Double-logarithmic asymptotics of quark scattering amplitudes with flavor exchange

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We propose simple equations in terms of the definite-signature partial waves of the quark scattering and annihilation amplitudes with quark-quark and quark-antiquark states in the exchange channel. We discuss the singularities in the complex angular momentum plane generated by the double-logarithmic contributions and point out their relation to the particle Regge trajectories.

Quark and gluon scattering appear as subprocesses in hadronic reactions with large transverse momenta in the final state and in exclusive two-body reactions with large momentum transfer.

In this Communication we present results on the quark scattering and annihilation amplitudes in the Regge region (see Fig. 1)

$$s \simeq -u >> \mu^2 \gtrsim |t| \quad \text{or} \quad s \simeq -t >> \mu^2 \gtrsim |u| \ . \qquad (1)$$

We introduce an infrared cutoff μ. In principle, this regularization can be introduced in a gauge-invariant way. The parameter μ is chosen large compared to the strong-interaction scale $\Lambda \simeq 100$ MeV:

$$\alpha(\mu^2) \equiv \frac{g^2(\mu^2)}{4\pi} << 1 \ .$$

We calculate the amplitudes in the double-logarithmic approximation applicable in the region

$$\frac{\alpha(\mu^2)}{\pi}\ln^2\left|\frac{s}{\mu^2}\right| \lesssim 1 \ , \quad \frac{\alpha(\mu^2)}{\pi}\ln\left|\frac{s}{\mu^2}\right| << 1 \ . \qquad (2)$$

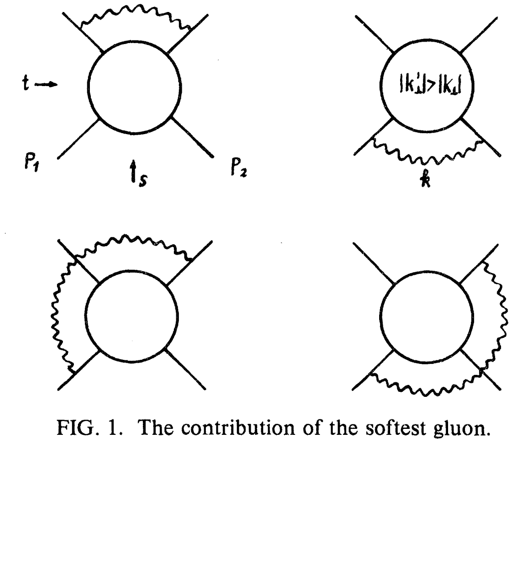

FIG. 1. The contribution of the softest gluon.

We consider the amplitudes of annihilation ($q\bar{q} \to Q\bar{Q}$ and $q\bar{q} \to \bar{Q}Q$) and backward scattering ($qQ \to qQ$ and $q\bar{Q} \to q\bar{Q}$). The corresponding graphs have two quark lines in the t channel or u channel, respectively. We distinguish the case of the exchange of a diquark state with baryon number $\frac{2}{3}$ $[D(s)]$ and the case of the exchange of a mesonlike state with baryon number 0 $[M(s)]$. With respect to the color group $SU_C(N)$ the amplitudes can be decomposed into two parts corresponding to a singlet $[M_0(s)]$ and a vector $[M_V(s)]$ state or an antisymmetric $[D_A(s)$, representation $\underline{\bar{3}}$ in the case $N=3]$ and a symmetric state $[D_S(s)$, representation $\underline{6}$ in the case $N=3]$ in the exchange channel.

Our approach differs from the usual one,[1,2] which is based on deriving Bethe-Salpeter integral equations and where one considers off-mass-shell amplitudes. The integral equations contain more information than one needs finally. In our approach we deal with on-mass-shell amplitudes only and maintain gauge invariance (in the approximation adopted). The resulting equations are much simpler: In terms of partial waves they are differential equations of Riccati type or just algebraic equations.

The main point in our approach consists in isolating the softest virtual particle with the lowest transverse momentum $\kappa_\perp$ in the Feynman graphs. It turns out that the integration over the momenta of the remaining virtual particles can be expressed in terms of on-mass-shell amplitudes with the replacement $\mu \to |\kappa_\perp|$. The physical idea is reminiscent of the renormalization group (separation of interactions at different scales).[3]

Consider an arbitrary graph contributing to the amplitude $M(s)$ or $D(s)$ and take the softest virtual particle. Consider first the case that the softest virtual particle is a gluon with the momentum k, $k = \alpha p_2 + \beta p_1 + \kappa_\perp$. The transverse component $\kappa_\perp$ is much smaller than the transverse component in the

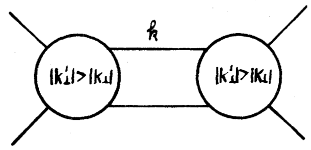

FIG. 2. The contribution of the softest fermion.

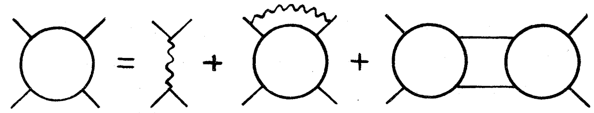

FIG. 3. The equation for the double-logarithmic amplitude.

remaining loops, $|\kappa_\perp| \ll |\kappa_\perp^j|$. It can be shown that in the integral over k, the pole terms in the invariants $(p_1-k)^2 \simeq -s\alpha$ and $(p_2+k)^2 \simeq s\beta$ dominate. In other words, the sum of all double-logarithmic contributions arising from the softest gluon can be represented by the graphs of Fig. 1. The soft gluon is attached to the external lines and the blob represents the amplitude on the mass shell with the cutoff μ replaced by $|\kappa_\perp|$.

In the electrodynamic case the analogous statement has first been proven by Gribov and other authors.[4] The generalization to Yang-Mills theories and quantum gravity has been used to verify gluon and graviton Reggeization.[5] The proof is based on dispersion relations with respect to the invariants $(p_1-k)^2$ and $(p_2+k)^2$ and uses gauge invariance.

Consider now the case that the softest virtual particle is a fermion. One shows in a similar way as for the soft gluon that the sum of the corresponding contributions can be represented by the graph of Fig. 2. The blobs are the amplitude on the mass shell with the cutoff μ replaced by $|\kappa_\perp|$. Besides the two fermion lines there are no further virtual lines connecting the blobs. Only the transverse parts of the fermion propagators contribute in the double-logarithmic approximation.[1]

Now it is not difficult to see that the amplitude obeys an equation represented in terms of graphs in Fig. 3. The first graph on the right-hand side is the Born term. The second graph stands for the sum of the graphs of Fig. 1. In the following we obtain this equation in terms of partial waves. A more extended discussion of the arguments with more technical details is given in a further paper.[6]

Using the fact that in double-logarithmic approximation the spinor structure of the Born term is maintained in higher orders[1] we write the amplitude as

$$b_0(s,t)\, M\left(\frac{s}{\mu^2}\right) \quad \text{or} \quad b_0(s,t)\, D\left(\frac{s}{\mu^2}\right) , \tag{3}$$

where $b_0(s,t)$ is the Born amplitude except for the color structure and, for convenience, the coupling g^2. Above we introduced the components of the amplitudes $M=(M_0,M_V)$ and $D=(D_A,D_S)$ with states of definite gauge-group quantum numbers in the exchange channel. The Born term corresponds to vector exchange in the direct channel. Projecting on states with definite gauge-group [SU(N)] quantum numbers in the exchange channel we have in the Born approximation

$$\begin{aligned} M_0|_{\text{Born}} &= \frac{N^2-1}{2N} g^2 , \quad M_V|_{\text{Born}} = -\frac{1}{2N} g^2 , \\ D_A|_{\text{Born}} &= -\frac{N+1}{2N} g^2 , \quad D_S|_{\text{Born}} = \frac{N-1}{2N} g^2 . \end{aligned} \tag{4}$$

The contribution of the softest gluon (Fig. 1) to the amplitude is given by

$$M\left(\frac{s}{\mu^2}\right)\Bigg|_{\text{gluon}} = \frac{g^2}{8\pi^2}\int_{\mu^2}^{|s|} \frac{d|\kappa_\perp^2|}{|\kappa_\perp^2|}\left[(\hat m_s - \hat m_u)\left(\ln\left|\frac{s}{\kappa_\perp^2}\right| - \frac{i\pi}{2}\right) + \left(-\frac{i\pi}{2}(\hat m_s + \hat m_u)\,\text{sgn}(s)\right)\right] M\left(\frac{s}{|\kappa_\perp^2|}\right) . \tag{5}$$

The first term in the brackets is even in s and hence does not change the signature of the amplitude $M(s/|\kappa_\perp^2|)$. The second term represents the signature-changing contribution. The matrices $\hat m_s$ and $\hat m_u$ are given by

$$\hat m_s = \begin{pmatrix} 0 & \dfrac{N^2-1}{2N} \\ \dfrac{1}{2N} & \dfrac{N^2-2}{2N} \end{pmatrix} , \quad \hat m_u = \begin{pmatrix} 0 & \dfrac{N^2-1}{2N} \\ \dfrac{1}{2N} & -\dfrac{1}{N} \end{pmatrix} . \tag{6}$$

The soft-fermion contribution (Fig. 2) is given by

$$M_i^P\left(\frac{s}{\mu^2}\right)\Bigg|_{\text{fermion}} = \frac{-i}{(2\pi)^4}\int \frac{|s|\, d\alpha\, d\beta\, d^2\kappa_\perp \kappa_\perp^2}{(s\alpha\beta + \kappa_\perp^2 + i\epsilon)^2} \frac{s}{-s\alpha} M_i^P\left(\frac{-s\alpha}{|\kappa_\perp^2|}\right) \frac{1}{s\beta} M\left(\frac{s\beta}{|\kappa_\perp^2|}\right) . \tag{7}$$

i labels the color-singlet and -vector channels 0 and V. The integration runs over the region

$$\infty > -\kappa_\perp^2 > \mu^2 , \quad -\infty < \alpha, \beta < \infty , \quad s\alpha\beta \simeq \kappa_\perp^2 , \quad |s\alpha| \gtrsim |\kappa_\perp^2|, \quad |s\beta| \gtrsim |\kappa_\perp^2| . \tag{8}$$

Here the conservation of the signature P (Ref. 7) and the conservation of the gauge-group quantum numbers have been used.

In double-logarithmic approximation the Sommerfeld-Watson transformation reduces to a Mellin transformation.[8] We write the soft-gluon contributions Eq. (5) and the soft-fermion contributions Eq. (7) in terms of the transformed amplitudes $f_i^P(\omega)$. For the soft-fermion contribution the calculation is analogous to the discussion of the enhancement problem in Reggeon calculus.[7] With the results we are able to write the equation of Fig. 3 in terms of $f_i^P(\omega)$.

For the positive-signature amplitudes we have

$$\begin{aligned} f_0^+(\omega) &= a_o\frac{g^2}{\omega}+\frac{1}{8\pi^2}\frac{1}{\omega}[f_0^+(\omega)]^2 , \\ f_i^+(\omega) &= a_i\frac{g^2}{\omega}+b_i\frac{g^2}{8\pi^2}\frac{1}{\omega}\frac{d}{d\omega}f_i^+(\omega) \\ &\quad +c_i\frac{1}{8\pi^2}\frac{1}{\omega}[f_i^+(\omega)]^2 \quad (i=V,A,S) \end{aligned} \tag{9}$$

with the boundary conditions

$$f_i^+(\omega)|_{\omega\to\infty}=a_i\frac{g^2}{\omega} \quad (i=0,V,A,S) . \tag{10}$$

The coefficients are given by

$$\begin{aligned} &a_0=\frac{N^2-1}{2N} , \\ &a_V=-\frac{1}{2N},\quad b_V=N,\quad c_V=+1 , \\ &a_A=-\frac{N+1}{2N},\quad b_A=\frac{(N+1)(N-2)}{N},\quad c_A=-1 , \\ &a_S=\frac{N-1}{2N},\quad b_S=\frac{(N-1)(N+2)}{N},\quad c_S=-1 . \end{aligned} \tag{11}$$

The resulting equations for the negative-signature amplitudes are more complicated due to the signature-changing contribution in Eq. (5):

$$\begin{aligned} f_0^-(\omega) &= a_0\frac{g^2}{\omega}-\frac{N^2-1}{N}\frac{g^2}{4\pi^2}\frac{1}{\omega^2}f_V^+(\omega)+\frac{1}{8\pi^2}\frac{1}{\omega}[f_0^-(\omega)]^2 , \\ f_V^-(\omega) &= a_V\frac{g^2}{\omega}-\frac{N^2-4}{2N}\frac{g^2}{4\pi^2}\frac{1}{\omega^2}f_V^+(\omega)-\frac{1}{N}\frac{g^2}{4\pi^2}\frac{1}{\omega^2}f_0^+(\omega)+b_V\frac{g^2}{8\pi^2}\frac{1}{\omega^2}\frac{d}{d\omega}[\omega f_V^-(\omega)]+\frac{1}{8\pi^2}\frac{1}{\omega}[f_V^-(\omega)]^2 , \\ f_A^-(\omega) &= a_A\frac{g^2}{\omega}-\frac{N+1}{2}\frac{g^2}{4\pi^2}\frac{1}{\omega^2}f_S^+(\omega)+b_A\frac{g^2}{8\pi^2}\frac{1}{\omega^2}\frac{d}{d\omega}[\omega f_A^-(\omega)]-\frac{1}{8\pi^2}\frac{1}{\omega}[f_A^-(\omega)]^2 , \\ f_S^-(\omega) &= a_S\frac{g^2}{\omega}-\frac{N-1}{2}\frac{g^2}{4\pi^2}f_A^+(\omega)+b_S\frac{g^2}{8\pi^2}\frac{1}{\omega^2}\frac{d}{d\omega}[\omega f_S^-(\omega)]-\frac{1}{8\pi^2}\frac{1}{\omega}[f_S^-(\omega)]^2 . \end{aligned} \tag{12}$$

The Born contributions a_ig^2/ω coincide for both signatures because there is only a s-channel singularity in lowest order.

$f_0^+(\omega)$ has a square-root cut starting from $\omega=\omega_0^+\equiv[g^2(N^2-1)4\pi^2N]^{1/2}$. $f_0^-(\omega)$ has singularities to the right of ω_0^+. This is clear for large N, where $f_V^+(\omega)$ can be replaced by its Born term. It can be checked that this property is maintained down to $N=2$.

The solution for the other positive-signature channels is given by

$$\begin{aligned} f_i^+(\omega) &= \frac{g^2}{P_i}\frac{d}{d\omega}\ln\left[e^{-(\omega/\omega_i)^2/2}\mathfrak{D}_{P_i}\left(\frac{\omega}{\omega_i}\right)\right] , \\ P_i &= \frac{a_i}{b_i},\quad \omega_i^2=\frac{g^2}{8\pi^2}b_i \qquad (i=V,A,S) . \end{aligned} \tag{13}$$

$f_i^+(\omega)$ has simple poles at the zeros of the parabolic cylinder function $\mathfrak{D}_P(x)$.[9] This solution holds for $N\geq 3$ if $i=V,S$ and for $N\geq 3$ if $i=A$. Notice that at $N=2$ $f_0^P=-f_A^P$ and $f_V^P=-f_S^P$. At $N\geq 3$ and for the symmetric (S) and the vector (V) channels at $N=2$ the zeros of $\mathfrak{D}_P(x)$ lie to the left of the imaginary axis. The negative-signature amplitudes of the same channels behave at $\omega\to 0$ like $\ln\omega$.

We see that in all cases the negative-signature amplitudes have singularities to the right of the rightmost singularity of the corresponding positive-signature amplitude, i.e., the negative-signature amplitudes dominate asymptotically.

The results can be extended to processes with gluons or photons in the initial or final states.

The results are easily extended to the case $\mu^2>>-t$. It is also possible to include phenomenologically the contribution from the confinement region $|\kappa_\perp|\sim\Lambda\simeq 100$ MeV. The color-singlet partial waves contain moving meson Regge poles. The proper generalization of a particle Regge-pole contribution including the perturbative double logarithms has the form (compare Ref. 10)

$$F^P(\omega,t)=\frac{b(t)}{g^2a_0[f_0^P(\omega)]^{-1}-\alpha^P(t)} . \tag{14}$$

$f_0^P(\omega)$ is the solution for the singlet channel with signature P. $\alpha^P(t)$ is some meson Regge trajectory. $b(t)$ is the residuum function.

We conclude that besides the Regge poles and cuts known from phenomenology there are further singularities of pure perturbative origin. In the double-logarithmic approximation in the color-singlet channel the most important perturbative singularities are fixed cuts starting in the right half plane. Because the particle Regge poles move to the left with increasing t there are regions where the perturbative singularities become important.

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