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CP-violating effects in heavy-meson systems

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We calculate the dilepton charge asymmetry in the neutral-conjugate-heavy-meson systems produced in e^+e^- annihilation. This asymmetry, which is a measure of the intrinsic CP violation in the mass matrix, is calculated in the Kobayashi-Maskawa (KM) model as well as the Higgs-boson model of CP nonconservation. While the charge asymmetry is small for the D^0-$\bar{D}^0$ and B^0-$\bar{B}^0$ systems in both models, it is predicted to be quite large ($10^{-1}-10^{-2}$) in the T^0-$\bar{T}^0$ system in the Higgs-boson scheme. Thus if experiments to measure the dilepton asymmetry are feasible (but could be considerably difficult in practice), only the T^0-$\bar{T}^0$ system can distinguish between the Higgs-boson-exchange model and the KM model. A T-violating polarization of the lepton in the semileptonic decay of a meson cannot arise in the KM model. However, even in the Higgs-boson model the transverse polarization of the muon normal to the decay plane in the semileptonic kaon decay although nonzero is still quite small (at the level $\sim 10^{-4}$) unless the ratio of the two vacuum expectation values $v_2{}^2/v_3{}^2$ is unexpectedly large. The T-odd τ-lepton polarization in B-meson decay is expected to be $\sim 10^{-2}$, thus conceivably measurable.

I. INTRODUCTION

To date CP violation has been experimentally observed only in the neutral-kaon system, and theoretically a number of different models have been proposed to explain the phenomenon of CP nonconservation. Current models can be cataloged into two classes. In one class of models the breakdown of CP invariance arises from the exchange of a gauge boson, as in the Kobayashi-Maskawa (KM) model[1] and the left-right-symmetric gauge model.[2] In another class of models CP violation originates in the Higgs-boson interaction, and two or three Higgs doublets are needed to spontaneously break the CP symmetry. In this paper we confine our attention to the KM model and the Higgs-boson model with natural flavor conservation, as originally stressed by Weinberg.[3] These two schemes are in some sense orthogonal.

Recently, the Weinberg model of CP nonconservation has been criticized on the ground that in the context of this model the magnitude of the ratio of two CP-violating parameter ϵ' and ϵ in the K^0-$\bar{K}^0$ system is unacceptably large.[4–6] However, there are sizable theoretical uncertainties associated with these calculations, having to do with penguin diagrams and long-distance effects (i.e., contributions from low-mass intermediate states) which may have important consequences for CP violation.[7–8] In fact, long-distance effects may reduce (ϵ'/ϵ) by a factor of $\frac{1}{2.5}$ using an estimate given by Hill.[7] It is, therefore, too early to conclude that a serious problem exists in the Weinberg model. Yet, it is important to have some tests to distinguish the Higgs-boson model from the KM model. Among such tests, in which the predictions of these two schemes could differ substantially, are the dilepton charge asymmetry of conjugate heavy-neutral-meson pairs in e^+e^- annihilation, CP-odd lepton polarization in the semileptonic decay of a meson, the neutron electric dipole moment, the decay-rate difference in semileptonic decays (for example, in $D\rightarrow K\pi l\nu$ decay[9]), and P- and T-violating effects in ψ' or ψ decay.[10] Among these possible tests, only the first two CP-violating effects will be considered in this paper because the charge asymmetry is a quantity which is less sensitive to the change of the Higgs-boson mixing parameters, while the T-odd polarization is predicted to be zero in the KM model (except for possible final-state electromagnetic interactions).

In Sec. II the CP phenomenology in the neutral-meson systems is briefly reviewed. For completeness the main features of the Weinberg model of CP violation are also reviewed in Sec. III, and several constraints on the Higgs-boson mixing parameters are given. Section IV is devoted to the computation of the charge asymmetry or *CP* impurity in both the KM and Higgs-boson-exchange models. We start from a gauge-invariant effective Hamiltonian, in which no assumptions about the relative magnitudes of the masses of particles are made. Section V gives a general treatment of the *T*-violating lepton polarization in the semileptonic decay of a meson, and some experimental considerations as well. Section VI presents our conclusions, and some calculational details are given in an Appendix.

II. *CP* VIOLATION OF THE NEUTRAL-MESON MIXING

In this section we describe briefly the main features of the phenomenology of *CP* violation in the neutral-meson P^0-$\bar{P}^0$ system. The proper-time evolution of the P^0-$\bar{P}^0$ wave function is described by a 2×2 non-Hermitian mass matrix

$$H\begin{pmatrix}P^0\\ \bar{P}^0\end{pmatrix}\equiv\left(M-\frac{i}{2}\Gamma\right)\begin{pmatrix}P^0\\ \bar{P}^0\end{pmatrix}=\begin{pmatrix}M_{11}-\frac{i}{2}\Gamma_{11} & M_{12}-\frac{i}{2}\Gamma_{12}\\ M_{21}-\frac{i}{2}\Gamma_{21} & M_{22}-\frac{i}{2}\Gamma_{22}\end{pmatrix}\begin{pmatrix}P^0\\ \bar{P}^0\end{pmatrix}, \tag{2.1}$$

where both M and Γ are 2×2 Hermitian matrices; M_{11}, M_{22}, Γ_{11}, and Γ_{22} are thus real, and $M_{21}=M_{12}^*$, $\Gamma_{21}=\Gamma_{12}^*$.

Assuming *CPT* invariance (which implies $M_{11}=M_{22}=m$ and $\Gamma_{11}=\Gamma_{22}=\Gamma$), and adopting the phase convention $CP\,|\,P^0\rangle=-\,|\,\bar{P}^0\rangle$, the eigenstates of definite mass $m_{1,2}$ and width $\Gamma_{1,2}$ are

$$\begin{aligned}P_1&=\frac{1}{[2(1+|\epsilon|^2)]^{1/2}}[(1+\epsilon)P^0+(1-\epsilon)\bar{P}^0)\ ,\\ P_2&=\frac{1}{[2(1+|\epsilon|^2)]^{1/2}}[(1+\epsilon)P^0-(1-\epsilon)\bar{P}^0]\ ,\end{aligned} \tag{2.2}$$

where the *CP*-violating parameter ϵ is given by

$$\frac{1-\epsilon}{1+\epsilon}=\left[\frac{M_{12}^*-\frac{i}{2}\Gamma_{12}^*}{M_{12}-\frac{i}{2}\Gamma_{12}}\right]^{1/2}, \tag{2.3}$$

or

$$\epsilon=\frac{i\,\mathrm{Im}M_{12}+\mathrm{Im}\Gamma_{12}/2}{[(M_{12}^*-i\Gamma_{12}^*/2)(M_{12}-i\Gamma_{12}/2)]^{1/2}+\mathrm{Re}M_{12}-i\,\mathrm{Re}\Gamma_{12}/2} \tag{2.4a}$$

$$=\frac{[(M_{12}^*-i\Gamma_{12}^*/2)(M_{12}-i\Gamma_{12}/2)]^{1/2}-\mathrm{Re}M_{12}+i\,\mathrm{Re}\Gamma_{12}/2}{-i\,\mathrm{Im}M_{12}-\mathrm{Im}\Gamma_{12}/2}\ . \tag{2.4b}$$

Because of the freedom of adjusting the phase of one of the neutral-meson states, ϵ is phase-dependent and thus not a physical quantity. If *CP* is good, P_1 and P_2 are *CP* eigenstates with eigenvalues -1 and $+1$, respectively. The corresponding eigenvalues for the mass eigenstates P_1 and P_2 are

$$m_{1,2}=m\pm\mathrm{Re}\left[\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)\left(M_{12}^*-\frac{i}{2}\Gamma_{12}^*\right)\right]^{1/2}, \tag{2.5a}$$

$$\tfrac{1}{2}\Gamma_{1,2}=\tfrac{1}{2}\Gamma\mp\mathrm{Im}\left[\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)\left(M_{12}^*-\frac{i}{2}\Gamma_{12}^*\right)\right]^{1/2}, \tag{2.5b}$$

respectively. The mass and decay-rate difference are then

$$\begin{aligned}\Delta\Gamma&=\Gamma_1-\Gamma_2\\ &=-4\,\mathrm{Im}\left[\left(M_{12}-\frac{i}{2}\Gamma_{12}\right)\left(M_{12}^*-\frac{i}{2}\Gamma_{12}^*\right)\right]^{1/2}\end{aligned} \tag{2.6a}$$

$$\Delta m = m_1 - m_2$$
$$= 2\,\mathrm{Re}\left[\left[M_{12} - \frac{i}{2}\Gamma_{12}\right]\left[M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*}\right]\right]^{1/2} . \tag{2.6b}$$

If $\epsilon \ll 1$, it follows that

$$\Delta\Gamma \approx 2\,\mathrm{Re}\Gamma_{12} \text{ and } \Delta m \approx 2\,\mathrm{Re}M_{12} . \tag{2.7}$$

The off-diagonal mass-matrix elements are related to those of the effective Hamiltonian H_{eff} by the perturbation expansion

$$M_{12} = \langle P^0 | H_{\mathrm{eff}}^{\Delta=2} | \bar{P}^0 \rangle + \sum_n \mathrm{P} \frac{\langle P^0 | H_{\mathrm{eff}}^{\Delta=1} | n \rangle \langle n | H_{\mathrm{eff}}^{\Delta=1} | \bar{P}^0 \rangle}{m_p - E_n} , \tag{2.8a}$$

$$\Gamma_{12} = 2\pi \sum_F \rho_F \langle P^0 | H_{\mathrm{eff}}^{\Delta=1} | F \rangle \langle F | H_{\mathrm{eff}}^{\Delta=1} | \bar{P}^0 \rangle , \tag{2.8b}$$

where Δ stands for $|\Delta S|, |\Delta C|, \ldots$ etc., depending on the $P^0-\bar{P}^0$ system under consideration, P denotes the principal value, ρ_F is the density of final states F, and m_p is defined by (*CPT* invariance is assumed)

$$m_p = \langle P^0 | H_{\mathrm{st}} | P^0 \rangle = \langle \bar{P}^0 | H_{\mathrm{st}} | \bar{P}^0 \rangle , \tag{2.9}$$

with H_{st} the Hamiltonian of the strong interaction. The second term on the right-hand side of Eq. (2.8a) represents large-distance contributions to M_{12} arising from low-mass intermediate states.

The PCAC (partial conservation of axial-vector current) relation under the normalization condition (2.9) is, for example,

$$\langle 0 | \bar{s}\gamma_\mu(1-\gamma_5)d | K^0 \rangle = i f_K q_\mu / \sqrt{2m_K} . \tag{2.10}$$

However, care must be taken when one adopts the opposite *CP* phase convention $CP|P^0\rangle = |\bar{P}^0\rangle$; the PCAC relation for $\bar{K}^0$ reads

$$\langle 0 | \bar{d}\gamma_\mu(1-\gamma_5)s | \bar{K}^0 \rangle = -i f_K q_\mu / \sqrt{2m_k} . \tag{2.11}$$

Finally, we note that heavy mesons have shorter lifetimes, thus the useful quantities measuring the mixing are the time-integrated ones. One such quantity is[11]

$$\rho = \frac{4(\Delta m/\Gamma)^2 + (\Delta\Gamma/\Gamma)^2}{2 + 4(\Delta m/\Gamma)^2 - (\Delta\Gamma/\Gamma)^2} . \tag{2.12}$$

The strength of the mixing depends on $\Delta m/\Gamma$ and $\Delta\Gamma/\Gamma$.

III. MODELS

There are essentially two different models of *CP* nonconservation which are most frequently used:[12]

(1). The Kobayashi-Maskawa (KM) *model.* In the standard SU(2)×U(1) model of electromagnetic and weak interactions with six quarks and one Higgs doublet, *CP* violation enters into the gauge sector through the complex phase δ in the unitary KM matrix needed to diagonalize the mass matrix

$$U = \begin{bmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{bmatrix} = \begin{bmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{bmatrix} , \tag{3.1}$$

where $s_i \equiv \sin\theta_i$ and $c_i \equiv \cos\theta_i$ ($i = 1,2,3$). In the KM model *CP* violation is imposed in the initial Lagrangian and is not a consequence of spontaneous *CP*-symmetry breaking. The smallness of *CP* noninvariance as compared to the ordinary weak interaction is ascribed to the small values of the quark mixing angles θ_2 and θ_3, and/or to the size of δ; but why these parameters are small is still unknown.

(2) Higgs-boson-mixing models. In this type of model *CP* violation arises either from a scalar self-interaction or from spontaneous symmetry breaking. The realization of spontaneous violation of *CP* symmetry could be different in character for different choices of the Higgs-boson masses:

(a) If the Higgs bosons (neutral and charged) are light, say of the order of 10 GeV, then conservation of natural flavor has to be imposed to eliminate the unwanted strangeness-changing current at the tree level by the neutral Higgs boson. As a result, at least three Higgs doublets are needed to implement *CP* violation, as originally shown by Weinberg.[3] In this model *CP* nonconservation arises from the exchange of light charged Higgs bosons and is suppressed by a factor ${m_q}^2/{m_H}^2$ for light quarks relative to the *CP*-even amplitude.

(b) The other possibility is that the Higgs bosons are very heavy, say of the order of several hundred GeV, so that the $\Delta S=2$ neutral current at the tree level is naturally suppressed. Within the context of this model spontaneous *CP* violation can be introduced by only two Higgs doublets[13,14] and occurs in both the gauge sector (i.e., the KM matrix is allowed to be complex) and the Higgs sector. This scheme is essentially a superweak theory at least for light hadrons.

In the following we will focus on the former version of spontaneous *CP* violation and for completeness we review simply the characteristic features of this scheme. In the Weinberg model of *CP* noninvariance there are three Higgs doublets:

$$\phi_i=\begin{bmatrix}\phi_i^+\\ \phi_i^0\end{bmatrix} \quad (i=1,2,3)\ . \tag{3.2}$$

Each Higgs doublet develops a vacuum expectation value,

$$\langle\phi_i\rangle_0=\begin{bmatrix}0\\ v_i e^{i\theta_i}\end{bmatrix}, \tag{3.3}$$

with v_i real. One may next invoke the following discrete symmetries to conserve the natural flavors:

$$\begin{aligned}&\phi_1\to-\phi_1;\ D_R\to-D_R\ ,\\ &\phi_2\to-\phi_2;\ U_R\to-U_R\ ,\\ &\phi_3\to-\phi_3;\ E_R\to-E_R\ ,\end{aligned} \tag{3.4}$$

where $D^T=(d,s,b)$, $U^T=(u,c,t)$, and $E^T=(e,\mu,\tau)$, so that the third Higgs doublet does not couple to quarks.

The most general quark—Higgs-boson and lepton—Higgs-boson Yukawa interactions consistent with the discrete symmetries (3.4) as well as SU(2)$\times$U(1) gauge invariance have the form

$$\begin{aligned}\mathscr{L}_Y=&g_{ij}^{(1)}\bar{D}_R^i(\phi_1^{+*}U_L^j+\phi_1^{0*}D_L^j)\\ &+g_{ij}^{(2)}\bar{U}_R^i(\phi_2^0U_L^j-\phi_2^+D_L^j)\\ &+g_{ij}^{(3)}\bar{E}_R^i(\phi_3^{+*}N_L^j+\phi_3^{0*}E_L^j)+\text{H.c.}\ ,\end{aligned} \tag{3.5}$$

where $N^T=(\nu_e,\nu_\mu,\nu_\tau)$. The Yukawa coupling constants $g_{ij}^{(n)}$ are chosen to be real, so that *CP* invariance is a good symmetry of the Lagrangian $\mathscr{L}_Y$. We may next expand (3.5) by defining

$$\phi_i^0=e^{i\theta_i}(v_i+\rho_i+i\chi_i)\ . \tag{3.6}$$

After spontaneous symmetry breaking, the mass matrices of quarks can be made to be real by readjusting the phases of the quarks. Indeed it has been shown that the gauge interactions always conserve *CP* invariance for an arbitrary number of quark generations.[15] In terms of the six new quarks, whose phases are fixed by the gauge interaction, the relevant Higgs-boson Yukawa interactions are

$$\begin{aligned}\mathscr{L}_Y^{\pm}=&\frac{\phi_1'^{\,+}}{v_1}\bar{U}_LKM_DD_R-\frac{\phi_2'^{\,+}}{v_2}\bar{U}_RKM_UD_L\\ &+\frac{\phi_3'^{\,+}}{v_3}\bar{N}_LM_EE_R+\text{H.c.}\end{aligned} \tag{3.7}$$

for charged Higgs bosons with $\phi_i'=\phi_ie^{i\theta_i}$ and where K is a real KM matrix, and

$$\begin{aligned}\mathscr{L}_Y^0=&\frac{\rho_1}{v_1}\bar{D}M_DD+i\frac{\chi_1}{v_1}\bar{D}M_D\gamma_5D+\frac{\rho_2}{v_2}\bar{U}M_UU\\ &-i\frac{\chi_2}{v_2}\bar{U}M_U\gamma_5U+\frac{\rho_3}{v_3}\bar{E}M_EE+i\frac{\chi_3}{v_3}\bar{E}M_E\gamma_5E\ ,\end{aligned} \tag{3.8}$$

where M_D, M_U, and M_E are diagonal mass matrices.

The Yukawa interactions $\mathscr{L}_Y^{\pm}$ and $\mathscr{L}_Y^0$ are still *CP*-invariant. The scalar fields ϕ_i, ρ_i, and χ_i are, however, still not the physical Higgs states with definite masses, and *CP* invariance of the Yukawa interactions does not necessarily imply the reality of the mass matrices of charged and neutral Higgs bosons. Deshpande and Ma[18] have shown that the Higgs potential gives a complex mass matrix for the ϕ_i'. The unitary matrix U_H^+, which relates the $\phi_i'^{\,+}$ to the physical charged Higgs states H_i^+,[15]

$$\begin{bmatrix}\phi_1'^{\,+}\\ \phi_2'^{\,+}\\ \phi_3'^{\,+}\end{bmatrix}=U_H^+\begin{bmatrix}G^+\\ H_1^+\\ H_2^+\end{bmatrix}, \tag{3.9}$$

with G^+ a charged Goldstone boson, is also complex. There are three arbitrary phases in U_H^+, and two of them can be removed by redefining H_1^+ and H_2^+. Hence U_H^+ can be parametrized exactly in the same way as the KM matrix (3.1). Because the charged Goldstone boson is given by

$$G^+=\frac{1}{v}(v_1\phi_1'^{\,+}+v_2\phi_2'^{\,+}+v_3\phi_3'^{\,+})\ , \tag{3.10}$$

and also

$$v\equiv(v_1{}^2+v_2{}^2+v_3{}^2)^{1/2}=(2\sqrt{2}G_F)^{-1/2} \tag{3.11}$$

with G_F a Fermi constant, it follows that

$$v_1=\bar{c}_1 v,\quad v_2=\bar{s}_1\bar{c}_2 v\ ,\quad v_3=\bar{s}_1\bar{s}_2 v\ , \tag{3.12}$$

where $\bar{s}_i\equiv\sin\bar{\theta}_i$, $\bar{c}_i\equiv\cos\bar{\theta}_i$, and $\bar{\theta}_i$ ($i=1,2,3$) are Higgs-boson mixing angles. From (3.12) we note that the mixing angles $\bar{\theta}_1$ and $\bar{\theta}_2$ are determined by the vacuum expectation values v_1, v_2, and v_3, but the magnitude of $\bar{\theta}_3$ and CP-violating phase δ_H depend on the parameters in the Higgs potential.

The Yukawa interaction $\mathscr{L}_Y^{\pm}$ can be recast in terms of the physical charged Higgs fields in the form (in the unitary gauge)[16,17]

$$\mathscr{L}_Y^{\pm}=(2\sqrt{2}G_F)^{1/2}\sum_{i=1}^{2}(\alpha_i\bar{U}_L K M_D D_R H_i^+ +\beta_i\bar{U}_R M_U K D_L H_i^+ +\gamma_i\bar{N}_L M_E E_R H_i^+ +\text{H.c.})\ , \tag{3.13}$$

with

$$\alpha_1=-\frac{\bar{s}_1\bar{c}_3}{\bar{c}_1}\ ,\quad \beta_1=\frac{-\bar{c}_1\bar{c}_2\bar{c}_3+\bar{s}_2\bar{s}_3e^{i\delta_H}}{\bar{s}_1\bar{c}_2}\ ,\quad \gamma_1=\frac{\bar{c}_1\bar{s}_2\bar{c}_3+\bar{c}_2\bar{s}_3e^{i\delta_H}}{\bar{s}_1\bar{s}_2}\ ,$$
$$\alpha_2=-\frac{\bar{s}_1\bar{s}_3}{\bar{c}_1}\ ,\quad \beta_2=\frac{-\bar{c}_1\bar{c}_2\bar{s}_3-\bar{s}_2\bar{c}_3e^{i\delta_H}}{\bar{s}_1\bar{c}_2}\ ,\quad \gamma_2=\frac{\bar{c}_1\bar{s}_2\bar{s}_3-\bar{c}_2\bar{c}_3e^{i\delta_H}}{\bar{s}_1\bar{s}_2}\ . \tag{3.14}$$

From (3.14) we observe that

$$\text{Im}(\alpha_2\beta_2^*)=-\text{Im}(\alpha_1\beta_1^*)\ ,\quad \text{Im}(\alpha_2\gamma_2^*)=-\text{Im}(\alpha_1\gamma_1^*)\ ,\quad \text{Im}(\beta_2\gamma_2^*)=-\text{Im}(\beta_1\gamma_1^*)\ , \tag{3.15}$$

and

$$\frac{\text{Im}(\alpha_1\gamma_1^*)}{\text{Im}(\alpha_1\beta_1^*)}=-\frac{v_2^2}{v_3^2}\ ,\quad \frac{\text{Im}(\beta_1\gamma_1^*)}{\text{Im}(\alpha_1\beta_1^*)}=\frac{v_1^2}{v_3^2}\ . \tag{3.16}$$

On the other hand, the mass matrix of the real scalar fields ρ_i and χ_i is real, and ρ as well as χ get mixing. It turns out that the unitary matrix U_H, that diagonalizes the mass matrix of the neutral scalar fields,

$$(\rho_1,\rho_2,\rho_3,\chi_1,\chi_2,\chi_3)=(G_0,H_1,H_2,H_3,H_4,H_5)U_H^T\ , \tag{3.17}$$

with G_0 a neutral Goldstone boson, is likewise real. The neutral-Higgs-boson Yukawa interaction (3.8) then becomes

$$\mathscr{L}_Y^0=(2\sqrt{2}G_F)^{1/2}\sum_{i=1}^{5}(\zeta_{1i}\bar{D}M_D DH_i+i\zeta_{2i}\bar{D}M_D\gamma_5 DH_i+\zeta_{3i}\bar{U}M_U UH_i$$
$$+i\zeta_{4i}\bar{U}M_U\gamma_5 UH_i+\zeta_{5i}\bar{E}M_E EH_i+i\zeta_{6i}\bar{E}M_E\gamma_5 EH_i)\ , \tag{3.18}$$

where the coupling constants ζ_{ij} ($i=1,\ldots,6$; $j=1,\ldots,5$) are real. Since $\bar{U}U$ and $i\bar{U}\gamma_5 U$ have opposite P, T, and CP transformation properties (similarly for leptons and charge $-\frac{1}{3}$ quarks), P and CP are violated through the exchange of the neutral Higgs boson.[18] However, unless the strong CP problem is solved, the experimental test of $\mathscr{L}_Y^0$ is subject to an ambiguity.

In some sense the Higgs-boson-exchange model is more attractive and appealing than the KM model because not only is CP violation generated spontaneously, but also the CP-odd effect is naturally suppressed by a factor m_q^2/m_H^2 at least for light quarks. Nonetheless, in view of the fact that there are too many free parameters it is usually difficult to extract any quantitative predictions from this model. A useful relation among the various coupling constants and the masses of charged Higgs bosons, however, could be obtained from consideration of the K^0-$\bar{K}^0$ system, since the K_L-K_S mass difference and the CP-violating parameters have been measured precisely there.

If the phase of K^0 is chosen so that the stationary $I=0$ component of $K^0\to 2\pi$ amplitude is real, the CP-violating parameter ϵ in the K^0-$\bar{K}^0$ system is then approximately given by[19]

$$\epsilon\simeq\frac{1}{2\sqrt{2}}\epsilon_m\left[1+\frac{2\xi}{\epsilon_m}\right]e^{i\phi}\ , \tag{3.19}$$

where

$$\epsilon_m = \text{Im}M_{12}/\text{Re}M_{12},$$

$$\tan\xi = \frac{\text{Im}\langle \pi\pi(I=0) \,|\, H_w \,|\, K^0\rangle}{\text{Re}\langle \pi\pi(I=0) \,|\, H_w \,|\, K^0\rangle},$$

and calculations are done with the KM matrix, which fixes the relative phases of the quarks. Deshpande and Sanda[4] have independently calculated ξ from the Higgs-boson penguin contribution to $K^0 \to 2\pi$ decays, and $\text{Im}M_{12}$ using the following effective Lagrangian in the four-quark model[20]:

$$\mathscr{L}_{\text{eff}} = i\tilde{g}[\bar{s}_i(x)\gamma_\mu(1-\gamma_5)d_j(x)]\partial_\nu \times[\bar{s}_j(x)\gamma_\nu\gamma_\mu(1-\gamma_5)d_i(x)] + \text{H.c.} \quad (3.20)$$

Here $\tilde{g} = (1/32\pi^2)\, G_F{}^2 m_c{}^2 m_s\, (\sin\theta_C\cos\theta_C)^2 A^*$, and $A = \sum_{i=1}^{2} \alpha_i\beta_i^*/m_{H_i}{}^2$ where i and j in (3.20) are color indices. From (3.20) one can calculate the imaginary part of M_{12} (Ref. 21),

$$\text{Im}M_{12} = \frac{1}{32\pi^2} G_F{}^2 m_c{}^2 m_K{}^3\, (\sin\theta_C\cos\theta_C)^2 f_K{}^2 \left[\frac{1}{3} + \frac{m_s{}^{*2} - m_d^{*2}}{m_K^2}\right] \text{Im}A\ . \quad (3.21)$$

Combining (3.21) with

$$\text{Re}M_{12} = \frac{G_F{}^2}{12\pi^2} m_c^2 m_K f_K{}^2 (\sin\theta_C\cos\theta_C)^2, \quad (3.22)$$

one finds

$$\epsilon_m = \tfrac{1}{8} m_K{}^2 \left[1 + 3\frac{m_s^{*2} - m_d^{*2}}{m_K{}^2}\right] \text{Im}A\ , \quad (3.23)$$

with m_s^* and m_d^* the constituent quark masses of the s and d quarks, respectively. Because of the uncertainties associated with the penguin diagram and the masses of the charged Higgs bosons, Deshpande and Sanda concluded that

$$10 < 2\xi/\epsilon_m < 30\ . \quad (3.24)$$

From Eqs. (3.15), (3.23), and (3.24), and the experimental result $\epsilon \approx 2.274\times 10^{-3} e^{i\pi/4}$ it follows that

$$2.8\times 10^{-3}\ \text{GeV}^{-2} < \frac{2\sqrt{2}}{m_0{}^2} < 8.0\times 10^{-3}\ \text{GeV}^{-2}\ , \quad (3.25)$$

where $2\sqrt{2}/m_0{}^2 \equiv \text{Im}A = \text{Im}(\alpha_1\beta_1^*)\,(1/m_{H_1}{}^2 - 1/m_{H_2}{}^2)$, and m_s^* and m_d^* are assumed to be 450 and 300 MeV, respectively. Previously m_0 has been estimated to be 2 GeV.[20]

Although the contributions from the t quark and from the two-Higgs-boson-exchange diagram have been neglected in deriving (3.25), the use of (3.25) is nonetheless justified. Following Anselm and Ural'tsev[22] we note that when $m_t < m_H$ both the real and imaginary components of M_{12} in the six-quark model are related to those in the four-quark model by

$$\text{Re}M_{12}^{(6)}(\text{Im}M_{12}^{(6)}) = \text{Re}M_{12}^{(4)}(\text{Im}M_{12}^{(4)}) \left[\left[\xi_t{}^2\frac{m_t{}^2}{m_c{}^2} + 2\xi_t\xi_c \ln\frac{m_t{}^2}{m_c{}^2} + \xi_c{}^2\right] \Big/ (\sin\theta_C\cos\theta_C)^2\right], \quad (3.26)$$

with $\xi_i = U_{id}U_{is}^*$. The terms in the square brackets should not be very different from unity because the four-quark model by itself can account for the $K_L - K_S$ mass difference quite well. However, the mass of the t quark is likely to be much larger than that of the charged Higgs boson, which is of the order of 10 GeV, Eq. (3.21) for $\text{Im}M_{12}$ thus should be modified. Fortunately, the results of detailed calculations indicate that it can only lead to a suppression of the contribution of the t quark (in particular the contributions from $\xi_t{}^2 m_t{}^2/m_c{}^2$) by letting $m_t > m_H$.[23] On the other hand contributions from the two-Higgs boson-exchange diagram are at most of the same order as that of the $W-H$ diagram for the neutral-kaon system, as we will see from the next section and also from Ref. 8; so we can conclude that the use of (3.25) is legitimate even in the six-quark model as long as our purpose is to estimate the order of magntiude of CP violation.

In terms of the Higgs-boson mixing angles we may write m_0 in the form

$$\frac{2\sqrt{2}}{m_0{}^2} = \frac{\bar{s}_2\bar{s}_3\bar{c}_3}{\bar{c}_1\bar{c}_2}\sin\delta_H \left[\frac{1}{m_{H_1}{}^2} - \frac{1}{m_{H_2}{}^2}\right]. \quad (3.27)$$

From Eqs. (3.14) and (3.27) we have

$$5.6\times10^{-3}<\frac{vv_3}{v_1v_2}\sin2\bar{\theta}_3\sin\delta_H\left[\frac{1}{m_{H_1}{}^2}-\frac{1}{m_{H_2}{}^2}\right]$$

$$<1.6\times10^{-2}\ (\text{in GeV}^{-2})\ . \qquad (3.28)$$

Assuming maximum CP violation (i.e., $\delta_H=\pi/2$), $\bar{\theta}_3=\pi/4$, $m_{H_2}\gg m_{H_1}$, and $v_1\sim v_2\sim v_3$, we find

$$m_{H_1}\sim10-17\ \text{GeV}\ .$$

Therefore in the Higgs-boson-exchange model of CP noninvariance with natural flavor conservation, at least one of the charged Higgs bosons must be light and is of the order of 10 GeV, otherwise the constraint (3.28) cannot be satisfied.

There are two other useful constraints on the Higgs mixing parameters, one of which arises from consideration of the K_L-K_S mass difference. In general there are a number of Higgs fields in a spontaneous CP-violating model and as a consequence it is quite possible to obtain a very large K^0-$\bar{K}^0$ mass mixing. Hence a restriction on the Higgs-boson mixing variables must be imposed in order that the real part of the H-W and H-H box diagrams do not give too large a K_L-K_S mass difference. A good discussion on this is given in Ref. 8.

In Sec. V we shall see that the T-violating polarization of the muon normal to the decay plane in kaon semileptonic decay is proportional to the factor $v_2{}^2/v_3{}^2$. The null result on $K^+_{\mu3}$ decays performed at Brookhaven sets a conservative bound that $v_2/v_3<8$.

IV. CHARGE ASYMMETRIES IN NEUTRAL-HEAVY-MESON SYSTEMS

While the various parameters in the KM and Higgs-boson models can be fine tuned to give the same K_L-K_S mass difference and CP violating ϵ of the neutral conjugate kaon system, the predictions of these schemes may differ in other systems. The size of CP nonconservation in the KM model is essentially governed by the intrinsic CP-odd phase δ, whereas in the Higgs-boson-exchange model the Higgs-boson—quark coupling strength increases with the mass of the quark. Hence, naively, one may expect CP violation in heavy mesons to provide a nice place to distinguish the Higgs-boson model from the KM model.

One place to look for CP violation in the heavy mesons is the dilepton charge asymmetry $\mathscr{A}$ in e^+e^- annihilation, which proceeds through the process $e^+e^-\to P^0\bar{P}^0\to l^\pm l^\pm+X$ (X is anything else)[24]:

$$\mathscr{A}=\frac{N^{++}-N^{--}}{N^{++}+N^{--}}\ . \qquad (4.1)$$

Here $\mathscr{A}$ measures the difference between equal-sign dilepton events coming from the semileptonic decays of P^0 ,$\bar{P}^0$ pairs. In terms of ϵ the charge asymmetry $\mathscr{A}$ is given by

$$\mathscr{A}=\frac{1-\left|\dfrac{1-\epsilon}{1+\epsilon}\right|^4}{1+\left|\dfrac{1-\epsilon}{1+\epsilon}\right|^4}=\frac{4\,\mathrm{Re}\epsilon(1+|\epsilon|^2)}{(1+|\epsilon|^2)^2+4(\mathrm{Re}\epsilon)^2}\ . \qquad (4.2)$$

Although ϵ is not a measurable quantity, the phase-convention-independent quantity $y=2\,\mathrm{Re}\epsilon/(1+|\epsilon|^2)$ indicates that the real part of ϵ measures the magnitude of intrinsic CP nonconservation residing solely in the mass matrix.[25] The dilepton asymmetry is, therefore, a measure of the CP impurity of the neutral meson rather than a CP-violating effect in the decay amplitude.

The charge asymmetry (or an equivalent quantity) for the heavy mesons has already been computed by a number of authors within the KM model,[17,25–27] and it is known that this asymmetry becomes smaller as the meson becomes heavier. A simple physical reason for this (see Ref. 28) is that in the B^0-$\bar{B}^0$ system, for instance, the masses of charm and up quarks are almost the same on the scale of the b quark. However, since the phase δ can be removed from the KM matrix by redefining the quark fields if the charm and up quarks have equal masses, it follows that the charge asymmetry is small in the B^0-$\bar{B}^0$ system.

In the following we recalculate the dilepton asymmetry for neutral heavy mesons which consist of one heavy and one light quark. The new features of the present calculations are shown below. To evaluate ϵ a knowledge of Γ_{12} is needed. The general wisdom for computing Γ_{12} is based on the assumption that the integration over the quark phase space approximates the integration over the hadron phase space. Hence summing over the physical intermediate hadron states is equivalent to summing over all possible intermediate quark states. Under this assumption Eq. (2.8) can be replaced by

$$M_{12}-i\tfrac{1}{2}\Gamma_{12}=\langle P^0\,|\,H_{\text{dis}}+iH_{\text{abs}}\,|\,\bar{P}^0\rangle\ , \qquad (4.3)$$

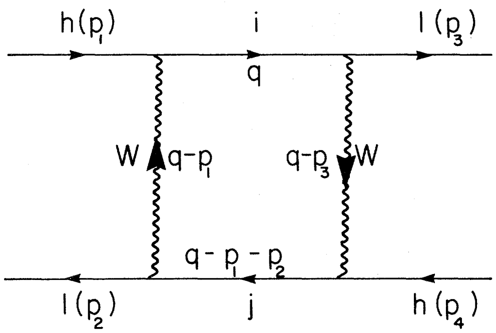

FIG. 1. The box diagram which contributes to P^0-$\bar{P}^0$ mixing with two W-boson exchanges where h (l) represents a heavy (light) quark.

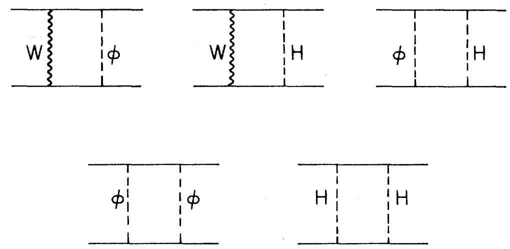

FIG. 2. The remaining box diagrams contributing to P^0-$\bar{P}^0$ mixing. H (ϕ) is a physical (unphysical) charged Higgs boson.

where the contribution from the low-mass intermediate state to M_{12} has been neglected. $H_{\rm dis}$ and $H_{\rm abs}$ are the dispersive and absorptive parts, respectively, of the effective Hamiltonian $H_{\rm eff}^{\Delta=2}$ arising from the box diagrams. In order to properly take into account all the effects of the heavy t and b quarks, we use a finite mass M_W ($=78.1$ GeV) for the W boson and make no assumptions about the relative magnitudes of the masses of particles. Since the 't Hooft—Feynman gauge is chosen for the evaluation of the box diagrams, the contribution from the unphysical charged Higgs boson $\phi^{\pm}$ with mass M_W must also be considered in addition to the two-W-exchange box graph. Summing over the contributions from the box diagrams with two W exchanges (Fig. 1), one W and one unphysical-Higgs-boson exchange, and two unphysical-Higgs-boson exchanges (Fig. 2), and neglecting the momenta of light quarks compared to those of the heavy quarks, we find the following gauge-invariant effective Hamiltonian (see the Appendix for the details),[29]

$$H_{\rm eff}=B\bar{l}_\alpha\gamma_\mu(1-\gamma_5)h_\alpha\bar{l}_\beta\gamma_\mu(1-\gamma_5)h_\beta+C\bar{l}_\alpha(1+\gamma_5)h_\alpha\bar{l}_\beta(1+\gamma_5)h_\beta+\text{H.c.} \tag{4.4}$$

with

$$B(C)=\frac{G_F{}^2}{8\pi^2}M_W{}^2\sum_{i,j}\xi_i\xi_j\int_0^1 dx\,B'(C')\,,$$

where

$$B'(2W+W\phi+2\phi)=\frac{1}{(1-\lambda_i)(1-\lambda_j)}\sum_k{}'[(1+\tfrac{1}{4}\lambda_i\lambda_j)\Lambda_k(W,W)-\lambda_i\lambda_j-\tfrac{1}{2}(\lambda_i+\lambda_j)\lambda_h x]\ln\Lambda_k(W,W) \tag{4.5a}$$

$$C'(2W+W\phi+2\phi)=\frac{1}{(1-\lambda_i)(1-\lambda_j)}\sum_k{}'[(2+\tfrac{1}{2}\lambda_i\lambda_j)\lambda_h x(1-x)+\lambda_i\lambda_j\lambda_h(\tfrac{1}{2}+x)]\ln\Lambda_k(W,W)\,, \tag{4.5b}$$

and

$$\Lambda_1(a,b)=\lambda_a(1-x)+\lambda_b x-\lambda_h x(1-x),\quad \Lambda_2(a,b)=\lambda_i(1-x)+\lambda_j x-\lambda_h x(1-x)\,,$$

$$\Lambda_3(a,b)=\lambda_a(1-x)+\lambda_j x-\lambda_h x(1-x),\quad \Lambda_4(a,b)=\lambda_i(1-x)+\lambda_b x-\lambda_h x(1-x)\,,$$

$$\sum_k{}'=\sum_{k=1}^{2}-\sum_{k=3}^{4},\quad \lambda_i=\frac{m_i{}^2}{M_W{}^2},\quad \lambda_a=\frac{m_a{}^2}{M_W{}^2},\quad \lambda_h=\frac{m_h{}^2}{M_W{}^2}\,. \tag{4.6}$$

In (4.4), h and l represent the heavy and light quarks, respectively, $\xi_i=U_{ik}U_{il}^*$ ($U_{li}U_{hi}^*$) when both h and l

are D-type (U-type) quarks, α and β are color indices, and the summation $\sum_{i,j}$ is carried out over all possible intermediate quark states i and j. We note that strictly speaking the effective Hamiltonian (4.4) is not a local operator because in deriving it we have already put the heavy quark h on its mass shell. It is, however, legitimate to do this for the purpose of calculating the mass matrix elements becuase the current and constituent masses of the heavy quark are very close. The reader may verify that when all the external momenta of the box diagram are neglected, the expression for B in (4.4) reduces to the well known result (see Appendix)

$$B=\frac{G_F{}^2}{16\pi^2}M_W{}^2\sum_{i,j}\xi_i\xi_j\left\{\frac{1}{(1-\lambda_i)(1-\lambda_j)}+\frac{1}{(\lambda_i-\lambda_j)}\left[\frac{\lambda_i{}^2\ln\lambda_i}{(1-\lambda_i)^2}-\frac{\lambda_j{}^2\ln\lambda_j}{(1-\lambda_j)^2}\right]\right\} \tag{4.7}$$

To proceed we note that by virtue of the vacuum-saturation method,

$$\langle\bar{P}^0\,|\,\bar{l}\gamma_\mu(1-\gamma_5)h\bar{l}\gamma_\mu(1-\gamma_5)h\,|\,P^0\rangle=\tfrac{8}{3}f_P{}^2m_P{}^2/2m_P\ , \tag{4.8a}$$

$$\langle\bar{P}^0\,|\,\bar{l}(1+\gamma_5)h\bar{l}(1+\gamma_5)h\,|\,P^0\rangle=-\tfrac{5}{3}f_P{}^2m_P{}^4/[(m_h+m_l)^2 2m_P]\approx-\tfrac{5}{3}f_P{}^2m_P{}^2/2m_P\ , \tag{4.8b}$$

where m_P is defined in Eq. (2.9) and f_P is the decay constant of the P^0 meson.

We are now in position to compute M_{12} and Γ_{12}. From Eqs. (4.4) and (4.8) it follows that

$$M_{12}=\frac{G_F{}^2f_P{}^2m_P}{6\pi}M_W{}^2\sum_{i,j}\frac{\xi_i^*\xi_j^*}{(1-\lambda_i)(1-\lambda_j)}$$
$$\times\sum_k{}'\int dx\,\ln\Lambda_k(W,W)\{\,(1+\tfrac{1}{4}\lambda_i\lambda_j)\Lambda_k(W,W)-\lambda_i\lambda_j-\tfrac{1}{2}(\lambda_i+\lambda_j)\lambda_h x$$
$$+\tfrac{5}{8}[(2+\tfrac{1}{2}\lambda_i\lambda_j)\lambda_h x(1-x)-\lambda_i\lambda_j\lambda_h(\tfrac{1}{2}+x)]\}, \tag{4.9}$$

where only the domain of positive argument of the logarithm is integrated over. Γ_{12} is obtained by replacing the logarithms in (4.9) by $-i\pi$ and integrating over the domain of negative argument.[27] Because of the complicated integrands, M_{12} and Γ_{12} are computed by numerical integration. We take the following values for the quark current masses: $m_u=3$ MeV, $m_d=8$ MeV, $m_s=180$ MeV, $m_c=1.5$ GeV, and $m_b=4.5$ GeV. The magnitude of the quark mixing angles θ_i and phase δ depend on the mass of the t quark, and the following four different sets of values for various parameters are used[30]:

$$\begin{aligned}&(\text{I})\ m_t=15\text{ GeV},\ \sin\theta_2=0.5,\ \sin\delta=0.005,\ \cos\delta<0\ ;\\&(\text{II})\ m_t=15\text{ GeV},\ \sin\theta_2=0.2,\ \sin\delta=0.011,\ \cos\delta>0\ ;\\&(\text{III})\ m_t=30\text{ GeV},\ \sin\theta_2=0.42,\ \sin\delta=0.003,\ \cos\delta<0\ ;\\&(\text{IV})\ m_t=30\text{ GeV},\ \sin\theta_2=0.12,\ \sin\delta=0.008,\ \cos\delta>0\ ;\end{aligned} \tag{4.10}$$

all with $\sin\theta_1=0.23$ and $\sin\theta_3=0.3$.

The results of calculation are displayed in Table I and one can see that the dilepton charge asymmetries $\mathscr{A}$ are of order 10^{-3}, $10^{-3}-10^{-4}$, $10^{-4}-10^{-5}$, and 10^{-7} for $m_t=15$ GeV, and 10^{-3}, $10^{-4}-10^{-5}$, $10^{-5}-10^{-6}$, and 10^{-6} for $m_t=30$ GeV for D^0-$\bar{D}^0$, B_d^0-$\bar{B}_d^0$, B_s^0-$\bar{B}_s^0$, and T_u^0-$\bar{T}_u^0$, respectively. The charge asymmetry is thus very small in the T^0-$\bar{T}^0$ system even though $|\epsilon|$ is of the same order as that of other neutral mesons. It is interesting to note that the real parts of M_{12} and Γ_{12} of the T^0-$\bar{T}^0$ system have the same sign in the KM model, which means that T_1^0 is more massive than T_2^0 but has a shorter lifetime.

Next we turn to the Weinberg model. In addition to the three diagrams already computed in the KM model, we also have to consider contributions from the H-ϕ, H-W, and H-H exchange box graphs (see Fig. 2). Using the same notation as in (4.4), we find the contributions to the effective Hamiltonian from the physical charged Higgs bosons are[31]

$$B'(WH+H\phi)=\sum_{n=1}^{2}\frac{1}{(\lambda_{H_n}-\lambda_i)(1-\lambda_j)}\sum_k{}'\{\tfrac{1}{2}\,|\beta_n|^2\lambda_i\lambda_j[\Lambda_k(H_n,W)-2]-\alpha_n\beta_n^*\lambda_i\lambda_h x\}\ln\Lambda_k(H_n,W)\,,$$

$$B'(2H)=\sum_{m,n,=1}^{2}\frac{1}{(\lambda_{H_m}-\lambda_i)(\lambda_{H_n}-\lambda_j)}\sum_k{}'\tfrac{1}{4}\,|\beta_m\beta_n|^2\lambda_i\lambda_j\Lambda_k(H_m,H_n)\ln\Lambda_k(H_m,H_n)\,,\tag{4.11}$$

$$C'(WH+H\phi)=\sum_{n=1}^{2}\frac{1}{(\lambda_{H_n}-\lambda_i)(1-\lambda_j)}\sum_k{}'[\,|\beta_n|^2\lambda_i\lambda_j\lambda_h x(1-x)+\alpha_n\beta_n^*\lambda_i\lambda_j\lambda_h(1+2x)]\ln\Lambda_k(H_n,W)$$

$$C'(2H)=\sum_{m,n=1}^{2}\frac{1}{(\lambda_{H_m}-\lambda_i)(\lambda_{H_n}-\lambda_j)}\sum_k{}'[\tfrac{1}{2}\,|\beta_m\beta_n|^2\lambda_i\lambda_j\lambda_h x(1-x)+(\alpha_m\beta_m^*)(\beta_n\beta_n^*)\lambda_i\lambda_j\lambda_h x$$
$$+\tfrac{1}{2}(\alpha_m\beta_m^*)(\alpha_n\beta_n^*)\lambda_i\lambda_j\lambda_h]\ln\Lambda_K(H_m,H_n)\,,$$

where $\lambda_{H_i}=m_{H_i}{}^2/M_W{}^2$, α_i, and β_i are the charged-Higgs-boson—quark coupling constants as given in Eq. (3.14). To proceed we have to specify the values of various parameters appearing in the Higgs-boson model. For the quark mixing angles we still use the four sets of values as given in (4.10) but with the CP-odd phase $\delta=0$. For the variables in the Higgs sector, Chang has suggested that a reasonable choice for v_3/v is 0.5, and he also showed a correlation between m_H/m_c and v_2/v_1 in Fig. 5 of Ref. 8. We have set $m_{H_1}=7$ GeV, $\theta_3=\pi/4$, and computed the off-diagonal mass matrix elements, as well as the dilepton asymmetry for a range of values of v_2/v_1, δ_H, and m_{H_2}. In general M_{12} and Γ_{12} are sensitive to the choice of the values of the Higgs parameters, while the magnitude of the dilepton asymmetry is relatively stable. We thus present in Tables II and III the results of calculations based on the following two sets of values,

TABLE I. Results for M_{12}, Γ_{12}, and charge asymmetry $\mathscr{A}$ for D^0-$\bar{D}^0$, B_d-$\bar{B}_d^0$, B_s^0-$\bar{B}_s^0$, and T_u^0-$\bar{T}_u^0$ systems in the KM model for various values of quark-mixing parameters as given in Eq. (4.10). A common factor $G_F{}^2f_p{}^2m_pM_W{}^2/6\pi^2$ is factored out for each neutral-conjugate-meson system.

		ReM_{12}	ImM_{12}	ReΓ_{12}	ImΓ_{12}	$\mathscr{A}$
(I)	D-$\bar{D}$	2.76×10^{-7}	-4.76×10^{-9}	-9.92×10^{-8}	3.19×10^{-9}	5.18×10^{-3}
	B_d-$\bar{B}_d$	1.61×10^{-4}	1.42×10^{-6}	-8.59×10^{-5}	-6.53×10^{-7}	6.08×10^{-4}
	B_s-$\bar{B}_s$	5.52×10^{-4}	1.17×10^{-5}	-2.06×10^{-4}	-4.45×10^{-6}	-1.75×10^{-4}
	T_u-$\bar{T}_u$	2.02×10^{-6}	-1.72×10^{-8}	1.65×10^{-6}	-1.40×10^{-8}	-5.24×10^{-7}
(II)	D-$\bar{D}$	5.34×10^{-7}	5.78×10^{-9}	-1.14×10^{-6}	-9.02×10^{-9}	2.93×10^{-3}
	B_d-$\bar{B}_d$	2.76×10^{-5}	-5.84×10^{-7}	-1.83×10^{-6}	1.28×10^{-7}	3.21×10^{-3}
	B_s-$\bar{B}_s$	2.44×10^{-3}	-2.44×10^{-5}	-9.71×10^{-4}	9.61×10^{-6}	-3.96×10^{-5}
	T_u-$\bar{T}_u$	1.64×10^{-6}	3.85×10^{-8}	1.34×10^{-6}	3.14×10^{-8}	-6.42×10^{-7}
(III)	D-$\bar{D}$	1.29×10^{-7}	-1.69×10^{-9}	-9.22×10^{-9}	7.35×10^{-10}	4.74×10^{-3}
	B_d-$\bar{B}_d$	5.87×10^{-4}	3.10×10^{-6}	-6.49×10^{-5}	-3.01×10^{-7}	7.19×10^{-5}
	B_s-$\bar{B}_s$	8.84×10^{-4}	1.69×10^{-5}	-6.87×10^{-5}	-1.36×10^{-6}	-4.92×10^{-5}
	T_u-$\bar{T}_u$	2.62×10^{-7}	-1.37×10^{-9}	1.90×10^{-7}	-9.97×10^{-10}	-2.68×10^{-6}
(IV)	D-$\bar{D}$	3.50×10^{-7}	2.06×10^{-9}	-8.57×10^{-7}	-3.42×10^{-9}	1.86×10^{-3}
	B_d-$\bar{B}_d$	4.38×10^{-5}	-7.04×10^{-7}	1.01×10^{-6}	2.22×10^{-8}	8.77×10^{-4}
	B_s-$\bar{B}_s$	9.08×10^{-3}	-4.51×10^{-5}	-7.67×10^{-4}	3.78×10^{-6}	-3.97×10^{-6}
	T_u-$\bar{T}_u$	2.22×10^{-7}	3.69×10^{-9}	1.61×10^{-7}	2.68×10^{-9}	-2.64×10^{-6}

TABLE II. Results for M_{12}, Γ_{12}, and charge asymmetry $\mathscr{A}$ for D^0-$\bar{D}^0$, B_d-$\bar{B}^0_d$, B^0_s-$\bar{B}^0_s$, and T^0_u-$\bar{T}^0_u$ systems in the Weinberg model for various quark-mixing parameters as given in Eq. (4.10). The various values of Higgs-sector parameters are taken to be $m_{H_1}=7$ GeV, $m_{H_2}=25$ GeV, $\bar{s}_1=0.74$, $\bar{s}_2=0.4$, $\bar{s}_3=1$, and $\sin\delta_H=0.3$ with $\cos\delta_H<0$. A common factor $G_F^2 f_p^2 m_p M_W^2/6\pi^2$ is factored out for each neutral-conjugate-meson system.

		ReM_{12}	ImM_{12}	ReΓ_{12}	ImΓ_{12}	$\mathscr{A}$
(I)	D-$\bar{D}$	3.69×10^{-7}	-1.16×10^{-9}	-9.90×10^{-8}	-1.34×10^{-10}	1.19×10^{-3}
	B_d-$\bar{B}_d$	3.58×10^{-4}	-3.38×10^{-6}	-8.53×10^{-5}	-2.00×10^{-7}	2.77×10^{-3}
	B_s-$\bar{B}_s$	1.16×10^{-3}	-9.89×10^{-6}	-2.03×10^{-4}	-1.37×10^{-6}	2.66×10^{-3}
	T_u-$\bar{T}_u$	5.53×10^{-7}	-3.02×10^{-6}	-1.03×10^{-6}	-2.76×10^{-6}	3.99×10^{-1}
(II)	D-$\bar{D}$	1.00×10^{-6}	-7.33×10^{-9}	-1.15×10^{-6}	4.90×10^{-10}	5.93×10^{-3}
	B_d-$\bar{B}_d$	5.05×10^{-5}	-2.20×10^{-7}	-1.80×10^{-6}	-6.10×10^{-8}	1.36×10^{-3}
	B_s-$\bar{B}_s$	5.16×10^{-3}	-3.21×10^{-5}	-9.56×10^{-4}	-5.72×10^{-6}	2.24×10^{-3}
	T_u-$\bar{T}_u$	4.61×10^{-7}	-2.43×10^{-6}	-8.20×10^{-7}	-2.23×10^{-6}	4.01×10^{-1}
(III)	D-$\bar{D}$	1.61×10^{-7}	-2.92×10^{-10}	-9.25×10^{-9}	-4.62×10^{-11}	3.89×10^{-4}
	B_d-$\bar{B}_d$	1.41×10^{-3}	-3.65×10^{-6}	-6.46×10^{-5}	-1.11×10^{-7}	1.98×10^{-4}
	B_s-$\bar{B}_s$	2.09×10^{-3}	-5.07×10^{-6}	-6.76×10^{-5}	-4.84×10^{-7}	3.10×10^{-4}
	T_u-$\bar{T}_u$	1.46×10^{-5}	1.31×10^{-6}	1.34×10^{-5}	9.52×10^{-7}	1.43×10^{-2}
(IV)	D-$\bar{D}$	6.88×10^{-7}	-5.27×10^{-9}	-8.59×10^{-7}	4.47×10^{-10}	6.40×10^{-3}
	B_d-$\bar{B}_d$	9.81×10^{-5}	-8.68×10^{-8}	9.25×10^{-7}	-6.08×10^{-9}	5.37×10^{-5}
	B_s-$\bar{B}_s$	2.15×10^{-2}	-5.33×10^{-5}	-7.65×10^{-4}	-4.50×10^{-6}	2.96×10^{-4}
	T_u-$\bar{T}_u$	1.24×10^{-5}	1.11×10^{-6}	1.15×10^{-5}	8.08×10^{-7}	1.47×10^{-2}

TABLE III. Same as Table II except that the various values of Higgs-sector parameters are taken to be $m_{H_1}=7$ GeV, $m_{H_2}=12$ GeV, $\bar{s}_1=0.79$, $\bar{s}_2=0.64$, $\bar{s}_3=1$, and $\sin\delta_H=1$.

		ReM_{12}	ImM_{12}	ReΓ_{12}	ImΓ_{12}	$\mathscr{A}$
(I)	D-$\bar{D}$	6.73×10^{-7}	-5.46×10^{-9}	-9.93×10^{-8}	-6.73×10^{-10}	2.18×10^{-3}
	B_d-$\bar{B}_d$	7.39×10^{-4}	-1.38×10^{-5}	-8.46×10^{-5}	-1.01×10^{-6}	3.49×10^{-3}
	B_s-$\bar{B}_s$	2.32×10^{-3}	-4.01×10^{-5}	-2.00×10^{-4}	-7.00×10^{-6}	4.51×10^{-3}
	T_u-$\bar{T}_u$	-1.11×10^{-5}	1.01×10^{-5}	-1.03×10^{-5}	1.34×10^{-5}	1.51×10^{-1}
(II)	D-$\bar{D}$	2.54×10^{-6}	-3.30×10^{-8}	-1.15×10^{-6}	2.45×10^{-9}	4.66×10^{-3}
	B_d-$\bar{B}_d$	9.28×10^{-5}	-7.45×10^{-7}	-2.38×10^{-6}	-3.23×10^{-7}	3.69×10^{-3}
	B_s-$\bar{B}_s$	1.04×10^{-2}	-1.83×10^{-4}	-9.42×10^{-4}	-2.91×10^{-5}	4.40×10^{-3}
	T_u-$\bar{T}_u$	-9.11×10^{-6}	8.10×10^{-6}	-8.35×10^{-6}	1.08×10^{-5}	1.57×10^{-1}
(III)	D-$\bar{D}$	2.66×10^{-7}	-1.20×10^{-9}	-9.97×10^{-9}	-2.32×10^{-10}	1.04×10^{-3}
	B_d-$\bar{B}_d$	2.53×10^{-3}	-1.34×10^{-5}	-6.41×10^{-5}	-5.65×10^{-7}	3.57×10^{-4}
	B_s-$\bar{B}_s$	3.73×10^{-3}	-1.84×10^{-5}	-6.66×10^{-5}	-2.47×10^{-6}	7.51×10^{-4}
	T_u-$\bar{T}_u$	-8.99×10^{-7}	-1.96×10^{-8}	-2.08×10^{-6}	2.08×10^{-7}	7.67×10^{-2}
(IV)	D-$\bar{D}$	1.80×10^{-6}	-2.37×10^{-8}	-8.61×10^{-7}	2.23×10^{-9}	4.75×10^{-3}
	B_d-$\bar{B}_d$	1.70×10^{-4}	-1.28×10^{-7}	2.88×10^{-7}	-4.15×10^{-8}	2.43×10^{-4}
	B_s-$\bar{B}_s$	3.85×10^{-2}	-1.94×10^{-4}	-7.45×10^{-4}	-2.29×10^{-5}	6.93×10^{-4}
	T_u-$\bar{T}_u$	-7.66×10^{-7}	1.64×10^{-8}	-1.76×10^{-6}	1.76×10^{-7}	7.73×10^{-2}

$$\frac{v_1}{v_2}=1,\ \frac{v_3}{v}=0.5,\ m_{H_2}=12\text{ GeV},$$
$$\sin\delta_H=1,\ \cos\delta_H>0,$$
$$\text{and} \tag{4.12}$$
$$\frac{v_1}{v_2}=1,\ \frac{v_3}{v}=0.3,\ m_{H_2}=25\text{ GeV},$$
$$\sin\delta_H=0.3,\ \cos\delta_H<0,$$

which satisfy the restriction given by (3.28).

From Tables II and III, it can be concluded that the charge asymmetries are of order 10^{-3}, 10^{-3}, 10^{-3}, and 10^{-1} at $m_t=15$ GeV, and $10^{-3}-10^{-4}$, $10^{-4}-10^{-5}$ 10^{-4}, and 10^{-2} at $m_t=30$ GeV for D^0-$\bar{D}^0$, B_d^0-$\bar{B}_d^0$, B_s^0-$\bar{B}_s^0$, and T_u^0-$\bar{T}_u^0$, respectively. The dilepton asymmetry or *CP* impurity is therefore very large in the T^0-$\bar{T}^0$ system. In passing we note that although the Higgs-boson—quark coupling strength is proportional to the mass of the quark, the *CP* impurity of the B^0-$\bar{B}^0$ system is not large, and in fact it may even be smaller than in the D^0-$\bar{D}^0$ system, as happens in the KM model. This has to do with the fact that in the B^0-$\bar{B}^0$ system the real parts as well as the imaginary parts of M_{12} and Γ_{12} are boosted by the increase of the quark mass, and as a result this yields a small Reϵ. Hence if an experimental measurement of the charge asymmetry in e^+e^- annihilation is feasible, it seems that only in the T^0-$\bar{T}^0$ system can one distinguish between the Higgs-boson-exchange model and the KM model.

Several remarks are in order.

(1) Contributions from both the *W-H* and *H-H* diagrams are quite important, and for a large range of Higgs-boson mixing angles M_{12} and Γ_{12} for the heavy neutral mesons are dominated by the *W-H* box contribution. In general the contribution from the *H-H* box graph is smaller than that of the *W-H* diagram by a factor of the order of 10, except in the case of the T^0-$\bar{T}^0$ system where the *H-H* graph could dominate for a certain range of Higgs-boson mixing variables. This also justifies the smallness of the *H-H* diagram in comparison with the *W-H* contribution in the K^0-$\bar{K}^0$ system as we mentioned in Sec. III. We also note that the quark mixing angles, in particular θ_2 and θ_3, are not necessarily the same in both the KM and Higgs-boson models and should be redetermined in principle.

(2) Possible QCD corrections, hadronic corrections,[32] and long-distance effects are not considered here in calculating the off-diagonal mass matrix elements. The magnitude of M_{12} and Γ_{12} could be changed significantly if these corrections (particularly the last two) are taken into account.

(3) In spite of the fact that the *CP* impurity of the T^0-$\bar{T}^0$ system is large ($10^{-1}-10^{-2}$), one's experimental ability to observe this intrinsic *CP*-odd effect is, unfortunately, offset to a limited extend by the fact that the T^0-$\bar{T}^0$ mixing is very weak. The possibility of measuring the wrong-sign-lepton event is determined by[11]

$$r=\frac{\Gamma(T^0\to l^-+x)}{\Gamma(T^0\to l^++x)}=\left|\frac{1-\epsilon}{1+\epsilon}\right|^2\rho\,;$$
$$\bar{r}=\frac{\Gamma(\bar{T}^0\to l^++x)}{\Gamma(\bar{T}^0\to l^-+x)}=\left|\frac{1+\epsilon}{1-\epsilon}\right|^2\rho\,,$$

where ρ is given in Eq. (2.12). In the T^0-$\bar{T}^0$ system Δm and $\Delta\Gamma$ are smaller than in the B^0-$\bar{B}^0$ system as one can see from Tables I—III. The short lifetime of T^0 compared to that of B^0 makes things even worse. Likewise, the primary leptons can be contaminated from the subsequent semileptonic decay. The experimental observation of the same-sign-dilepton events is thus difficult in the T^0-$\bar{T}^0$ meson; indeed, this is a general feature for all neutral heavy mesons. Perhaps one should look at *CP*-violating effects in the decay amplitudes of heavy-meson systems to search for the difference between the KM and Higgs-boson-exchange models of *CP* noninvariance. A full discussion of this will be presented elsewhere.

V. TRANSVERSE POLARIZATION OF LEPTONS

One of the salient features of the Higgs-boson model of *CP* nonconservation is that not only quarks but also leptons have a *CP*-violating Higgs-boson interaction. Hence an observation of *CP* violation in the leptonic decays of hadrons is one of the best areas to distinguish between the Higgs-boson and KM models; in the latter scheme no such *CP*-violating effects can occur.[33]

It is well known that the breakdown of *CP* invariance has no consequences in the P_{l2} decay modes (as before *P* denotes a neutral pseudoscalar meson). One may then look for the *T*- (or *CP*-) violating effects in the semileptonic decays of hadrons, such as the lepton polarization transverse to the decay plane in P_{l3} decays, and the decay rate difference in P_{l4} decays. Since the Higgs-boson—lepton coupling strength is proportional to the mass of the lepton, such *T*-odd effects will be manifest mostly in the semileptonic decay of heavy hadrons involving the τ lepton, and do not arise in processes such as β decay.

Zhitnitskii[34] has calculated the transverse polarization of the muon in kaon and D-meson decays. Here we attempt to generalize this calculation to general P_{l3} decays. We begin by considering the decay $P_i^- \to P_f^0 l^- \bar{\nu}_l$ and assume the spectator model for such a semileptonic decay.[35] Referring to Fig. 3, we let quark a be a D-type quark, and neglect the momentum transfer squared t compared to $M_W{}^2$. We then find the amplitude has the form

$$M=\frac{G_F}{\sqrt{2}}U_{ab}\bar{l}\gamma_\mu(1-\gamma_5)\nu\langle P_f\,|\,\bar{b}\gamma_\mu(1-\gamma_5)a\,|\,P_i\rangle+\frac{G_F}{\sqrt{2}}U_{ab}m_l m_a\left[\sum_{i=1}^{2}\frac{\alpha_i\gamma_i^*}{m_{H_i}{}^2-t}\right]\bar{l}(1-\gamma_5)\nu\langle P_f\,|\,\bar{b}(1+\gamma_5)a\,|\,P_i\rangle$$
$$+\frac{G_F}{\sqrt{2}}U_{ab}m_l m_b\left[\sum_{i=1}^{2}\frac{\beta_i\gamma_i^*}{m_{H_i}{}^2-t}\right]\bar{l}(1-\gamma_5)\nu\langle P_f\,|\,\bar{b}(1-\gamma_5)a\,|\,P_i\rangle\ , \qquad (5.1)$$

where $t=(p_i-p_f)^2=(p_a-p_b)^2$.

The relevant hadronic matrix elements are

$$\langle P_f\,|\,\bar{b}\gamma_\mu(1-\gamma_5)a\,|\,P_i\rangle=\frac{1}{\sqrt{2}}[(p_i+p_f)_\mu f_+(t)+(p_i-p_f)_\mu f_-(t)]\ , \qquad (5.2)$$

and

$$\langle P_f\,|\,\bar{b}(1\pm\gamma_5)a\,|\,P_i\rangle=\frac{i}{m_a-m_b}\langle P_f\,|\,\partial_\mu(\bar{b}\gamma_\mu a)\,|\,P_i\rangle=\frac{f_+(t)(m_i{}^2-m_f{}^2)+f_-(t)t}{\sqrt{2}(m_a-m_b)}\ , \qquad (5.3)$$

where the ratio of the two form factors $f_+(t)$ and $f_-(t)$ is real. Then M becomes

$$M=\frac{G_F}{2}U_{ab}\bar{l}\gamma_\mu(1-\gamma_5)\nu(p_1+p_2)_\mu f'_+(t)+\frac{G_F}{2}U_{ab}m_l\bar{l}(1-\gamma_5)\nu f'_-(t)\ , \qquad (5.4)$$

with

$$f'_+(t)=f_+(t)\ , \qquad (5.5a)$$

$$f'_-(t)=f_-(t)+\frac{f_+(t)(m_i{}^2-m_f{}^2)+f_-(t)t}{m_a-m_b}\left[m_a\sum_{i=1}^{2}\frac{\alpha_i\gamma_i^*}{m_{H_i}{}^2-t}+m_b\sum_{i=1}^{2}\frac{\beta_i\gamma_i^*}{m_{H_i}{}^2-t}\right]\ . \qquad (5.5b)$$

The imaginary part of $\xi'\equiv f'_-(t)/f'_+(t)$, which governs the size of the transverse polarization of the lepton, is given by

$$\mathrm{Im}\xi'(t)=\frac{m_i{}^2-m_f{}^2+\xi(t)t}{m_a-m_b}\,\frac{\Delta m_H{}^2\,\mathrm{Im}(\alpha_1\beta_1^*)}{(m_{H_1}{}^2-t)(m_{H_2}{}^2-t)}\left[m_a\frac{v_2{}^2}{v_3{}^2}-m_b\frac{v_1{}^2}{v_3{}^2}\right]\ , \qquad (5.6)$$

where we have made use of Eq. (3.16), $\xi(t)=f_-(t)/f_+(t)$, and $\Delta m_H{}^2=m_{H_1}{}^2-m_{H_2}{}^2$. For the U-type quark a, $\mathrm{Im}\xi'$ is obtained from (5.6) by the replacement $a\rightleftarrows b$. For P_{l3}^+ decay, $\mathrm{Im}\xi'$ has the same magnitude as that of (5.6) but with different sign, i.e.,

$$\mathrm{Im}\xi'(P_{l3}^+)=-\,\mathrm{Im}\xi'(P_{l3}^-)\ . \qquad (5.7)$$

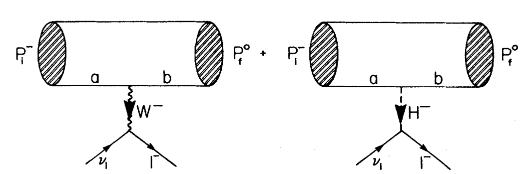

FIG. 3. The spectator model of the semileptonic decay $P_i^- \to P_f^0 l^- \bar{\nu}_l$.

Using the same notation as in Refs. 34 and 40, the T-violating four-vector of transverse polarization is then given by

$$P_{\perp}^{\alpha}=m_l\,\mathrm{Im}\xi'(t)\frac{\epsilon_{\alpha\beta\gamma\delta}p_l^{\beta}p_{\nu}^{\gamma}p_i^{\delta}}{\Phi}\,, \tag{5.8}$$

with

$$\Phi=2(p_l\cdot p_i)(p_\nu\cdot p_i)-m_i^2(p_l\cdot p_\nu)+2\,\mathrm{Re}\chi m_l^2(p_\nu\cdot p_i)+m_l^2|\chi|^2(p_l\cdot p_\nu)\,,$$

and (5.9)

$$\chi=(f'_- - f'_+)/2f'_+=(f'_- - f_+)/2f_+\,.$$

The four-vector of the lepton polarization s is related to $\vec{\xi}$ by

$$\vec{s}=\vec{\xi}+\frac{\vec{p}_l(\vec{p}_l\cdot\vec{\xi})}{m_l(E_l+m_l)}\,,\quad s_0=\frac{\vec{p}_l\cdot\vec{\xi}}{m_l}\,, \tag{5.10}$$

where $\vec{\xi}$ is a unit polarization vector of the lepton selected by the detector in its rest frame. The evaluation of the transverse polarization is usually carried out in the center-of-mass frame, i.e., P_i rest frame. Hence

$$t=m_l^2+2E_\nu(E_l-\vec{n}_l\cdot\vec{n}_\nu|\vec{p}_l|)\,, \tag{5.11a}$$

$$\Phi=m_i^2E_\nu(E_l+\vec{n}_l\cdot\vec{n}_\nu|\vec{p}_l|)+2\,\mathrm{Re}\chi\frac{m_l^2}{m_i}+\frac{m_l^2}{m_i^2}|\chi|^2(E_l-\vec{n}_l\cdot\vec{n}_\nu|\vec{p}_l|) \tag{5.11b}$$

with $\vec{n}_l$ and $\vec{n}_\nu$ unit vectors along $\vec{p}_l$ and $\vec{p}_\nu$. Because

$$\epsilon_{\alpha\beta\gamma\delta}s^{\alpha}p_l^{\beta}p_\nu^{\gamma}p_i^{\delta}=m_iE_\nu|\vec{p}_l|\,\vec{\xi}\cdot(\vec{n}_l\times\vec{n}_\nu)\,,$$

we have

$$\vec{P}_\perp=\vec{n}_l\times\vec{n}_\nu\,\mathrm{Im}\xi'(t)\frac{m_l}{m_i}\,\frac{|\vec{p}_l|}{E_l+\vec{n}_l\cdot\vec{n}_\nu|\vec{p}_l|+2\,\mathrm{Re}\chi\frac{m_l^2}{m_i}+\frac{m_l^2}{m_i^2}|\chi|^2(E_l-\vec{n}_l\cdot\vec{n}_\nu|\vec{p}_l|)} \tag{5.12}$$

in the center-of-mass frame. For P_{l3}^0 and $\bar{P}_{l3}^0$ decays, Imξ' has the same form as that given by (5.6) with an appropriate sign. However, a troublesome background problem may enter into the neutral-meson decays, as we shall discuss below.

Neglecting the contribution from the up quark and assuming $f_-/f_+\approx 0$, we find from Eq. (5.6) that Imξ' for K_{l3}^+ and K_{l3}^0 decays is[34]

$$\mathrm{Im}\xi'=-\frac{m_K^2}{m_0^2}\frac{v_2^2}{v_3^2}\,. \tag{5.13}$$

Combining with (3.25) then gives[36]

$$-6.8\times10^{-4}\frac{v_2^2}{v_3^2}<\mathrm{Im}\xi'<-2.0\times10^{-3}\frac{v_2^2}{v_3^2}\,, \tag{5.14}$$

which contains an unknown factor v_2^2/v_3^2. The significance of (5.14) is that if v_2^2/v_3^2 is of the order of unity, it then follows that a measurement of Imξ' should be done at the level of 10^{-3} in order to provide unambiguous evidence of CP violation associated with leptons in the semileptonic decay of kaons.

A recent experiment on the transverse polarization of muons in $K_{\mu3}^+$ decays gives

$$\mathrm{Im}\xi'=-0.016\pm0.025\quad(\text{with }\mathrm{Re}\xi'=0)$$

(Campbell *et al.*[37]). There are also two recent experimental results for $K_{\mu3}^0$ decays:

$$\mathrm{Im}\xi'=-0.060\pm0.045\quad(\text{with }\mathrm{Re}\xi'=-0.655\pm0.127)$$
$$=-0.085\pm0.064\quad(\text{with}\mathrm{Re}\xi'=0)$$

(Sandweiss *et al.*,[38]);

$$\mathrm{Im}\xi'=0.009\pm0.030\quad(\text{with }\mathrm{Re}\xi'=0)$$

(Morse *et al.*[39]) .

Hence measurements of Imξ' at Brookhaven give a null result for T violation at the one-percent level. The conservative bound on v_2/v_3 set by the data on $K_{\mu3}^+$ decays is $v_2/v_3\lesssim 8$.

A transverse polarization of the muon can also arise from the final-state electromagnetic interaction between the pion and muon.[40,41] For $K_{\mu3}^0$ and $\bar{K}_{\mu3}^0$ decays, Imξ'(EM) is typically of the order of

0.008,[39] which is comparable to or even larger than the effect of direct *CP* violation. Fortunately, as emphasized by Okun and Khriplovich,[40] the transverse polarization caused by the electromagnetic interaction is the same in $K^0_{\mu 3}$ and $\bar{K}^0_{\mu 3}$ decays, whereas the polarization due to the *CP* nonconservation has different signs in these cases. Hence the absolute magnitude of the transverse polarizations of muons should be different in $K^0_{\mu 3}$ and $\bar{K}^0_{\mu 3}$ decays. Of course, if $\mathrm{Im}\xi'(\mathrm{EM}) \gg \mathrm{Im}\xi'$ the *CP*-violating effect would be totally contaminated by the background.

For $K^{\pm}_{\mu 3}$ decays Zhitnitskii has calculated the effect due to the electromagnetic interaction and found $P_\perp^{(\mathrm{EM})} \le 10^{-6}$,[34] which effect can thus be neglected.

Because experimentally no positive result for the *CP*-nonconserving muon polarization in kaon decays has been found, and theoretically such a *CP*-odd effect is very small as indicated by Eq. (5.14) (unless the ratio ${v_2}^2/{v_3}^2$ is unusually large), a transverse polarization of the lepton may be manifest only in the semileptonic decay of heavy mesons, such as $B_u^- \to D^0 \tau^- \bar{\nu}_\tau$. To obtain a numerical estimate of the transverse τ polarization in B_u^- semileptonic decays, a knowledge of the form factors $f_\pm(t)$ is required. Unfortunately, little is known about these form factors, although some useful information on $f_\pm(t)$ may be extracted from the scheme developed in Ref. 42. For our purposes we simply calculate the *T*-violating polarization by assuming a set of three different "orthogonal" values for $\xi(t)=f_-(t)/f_+(t)$, namely, 1, 0, and -1. For the kinematic region in which the momenta of τ and $\bar{\nu}_\tau$ obey $|\vec{p}_\tau| = |\vec{p}_\nu|$ and $\vec{p}_\tau \cdot \vec{p}_\nu = 0$, t has the value 5.78 $(\mathrm{GeV})^2$ and $|\vec{p}_\tau| = |\vec{p}_\nu| = 0.68$ GeV. The magnitudes of $\mathrm{Im}\xi'$ and $P_\perp$ are sensitive to the change of δ_H but less sensitive to the relative magnitude of v_1, v_2, and v_3, and the choice of $\xi(t)$. We set $m_{H_1}=7$ GeV, $m_{H_2}=25$ GeV, $\sin\delta_H=0.3$ (so as not to maximize the *CP* violation), and choose two sets of v_i' $(=v_i/v)$: (1) $v_1'=v_2'=0.61$, $v_3'=0.5$; (2) $v_1'=0.91$, $v_2'=v_3'=0.3$, for illustrative purposes. Then averaging over v_i', $\xi(t)$, and two different signs of $\cos\delta_H$, we find

$$\langle |\mathrm{Im}\xi'| \rangle = 2.6\times 10^{-1} , \qquad \langle |P_\perp| \rangle = 4.1\times 10^{-2} . \tag{5.15}$$

It may be conceivable to search for such a large *T*-violating effect in future experiments. The observation of a nonvanishing polarization normal to the decay plane of the heavy lepton with a magnitude of the order of 10^{-2} would be strong evidence against the KM model.

VI. CONCLUSIONS

Both theoretical and experimental improvements in the very near future will determine whether the ratio of the two *CP*-violating parameters $|\epsilon'/\epsilon|$ is a serious problem in the Weinberg model of *CP* violation. Meanwhile, several other tests are needed to distinguish between the Higgs-boson-exchange model and the KM model. The dilepton charge asymmetry is a measure of the intrinsic *CP* nonconservation in the mass matrix of the heavy-neutral-conjugate-meson system, and is less sensitive to the change of quark and Higgs-boson mixing parameters. Keeping the mass of the *W* boson finite, and taking into account the full effects of the masses of quarks, we obtain a gauge-invariant effective Hamiltonian, which can then be used to calculate the off-diagonal mass matrix elements. The dilepton asymmetries are of order 10^{-3}, $10^{-3}-10^{-5}$, $10^{-4}-10^{-6}$, and $10^{-6}-10^{-7}$ in the KM model, and $10^{-3}-10^{-4}$, $10^{-3}-10^{-5}$, $10^{-3}-10^{-4}$, and $10^{-1}-10^{-2}$ in the Higgs-boson model for D^0-$\bar{D}^0$, B_d^0-$\bar{B}_d^0$, B_s^0-$\bar{B}_s^0$, and T_u^0-$\bar{T}_u^0$ respectively. Though the measurement of the dilepton charge asymmetry in the $T^0-\bar{T}^0$ system is one of the best means to distinguish between two schemes, such an experiment is made difficult by the fact that T^0-$\bar{T}^0$ mixing is very small.

The KM and Higgs-boson models of *CP* violation are quite different in their predictions for *CP*-odd effects associated with the leptons in semileptonic decays. At the tree level a *CP*-violating lepton polarization cannot arise in the KM scheme. However, in the Higgs-boson-exchange model the *T*-odd polarization of the muon in the kaon decay is also quite small and is of order 10^{-4} (the corresponding $\mathrm{Im}\xi'$ is of order 10^{-3}) unless the ratio ${v_2}^2/{v_3}^2$ is unusually large. The *T*-violating polarization is, nonetheless, large for the heavy lepton produced in the semileptonic decay of a heavy meson. For instance, in the decay $B_u^- \to D^0 \tau^- \bar{\nu}$, $P_\perp$ is expected to be of order $\sim 10^{-2}$ (while $\mathrm{Im}\xi' \sim 10^{-1}$), which may be practicably measurable in the future.

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APPENDIX

We evaluate in this appendix the box diagram of Fig. 1; the other box diagrams in Fig. 2 can be treated in a similar way. The neutral heavy mesons in the box diagrams contain one heavy (h) and one light (l) quark. In the 't Hooft–Feynman gauge the amplitude for the box diagram in Fig. 1 has the form

$$M=i\left[\frac{-ig}{2\sqrt{2}}\right]^4\sum_{i,j}\xi_i\xi_j\int\frac{d^4q}{(2\pi)^4}\bar{l}(p_3)\gamma_\mu(1-\gamma_5)\frac{1}{\not{q}-m_i}\gamma_\nu(1-\gamma_5)h(p_1)\bar{l}(p_2)\gamma_\nu(1-\gamma_5)$$
$$\times\frac{1}{\not{q}-\not{p}_1-\not{p}_2-m_j}\gamma_\mu(1-\gamma_5)h(p_4)\frac{1}{(q-p_1)^2-M_W{}^2}\frac{1}{(q-p_3)^2-M_W{}^2}\,, \tag{A1}$$

where $\xi_i=U_{ih}U_{il}^*$ ($U_{li}U_{hi}^*$) when both h and l are D- (U-) type quarks and the U_{ij}'s are the Kobayashi-Maskawa matrix elements as given in (3.1). Assuming $p_1\gg p_2$, $p_4\gg p_3$ and noting that

$$[\gamma_\mu\gamma_\alpha\gamma_\nu(1-\gamma_5)][\gamma_\nu\gamma_\beta\gamma_\mu(1-\gamma_5)]=4[\gamma_\beta(1-\gamma_5)][\gamma_\alpha(1-\gamma_5)]\,, \tag{A2}$$

we find

$$M=i\frac{g^4}{4}\sum_{i,j}\xi_i\xi_j\int\frac{d^4q}{(2\pi)^4}\bar{l}(p_3)\gamma_\nu(1-\gamma_5)h(p_1)\bar{l}(p_2)\gamma_\mu(1-\gamma_5)h(p_4)q_\mu(q-p_1)_\nu$$
$$\times\frac{1}{(q^2-M_W{}^2)(q^2-m_i{}^2)[(q-p_1)^2-M_W{}^2][(q-p_1)^2-m_j{}^2]}\,. \tag{A3}$$

The momentum integral

$$I=\int d^4q\frac{q_\mu(q-p_1)_\nu}{(q^2-M_W{}^2)(q^2-m_i{}^2)[(q-p_1)^2-M_W{}^2][(q-p_1)^2-m_j{}^2]}$$

can be written as

$$I=\frac{1}{(M_W{}^2-m_i{}^2)(M_W{}^2-m_j{}^2)}$$
$$\times\int d^4q q_\mu(q-p_1)_\nu\left\{\frac{1}{(q^2-M_W{}^2)[(q-p_1)^2-M_W{}^2]}+\frac{1}{(q^2-m_i{}^2)[(q-p_1)^2-m_j{}^2]}\right.$$
$$\left.-\frac{1}{(q^2-M_W{}^2)[(q-p_1)^2-m_j{}^2]}-\frac{1}{(q^2-m_i{}^2)[(q-p_1)^2-M_W{}^2]}\right\}. \tag{A4}$$

The integral I can be easily evaluated using the method of dimensional regularization and all terms which are not functions of $m_i{}^2$, $m_j{}^2$, or $M_W{}^2$ can be thrown away. With the definitions $\lambda_i=m_i{}^2/M_W{}^2$, $\lambda_j=m_j{}^2/M_W{}^2$, and $\lambda_h=m_h{}^2/M_W{}^2$ (m_h is the mass of the heavy quark h), we find

$$I=-i\frac{\pi^2}{2}\frac{1}{M_W{}^2}\frac{1}{(1-\lambda_i)(1-\lambda_j)}\sum_{n=1}^{4}{}'\int_0^1dx\left[q_{\mu\nu}\Lambda_n-\frac{2p_{1\mu}p_{1\nu}}{M_W{}^2}x(1-x)\right]\ln\Lambda_n\,, \tag{A5}$$

where

$$\Lambda_1=1-\lambda_hx(1-x)\,,\quad \Lambda_2=\lambda_i(1-x)+\lambda_jx-\lambda_hx(1-x)\,,$$

$$\Lambda_3=\lambda_i(1-x)+x-\lambda_hx(1-x)\,,\quad \Lambda_4=1-x+\lambda_jx-\lambda_hx(1-x)\,,$$

and

$$\sum_{n=1}^{4}{}'=\sum_{n=1}^{2}-\sum_{n=3}^{4}\,.$$

Substituting (A5) into (A3) we obtain

$$M=\frac{G_F{}^2}{4\pi^2}M_W{}^2\bar{l}(p_3)\gamma_\nu(1-\gamma_5)h(p_1)\bar{l}(p_2)\gamma_\mu(1-\gamma_5)h(p_4)$$

$$\times\sum_{i,j}\frac{\xi_i\xi_j}{(1-\lambda_i)(1-\lambda_j)}\int_0^1 dx\sum_n{}'\left[g_{\mu\nu}\Lambda_n-\frac{2p_{1\mu}p_{1\nu}}{M_W{}^2}x(1-x)\right]\ln\Lambda_n\ . \tag{A6}$$

Summing the contributions from both s and t channels yields the effective Hamiltonian

$$H_{\rm eff}=B\bar{l}_\alpha\gamma_\mu(1-\gamma_5)h_\alpha\bar{l}_\beta\gamma_\mu(1-\gamma_5)h_\beta+C\bar{l}_\alpha(1+\gamma_5)h_\alpha\bar{l}_\beta(1+\gamma_5)h_\beta+\text{H.c.} \tag{A7}$$

with

$$B=\frac{G_F{}^2}{8\pi^2}M_W{}^2\sum_{i,j}\frac{\xi_i\xi_j}{(1-\lambda_i)(1-\lambda_j)}\int_0^1 dx\sum_n{}'\Lambda_n\ln\Lambda_n\ ,$$

$$C=\frac{G_F{}^2}{8\pi^2}M_W{}^2\sum_{i,j}\frac{\xi_i\xi_j}{(1-\lambda_i)(1-\lambda_j)}\int_0^1 dx\,2\pi x(1-x)\lambda_h\sum_n{}'\ln\Lambda_n\ ,$$

where α and β are color indices. Equation (A7) is the effective Hamiltonian arising from the box diagrams with the exchange of two W bosons. Equation (A7) reduces to (4.7) which was originally derived in Ref. 43.

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