# New evaluation of the strong $\Delta\Delta\pi$ coupling constant and the weak axial-vector coupling constant $[g_A(0)]_{\Delta\Delta}$

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We calculate the strong $\Delta\Delta\pi$ coupling constant $g^2/4\pi$ and the weak axial-vector coupling constant $[g_A(0)]_{\Delta\Delta}$ without using SU(6) symmetry. The method is based on the dynamical ansatz of level realization of asymptotic flavor SU(2) symmetry, applied to the charge-charge and charge-current SU(2) $\otimes$ SU(2) chiral algebras. We predict that $g^2/4\pi \simeq 125$ and $[g_A(0)]_{\Delta^{++}\Delta^+} \simeq 1.30$.

Recently an elaborate isobar-model partial-wave analysis of $\pi p \to \pi^+\pi^- n$ bubble-chamber events yielded[1,2] a value of the strong $\Delta\Delta\pi$ coupling constant $(g^2/4\pi) \simeq 40 \pm 20$, a value which is considerably smaller than the several previous theoretical estimates based on SU(6),[3] U(12),[4] the quark model,[5] superconvergence relations,[6,7] the MIT bag model with the Goldberger-Treiman relation as discussed in Ref. 1, and others.[8]

In view of this, we wish to present in this paper a new evaluation of the coupling constant by extending the result of previous work[9] (referred to as I throughout this paper), which has been successfully applied to closely related problems. The work, without introducing the concept of SU(6) symmetry, has produced correct values for the weak axial-vector coupling constants $[g_A(0)]_{pn}$ and $[g_A(0)]_{\Delta p}$ and a good nucleon-anomalous-magnetic-moment relation $k_p = -k_n$, etc. As a by-product we also calculate the weak axial-vector coupling constant $[g_A(0)]_{\Delta\Delta}$.

In the realization of a certain class of chiral-$SU_L(2) \otimes SU_R(2)$-type charge-charge and charge-current equal-time commutators by observable hadrons, our technique utilizes the dynamical ansatz that flavor SU($N$) symmetry should be secured levelwise in the asymptotic infinite-momentum limit. In this approach, the quark chiral algebras [$A_\alpha$, $\alpha=\pi^+, 3, \pi^-$, is the SU(2) axial charge] such as

$$[A_{\pi^+},A_{\pi^-}] = 2V_3, \quad [[A_3,A_{\pi^+}],A_{\pi^-}] = 2A_3 \ , \tag{1}$$

$$[[V_3^\mu(0),A_{\pi^+}],A_{\pi^-}] = 2V_3^\mu(0) \ , \qquad [[A_3^\mu(0),A_{\pi^+}],A_{\pi^-}] = 2A_3^\mu(0) \ , \tag{2}$$

are regarded simply as indispensable constraints imposed by (confined) quarks and gluons on the world of observable hadrons with which we solely deal. In response to the requirements of SU(3) color symmetry, hadrons are assumed to obey the level scheme of (mainly $q\bar{q}$ and $qqq$) the constituent quark model. One can then derive (nonperturbative) constraints—without assuming SU(6) symmetry—among the asymptotic hadronic matrix elements of the vector $V_\alpha^\mu(x)$ and axial-vector $A_\alpha^\mu(x)$ currents ($\mu=0,1,2,3$),

$$\langle B(\beta,\vec{s},\lambda)|\, V_\alpha^\mu(x)\ [A_\alpha^\mu(x)]\,|B(\gamma,\vec{t},\lambda')\rangle$$

with $\vec{s}\to\infty$ and $\vec{t}\to\infty$, as well as among those of the axial-vector charges, i.e.,

$$\langle B(\beta,\vec{s},\lambda)|A_\alpha|B(\gamma,\vec{s},\lambda)\rangle$$

with $\vec{s}\to\infty$. $\lambda$ denotes the helicity and $\alpha$, $\beta$, and $\gamma$ the physical SU(2) indices $(\pi^+,3,\pi^-)$.

As in I, we define the relevant asymptotic axial-vector ground-state baryon $(\lambda=\frac{1}{2})$ matrix elements as (suppressing $\vec{s}\to\infty$)

$$\langle p,\tfrac{1}{2}|A_{\pi^+}|n,\tfrac{1}{2}\rangle \equiv f \ ,$$
$$\langle \Delta^{++},\tfrac{1}{2}|A_{\pi^+}|\Delta^+,\tfrac{1}{2}\rangle \equiv -(\tfrac{3}{2})^{1/2}\tilde{g} \ , \tag{3}$$
$$\langle \Delta^{++},\tfrac{1}{2}|A_{\pi^+}|p,\tfrac{1}{2}\rangle \equiv -\sqrt{6}h \ .$$

In addition, we also consider, in this paper, the helicity-$\frac{3}{2}$ asymptotic axial-vector matrix element

$$\langle \Delta^{++},\tfrac{3}{2}|A_{\pi^+}|\Delta^+,\tfrac{3}{2}\rangle \equiv -(\tfrac{3}{2})^{1/2}j \ . \tag{4}$$

In I, we have derived, along with other relations, the following constraints among the above asymptotic matrix elements $f$, $\tilde{g}$, and $h$, namely,

$$f=\tfrac{5}{3}\sqrt{k}, \quad h=\pm(\tfrac{2}{3}\sqrt{k}), \quad \tilde{g}=-\frac{\sqrt{2}}{3}\sqrt{k} \ . \tag{5}$$

Here, among the sum over the whole set of single-particle hadron intermediate states inserted between the two axial charges of one of the algebras of Eq. (1), i.e., $[A_{\pi^+},A_{\pi^-}]=2V_3$, we have denoted the fractional contribution from the *ground*-state baryons (which consist of the $L=0$ $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet baryons) as $k$. The ansatz implies that the fraction is invariant under flavor SU($N$) rotation in the asymptotic limit $\vec{s}\to\infty$. Actually,

$$f\equiv\langle p,\vec{s},\tfrac{1}{2}|\int A^{0}_{\pi^+}(x)\,d^3x\,|n,\vec{t},\tfrac{1}{2}\rangle$$

with $\vec{s}\to\infty$ is equal to $[g_A(0)]_{pn}$ up to the factor $\delta^3(\vec{s}-\vec{t})$. Therefore, $[g_A(0)]_{pn}=\frac{5}{3}\sqrt{k}$ by Eq. (5). If we take the unrealistic value of $k$, $k=1$ (i.e., the saturation assumption by the ground-state baryons), we obtain the SU(6) result $[g_A(0)]_{pn}=\frac{5}{3}$. However, in the present approach one can choose the value of the fraction $k$ ($k$ is positive and $k$ is less than or equal to one) in such a way that $[g_A(0)]_{pn}$ yields the experimental value $[g_A(0)]_{pn}\simeq 1.25$. The fact that one can choose the value of $k$ *consistently* can be tested by using the second constraint in Eq. (5). By using PCAC (partial conservation of axial-vector current) for $h$, one can relate $h$ to the $\Delta\to p\pi$ coupling constant (neglecting the off-mass-shell extrapolation $m_\pi{}^2\to 0$). We find[8] that the value of $k$ determined from the value of $[g_A(0)]_{pn}$ explains the magnitude of the $\Delta p\pi$ coupling or the width of the $\Delta\to p\pi$ decay well. The same procedure applied for the third constraint of Eq. (5) predicts the value of the $\Delta\Delta\pi$ coupling (again tolerating the off-mass-shell extrapolation $m_\pi{}^2\to 0$), which will be discussed below in some detail. As a matter of fact, the constraints in Eq. (5) fix the ratios $h/f$ and $\tilde{g}/f$ independently of the value of $k$. Therefore, we expect that the predictions based on these ratios (i.e., the ratios $g_{\Delta p\pi}/g_{pn\pi}$ and $g_{\Delta\Delta\pi}/g_{pn\pi}$) by using PCAC will be close to the ones obtained by SU(6) symmetry.

The PCAC relation is written as $\partial_\mu A^{\mu}_{\pi^+}(x)=f_\pi m_\pi{}^2\phi_{\pi^+}(x)$ and the pion source function is defined by $(\Box+m_\pi{}^2)\phi_{\pi^+}(x)=j_{\pi^+}(x)$.[10] We now define

$$\langle\Delta^{++}(\vec{s})|A^{\mu}_{\pi^+}(x)|\Delta^{+}(\vec{t})\rangle\equiv e^{iqx}\bar{u}^{\alpha}_{\Delta}(\vec{s})\{[-g_A(q^2)]_{\Delta^{++}\Delta^{+}}g_{\alpha\beta}\gamma^{\mu}\gamma^{5}+\text{terms proportional to } q^{\mu}\}u^{\beta}_{\Delta}(\vec{t})\ . \tag{6}$$

We also define

$$\langle\Delta^{++}(\vec{s})|j_{\pi^+}(x)|\Delta^{+}(\vec{t})\rangle\equiv e^{iqx}\bar{u}^{\alpha}_{\Delta}(\vec{s})[g_{\Delta^{++}\Delta^{+}\pi^{+}}K_{\Delta^{++}\Delta^{+}\pi^{+}}(q^2)i\gamma^5 g_{\alpha\beta}+g'_{\Delta^{++}\Delta^{+}\pi^{+}}K'_{\Delta^{++}\Delta^{+}\pi^{+}}(q^2)i\gamma^5 t_\alpha s_\beta]u^{\beta}_{\Delta}(\vec{t})\ . \tag{7}$$

Here $q_\mu=(s-t)_\mu$ ($\mu=0,1,2,3$), $K_{\Delta^{++}\Delta^{+}\pi^{+}}(q^2)$ and $K'_{\Delta^{++}\Delta^{+}\pi^{+}}(q^2)$ are normalized by $K_{\Delta^{++}\Delta^{+}\pi^{+}}(m_\pi{}^2)=1$, and $u^{\alpha}_{\Delta}$ ($\alpha=0,1,2,3$) denotes the Rarita-Schwinger spin-$\frac{3}{2}$ $\Delta$ field. $g_{\Delta^{++}\Delta^{+}\pi^{+}}$ and $g'_{\Delta^{++}\Delta^{+}\pi^{+}}$ are the two independent on-shell $\Delta\Delta\pi$ couplings. (According to the general assignment,[11] there are two independent $\Delta\Delta\pi$ vertices.)

We now study the implication of the constraint $\tilde{g}=(-\sqrt{2}/5)f$ obtained from Eq. (5). Using Eq. (6), we then obtain [up to the factor of $\delta^3(\vec{s}-\vec{t})$]

$$\begin{aligned}\tilde{g}&\equiv-(\tfrac{2}{3})^{1/2}\langle\Delta^{++},\tfrac{1}{2}|A_{\pi^+}|\Delta^{+},\tfrac{1}{2}\rangle\\&=[-(\tfrac{2}{3})^{1/2}]\tfrac{1}{3}[g_A(0)]_{\Delta^{++}\Delta^{+}}\ ,\end{aligned} \tag{8}$$

whereas

$$f\equiv\langle p,\tfrac{1}{2}|A_{\pi^+}|n,\tfrac{1}{2}\rangle=[g_A(0)]_{pn}\ .$$

Therefore we predict via $\tilde{g}=(-\sqrt{2}/5)f$,

$$[g_A(0)]_{\Delta^{++}\Delta^{+}}=\frac{3\sqrt{3}}{5}[g_A(0)]_{pn}=1.30\ , \tag{9}$$

where we have used $[g_A(0)]_{pn}\simeq 1.25$ from experiment. The value of Eq. (9) may be compared with the prediction[1] based on the MIT bag model $[g_A(0)]_{\Delta^{++}\Delta^{+}}=1.13$. Now from Eq. (6), we obtain

$$\lim_{q_\mu\to 0}\langle\Delta^{++}(\vec{s})|\partial_\mu A^{\mu}_{\pi^+}(x)|\Delta^{+}(\vec{t})\rangle=i2m_\Delta\bar{u}^{\alpha}_{\Delta}(\vec{s})[-g_A(0)]_{\Delta^{++}\Delta^{+}}\gamma^5 u_{\Delta\alpha}(\vec{s})\ . \tag{10}$$

Via PCAC the left-hand side of this equation becomes, using Eq. (7),

$$f_\pi\langle\Delta^{++}(s)|j_{\pi^+}(0)|\Delta^{+}(t)\rangle=f_\pi g_{\Delta^{++}\Delta^{+}\pi^{+}}K_{\Delta^{++}\Delta^{+}\pi^{+}}(0)\bar{u}^{\alpha}_{\Delta}(\vec{s})i\gamma_5 u_{\Delta\alpha}(\vec{s})\ . \tag{11}$$

Note that in the limit $q_\mu\to 0$ (i.e., $\vec{s}\to\vec{t}$), the term involving the second coupling $g'_{\Delta\Delta\pi}$ in Eq. (7) *vanishes,* since $s_\alpha u^{\alpha}_{\Delta}(\vec{s})=0$. If we assume smooth extrapolation of $K_{\Delta\Delta\pi}(q^2)$, i.e., $K_{\Delta\Delta\pi}(0)\simeq K_{\Delta\Delta\pi}(m_\pi{}^2)=1$, we then obtain from Eqs. (10) and (11),

$$g_{\Delta^{++}\Delta^{+}\pi^{+}}=\left(\frac{1}{f_\pi}\right)(-2m_\Delta)[g_A(0)]_{\Delta^{++}\Delta^{+}}=\left(\frac{m_\Delta}{f_\pi}\right)\left(\frac{-6\sqrt{3}}{5}\right)[g_A(0)]_{pn}\ . \tag{12}$$

Using the pole value of $m_\Delta$, $m_\Delta=1.211$ GeV, and $f_\pi\simeq 0.93m_\pi$ (determined from the rate of $\pi\to\mu\nu$ decay), we obtain $g_{\Delta^{++}\Delta^{+}\pi^{+}}{}^2/4\pi \simeq 50$ (or $\simeq 51.4$, if we use the resonance position $m_\Delta = 1.236$ GeV). In order to compare with the value of the $\Delta\Delta\pi$ coupling obtained in Ref. 1, we define according to Ref. 1 a $\Delta\Delta\pi$ coupling $g$ by factoring out the Clebsch-Gordan coefficient, i.e.,

$$g^2 = [C(\tfrac{3}{2},\tfrac{3}{2},1;\tfrac{3}{2},\tfrac{1}{2},1)]^{-2} g_{\Delta^{++}\Delta^{+}\pi^{+}}{}^2 = \tfrac{5}{2} g_{\Delta^{++}\Delta^{+}\pi^{+}}{}^2 \ .$$

We then find

$$g^2/4\pi \simeq 125 \text{ (or 128 for } m_\Delta = 1.236 \text{ GeV)} \ , \tag{13}$$

whereas the MIT-bag-model estimate of $[g_A(0)]_{\Delta^{++}\Delta^{+}}$ predicts[1] $g^2/4\pi \simeq 100$. SU(6) (using the average mass of meson octet and baryon decuplet) and U(12) predict $g^2/4\pi \simeq 130$.

As summarized in Refs. 1, 2, and 7, other theoretical estimates generally lie in the range $g^2/4\pi \simeq 125-225$. Therefore, the theoretical values of $g$ are considerably larger than the value $g^2/4\pi \simeq 40 \pm 20$ obtained recently in Ref. 1 or Ref. 2.

The discrepancy may be due[12] to the neglect of momentum-dependent effects in the rather difficult evaluation of certain Feynman diagrams for the process $\pi^- p \to \pi^+\pi^- n$ in Ref. 1. These effects include the neglect of one of the two independent $\Delta\Delta\pi$ couplings as well as the explicit $q^2$ form-factor dependence of the couplings. In order to minimize the impact of the effect of $g'_{\Delta\Delta\pi}$ coupling constant, one may need to add further kinematical constraints on the analysis.

Finally, we briefly add the result of the study of the level realization of flavor SU(2) symmetry in the algebra, Eq. (2), when it is sandwiched between the ground-state baryons $\langle B(\alpha, \vec{s}, \lambda = \frac{1}{2})|$ and $|B(\beta, \vec{t}, \lambda = \frac{3}{2})\rangle$ with $\vec{s} \to \infty$ and $\vec{t} \to \infty$. This helicity combination was not studied in Ref. 8. The choices of $(\alpha, \beta)$ are $(\Delta^{++}, \Delta^{++})$, $(\Delta^{+}, \Delta^{+})$, $(\Delta^{0}, \Delta^{0})$, and $(\Delta^{-}, \Delta^{-})$ and also $(p, \Delta^{+})$ and $(n, \Delta^{0})$. The same procedure as used in Ref. 8 turned out to produce a new constraint only on the strong couplings, i.e.,

$$j^2 = 9\tilde{g}^2 \ , \tag{14}$$

where $j$ is defined by Eq. (4).

However, the actual evaluation (in the limit $\vec{s} \to \infty$) using Eq. (6) produces,

$$\langle \Delta^{++}, \tfrac{3}{2} | A_{\pi^+} | \Delta^{+}, \tfrac{3}{2} \rangle = 3 \langle \Delta^{++}, \tfrac{1}{2} | A_{\pi^+} | \Delta^{+}, \tfrac{1}{2} \rangle \ . \tag{15}$$

Therefore Eq. (14) is automatically satisfied and it does not lead to new information, although it does demonstrate the consistency of the ansatz of level realization.

As discussed in Ref. 2, several other possibilities exist which could account for the apparent disagreement between theory[3–8] and experiment.[1,2] We strongly concur with the authors of Ref. 2 that resolution of this problem be given strong attention.

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[10] States are normalized covariantly such that $\langle \vec{p}'\lambda' | \vec{p}\lambda \rangle = (2\pi)^3 2p^0 \delta(\vec{p}' - \vec{p}) \delta_{\lambda'\lambda}$ ($\lambda'$, $\lambda$ are helicities). The metric tensor is $g_{\alpha\beta} = (1, -1, -1, -1)$. Spinors are normalized such that $\bar{u}(\vec{p}, \lambda') u(\vec{p}, \lambda) = 2m\delta_{\lambda'\lambda}$.

$$\gamma_5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \ , \quad \gamma_5^\dagger = \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ ,$$

$$\gamma_k^\dagger = -\gamma_k \ , \text{ and } [\gamma^\mu, \gamma^\nu]_+ = 2g^{\mu\nu} \ .$$

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[12] It is desirable to choose the kinematic domain in which the contribution of the $g'_{\Delta\Delta\pi}$ coupling becomes small.