilde{M}^R_{\nu\beta}\left[-g_{\alpha\beta}+\frac{P_\alpha P_\beta}{m^2}\right]. \tag{2.10}$$

Here $\tilde{M}^R$ is the γ matrix string of (2.5) read in reversed order. The phase-space differential is

$$d\Omega=\frac{4d^3P}{2P_0}\frac{d^3P_1}{2P_{10}}\frac{d^3P_2}{2P_{20}}\frac{\delta^4(q-P_1-P_2-2P)}{(2\pi)^5}\,. \tag{2.11}$$

Relative to $\mu^+\mu^-$ production we have

$$\frac{d\sigma}{\sigma_{\mu\mu}}=\tfrac{32}{3}\alpha_s{}^2\frac{A^2}{s}(2\pi)^3\frac{d\Omega}{s}\left[\frac{3(N_c{}^2-1)}{8N_c}\right]^2L_{\mu\nu}\tilde{H}_{\mu\nu}\,, \tag{2.12}$$

where

$$\sigma_{\mu\mu}\equiv\sigma(e^+e^-\to\mu^+\mu^-)=\frac{4\pi\alpha^2}{3s}\,. \tag{2.13}$$

The reduced hadronic tensor $\tilde{H}_{\mu\nu}$ is given in Appendix A. In the following we will neglect the factor in square brackets in Eq. (2.12) which is unity for the standard three-color theory. The phase-space integrals are performed using

$$(2\pi)^3\frac{d\Omega}{s}=\frac{1}{4}\frac{dH_p}{s}\frac{dH_1}{s}d\cos\theta\frac{d\phi_p d\phi_1}{(2\pi)^2}\,. \tag{2.14}$$

All of the integrals except one can be performed analytically and in the limit $s/m^2\to\infty$ the total rate can be written as a function of the constituent masses. Asymptotically this rate approaches a constant relative to the μ-pair-production rate:

$$R(^3S_1(q_1\bar{q}_2))=\int \frac{d\sigma}{\sigma_{\mu\mu}} \underset{s/m^2\to\infty}{=} \frac{16}{3}\frac{A^2}{m^2}\alpha_s^2 f\left[\frac{m_1}{m},\frac{m_2}{m},Q_1^2,Q_2^2\right] . \tag{2.15}$$

The function f is conveniently expressible in terms of

$$\langle Q^2\rangle=\frac{Q_1^2+Q_2^2}{2} , \tag{2.16}$$

$$\Delta_{Q^2}=\frac{Q_1^2-Q_2^2}{2} , \tag{2.17}$$

$$R_1=m_1/m , \tag{2.18}$$

$$R_2=m_2/m , \tag{2.19}$$

$$R=R_1R_2 , \tag{2.20}$$

$$Q_\pm=2Q_1^2\ln(2/R_2)\pm 2Q_2^2\ln(2/R_1) , \tag{2.21}$$

$$m_r=\frac{R_1-R_2}{2}=1-R_2=R_1-1 , \tag{2.22}$$

$$\begin{aligned} f(R_1,R_2,Q_1^2,Q_2^2)=&\frac{384}{R^6}(Q_-m_r-Q_+)+\frac{1}{R^5}(\tfrac{1792}{3}Q_+ +384\langle Q^2\rangle-\tfrac{1216}{3}Q_-m_r-384m_r\Delta_{Q^2})\\ &+\frac{1}{R^4}(-268Q_+ -\tfrac{1360}{3}\langle Q^2\rangle+\tfrac{340}{3}Q_-m_r+\tfrac{784}{3}m_r\Delta_{Q^2})\\ &+\frac{1}{R^3}(36Q_+ +\tfrac{1892}{15}\langle Q^2\rangle-6Q_-m_r-\tfrac{196}{5}m_r\Delta_{Q^2})+\frac{1}{R^2}\left[-\frac{Q_+}{3}-\frac{178}{45}\langle Q^2\rangle\right] . \end{aligned} \tag{2.23}$$

For flavor-neutral vectors such as ϕ, ψ, Υ, etc., the expression simplifies considerably since then

$$\Delta_{Q^2}=Q_-=m_r=0 , \tag{2.24a}$$

$$R_1=R_2=R=1 , \tag{2.24b}$$

$$Q_+=4Q^2\ln 2 , \tag{2.24c}$$

$$\langle Q^2\rangle=Q^2 \tag{2.24d}$$

and therefore in this flavor-neutral case

$$\lim_{(s/m_V^2)\to\infty} R(^3S_1(q\bar{q})) =\frac{A^2}{m_V^2}\alpha_s^2Q^2\frac{4864}{3}[\tfrac{1189}{1710}-\ln 2] . \tag{2.25}$$

The terms in the square brackets cancel to within 0.3%. This is strongly reminiscent of a similar approximate cancellation in the total hadronic decay rate of the $^3S_1$ state:

$$\Gamma(^3S_1(q\bar{q})\to\text{hadrons})=\frac{80}{81}\alpha_s^3\frac{A^2}{m_V}[\pi^2-9] . \tag{2.26}$$

In (2.26), however, the terms in the square brackets only cancel to within 10%.

Because of the approximate cancellation in (2.25), the numerical integration of the matrix element squared requires some extra care, especially at high c.m. energies.

In spite of the terms in square brackets in (2.25), this contribution to ψ production dominates over the mechanisms of (1.1) and (1.2) above 20 GeV c.m. energy. However, as a further consequence of the remarkable accidental cancellation in (2.25), one might expect that higher-order QCD corrections and/or a relaxation of the zero-binding approximation might greatly increase the theoretical ψ production rate. It is even therefore conceivable that inclusive ψ and Υ production could become one of the most sensitive probes of charmonia potentials.

If on the other hand these, and other possible nonperturbative, corrections do not render (2.25) irrelevant, an interesting pattern of heavy-vector-meson production rates is predicted. This follows from the fact that the cancellation in (2.25), although independent of the quark mass and charge, is very sensitive to the flavor neutrality of ϕ, ψ, Υ, etc., i.e., to the relative quark and antiquark masses and charges. The rates for inclusive production of $^3S_1$ states of unlike quarks, given by the more general formulas of Eqs. (2.23) and (2.15) do not show such a suppression. In Fig. 4 we show the energy dependence of R_V/α_s^2 in the cases $V={}^3S_1(s\bar{s},c\bar{c},b\bar{b},s\bar{c},s\bar{b},c\bar{b})$ using $m_s=0.5$ GeV, $m_c=1.5$ GeV, and $m_b=4.5$ GeV. It is commonly thought[3,4] that the value of α_s relevant to such lowest-order QCD calculations is

$$\alpha_s=\frac{12\pi}{(33-2n_f)\ln M_V^2/\Lambda^2} \tag{2.27}$$

with $\Lambda\sim 500$ MeV although a fit to charmonium decay rates with higher-order QCD contributions included[10] brings Λ down to the vicinity of 100 MeV. In Fig. 5 we show the inclusive differential cross section as a function of the c.m. scaled energy x_V of the vector meson V for $V=s\bar{c}$, $c\bar{c}$, $b\bar{c}$, and $s\bar{s}$ at 34 GeV c.m. energy,

$$x_V=\frac{2E_V}{\sqrt{s}} . \tag{2.28}$$

The heavy-quark masses keep the gluons of Fig. 3 far off shell, thus motivating the use of perturbation theory. Nevertheless the QCD distributions favor gluons of the softest possible momentum

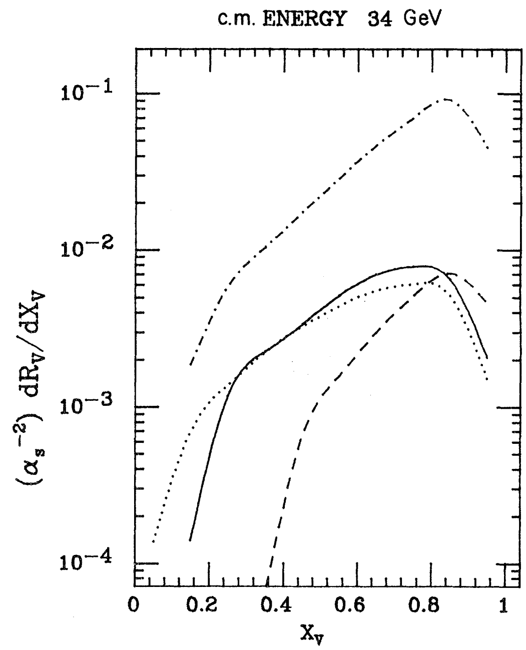

FIG. 5. Differential production rate in the scaled energy, $x_V=2E_V/\sqrt{s}$, for $s\bar{c}$ (dot-dash), $c\bar{c}$ (solid), $s\bar{s}$ (dotted), and $b\bar{c}$ (dashed) vector mesons at a c.m. energy of 34 GeV.

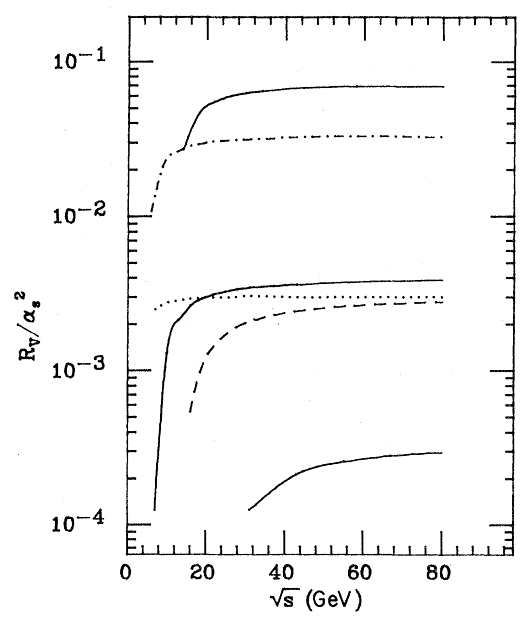

FIG. 4. Energy dependence of associated vector-meson inclusive production in single-photon e^+e^- annihilation. The solid curves in order of decreasing asymptotic production rates correspond to $s\bar{b}$, $c\bar{c}$, and $b\bar{b}$ vector mesons. Also shown are the rates for $s\bar{c}$ (dot-dashed curve), $s\bar{s}$ (dotted curve), and $b\bar{c}$ (dashed curve).

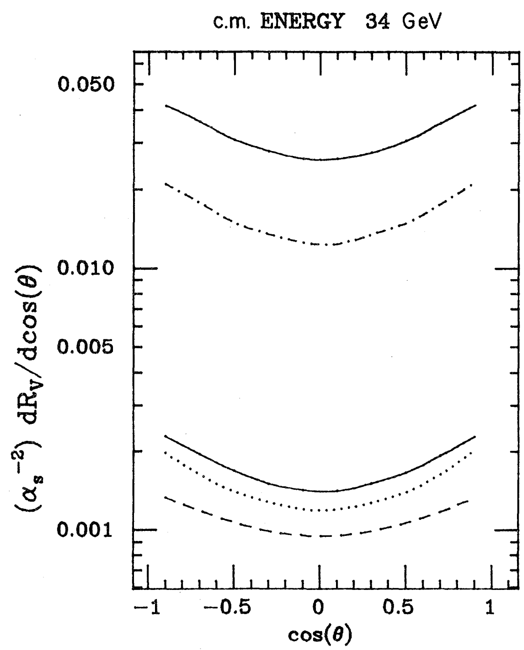

FIG. 6. Angular distribution of vector-meson production relative to the beam direction at 34 GeV. The upper and lower solid curves correspond to $s\bar{b}$ and $c\bar{c}$ states. The remaining curves correspond to $s\bar{c}$ (dot-dash), $s\bar{s}$ (dotted), and $b\bar{c}$ (dashed) vector mesons.

squared. The distribution in x_V peaks therefore near

$$(x_V)_{\rm peak}=\frac{M_V}{M_V+m_2}\left[1+\frac{4m_Vm_2}{s}\right] \tag{2.29}$$

with m_2 being the mass of the lighter constituent of the $^3S_1$ bound state. Hadronization of the associated quarks will be expected to smear this peak.

The angular distribution of the produced vector relative to the beam direction is shown in Fig. 6 again for $V=s\bar{c}$, $s\bar{b}$, $c\bar{c}$, $c\bar{b}$, and $b\bar{b}$ at 34 GeV.

III. CONCLUSION

A major question to be decided in inclusive ψ (or other heavy-meson) production is whether, at energies large compared to the meson mass, ψ production becomes qualitatively indistinguishable from the production of light-quark states such as ρ or ω whose multiplicity presumably grows at least logarithmically with energy due to nonperturbative effects or whether, for quarks of mass much greater than the QCD scale parameter Λ, the production mechanism is perturbative and therefore qualitatively different from that of ρ's and ω's.

In the former case at energies far above the ψ mass, one might find an R_ψ of the order of unity.[5] In the latter case, for which the calculations of this article are relevant, R_ψ remains very small and the differential cross section in ψ energy reflects a parton-level kinematic enhancement at a relatively large energy.

The associated-production mechanism discussed here dominates over the previously calculated ψ-gluon-gluon final state and over the prediction of the ψ-gluon color-evaporation model. The production rate of flavor-neutral vector mesons satisfies a universality of the form

$$M_VR_V(s)=\alpha_s{}^2Q^2f(M_V{}^2/s)\ ,$$

where Q is the quark charge.

The asymptotic dominance of associated production over unaccompanied heavy vectors may be a general prediction of QCD irrespective of the incident beams. This follows from the fact that the matrix element of onium production without associated open flavor is the trace of an odd number of vector vertices and quark propagators. It is therefore proportional to quark masses which may become negligible at high energy.

In the charmonium zero-binding approximation which we have treated, a very near cancellation between a rational number and the logarithm of 2 suppresses production of $^3S_1$ states of like quarks, leading to a relative enhancement of heavier states of dissimilar quarks. Thus the $c\bar{b}$ state is produced at a rate comparable to ψ production and the $s\bar{b}$ rate is significantly greater than that of $s\bar{c}$. The terms in square brackets in Eq. (2.25), suppressing production of the ψ and other flavor-neutral vector mesons, occur also with γ_5 coupling of the current to the quarks. Thus the decay of the Z^0 into $\psi+c+\bar{c}$ is similarly suppressed.

We rely on the heavy-quark mass to block the infrared singularities of QCD. For this reason our predictions of strange-quark bound states are, perhaps, somewhat more speculative than those for charm and bottom. We have also neglected neutral-current contributions which could become large at high energies.

The distributions presented here in Figs. 4–6 were generated using the hadronic tensor of Appendix A and the Monte Carlo integration routine of T. Gottschalk. The analytic results were obtained using the SCHOONSHIP program of algebraic manipulation.

After this work was completed we received an advance copy of a paper by Grayson and Tuite[11] which treats the production of like-quark vector states (ψ and Υ) by the mechanism of Fig. 3. We differ from their results in that we find an asymptotically constant rate relative to μ pairs whereas they seem to find a relative rate decreasing at high energy. Our analytic results and extension of the model to flavor-nonsinglet $^3S_1$ states go beyond their treatment.

Note added in proof. The color-evaporation model of Ref. 1 was in fact considered earlier in Ref. 5.

ACKNOWLEDGMENTS

We acknowledge frequent discussions on ψ production with Dr. E. L. Berger, Dr. D. Jones, and Dr. N. Wright at Argonne. We thank Dr. E. L. Berger, in particular, for reminding us of the empirical regularity (2.3) in leptonic-decay widths.

APPENDIX

In the case of inclusive production of a general $^3S_1(q_1,\bar{q}_2)$ state, the reduced hadronic tensor of (2.10) is rather bulky. The quark q_1 has mass m_1 and charge Q_1; the antiquark $\bar{q}_2$ has mass m_2 and charge $(-Q_2)$. The bound state has momentum, mass, and charge $2P_\mu$, $m=m_1+m_2$, and $Q_V=Q_1-Q_2$. The external quark and antiquark have momenta P_2 and P_1, respectively, as shown in Fig. 3. The incoming virtual photon has four-momentum q ($q^2=s$) and we use the kinematic relations of (2.6) and (2.18) to (2.20) and (2.22). In terms of these variables we define

$$C_1=\frac{mQ_1}{m_2H_1}\,, \tag{A1}$$

$$C_2=\frac{mQ_2}{m_1H_2}\,, \tag{A2}$$

$$C_4=\frac{C_1}{H_1}+\frac{C_2}{H_2}\,, \tag{A3}$$

$$A_0=\frac{C_1}{H_b}+\frac{C_2}{H_c}\,, \tag{A4}$$

$$C_-=\frac{C_1}{H_1}-\frac{C_2}{H_2}+\frac{C_1}{H_b}-\frac{C_2}{H_c}\,, \tag{A5}$$

$$y=s-4p\cdot q\,. \tag{A6}$$

We also define

$$q_{5\mu}=(P_{1\mu}-P_{2\mu})/2 \tag{A7}$$

and the tensors

$$D_V(P_i,P_j)=\frac{1}{2}\left[P_{i\mu}-\frac{q\cdot P_i}{q^2}q_\mu\right]\left[P_{j\nu}-\frac{q\cdot P_j}{q^2}q_\nu\right]+(\mu-\nu)\,, \tag{A8}$$

$$D_H(\mu,\nu)=\left[g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right]. \tag{A9}$$

The reduced hadronic tensor of (2.10) is then given by

$$\begin{aligned}
H_{\mu,\nu}=D_H(\mu,\nu)\{&C_-^2[-8m^2(p\cdot q)^2+8m^2(q\cdot q_5)^2-8Rm^2(p\cdot q)^2+16Rsm^4+2sym^2-2y(p\cdot q)^2]\\
&+A_0C_-(-48m^2p\cdot qq\cdot q_5+48H_pm^2m_rp\cdot q-4yp\cdot qq\cdot q_5)\\
&+A_0^2[16m^2(p\cdot q)^2-24Rm^2(p\cdot q)^2+8Rsm^4+6sym^2\\
&\quad-16H_pm^2m_rq\cdot q_5-16y(p\cdot q)^2-2y(q\cdot q_5^2)]\\
&+C_4C_-(16m^2p\cdot qq\cdot q_5-16H_pm^2m_rp\cdot q+4yp\cdot qq\cdot q_5)\\
&+C_4A_0[-16Rm^2(p\cdot q)^2+16Rsm^4+4sym^2+4y(q\cdot q_5)^2]\\
&+C_4^2[-8m^2(p\cdot q^2)+8Rm^2(p\cdot q)^2+8Rsm^4+16H_pm^2m_rq\cdot q_5-8H_p^2m^2-2y(q\cdot q_5)^2]\}\\
+D_V(p,p)\{&C_-^2[96Rm^4-8Rsm^2+16ym^2+8(p\cdot q)^2]\\
&+A_0C_-(-96m^2q\cdot q_5-32Rm^2q\cdot q_5+192Rm^4m_r\\
&\quad-16H_pm_rp\cdot q+32ym^2m_r+16p\cdot qq\cdot q_5)\\
&+A_0^2[-96m^2m_rq\cdot q_5-64m^2p\cdot q-32Rm^2m_rq\cdot q_5\\
&\quad+64Rm^2p\cdot q+96Rm^4-80Rsm^2-8R(p\cdot q)^2+8R(q\cdot q_5)^2-96R^2m^4\\
&\quad+72sm^2-16sy-16H_pm_rq\cdot q_5+8(p\cdot q)^2]\\
&+C_4C_-(64m^2q\cdot q_5-16p\cdot qq\cdot q_5)\\
&+C_4A_0[64m^2m_rq\cdot q_5-16m_rp\cdot qq\cdot q_5-16sm^2+8sm_rq\cdot q_5-16(q\cdot q_5)^2]\\
&+C_4^2[8Rsm^2-16sm^2+8(q\cdot q_5)^2]\}\\
+D_V(q_5,p)\{&C_-^2(32m^2m_rp\cdot q)\\
&+A_0C_-(32m^2p\cdot q-16m_rp\cdot qq\cdot q_5-32Rm^2p\cdot q-128H_pm^2+16H_pp\cdot q)\\
&+A_0^2[128m^2m_rp\cdot q-16m_r(p\cdot q)^2+16H_pq\cdot q_5]\\
&+C_4C_-(-32m^2m_rq\cdot q_5+64m^2p\cdot q-192Rm^4)\\
&+C_4A_0[64m^2m_rp\cdot q+64m^2q\cdot q_5+16m_r(q\cdot q_5)^2+64Rm^2q\cdot q_5-192Rm^4m_r
\end{aligned}$$

$$-48sm^2m_r-16H_p q\cdot q_5]+C_4{}^2(-64m^2q\cdot q_5)\}$$

$$+D_V(q_5,q_5)\{C_-{}^2[8sm^2-8(p\cdot q)^2]+8(p\cdot q)^2A_0{}^2+C_4C_-(-32m^2m_r p\cdot q+16p\cdot qq\cdot q_5)$$

$$+C_4A_0(-128m^2p\cdot q+16sm^2)$$

$$+C_4{}^2[32m^2m_r q\cdot q_5+96Rm^4-8sm^2-8(q\cdot q_5)^2]\}\ . \tag{A10}$$

---

[1]H. Fritzsch and J. H. Kuhn, Phys. Lett. 90B, 164 (1980).

[2]M. Strovink, in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn,* edited by W. Pfeil (Universität Bonn, Bonn, 1981).

[3]E. L. Berger and D. Jones, Phys. Rev. D 23, 1521 (1981); W.-Y. Keung, in Proceedings of Cornell Z^0 Workshop, 1981 (unpublished).

[4]W.-Y. Keung, Phys. Rev. D 23, 2072 (1981); J. H. Kuhn and H. Schneider, Z. Phys. C 11, 263 (1981); Phys. Rev. D 24, 2996 (1981).

[5]G. P. Kane, J. P. Leveille, and D. M. Scott, Phys. Lett. 85B, 115 (1979).

[6]G. C. Branco, H.-P. Nilles, and K. H. Streng, Phys. Lett. 85B, 269 (1979).

[7]T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975); Phys. Rev. D 12, 1404 (1975). V. A. Novikov *et al.*, Phys. Rep. 41C, 1 (1978).

[8]D. R. Yennie, Phys. Rev. Lett. 34, 239 (1975). G. J. Gounaris, Phys. Lett. 62B, 213 (1976).

[9]Similar diagrams for higher-twist pion production in $e^+e^-\rightarrow\pi X$ and other processes were first discussed by E. L. Berger in, e.g., Z. Phys. C 4, 289 (1980). See also E. L. Berger and S. J. Brodsky, Phys. Rev. D 24, 2428 (1981).

[10]P. Mackenzie and P. Lepage, Phys. Rev. Lett. 47, 1244 (1981).

[11]S. L. Grayson and M. P. Tuite, Z. Phys. C (to be published).