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Observer dependence of quantum states in relativistic quantum field theories

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Quantum states can be understood as either (i) describing quantum systems or (ii) representing observers' knowledge about quantum systems. These different meanings are shown to imply different transformation properties in relativistic field theories. The rules for the reduction of quantum states and the transformation properties of quantum states under Lorentz transformations are derived for case (ii). The results obtained are applied to a quantum system recently presented and analyzed by Aharonov and Albert. It is shown that the present results, combined with Aharonov and Albert's, amount to a proof of Bohr's view that quantum states represent observers' knowledge about quantum systems.

I. INTRODUCTION

In recent Phys. Rev. articles[1,2] Aharonov and Albert presented a new and poignant analysis of the measurement process in relativistic quantum mechanics. The reduction of the wave function was one of the issues discussed. Aharonov and Albert have proved[2] that quantum states do not transform covariantly because the reduction of a quantum state occurs in each frame of reference on a constant-time hypersurface of the frame.

The noncovariance of quantum states is a surprising result, if one believes that quantum states describe quantum systems. In the present paper we explore the alternative to this assumption: the view apparently adhered to by Niels Bohr, that states represent not quantum systems, but observers' knowledge of them.[3]

The distinction between describing systems and representing observers' knowledge may seem to be merely a philosophical subtlety. We show in Secs. II and III, however, that this distinction has immediate mathematical implications in regards to the formalism of relativistic quantum field theories.

The results of Secs. II and III can be summarized as follows. (i) For each observer, the reduction of a quantum state which corresponds to a given measurement occurs when the information about the measurement can become available to him, i.e., at a time $t+t'$, where t is the time of measurement and t' is the time it takes a light signal to reach him. The reduction occurs for each observer on *his* constant-time hyperplane. (ii) When the information about a system which is available to two observers is identical, their quantum states of the system, i.e., the wave functions of the system in their respective coordinate systems, are covariantly related in that region of space-time which is future for both. No covariant relationship is expected in general in other regions of space-time.

Section IV is devoted to an application of these results to the case of a scalar boson in an infinitely deep double-well potential, presented in Ref. 1. We explicitly prove that whenever there is a discrepancy between describing the state and describing observers' knowledge of it, the quantum state contains the latter information.

Section V is devoted to a comparison of these results with Aharonov and Albert's analysis. We finally conclude that the combined results of Ref. 2 and the present paper amount to a proof that within the conventional framework of quantum mechanics the only meaning of quantum states which is free of inconsistencies is Bohr's—viewing them as representing observers' knowledge of quantum systems.

II. THE DEPENDENCE OF QUANTUM STATES ON OBSERVERS' POSITION AND VELOCITIES

Consider the following statements:

(I) "A quantum state describes a system."

(II) "A quantum state represents an observer's knowledge of a system." What precisely is the difference between them?

If we consider as a simple example of a system a scalar boson field, then statement (I) implies that at each point of space-time the value of the wave function[4] is independent of the coordinate system.[5] Statement (II) carries no such implication. Different observers' information about the scalar boson field may well be different, with corresponding differences in the values of the wave functions. The implications which statement (II) does carry are the subject of the present section and of Sec. III.

The phrase "an observer's knowledge of a system at time t" is taken to mean "the information about the system which can be available at the spatial origin of a given frame of reference at the time t by appropriate lightlike signals."

Consider a case in which a measurement which causes the quantum state of a given system to collapse[4] has taken place at the spatial origin of an (x,y,z,t) system at time $t=0$. Consider another observer, located at $x=d$, $y=z=0$ and let his coordinate system (x',y',z',t') be defined by $x'=x-d$, $y'=y$, $z'=z$, $t'=t$.

The information about the measurement will become available to the primed observer at $t'=d/c$. If we accept that for this primed observer the quantum state describes *his* knowledge about the system, then the collapse occurs for him at $t'=d/c$.

This example demonstrates that if a collapse occurs for a given observer when the information about a measurement becomes available to him, then quantum states depend, in an essential way, on the position of the observer. Furthermore, because the collapse occurs for each observer on *his* hyperplane $t=\text{const}$,[2] the quantum state depends, in an essential way, on the observers' velocity as well.

Even when one does not consider collapse, quantum states have observers' dependence. This has been routinely taken into account by standard mathematical procedures, e.g., the spinor transformations for Dirac wave functions. Such procedures represent *the different mathematical expressions of the same set of statistical predictions about a given system.* When collapses are taken into account, however, *different observers' quantum states may well represent different probabilistic predictions about the system.* In the case discussed at the beginning of this section, for example, during the time interval $0 \le t < d/c$, $\psi(x,y,z,t)$ is collapsed, and a position measurement is predicted with certainly to result in a value close to x_1; $\psi(x',y',z',t')$ is uncollapsed. The corresponding prediction is that there are equal probabilities for the measurement to result in a value close to x_1 and in a value close to x_2.

To summarize, for a given measurement of a given system and for a given observer it follows from statement (II) that the collapse of the quantum state occurs on the observer's constant-time hyperplane at time t' after the time of measurement in his frame of reference, where t' is the time it takes a lightlike signal carrying the information about the measurement to reach the spatial origin of his frame.

III. TRANSFORMATION PROPERTIES OF RELATIVISTIC WAVE FUNCTIONS

The present section is devoted to an analysis of the implications of Bohr's view about the meaning of quantum states as far as transformation properties are concerned.

Consider the simple example discussed in Sec. II, or, more dramatically, Aharonov and Albert's[1] particle in a double-well potential (to be fully analyzed in Sec. IV). These examples show that if a reduction of a given observer's wave function occurs on his $t=\text{const}$ hyperplane at the instant t when information about a measurement can reach his spatial origin, then the histories of two observers' wave functions are not related by covariant Lorentz transformations.

To appreciate the origin of this result consider Wheeler's statement "No phenomenon is a phenomenon until it is observed as a phenomenon"[6] together with the fact that the reduction of a wave function is not, in itself, a phenomenon. The measurement which caused the reduction is a phenomenon—a physical event. The reduction is not. Away from the measuring apparatus which caused the reduction, there can be no measuring device which merely registers that a reduction has occurred. Such a hypothetical device cannot exist, because any interaction between a measuring instrument and the system is, in general, a cause for a new reduction.

Classically, Lorentz transformations relate the same event as viewed by different observers. Quantum mechanically Lorentz-covariant transformations among quantum states relate different observers' proabability distributions for the same system *if the same information about the system is available to them.* They do not and need not refer to the relationship between the *histories of different observers' knowledge* of a given system. It follows that if states represent observers' knowledge about systems, their past values will not be, in general, covariantly related.

On the other hand, when the same information about a system is available to two observers we do expect their statistical predictions for results of all measurements to be the same. Consequently, if the information available at the origin of an unprimed coordinate system at $t=t_A$ is the same as the information available at the origin of a primed coordinate at $t'=t'_B$, then the wave functions describing the two observers' knowledge of a given system *should transform covariantly in that region of space-time which is common to both regions* $t>t_A$ *and* $t'>t'_B$.

To summarize, if quantum states represent observers' knowledge of quantum systems rather than the systems themselves, then different observers' states of the same system will be covariantly related in their common future, provided the same information is available to them. Regardless of whether or not the same information is available to different observers, their wave-function histories will not be, in general, covariantly related.

IV. AHARONOV AND ALBERT'S PARTICLE IS A DOUBLE-WELL POTENTIAL

In the present section we re-examine a particular case, presented and analyzed in Ref. 1. We will show (i) that the noncovariance of quantum state rules out their interpretation as describing quantum systems and (ii) that the information contained in the quantum state is the observer's knowledge about the system. The term "observer's knowledge" is used as defined in Sec. II.[7]

Consider the case of a charged spin-0 particle in the space of a double-well time-independent potential (Fig. 1). Its state is given in an unprimed coordinate frame as

$$|\psi\rangle=|\beta\rangle=\frac{1}{\sqrt{2}}(|x_1\rangle+|x_2\rangle)\,,\quad t<0\,,$$
$$|\psi\rangle=|x_1\rangle\,,\quad t>0\,. \tag{1}$$

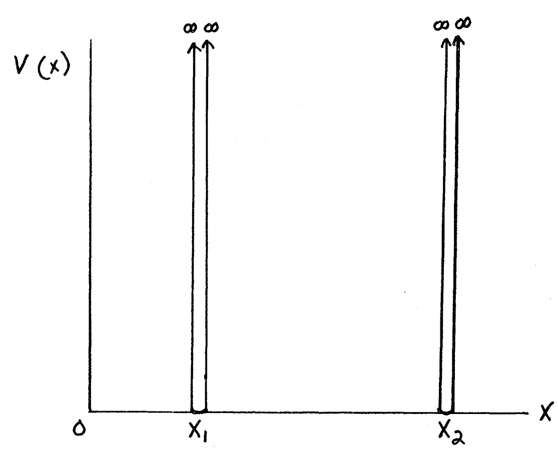

FIG. 1. An infinite double-well potential (Ref. 1).

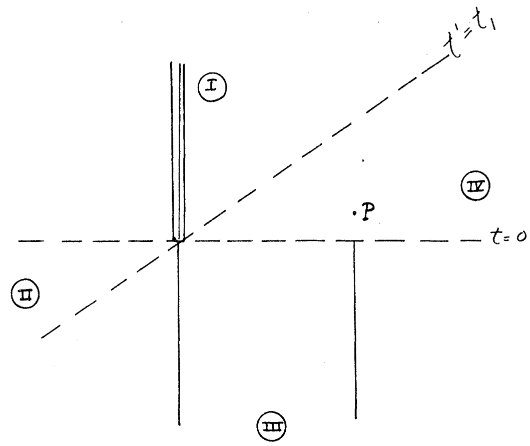

FIG. 2. The collapse of $|\beta\rangle$ to $x=x_1$.

The current density in this frame is

$$j^\mu(\vec{x},t)=\langle\psi\,|\,\Psi^*\nabla^\mu\Psi-\Psi\nabla^\mu\Psi^*\,|\,\Psi\rangle e \tag{2}$$

and the charge density is, therefore,

$$\rho(\vec{x},t)=\langle\Psi\,|\,\Psi^*\dot{\Psi}-\Psi\dot{\Psi}^*\,|\,\Psi\rangle e\,, \tag{3}$$

where ψ is the scalar field operator. It was shown in Ref. 1 that if this $|\psi\rangle$ is covariantly transformed to another (primed) frame,

$$x'^\mu=\Lambda^\mu{}_\nu x^\nu\,, \tag{4}$$

where $\Lambda^\mu{}_\nu$ is the Lorentz transformation matrix, then the corresponding charge density $\rho'(x'^\mu,t')$ has unphysical properties, such as its integral over all space being less than e at certain times t'.

Applying the results of Ref. 2 to this case one concludes that the source of the apparent paradox is in the assumption that the states transform covariantly. In fact, the reduction of the state occurs for each observer on *his* equal-time hypersurface.

Let us analyze this system using the results of Secs. II and III.

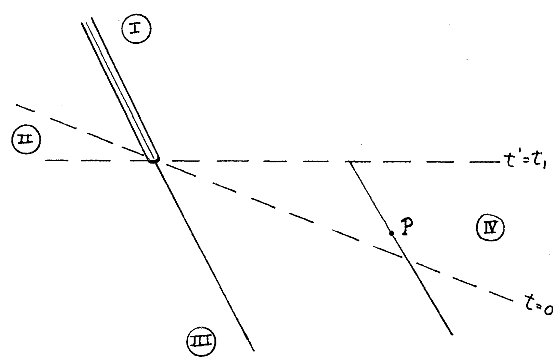

FIG. 3. The history of Fig. 2 as seen in the primed frame.

Figures 2 and 3 represent, respectively, the histories of the states from the (unprimed) frame in which the potential wells are at rest, and from a (primed) frame which is related to the unprimed frame by Eq. (4). Let us look at space-time as divided into 4 regions:

$$\begin{aligned}&\text{I.}\quad t\geq 0 \text{ and } t'\geq t_1\ ,\\&\text{II.}\quad t<0 \text{ and } t'\geq t_1\ ,\\&\text{III.}\quad t<0 \text{ and } t'<t_1\ ,\\&\text{IV.}\quad t\geq 0 \text{ and } t'<t_1\ .\end{aligned}\tag{5}$$

The reduction of the unprimed observer's quantum state occurs on his hyperplane $t=0$. The reduction of the primed observer's quantum states occurs on his hyperplane $t'=t_1$. Comparing Figs. 2 and 3, the two wave functions are related covariantly in regions I and III, and not related covariantly in regions II and IV.

Region I is that part of space-time which is the common future for both frames after reduction has occurred in both. In region I the quantum states are covariantly related, as expected (Sec. III). Noncovariant relationship is confined to regions II and IV, which are past for one of the observers. If past history of a wave function is to represent the history of an observer's knowledge, no covariant relationship need exist between two past histories, or between past history of the knowledge of one observer and future predictions of another.

The covariant relationship on the hypersurface

$$\begin{aligned}&t=0 \text{ for } x\leq x_1\ ,\\&t'=t_1 \text{ for } x\geq x_1\end{aligned}\tag{6}$$

ensures that when the values of the wave functions on the hypersurface (6) are taken as initial values both observers will derive the same sets of probabilities for measurements in region I.

The significance of these results in relation to the meaning of quantum states will be apparent if we consider the following. Let the event P (Figs. 2 and 3) indicate the location and time of a measurement designed to answer the question "Is the particle at the location x_2?" Let P be at a spacelike separation from the event A (the coordinate of A are $x=x_1$, $t=0$ in the unprimed frame). As we see from Fig. 2, the probability to find the particle at P is zero. This fact is known in both frames at $x=x_1$, $t>0$ and $x'=x_1'$, $t'>t_1$, respectively. Nevertheless, as Fig. 3 shows, the quantum state of the primed frame at P indicates a probability of $\frac{1}{2}$ to find the particle at x_2.

The quantum state at P in the primed frame describes neither the state of the system nor the knowledge which the primed observer has about it at later times, such as $t'=t_1+\epsilon$. It describes, rather, *the knowledge which the primed observer has about the system at the time of the event P.*

V. DISCUSSION

How do the present results compare with the results of Ref. 2? There is no question, in the view of the present author, that Aharonov and Albert have proved that the instantaneity of state reduction does not restrict the set of observables to purely local ones. Clearly, they have also proved that state reduction is instantaneous in each frame and that in general states do not transform covariantly. The points of a possible discrepancy between their views and the views expressed in the present paper are the following.

(1) The dependence of quantum states on the location of the observer (the location of the origin of the frame of reference) is not discussed in Refs. 1 and 2. In discussing nonlocal simultaneous measurements, the reduction is apparently taken to occur at the time of these measurements. According to the view presented here the reduction will take place, for a given observer, at the time at which the results of such measurements can reach him by lightlike signals, e.g., in the case of the measurements of nonlocal observables which take place by two simultaneous local interactions at x_1 and x_2, the reductions referred to in Ref. 2, Sec. II apply to an observer located at $(x_1+x_2)/2$ at a time $t=t_0+|x_1-x_2|/2c$, where t_0 is the time of interaction. For other observers the reduction will occur when lightlike signals from both x_1 and x_2 can reach them, following the measurement at t_0.

Let us point out that this view of the reduction makes operational sense. It takes into account the dependence of the information about systems on the location of the observers. This kind of observer dependence of states clearly entails the possibility of differences among observers concerning probabilistic predictions.

(2) There is another point of possible discrepancy concerning the statement "covariance of relativistic quantum states reside exclusively in experimental probabilities,"[2] which is unrelated to the dependence of the reduction on the location of observers. If the term "probability" indicates a reference to future measurements, it is in accord with the results of Sec. III. If, however, it may refer to mea-

surements which are in the future for one observer and in the past for another, there may be a discrepancy. The analysis of Sec. IV provides an example in which if the quantum state is interpreted as providing the probability for the result of a past experiment at spacelike separation, the probability is wrong, and not a covariant transform. In our view past quantum states do not provide probabilities of past measurements on the basis of present knowledge. They provide, rather, probabilities of these measurements as calculated from the information about the systems available at the time of these measurements.

While the present view on these points seems to be different from Aharonov and Albert's, it is in no way contradictory to their main new results. The changes in Ref. 2 which the present view calls for are fairly obvious.

Having shown, then, on the basis of the results of Ref. 2, that quantum states do not describe quantum systems, we can conclude that the combined results of Ref. 2 and the present paper amount to a proof that within the conventional framework of quantum mechanics the only meaning of quantum states which is free of inconsistencies is Bohr's—viewing them as representing observers' knowledge of quantum systems.[8]

ACKNOWLEDGMENTS

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[1]Y. Aharonov and D. Z. Albert, Phys. Rev. D 21, 3316 (1980).

[2]Y. Aharonov and D. Z. Albert, Phys. Rev. D 24, 359 (1981).

[3]"When asked whether the algorithm of quantum mechanics could be considered as somehow mirroring an underlying quantum world, Bohr would answer 'There is no quantum world. There is only an abstract quantum description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature'." [A. Petersen, Bull. At. Sci. 19, 8 (1963).]

[4]The terms "states," "quantum states," and "wave functions" will be used interchangeably. "Likewise, the terms "reduction of a quantum state" and "collapse of a quantum state" will be used interchangeably."

[5]In the general case statement (I) implies a Lorentz-covariant transformation between the values of any two observers' wave functions at all points of space-time.

[6]J. A. Wheeler, in *Mathematical Foundations of Quantum Theory,* edited by A. R. Marlow (Academic, New York, 1978).

[7]The same analysis can be analogously made using the two-spins system of Ref. 2.

[8]If one accepts Einstein, Podolsky, and Rosen's view that quantum mechanics is incomplete, the analysis of the present paper leads to the conclusion that the concept of wave functions can be retained as representing the (incomplete) knowledge of an observer. For a recent review of the Einstein-Podolsky-Rosen paradox by one of its authors see N. Rosen, in *Albert Einstein, His Influence on Physics, Philosophy and Politics,* edited by P. C. Aichelburg and U. Sexl (Vieweg and Shon, Braunschuig/Wiesbaden, 1979).