Parametrization of $\bar{p}$ invariant cross section in p-p collisions using a new scaling variable

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It is found that to parametrize the $\bar{p}$ invariant cross section in low-energy p-p collisions, a better scaling variable $\Delta X_R = X_R - X_{R\,\min}$ can be used instead of the usual radial variable X_R, where $X_{R\,\min}$ is the minimum of X_R for a given s and p_t. The ratio of the $\bar{p}$ invariant cross section to the corresponding radial-scaling limit is seen to be independent of p_t explicitly, and is monotonically and quickly approaching to one from above for increasing s.

Recently, Buffington *et al.*[1] reported their observation of an unexpected large interstellar $\bar{p}$ flux in the low-energy range, where the median energy of the parent protons should be between 20 and 30 GeV (Ref. 2), if these $\bar{p}$'s are produced in the collision of cosmic rays with the interstellar medium. It is known that at low energies ($\sqrt{s} < 10$ GeV), the radial-scaling (RS) limit is not reached and the existing accelerator data are insufficient. Hence a reexamination of the $\bar{p}$ invariant cross section used in the calculation of interstellar $\bar{p}$ flux is essential.

In order to avoid any distortion purely caused by kinematic effects near the $\bar{p}$ production threshold, we have used a precise expression for the radial variable X_R in calculating the invariant cross section for $p+p \to \bar{p}+X$, i.e.,

$$X_R = E^*/E^*_{\max} = E^* 2\sqrt{s}/(s - \bar{M}_X^2 + m_p^2) \ , \qquad (1)$$

where m_p is the proton mass and $\bar{M}_X = 3m_p$ is the minimum mass of the undetected particle system (X) consistent with quantum-number conservation.

Taylor *et al.*[3] have pointed out that the radial-scaling limit for this inclusive reaction is always approached from the above for increasing s and is reached by $\sqrt{s} \sim 10$ GeV. Hence we have parameterized the $\bar{p}$ invariant cross section $(E\, d^3\sigma/d^3p)_{\rm RS}$ at $\sqrt{s}$ greater than 10 GeV. In the low-transverse-momentum ($p_t = 0$–0.8 GeV/c) region, we have obtained

$$(E\, d^3\sigma/d^3p)_{\rm RS} = f(X_R)\exp[-(Ap_t + Bp_t^2)] \ , \qquad (2)$$

where [f in units of mb GeV$^{-2}c^3$, A in (GeV/c)$^{-1}$, B in (GeV/c)$^{-2}$]

$$f(X_R) = 3.34\exp(-17.6X_R)\theta(0.5 - X_R) + 2.10(1 - X_R)^{7.80} \ ,$$

$$A = 3.95\exp(-2.76X_R) \ ,$$

$$B = 40.5\exp(-3.21X_R)X_R^{2.13} \ ,$$

and

$$\theta(U) = \begin{cases} 0, & U < 0 \\ 1, & U \geq 0 \ . \end{cases}$$

The behavior of $f(X_R)$ is a characteristic of several fragmentation models,[4] and an exponential correction term is added to account for a possible increase of the $\bar{p}$ cross section in the center region. The functional form for p_t is similar to those of other authors (e.g., Alper *et al.*[5] and Guettler *et al.*[6]). Our parametrized result is shown in Fig. 1 together with the experimental data. The error for the important part of the experimental data has been given[7] to be 15%. Figure 1 also contains the fitted curves given by Stephens[8] and by Hillas[9] for comparison.

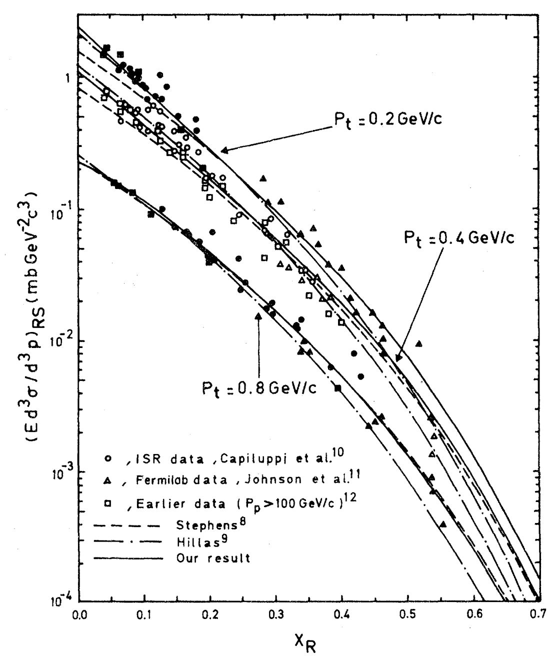

FIG. 1. The $\bar{p}$ invariant cross section at $\sqrt{s} \geq 10$ GeV. See Eq. (2) for our parametrized result.

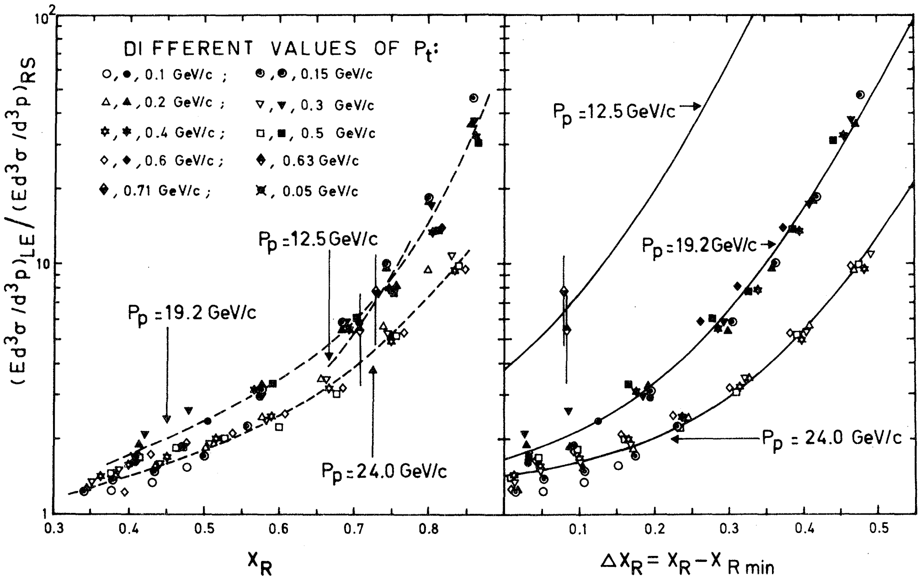

FIG. 2. Relation of the low-energy $\bar{p}$ invariant cross section at $\sqrt{s} < 10$ GeV with its corresponding radial-scaling limit, (a) as a function of X_R, and (b) as a function of $\Delta X_R = X_R - X_{R\,\min}$. Experimental data for $P_p = 12.5$ GeV/c from Ref. 13, 19.2 GeV/c from Ref. 14, and 24.0 GeV/c from Ref. 15.

Further, based on the experimental data for $\sqrt{s} < 10$ GeV we have examined the relation of the low-energy (LE) invariant cross section, $(E\,d^3\sigma/d^3p)_{\rm LE}$ with its corresponding radial-scaling limit. This is done by plotting the ratio $(E\,d^3\sigma/d^3p)_{\rm LE}/(E\,d^3\sigma/d^3p)_{\rm RS}$ against X_R as shown in Fig. 2(a). It is seen that at the same s (corresponding to the same incident momentum of a parent proton, P_p), the data taken at different p_t are lying at the vicinity of a curve (i.e., each dashed line). This fact indicates that if we compare the variation of the longitudinal-momentum distribution with s, we may find that the transverse-momentum distribution will approach the radial-scaling limit earlier. From Fig. 2(a), this indication is not convincing, because the scattering of data points apparently exceeds the experimental error. Furthermore, the two data points taken at $P_p = 12.5$ GeV/c fall on the curve representing $P_p = 19.2$ GeV/c, and this is a serious obstacle for extrapolating the $\bar{p}$ invariant cross section to the $\bar{p}$ production threshold.

On the other hand, a trend can be observed from Fig. 2(a) that the ratio $(E\,d^3\sigma/d^3p)_{\rm LE}/(E\,d^3\sigma/d^3p)_{\rm RS}$ decreases systematically for increasing p_t at the same X_R. This indicates an important behavior of X_R, i.e., for a given p_t and s, X_R should possess a minimum value,

$$X_{R\,\min} = (p_t^2 + m_p^2)^{1/2}/E^*_{\max} \ . \tag{3}$$

Thus $X_{R\,\min}$ increases with increasing p_t and m_p and with decreasing s. Therefore, the effect of $X_{R\,\min}$ would be more pronounced for massive-particle production (i.e., $\bar{p}$ production in this case) in the low-energy range.

Consequently, a new variable $\Delta X_R = X_R - X_{R\,\min}$ is introduced in place of X_R, and Fig. 2(a) is replotted as shown in Fig. 2(b). From the latter, it is clear that the data sets taken at different s are well separated, and each set is consistent with a specified curve as shown.[16] This shows that there is a correct threshold behavior, i.e., the ratio $(E\,d^3\sigma/d^3p)_{\rm LE}/(E\,d^3\sigma/d^3p)_{\rm RS}$ is monotonically and quickly approaching 1 from the above for increasing s. As this ratio is explicitly independent of p_t, parametrization becomes much easier. If we define $Q = \sqrt{s} - 4m_p$, where $4m_p$ is the $\sqrt{s}$ value at $\bar{p}$ production threshold, then the expression for the ratio becomes as follows:

$$(E\,d^3\sigma/d^3p)_{\rm LE}/(E\,d^3\sigma/d^3p)_{\rm RS} - 1 = 6.25\times 10^{-3}\times[\exp(-0.592Q) + 493\exp(-5.40Q)]$$
$$\times[\exp(6.08 + 2.57\Delta X_R + 7.95\Delta X_R^{\,2}) - 1]\exp[3.00\Delta X_R(3.09 - Q)] \ . \tag{4}$$

The solid curves presented in Fig. 2(b) have been obtained from the above equation.

In the present parametrization, we have not used the Serpukhov data,[17] because their measurement was made relative to the pions on aluminum targets for the extreme forward direction. Instead, we require that at $\sqrt{s}$

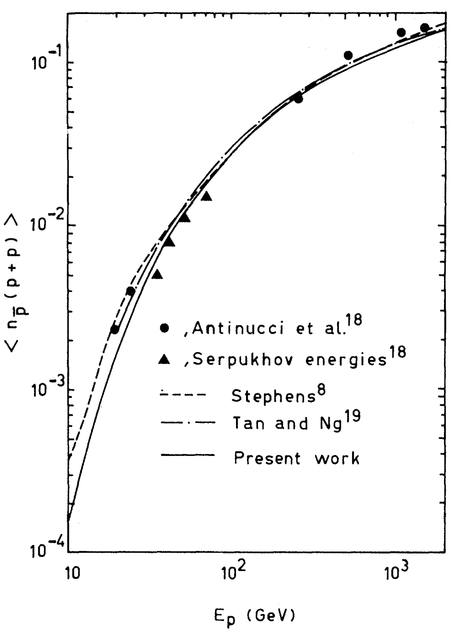

FIG. 3. The average multiplicity of $\bar{p}$'s in $p+p$ collisions. The solid curve is obtained by using the present parametrized invariant cross section and $\sigma_{\rm inel}$ from Ref. 9.

$=10$ GeV, the ratio $(E\, d^3\sigma/d^3p)_{\rm LE}/(E\, d^3\sigma/d^3p)_{\rm RS}$ reaches one by a 15% margin.

From our parameterized $\bar{p}$ invariant cross section, we have calculated the average multiplicity of $\bar{p}$'s in $p+p$ collisions,

$$\langle n_{\bar{p}}(p+p)\rangle = \frac{2\pi}{\sigma_{\rm inel}} \int\int_{p_t} \left| E\frac{d^3\sigma}{d^3p} \right| d\theta^*\, dE^* \ , \qquad (5)$$

where the inelastic cross section $\sigma_{\rm inel}$ is taken from Ref. 9. In Fig. 3, we compare $\langle n_{\bar{p}}(p+p)\rangle$ values obtained by various authors. It is shown that our values are lower than those of Antinucci *et al.* [18] by about 10% (except for Serpukhov energies), though

TABLE I. The average multiplicity of $\bar{p}$'s in $p+p$ collisions.

E_p (GeV)	$\langle n_{\bar{p}}(p+p)\rangle$
10	1.40×10^{-4}
20	1.81×10^{-3}
40	8.05×10^{-3}
70	1.81×10^{-2}
100	2.75×10^{-2}
200	5.20×10^{-2}
400	8.23×10^{-2}
700	0.108
1000	0.124
2000	0.153

the latter is usually taken as a standard. However, Whitmore[20] has already pointed out that the average $\bar{p}$ multiplicity given by Antinucci *et al.* may have an ~10 % overestimation because of the inaccuracies in extrapolating to small p_t. Consequently, what we have obtained as given in Table I should be closer to reality.

Some success for explaining the low-p_t behavior of hadron collisions on the basis of the quark or parton models has been reported. However, detailed discussion on the threshold characteristics of hadron collisions, particularly for massive-hadron production, is still lacking. Hence, it is of interest to explore the theoretical implication of this new variable ΔX_R and its applicability to other hadrons.

We have used the present parametrized $\bar{p}$ invariant cross section to calculate the interstellar $\bar{p}$ production spectrum, and the result will be published elsewhere.[2]

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[16]The consistency of the data points in Fig. 2(b) with the P_p curves is quite good at larger ΔX_R. At smaller ΔX_R, the spread of data at various p_t may be due to the correction term added to $f(X_R)$ in Eq. (2), which has not achieved the required accuracy for low-energy $\bar{p}$'s. However, in view of the large error in the existing data, no further improvement has been made.

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