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Hypercolor, extended hypercolor, and the generation problem

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We analyze some of the detailed consequences of the recently proposed grand unified theory based on $SO(10)_V \times SO(10)_H$ subject to a discrete $V \leftrightarrow H$ symmetry. The model attempts to unify flavor, color, and hypercolor, and provide a multigenerational grand unified theory. It predicts SU(4) as the unique unitary group for hypercolor. With the fundamental fermions belonging to the $(\underline{16},\underline{10}) \oplus (\underline{10},\underline{16})$ irreducible representation of $SO(10)_V \times SO(10)_H$, there are exactly four generations of ordinary fermions (hypercolor singlets). The dynamical symmetry breaking which gives masses to the vector bosons of the standard electroweak theory is accomplished through the condensates of a single generation of hyperfermions belonging to the sextet of SU(4). We show that the Weinberg relation $M_W = M_Z \cos\theta$ will be satisfied provided the hypercolor dynamics satisfies certain constraints. Present in the model are the much desired extended-hypercolor gauge bosons whose radiative transitions between the ordinary fermions and the hyperfermions give rise to both masses and mixing of the ordinary fermions; hence the generalized Cabibbo angles are in principle calculable. We also analyze the pseudo-Goldstone bosons and the rare decay modes of the K and D mesons. With the hyperfermions in the TeV range and the extended gauge bosons in the 1000-TeV range, we show that there is no conflict in the model with any known experimental bounds on such decays. We establish a hypercolor—Pati-Salam-color symmetry at medium energies.

I. INTRODUCTION

In this paper we combine two currently popular ideas in particle theory, namely grand unification of the fundamental strong, electromagnetic, and weak interactions, and the possible existence of a new superstrong hypercolor gauge interaction. We shall be particularly interested in hypercolor not only as a possible origin for a superstrong force which could lead to the dynamical symmetry breaking of unified theories, but also as an extra group-theoretical degree of freedom which will provide us with a new group-theoretical structure to both label and relate the various generations of fundamental fermions. Thus we shall seek to enlarge grand unifying theories to include hypercolor interactions as well.

With regard to grand unification we remark that while there is as of yet no theoretical unanimity or experimental preference regarding the choice of grand unifying group there is a lot of support in the current literature for grand unification based on the group SU(5).[1] This support derives from the fact that SU(5) involves the smallest possible number of additional vector bosons beyond those of $SU(3)_C \times SU(2)_L \times U(1)$, thus making SU(5) the simplest choice. Its immediate shortcoming is that it puts the fundamental fermions into many differing $\underline{5}^*$ and $\underline{10}$ representations. It is possible to partially remedy this by embedding the theory[2] in SO(10) provided some so far unobserved right-handed neutrinos which are to be singlets under SU(5) are introduced. The $\underline{16}$ representation of SO(10) then nicely accommodates 16 two-component fermions which transform as $\underline{5}^* \oplus \underline{10} \oplus \underline{1}$ under SU(5). Each set of 16 fermions of this type is known as a *generation* or *family*. Thus SO(10) puts all the members of a given generation into a single irreducible representation. However it still treats the different generations as separate irreducible representations under the group. Consequently SO(10), like its SU(5) subgroup, is strictly a single-generation grand unifying theory which can thus not adequately address questions such as how many generations there are or in what way they mix with each other. With regard to this generation problem two points of view can be adopted. One is to say that the very fact that SO(10) [or SU(5)] is a single-generation theory could be an indication that it is simply the wrong group for grand unification altogether; while the other is to take SO(10) as a good starting point with the generation structure then hopefully emerging after SO(10) is itself embedded into a yet larger group again. In the absence of an equally popular alternative to SO(10) we shall thus only explore the latter alternative in this paper, though we believe that the former alternative merits further study.

With regard to hypercolor we recall that the primary motivation for hypercolor was to understand the origin for a force which could be strong enough to produce a typical weak-interaction mass scale of the order of 100 GeV.[3,4] It was argued that hypercolor might act as a scaled-up version of quantum chromodynamics (QCD) which is itself thought of as being responsible for producing a scale of the order of 100 MeV for the usual strong interactions by breaking the strong-interaction chiral symmetry dynamically. Using this as-yet-poorly-understood analogy hypercolor would then be expected to break the weak-interaction flavor symmetry dynamically and provide a mass scale for the intermediate vector bosons of the $SU(2)_L \times U(1)$ theory. Indeed it was noted that the Weinberg mixing formula $M_W = M_Z \cos\theta$ would follow naturally (i.e., group theoretically) if the hypercolor force caused hypercolor-carrying fermions (hyperfermions) to form fermion bilinear condensates which acquired nonvanishing vacuum expectation values.

While these features of the hypercolor scheme are very attractive, the scheme nonetheless fails in one significant respect. In the conventional Weinberg-Salam theory a fundamental Higgs field gives masses to both the intermediate vector bosons and the usual fermions (now the hypersinglet fermions of the hypercolor scheme). In the dynamical hypercolor scheme the hyperfermion condensates which are to replace the fundamental Higgs fields give 100-GeV masses to the intermediate vector bosons and TeV masses to the hyperfermions, leaving the usual fermions massless. While this already constitutes a considerable though acceptable departure from the structure of the conventional Weinberg-Salam theory, it also leaves open the question of where do the usual fermions in fact get their masses from. To this end yet another even stronger interaction has been proposed, extended hypercolor,[5] which would then serve to mediate processes in which the now massive hyperfermions could then contribute radiatively to the self-energies of the usual fermions. The hypercolor theory as currently formulated is thus still somewhat unsatisfying in that it appeals to a still poorly understood dynamics, gives no indication of what the hypercolor group actually is [save that it be larger than SU(3) so that renormalization-group effects would make it stronger than QCD in the TeV region], and needs to be augmented by an additional extended hypercolor interaction. Thus further guiding principles are needed in order to make hypercolor a more restricted and hence more predictive theory. In this paper we shall provide such principles by using the group structure of hypercolor and extended hypercolor in order to solve the generation problem. We believe that this now puts the whole hypercolor idea on a more secure footing.

Our aim is thus to enlarge the SO(10) grand unifying theory to include hypercolor interactions as well. The immediate possibility of course would be to embed SO(10) into $SO(10+4n)$ groups. However this choice has been shown to be afflicted with serious phenomenological difficulties since it has to introduce unobserved conjugate $\underline{16}^*$ spinor representations and can only admit of two separate generations of usual fermions.[6] Motivated by our recent work in semisimple grand unification[7] we have instead proposed to embed SO(10) into the semisimple $SO(10)_V \times SO(10)_H$ group.[8] Here $SO(10)_V$ is our initial "vertical" single generation SO(10) while $SO(10)_H$ is a new "horizontal" SO(10) symmetry which as we will see will connect the various generations. Further we will identify $SO(10)_H$ as the complete hypercolor group, so that $SO(10)_H$ will contain both hypercolor-carrying and flavor-carrying (generation-changing) gauge bosons. Thus one single group structure restricts the hypercolor- and the flavor-changing interactions simultaneously. There is of course initially some freedom in specifying the horizontal group factor. However the simple imposition of a discrete vertical-horizontal symmetry, which we make, then unambiguously forces the horizontal group to be SO(10) just like its vertical counterpart, while also fixing the otherwise unknown horizontal coupling g_H to be equal to the vertical g_V at the grand unification mass scale. Moreover, as we will see below, there will also be vestiges of the discrete symmetry in the low-energy structure of our theory following the symmetry breaking. This will then lead to an attractive approximate low-energy color-hypercolor symmetry.

While we have now fixed the horizontal symmetry, our model is not yet completely specified since we still have to classify the fermions according to an appropriate irreducible representation of $SO(10)_V \times SO(10)_H$ and still have to identify which subgroup of $SO(10)_H$ is to serve as the hypercolor group. Whatever generators of $SO(10)_H$ are then left over will serve to count generations. Thus because of the embedding into an extended hypercolor $SO(10)_H$ group the size of the hypercolor group and the number of independent generations of fermions are simultaneously fixed.

In classifying the fermions the obvious embeddings of the $\underline{16}$ of $SO(10)_V$ would be in $(\underline{16},\underline{16}^*)$ or $(\underline{16},\underline{16})$ under $SO(10)_V \times SO(10)_H$. Because of the discrete symmetry however the $(\underline{16},\underline{16}^*)$ would have to be accompanied by a $(\underline{16}^*,\underline{16})$ to thus give a real representation which is not appropriate for fermions. The $(\underline{16},\underline{16})$ would be acceptable on this score but was found not to be acceptable phenomenologically as it could only lead to two ordinary generations of usual fermions.[8] Because of this we instead proposed to classify the fermions according to the $(\underline{16},\underline{10})$ representation which would then be accompanied by a $(\underline{10},\underline{16})$ representation to maintain irreducibility under the discrete symmetry. The $\underline{10}$ would not ordinarily be considered in grand unified models since by itself it would be real. However when combined with the $\underline{16}$ it leads to a complex $(\underline{16},\underline{10}) \oplus (\underline{10},\underline{16})$ representation which now is appropriate for fermions. Given this classification of fermions it was found that there was only one choice for the hypercolor subgroup which could lead to both a unitary hypercolor group and to more than two ordinary generations of fermions. Specifically, we decomposed $SO(10)_H$ according to $SO(6)_H \times SO(4)_H$ to obtain

$$\begin{aligned} \underline{16} &= (\underline{4},\underline{2}) + (\underline{4}^*,\underline{2}') , \\ \underline{10} &= (\underline{6},\underline{1}) + (\underline{1},\underline{4}) \end{aligned} \tag{1.1}$$

with the $(\underline{16},\underline{10}) \oplus (\underline{10},\underline{16})$ then containing altogether four generations of fermions [i.e., fermions which transform as the $\underline{16}$ under $SO(10)_V$] which were $SO(6)_H$ singlets. Thus identifying $SO(6)_H$ as the hypercolor group enables us to obtain the previously *ad hoc* choice of SU(4) uniquely as the hypercolor symmetry while yielding at the same time exactly four generations of usual fermions. As we noted in Ref. 8, the emergence of four generations occurred only because we identified SU(4) with the $SO(6)_H$ subgroup of $SO(10)_H$ using the SU(4), SO(6) homomorphy. Had we instead identified SU(4) through the $SO(8)_H$ subgroup of $SO(10)_H$ [using the connection between SU(N) and SO($2N$) groups] we would only have obtained two generations. Thus in our model the hypercolor group is special because it is both a unitary and an orthogonal group, a situation which only occurs for SU(4). It is this feature of our model which makes SU(4) special and not merely the fact that because SU(4) is larger than SU(3) it becomes strong at a higher energy than QCD.

Having chosen to identify $SO(6)_H$ as the hypercolor subgroup and $SO(4)_H$ as the generation-counting subgroup of $SO(10)_H$, we can decompose the adjoint representation of $SO(10)_H$ according to $SO(6)_H \times SO(4)_H$ to obtain

$$\underline{45} = (\underline{15},\underline{1}) + (\underline{1},\underline{6}) + (\underline{6},\underline{4}) . \tag{1.2}$$

The $(\underline{15},\underline{1})$ and $(\underline{1},\underline{6})$ gauge bosons are the generators of $SO(6)_H$ and $SO(4)_H$, respectively, while the additional $(\underline{6},\underline{4})$ gauge bosons carry both hypercolor and flavor. These latter gauge bosons couple the hyperfermions to the usual fermions, and thus enable the usual fermions to acquire radiative masses.[9] Our $SO(10)_V \times SO(10)_H$ model thus automatically contains the previously *ad hoc* extended-hypercolor gauge bosons of Ref. 5. Moreover, since the different generations of fermions all belong to one irreducible representation of $SO(10)_V \times SO(10)_H$, the spontaneous breakdown of the model leads not merely to fermion masses but also to fermion mass mixing. Thus we naturally obtain Cabibbo mixing in our model[9] by taking advantage of the correlation between hypercolor and the generation structure which our model possesses. Hence we exploit the group-theoretical structure of hypercolor in a nontrivial manner to make the hypercolor concept highly predictive.

In this present work we shall report on all the various interesting aspects of our theory, to complete our analysis of the central features of the model. This paper is organized as follows. In Sec. II we present the general group-theoretical structure, and the classification of all the fermions and gauge bosons of the model. We identify the relevant hyperfermion condensates needed for the dynamical symmetry breaking. In Sec. III we show how the Weinberg mixing relation $M_W = M_Z \cos\theta$ arises in the model. Since the weak-interaction local flavor group is the chiral $SU(2)_L \times SU(2)_R \times U(1)$ group the analysis needed to obtain this mixing relation dynamically is substantially different from that given in Refs. 3 and 4 where only the Weinberg-Salam weak interactions are local. In Sec. IV we discuss the generation of the light-fermion self-energies and of Cabibbo mixing. In Sec. V we identify the Goldstone and pseudo-Goldstone bosons of our model, and in Sec. VI we show how the model successfully meets the constraints due to rare flavor-changing decay processes. In Sec. VII we catalog all the various symmetry-breaking scales, and in Sec. VIII we present our conclusions. Finally for the sake of completeness we discuss briefly, in an appendix, the other alternatives for the choice of hypercolor and generation-counting subgroups of $SO(10)_H$. Some of these alternatives are theoretically permissible, but not as attractive as the one we have discussed in detail in this paper.

II. STRUCTURE OF THE MODEL

Our model is an $SO(10)_V \times SO(10)_H$ gauge theory in which the fermions belong to a left-handed $(\underline{16},\underline{10}) \oplus (\underline{10},\underline{16})$ representation and the gauge bosons to the $(\underline{45},\underline{1}) \oplus (\underline{1},\underline{45})$ representation. It is useful to decompose each SO(10) according to $SU(4)\times SU(2)\times SU(2)$. Under this decomposition

$$\begin{aligned}
\underline{10} &= (\underline{6},\underline{1},\underline{1}) + (\underline{1},\underline{2},\underline{2}) ,\\
\underline{16} &= (\underline{4},\underline{2},\underline{1}) + (\underline{4}^*,\underline{1},\underline{2}) ,\\
\underline{45} &= (\underline{15},\underline{1},\underline{1}) + (\underline{6},\underline{2},\underline{2}) + (\underline{1},\underline{3},\underline{1}) + (\underline{1},\underline{1},\underline{3})
\end{aligned} \tag{2.1}$$

with the complete classification of the fermions and gauge bosons being given in Table I. Physically on the vertical side we identify the two SU(2) groups as $SU(2)_L \times SU(2)_R$ and the SU(4) as the Pati-Salam vector color group which extends QCD to include the leptons as a fourth color.[10] Thus for the usual fermions of the first generation we identify

$$\begin{aligned}
&\begin{bmatrix} \nu_e \\ u_R \\ u_G \\ u_B \end{bmatrix}_L , \begin{bmatrix} e \\ d_R \\ d_G \\ d_B \end{bmatrix}_L \in (\underline{4},\underline{2},\underline{1};\underline{1},\underline{2},\underline{2}) ,\\
&\begin{bmatrix} \nu_e^C \\ u_R^C \\ u_G^C \\ u_B^C \end{bmatrix}_L , \begin{bmatrix} e^C \\ d_R^C \\ d_G^C \\ d_B^C \end{bmatrix}_L \in (\underline{4}^*,\underline{1},\underline{2};\underline{1},\underline{2},\underline{2}) ,
\end{aligned} \tag{2.2}$$

where R, G, and B denote the three QCD colors red, green, and blue. The 15 generators of SU(4) are then the 8 $SU(3)_C$ generators, 6 generators which mix quarks and leptons and one extra diagonal generator $Y = (B - L)/2$ with current

TABLE I. Classification of the fermions and gauge bosons.

	$[SU(4)\times SU(2)\times SU(2)]_V$			$\times$ $[SU(4)\times SU(2)\times SU(2)]_H$		
$(\underline{16},\underline{10})$	4	2	1	6	1	1
	4*	1	2	6	1	1
	4	2	1	1	2	2
	4*	1	2	1	2	2
$(\underline{10},\underline{16})$	6	1	1	4	2	1
	6	1	1	4*	1	2
	1	2	2	4	2	1
	1	2	2	4*	1	2
$(\underline{45},\underline{1})$	15	1	1	1	1	1
	6	2	2	1	1	1
	1	3	1	1	1	1
	1	1	3	1	1	1
$(\underline{1},\underline{45})$	1	1	1	15	1	1
	1	1	1	6	2	2
	1	1	1	1	3	1
	1	1	1	1	1	3

$$Y_\lambda = \sum_{i=R,G,B} \tfrac{1}{6}(\bar{u}_i\gamma_\lambda u_i + \bar{d}_i\gamma_\lambda d_i) - \tfrac{1}{2}(\bar{\nu}_e\gamma_\lambda\nu_e + \bar{e}\gamma_\lambda e) \ . \tag{2.3}$$

Denoting by T_L^3 and T_R^3 the third components of $SU(2)_L$ and $SU(2)_R$, respectively, we identify the electric-charge generator as

$$Q = T_L^3 + T_R^3 + \tfrac{1}{2}(B-L) \ , \tag{2.4}$$

so that

$$Y_{WS} = T_R^3 + \tfrac{1}{2}(B-L) \tag{2.5}$$

is the $U(1)_{WS}$ generator of the Weinberg-Salam group $SU(2)_L \times U(1)_{WS}$. With regard to the vertical gauge bosons we note also that the $(\underline{6},\underline{2},\underline{2};\underline{1},\underline{1},\underline{1})$ gauge bosons are the leptoquark bosons which lead to proton decay and hence possess masses of the order of the grand unification 10^{15}-GeV mass scale.

On the horizontal side we identify the two SU(2) groups together as an SO(4) which counts and links the generations. The SU(4) group is the hypercolor group. Thus the $(\underline{16},\underline{10})$ contains the following particles:

$$\begin{aligned} \text{hyperfermions} &\sim (\underline{4},\underline{2},\underline{1};\underline{6},\underline{1},\underline{1}) \\ &\quad + (\underline{4}^*,\underline{1},\underline{2};\underline{6},\underline{1},\underline{1}) \ , \\ \text{usual fermions} &\sim (\underline{4},\underline{2},\underline{1};\underline{1},\underline{2},\underline{2}) \\ &\quad + (\underline{4}^*,\underline{1},\underline{2};\underline{1},\underline{2},\underline{2}) \ , \end{aligned} \tag{2.6}$$

i.e., hyperfermions in the $\underline{6}$ of SU(4) and four generations of usual fermions (i.e., hypercolor singlets). In the following we shall denote the hyperfermions, whose $SO(10)_V$ content is the same as that of the usual fermions, by N, U_i, E, D_i. It is important to note that the hyperfermions are generation singlets. Thus, as we shall see in the following, hyperfermion condensation will not lead to strangeness-changing processes. Further, the $(\underline{10},\underline{16})$, whose presence is required by the discrete symmetry, contains no hypercolor singlets at all, and hence leads to no other observable particles, assuming of course that hypercolor confines all its hypercolor-nonsinglet particles.

With regard to the horizontal gauge bosons we note that the $(\underline{1},\underline{1},\underline{1};\underline{6},\underline{2},\underline{2})$ gauge bosons carry both hypercolor and flavor and hence connect a hyperfermion with a usual fermion to mediate the self-energy diagram of Fig. 1. The model thus naturally possesses the extended hypercolor gauge bosons which had previously been introduced by hand. The $(\underline{1},\underline{1},\underline{1};\underline{1},\underline{3},\underline{1})$ and $(\underline{1},\underline{1},\underline{1};\underline{1},\underline{1},\underline{3})$ gauge bosons lead to flavor-changing processes in the tree approximation. These will readily be seen to be easily suppressible in the following.

The symmetry breaking of the model is achieved first by fundamental Higgs fields (which might be an expression of our ignorance of yet stronger and stronger forces), and then in the TeV region by dynamical symmetry breaking. The force responsible for dynamical symmetry breaking is the exchange of the $(\underline{1},\underline{1},\underline{1};\underline{15},\underline{1},\underline{1})$ hypergluons. In our left-handed theory they can only produce Majorana-type masses $\psi_L C \psi_L$ which are symmetric in the internal $SO(10)_V \times SO(10)_H$ symmetry indices. When viewed from the classification according to the vertical $SU(2)_L \times SU(2)_R$ subgroup this mass term will include both Majorana and Dirac masses. Since hypergluon exchange is not to lead to dynamical symmetry breaking of hypercolor itself there can be no condensation of a $(\underline{16},\underline{10})$ fermion with a $(\underline{10},\underline{16})$ fermion. Further, bilinears built out of a pair of $(\underline{10},\underline{16})$ fermions will play no significant role in the following. (See however Sec. VII.) The key condensates are contained in a pair of $(\underline{16},\underline{10})$ bilinears and are

$$\begin{aligned} &(\underline{4},\underline{2},\underline{1};\underline{6},\underline{1},\underline{1}) \times (\underline{4},\underline{2},\underline{1};\underline{6},\underline{1},\underline{1}) \to (\underline{10},\underline{3},\underline{1};\underline{1},\underline{1},\underline{1}) + (\underline{6},\underline{1},\underline{1};\underline{1},\underline{1},\underline{1}) \ , \\ &(\underline{4}^*,\underline{1},\underline{2};\underline{6},\underline{1},\underline{1}) \times (\underline{4}^*,\underline{1},\underline{2};\underline{6},\underline{1},\underline{1}) \to (\underline{10}^*,\underline{1},\underline{3};\underline{1},\underline{1},\underline{1}) + (\underline{6},\underline{1},\underline{1};\underline{1},\underline{1},\underline{1}) \ , \\ &(\underline{4}^*,\underline{1},\underline{2};\underline{6},\underline{1},\underline{1}) \times (\underline{4},\underline{2},\underline{1};\underline{6},\underline{1},\underline{1}) \to (\underline{15},\underline{2},\underline{2};\underline{1},\underline{1},\underline{1}) + (\underline{1},\underline{2},\underline{2};\underline{1},\underline{1},\underline{1}) \ . \end{aligned} \tag{2.7}$$

On the vertical side we decompose SU(4) according to $SU(3)_C$ to obtain

$$\begin{aligned} \underline{10} &= \underline{6} + \underline{3} + \underline{1} \ , \\ \underline{6} &= \underline{3} + \underline{3}^* \ , \\ \underline{15} &= \underline{1} + \underline{3} + \underline{3}^* + \underline{8} \ . \end{aligned} \tag{2.8}$$

Thus the only fermion bilinears which can leave $SU(3)_C$ unbroken are

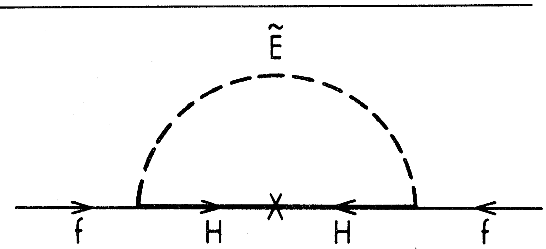

FIG. 1. Radiative graph that generates the light-fermion masses.

$$\Delta_L=(\underline{10},\underline{3},\underline{1};\underline{1},\underline{1},\underline{1})\,,$$
$$\Delta_R=(\underline{10}^*,\underline{1},\underline{3};\underline{1},\underline{1},\underline{1})\,, \tag{2.9}$$
$$X=(\underline{15},\underline{2},\underline{2};\underline{1},\underline{1},\underline{1})+(\underline{1},\underline{2},\underline{2};\underline{1},\underline{1},\underline{1})\,.$$

With electric charge to also be unbroken we recognize Δ_L and Δ_R as Majorana masses for the hyperneutrinos N, while X is a Dirac mass for the hyperfermions N, U_i, E, D_i. Since the hyperfermions have the same $\mathrm{SO}(10)_V$ properties as the usual fermions we note that the hypercolor-singlet condensates Δ_L, Δ_R, and X have the same group-theoretical content as condensates built out of the regular fermions. Now precisely such condensates (at the usual fermion level) were introduced by one of us recently to obtain the Weinberg-Salam theory by breaking chiral theories dynamically.[11] Thus all of the group theory of those works will carry over into the present theory with the bonus that now a new hypercolor dynamics has been introduced to provide a basis for the dynamical symmetry breaking in the first place. Thus Δ_L and Δ_R which were previously introduced as neutrino-pairing terms are now reinterpreted as hyperneutrino-pairing terms with all the group-theoretical structure remaining intact. Since that group theory is somewhat different than that discussed by Susskind and by Weinberg in their attempt to obtain $M_W=M_Z\cos\theta$ dynamically we shall now discuss the $M_W=M_Z\cos\theta$ problem in our model in some detail.

III. DERIVATION OF THE WEINBERG MIXING RELATION

In their original analyses Susskind[3] and Weinberg[4] identified a possible origin for the Weinberg mixing relation

$$\rho=\frac{M_W}{M_Z\cos\theta}=1 \tag{3.1}$$

in a dynamically broken Weinberg-Salam theory of the weak interactions. They discussed the situation in which the condensate potential possesses a global chiral flavor $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{L+R}$ invariance [the $\mathrm{U}(1)_{L+R}$ generator is the $(B-L)/2$ current of Eq. (2.3)] with its Weinberg-Salam $\mathrm{SU}(2)_L\times\mathrm{U}(1)_{\mathrm{WS}}$ subgroup being given a local extension [the $\mathrm{U}(1)_{\mathrm{WS}}$ generator is that of Eq. (2.5)]. The fermion Dirac mass term (to be labeled X) transforms as the $(\underline{2},\underline{2}^*,\underline{0})\oplus(\underline{2}^*,\underline{2},\underline{0})$ representation of the chiral group. If this mass term breaks diagonally down to a residual $\mathrm{SU}(2)_{L+R}\times\mathrm{U}(1)_{L+R}$ subgroup [i.e., if $\langle\bar{U}U\rangle=\langle\bar{D}D\rangle$], then the dynamical potential produces just the right three Goldstone bosons required to give masses to three of the four intermediate vector bosons. Since the $\mathrm{SU}(2)_L$ Weinberg-Salam gauge bosons transform as a triplet under this residual unbroken $\mathrm{SU}(2)_{L+R}$ subgroup they acquire degenerate masses by the Higgs mechanism. This then yields Eq. (3.1) after the mixing with the $\mathrm{U}(1)_{\mathrm{WS}}$ gauge boson. The residual global $\mathrm{SU}(2)_{L+R}$ symmetry of the condensate potential thus provides a group-theoretical origin for the Weinberg mixing relation.[3,4]

The above standard discussion is deficient in three respects. The first is that the same residual $\mathrm{SU}(2)_{L+R}$ symmetry which enforces Eq. (3.1) also entails an isospin invariance for the up and down hyperquarks to yield $M_U=M_D$ and hence (as we shall see in Sec. IV) $m_u=m_d$, a mass relation which is known to be violated experimentally. The second difficulty is that any attempt to split M_U and M_D would not just spoil Eq. (3.1) but would also reduce the residual symmetry of the condensate potential to $T^3_{L+R}\times\mathrm{U}(1)_{L+R}$. Then the potential would generate more Goldstone bosons than could be removed by the Weinberg-Salam gauge bosons. Finally, the above analysis does not by itself even yield the usual Weinberg-Salam phenomenology at all in the event that the full $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{L+R}$ group is given a local extension, since then both the fermion mass term and the gauge-boson sector would be parity conserving. The grand unified theory under discussion contains the full $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{L+R}$ as a local vertical subgroup of $\mathrm{SO}(10)_V$. Thus all of the above problems need to be addressed on our model. It turns out that the general resolution of all of these difficulties has been proposed recently.[11,12] Since the results of this analysis are applicable to our model we shall briefly review the important aspects.

The key point of Ref. 12 was to note that while a residual symmetry yields both $\rho=1$ and $M_U=M_D$, we cannot immediately say by how much these relations are not satisfied when the symmetry is only approximate, this being a model-dependent statement. Reference 12 then constructed an explicit model in which Eq. (3.1) was found to be approximately valid while M_U and M_D were free to be very different from each other. For our purposes here we note that the model considered in Ref. 12 was none other than a local chiral $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{L+R}$ weak-interaction theory.

With regard to this local chiral theory we have to introduce some new additional symmetry breaking which first breaks the local chiral flavor group down to $SU(2)_L \times U(1)_{WS}$. Now precisely such a breaking has recently been introduced through the idea of neutrino pairing[11] in which a Higgs field which transforms as a pair of right-handed neutrinos acquires a vacuum expectation value. This neutrino pairing term (to be labeled Δ_R) transforms as the $(\underline{1},\underline{3},-\underline{1})$ representation of $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ and hence breaks both $SU(2)_R$ and $U(1)_{L+R}$ (since it breaks lepton number), while leaving $SU(2)_L$ and Q unbroken. Hence according to Eqs. (2.4) and (2.5) it leaves $SU(2)_L \times U(1)_{WS}$ unbroken, and thus exactly breaks the local chiral theory down to the Weinberg-Salam model. Moreover, since the neutrino-pairing term transforms as a difermion, another fermion bilinear, this breaking can also be achieved dynamically.

This neutrino-pairing phenomenon has been studied in detail in Ref. 11 not in the dynamical-symmetry-breaking situation itself, but rather in a fundamental-Higgs-field theory. The model considered in Ref. 11 contained Higgs fields which transform as all the available fermion bilinears, namely χ and Δ_R given above and a left-handed counterpart Δ_L which transforms as the $(\underline{3},\underline{1},-\underline{1})$ representation. In Ref. 11 (and also Ref. 13) a tree-approximation minimum to the Higgs potential was found in which

$$\Delta_R \gg \chi \gg \Delta_L \tag{3.2}$$

and in which the fermion masses were split by an arbitrarily large amount. Explicit counting shows that because there are now seven gauge bosons we now have just the right number of gauge bosons to remove the six Goldstone bosons generated by the breaking pattern of Eq. (3.2) while simultaneously leaving the photon massless. Thus in a local chiral theory we can split the fermion masses without having any superfluous Goldstone bosons. Finally, evaluating the gauge-boson mass matrix we find all the usual Weinberg-Salam phenomenology with Eq. (3.1) being accurate to order

$$\frac{M_W}{M_Z \cos\theta} = 1 + O\left[\frac{\chi^2}{{\Delta_R}^2}\right] + O\left[\frac{{\Delta_L}^2}{\chi^2}\right]. \tag{3.3}$$

Thus in the tree approximation there is no residual symmetry at all and yet ρ only deviates from unity by a small amount dependent on the relative strengths of the left- and right-handed currents of the model.

Having now obtained a good limit in which ρ is close to unity and in which the fermions are far from degenerate, Ref. 12 then studied the radiative corrections to the tree approximation. It was found that there was sufficient approximate residual symmetry in the theory so that Eq. (3.3) continued to hold while the fermions stayed far from degenerate, no matter how big the coupling constants were. Thus the Weinberg mixing relation can be obtained in a local chiral weak-interaction theory to any required degree of accuracy with the fermion masses being unconstrained.

Returning now to the $SO(10)_V \times SO(10)_H$ model, we observe that the hyperfermion Dirac mass term χ of Eq. (2.9) contains exactly one $(\underline{2},\underline{2}^*,\underline{0}) \oplus (\underline{2}^*,\underline{2},0)$ piece under $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ which is an $SU(3)_C$ singlet, while the hyperfermion Majorana mass terms Δ_L and Δ_R of Eq. (2.9) contain exactly one $(\underline{3},\underline{1},-\underline{1}) \oplus (\underline{1},\underline{3},-\underline{1})$ piece under $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ which is an $SU(3)_C$ singlet. Thus our $SO(10)_V \times SO(10)_H$ model contains just the right condensates required for the analyses of Refs. 11 and 12, this therefore being a highly nontrivial feature of our theory. Since dynamical symmetry breaking is equivalent in content to a fundamental theory plus its radiative corrections the analysis of Ref. 12 will also hold dynamically. Thus if the hypercolor interactions of the horizontal sector of our theory are able to produce a hyperfermion bilinear condensate breaking pattern of the form of Eq. (3.2) dynamically (which of course we can make no comment on without a detailed knowledge of the dynamics of the actual breaking mechanism), then all the usual Weinberg-Salam phenomenology (and especially $\rho \simeq 1$ and m_u/m_d far from one) will obtain dynamically in the vertical sector of the theory.

We would like to make three additional remarks about our breaking pattern. First, we note that Eq. (3.2) has a clear experimental signal in that the right-handed and left-handed neutrinos acquire very different Majorana masses, with this disparity between the left- and right-handed sectors of the theory being due to the way we have spontaneously broken parity in Eq. (3.2). Further, since Δ_R and χ both arise dynamically via the exchange of the same hypergluons we would not expect their vacuum expectation values to be overwhelmingly different. Hence we may anticipate that the right-handed neutrinos will acquire GeV region masses. Thus in models in which local chiral theories are broken dynamically by hypercolor interactions there will be relatively light Majorana neutrinos, so that such models can be tested at normal energies way below the 10^{15}-GeV grand unification mass scale. Moreover, our neutrino mass spectrum stands in sharp contrast to that obtained in the standard treatment of the $SO(10)_V$ model. There $SO(10)_V$ is first broken down to SU(5) by a right-handed neutrino Majorana mass, which has to be of order 10^{15} GeV.[14] As we shall see below in more detail, in our model we break $SO(10)_V$ down to $SU(4)\times SU(2)_L\times SU(2)_R$ instead and never go via SU(5) at all. Hence in our scheme the right-handed neutrinos are decoupled from the 10^{15}-GeV region physics altogether and are thus able to be relatively light particles.

Our second remark concerns a subtle interplay between parity nonconservation and hypercolor conservation in our model. As we have just seen, in order to break $SU(2)_L\times SU(2)_R\times U(1)_{L+R}$ dynamically we have to break with Δ_R, a dihyperfermion. On the other hand, we do not want to break hypercolor itself, so that the dihyperfermion must be a hypercolor singlet. Since the product of two fundamentals of SU(4) contains no singlet, we see that we must not put the hyperfermions into the $\underline{4}$ of SU(4) of hypercolor, the "obvious" choice. Indeed, in our model we did not put the hyperfermions into the $\underline{4}$ but rather into the $\underline{6}$ [as they go into the $\underline{10}$ of $SO(10)_H$], so that Δ_R does nicely contain a hypercolor-singlet piece. Now our original choice of the $\underline{10}$ was determined for completely different phenomenological reasons [the classification according to the $(\underline{16},\underline{10})\oplus(\underline{10},\underline{16})$ representation is the only possible one for the fermions in our model which is complex, which contains no particles with bizarre quantum numbers, and which possesses the usual $SO(10)_V$ structure. See Ref. 8]; thus we see that the fact that Δ_R is a hypercolor singlet is then a highly nontrivial property of the model. Moreover, we will see below that the radiative corrections due to extended hypercolor are also only nonvanishing because the fermions are classified according to the $\underline{10}$ of $SO(10)_H$. Thus the model provides strong support for the use of the vector representation of orthogonal groups in grand unified theories, rather than just of the overwhelmingly popular spinor representation.

Our third and final remark concerns a subtle interplay between electric charge conservation and color conservation in the model. As well as serving to break $SU(2)_L\times SU(2)_R$ on the vertical side the difermion condensates Δ_R and Δ_L also break the vertical $SU(4)_V$ color group of Pati and Salam,[10] as they both transform according to the $\underline{10}$ of $SU(4)_V$. Now we note that the $\underline{10}$ of $SU(4)_V$ only contains one $SU(3)_C$ singlet. Thus there is only one breaking pattern which does not lead to QCD breaking. However, within each $\underline{10}$ of $SU(4)_V$ the $SU(3)_C$ singlet is the unique piece which is electrically neutral. Thus if the difermions do not break electric charge then only the dihyperneutrinos can acquire an expectation value, so that QCD is then necessarily not broken. Thus we correlate the conservation of electric charge with the lack of spontaneous breaking of QCD which is very interesting. Finally, to summarize we note that if hypercolor exchange is clever enough to break according to Eq. (3.2) we will obtain the relation $M_W=M_z\cos\theta$ in our theory to any required degree of accuracy of order $\chi^2/\Delta_R{}^2$. In this way the hypercolor dynamics produces all the usual $SU(3)_C\times SU(2)_L\times U(1)_{WS}$ phenomenology.

IV. THE LIGHT-FERMION SELF-ENERGIES AND THE CABIBBO ANGLE

We turn now to a discussion of the mass-generation mechanism for the usual fermions in our model. As we have seen, the hyperfermion condensates are responsible for the dynamical symmetry breaking. They give masses to the Weinberg-Salam gauge bosons by giving Dirac masses M_U, M_D, M_E, and M_N (to be referred to collectively as M_{HF}) to the hyperfermions. Specifically, these condensates produce a triplet of hyperpion Goldstone bosons with decay constant F. These hyperpions then become the longitudinal components of the gauge bosons and give them masses of order eF by the Higgs mechanism so that F is typically of order 125 GeV. Further, these hyperpions give a mass of order κF to the hyperfermions via the Goldberger-Treiman relation, where κ is the coupling of the hyperpions to the hyperfermions. If κ takes the same value here as it does for the coupling of ordinary pions to ordinary fermions we would expect M_{HF} to be of order 1 TeV, so that 1 TeV is the typical mass scale of hypercolor theories.

While the hyperfermion condensates give masses to the hyperfermions, we note that in a pure hypercolor theory these condensates do not give masses to the usual fermions, since these fermions carry no hypercolor and hence undergo no hyperstrong interaction in the first place. However, in our extended hypercolor model, the usual fermions can get masses through the radiative corrections of Fig. 1 which involve the hyperfermions and the additional $(\underline{1},\underline{1},\underline{1};\underline{6},\underline{2},\underline{2})$ extended hypergluons of the model. Specifically these extended hypergluons (to be labeled $E_{\alpha i}$, $\alpha=1,\ldots,6$, $i=1,\ldots,4$) can couple a $(\underline{4},\underline{2},\underline{1};\underline{6},\underline{1},\underline{1})$ hyperfermion H_α to a usual

(4,2,1;1,2,2) fermion f_i with an interaction Lagrangian $\mathscr{L}$ of the form [suppressing all $SO(10)_V$ indices]

$$\mathscr{L}=g_H \sum_{\alpha i} E^{\lambda}_{\alpha i}(\bar{f}_i\gamma_\lambda H_\alpha-\bar{H}_\alpha\gamma_\lambda f_i) \tag{4.1}$$

since the extended hypergluons carry both hypercolor and a generation index. We thus recognize $\mathscr{L}$ as the required interaction of Ref. 5, with the intermediate hyperfermions in the graph of Fig. 1 then providing a mass scale for the usual fermions. Since the dynamical symmetry breaking is only an effective TeV region phenomenon we evaluate the graph of Fig. 1 by cutting off the loop integration at $M_{\rm HF}$. Momentarily ignoring $SO(10)_H$ indices we thus expect light-fermion self-energies m of order

$$m=g_H{}^2\frac{M_{\rm HF}{}^3}{M_{\rm EHG}{}^2} \tag{4.2}$$

up to kinematic factors. Here $M_{\rm EHG}$ denotes the mass of an extended hypergluon. From Eq. (4.2) we thus typically expect $M_{\rm EHG}$ to be in the 1000-TeV region. Thus given the interaction of Eq. (4.1) the usual fermons acquire their masses by radiative corrections in a straightforward manner in our model.

We would like to stress the group-theoretical structure of our model which permits the interaction $\mathscr{L}$ to exist in the first place. On the vertical side we recall that the extended hypergluons are singlets. Consequently the hyperfermions and the usual fermions must both transform the same way under $SO(10)_V$ in order to couple in $\mathscr{L}$, and this exactly occurs in our model as they both transform as the 16 of $SO(10)_V$. Further, on the horizontal side we note that the extended hypergluons transform according to the vector representations of $SO(6)_H$ and $SO(4)_H$, and hence can only couple H_α to f_i if the fermions also transform according to a horizontal vector representation. Moreover, since the extended hypergluons have to belong to the adjoint representation of $SO(10)_H$ their classification according to the $SO(6)_H$ hypercolor vector representation is unique. With the usual fermions necessarily being hypercolor singlets, hypercolor conservation then forces the hyperfermions to also be in the vector representation of $SO(6)_H$ if they are to couple in $\mathscr{L}$ at all. Thus the very existence of the interaction of Eq. (4.1) is seen to be a highly nontrivial property of the model since we had already been obliged to introduce the vector 10 in the classification of the fermions according to the (16,10) ⊕ (10,16) representation of $SO(10)_V \times SO(10)_H$ for the completely different set of reasons previously outlined.

When $SO(10)_V \times SO(10)_H$ is spontaneously broken the $SO(4)_H$ group will necessarily have to be broken since there are no known massless gauge bosons associated with it. This will then both give masses to and mix the $E_{\alpha i}$ gauge bosons into new mass eigenstates

$$\tilde{E}_{\alpha i}=\sum_j A_{ij}E_{\alpha j} \tag{4.3}$$

so that the gauge-boson squared-mass matrix B can be written in terms of its eigenvalues $(\tilde{M}_k)^2$ as follows:

$$\sum_{\alpha ij} E_{\alpha i}(B)_{ij}E_{\alpha j}=\sum_{\alpha k}\tilde{E}_{\alpha k}(\tilde{M}_k)^2\tilde{E}_{\alpha k} \; . \tag{4.4}$$

Thus

$$(B)_{ij}=\sum_k (A^{-1})_{ik}(\tilde{M}_k)^2 A_{kj} \; . \tag{4.5}$$

With this diagonalization the interaction of Eq. (4.1) is replaced by

$$\mathscr{L}=g_H \sum_{\alpha ij}(A^{-1})_{ij}\tilde{E}^{\lambda}_{\alpha j}(\bar{f}_i\gamma_\lambda H_\alpha-\bar{H}_\alpha\gamma_\lambda f_i) \; . \tag{4.6}$$

In terms of a new basis for the usual fermions

$$g_i=\sum_j A_{ij}f_j \; , \tag{4.7}$$

we can completely rediagonalize $\mathscr{L}$ as

$$\mathscr{L}=g_H \sum_{\alpha i}\tilde{E}^{\lambda}_{\alpha i}(\bar{g}_i\gamma_\lambda H_\alpha-\bar{H}_\alpha\gamma_\lambda g_i) \; . \tag{4.8}$$

From Fig. 1 the interaction of Eq. (4.8) leads to a light-fermion mass matrix

$$\sum_{ij}\bar{f}_i m_{ij} f_j=g_H{}^2 M_{\rm HF}{}^3\sum_{ijk}\bar{f}_i\frac{(A^{-1})_{ik}A_{kj}}{\tilde{M}_k{}^2}f_j \; . \tag{4.9}$$

From Eq. (4.5) we see that

$$[m]_{ij}=g_H{}^2M_{\rm HF}{}^3[(B)^{-1}]_{ij} \tag{4.10}$$

to thus relate the fermion mass matrix to the gauge-boson squared-mass matrix. This is a particularly useful relation since the gauge-boson squared-mass matrix B is given directly by the symmetry-breaking mechanism. In terms of the basis of Eq. (4.7) we can also write the fermion mass matrix as

$$\sum_{ij} \bar{f}_i m_{ij} f_j = \sum_k \bar{g}_k \tilde{m}_k g_k \;, \tag{4.11}$$

with the mass eigenvalues $\tilde{m}_k$ satisfying

$$\tilde{m}_k = g_H{}^2 \frac{M_{\text{HF}}{}^3}{\tilde{M}_k{}^2} \;. \tag{4.12}$$

Thus we see that in our model the spontaneous breakdown of $\text{SO}(4)_H$ leads not only to light-fermion masses but also to fermion mass mixing, and necessarily so in fact since the four generations are all in the same irreducible representation of $\text{SO}(10)_H$ to thus share a common group theory. The light fermions mix through the same angles as do the gauge bosons according to Eq. (4.7) while the diagonal light-fermion masses stand in the same ratios as the gauge-boson inverse-squared masses according to Eq. (4.12). The model thus provides an in-principle mechanism to calculate intergenerational mass splittings and the generalized Cabibbo angles which mix the generations.

We shall refer to the eigenstates of Eq. (4.7) by the sets of generations (u,d,e,ν_e), (c,s,μ,ν_μ), (t,b,τ,ν_τ), and (h,l,σ,ν_σ). The masses of Eq. (4.12) are the so-called *weak-interaction current masses* and writing out Eq. (4.12) in detail, we see that the masses satisfy

$$\begin{aligned}
g_H{}^2 M_U{}^3 &= m_u \tilde{M}_1{}^2 = m_c \tilde{M}_2{}^2 = m_t \tilde{M}_3{}^2 = m_h \tilde{M}_4{}^2 \;, \\
g_H{}^2 M_D{}^3 &= m_d \tilde{M}_1{}^2 = m_s \tilde{M}_2{}^2 = m_b \tilde{M}_3{}^2 = m_l \tilde{M}_4{}^2 \;, \\
g_H{}^2 M_E{}^3 &= m_e \tilde{M}_1{}^2 = m_\mu \tilde{M}_2{}^2 = m_\tau \tilde{M}_3{}^2 = m_\sigma \tilde{M}_4{}^2 \;, \\
g_H{}^2 M_N{}^3 &= m(\nu_e) \tilde{M}_1{}^2 = m(\nu_\mu) \tilde{M}_2{}^2 = m(\nu_\tau) \tilde{M}_3{}^2 \\
&= m(\nu_\sigma) \tilde{M}_4{}^2 \;,
\end{aligned} \tag{4.13}$$

with each generation having its own associated mass scale $\tilde{M}_k{}^2$. Hence the generations will be reasonably separated in mass.

Typical mass scales can be inferred from the lepton masses. With M_E of order 1 TeV and g_H of order unity we expect $\tilde{M}_1$ to be of order 1000 TeV, $\tilde{M}_2$ of order 100 TeV, $\tilde{M}_3$ of order 10 TeV, which suggests $\tilde{M}_4$ is of order 1 TeV. With m_u and m_d thought to be in the few-MeV region, we find M_D and M_U to both be of order 1 TeV (but not equal to each other as we discussed in Sec. III). Consequently we also take M_N to be of order 1 TeV. With these mass values we can now estimate the other fermion masses. From Eq. (4.13) we obtain many mass formulas. The most interesting ones involve the charged fermions of the three so far observed generations, viz.,

$$\begin{aligned}
\frac{\tilde{M}_2{}^2}{\tilde{M}_1{}^2} &= \frac{m_u}{m_c} = \frac{m_d}{m_s} = \frac{m_e}{m_\mu} \;, \\
\frac{\tilde{M}_3{}^2}{\tilde{M}_1{}^2} &= \frac{m_u}{m_t} = \frac{m_d}{m_b} = \frac{m_e}{m_\tau} \;.
\end{aligned} \tag{4.14}$$

We will discuss the validity of these relations below after we discuss Cabibbo mixing in detail.

In the neutral fermion sector the right-handed hyperneutrino acquires a Majorana mass $\Delta_R(N)$ from the Δ_R condensate. [For simplicity we set $\Delta_L=0$ in Eq. (3.2)]. Through the radiative corrections of Fig. 1 the four ordinary neutrinos then also acquire right-handed Majorana masses which are diagonal in the same generation-space basis as the Dirac masses of Eq. (4.13) and are given by

$$\begin{aligned}
g_H{}^2[\Delta_R(N)]^3 &= \Delta_R(\nu_e)\tilde{M}_1{}^2 = \Delta_R(\nu_\mu)\tilde{M}_2{}^2 \\
&= \Delta_R(\nu_\tau)\tilde{M}_3{}^2 = \Delta_R(\nu_\sigma)\tilde{M}_4{}^2 \;,
\end{aligned} \tag{4.15}$$

For each ordinary neutrino its Dirac and right-handed Majorana masses will mix in its mass matrix to induce a left-handed Majorana mass. As noted in Sec. III, $\Delta_R(\nu_e)$ is in the GeV region [since $\Delta_R(N)$ is in the TeV region] and so these final observable left-handed Majorana masses such as $\Delta_L(\nu_e)$ will typically be of order 1 eV (see, e.g., Ref. 13). This is many orders of magnitude larger than the value of perhaps 10^{-5} eV expected in the usual treatment of the $\text{SO}(10)_V$ theory where $\Delta_R(\nu_e)$ is in the 10^{15}-GeV region.[14] Hence our model differs from the standard $\text{SO}(10)_V$ picture in its predictions for both $\Delta_R(\nu_e)$ and $\Delta_L(\nu_e)$.

We turn now to a discussion of Cabibbo mixing in our model. As we have already noted the very fact that $\text{SO}(4)_H$ is spontaneously broken forces the fermions to mix, and indeed if we had a detailed knowledge of the gauge-boson squared-mass matrix B we could determine all the mixing angles directly from Eq. (4.10). In the absence of such knowledge we must instead make some assumptions, and though we shall make what we regard as reasonable assumptions our analysis should only be taken as illustrative of the mixing phenomenon. To simplify we shall restrict ourselves to the first two generations of quarks by taking $\tilde{E}_{\alpha 1}$ and $\tilde{E}_{\alpha 2}$ to be heavier than $\tilde{E}_{\alpha 3}$ and $\tilde{E}_{\alpha 4}$. In this case $E_{\alpha 1}$ and $E_{\alpha 2}$ mix through an angle ϕ so that f_1 and f_2 also mix through the same angle ϕ according to Eq. (4.7).

To determine the gauge-boson mixing angle ϕ we need to make a model for the gauge-boson mass matrix. The simplest possibility is to break $SO(4)_H$ in the quartet representation by fundamental Higgs fields in the requisite 1000-TeV region. (The dynamical hyperfermion condensates only act at the 1-TeV level.) Since we are dealing with two generations we shall need two quartets which we label ϕ_i^a and ϕ_i^b ($i=1,\ldots,4$). To implement the breaking we note that the extended hypergluons are minimally coupled to the $\underline{10}$ of $SO(10)_H$ which contains the $\underline{4}$ of $SO(4)_H$, so that the gauge-boson mass matrix is obtained as

$$\mathscr{L}_{\text{mass}}=g_H{}^2\sum_\alpha\left[\left|\sum_i E_{\alpha i}\phi_i^a\right|^2+\left|\sum_i E_{\alpha i}\phi_i^b\right|^2\right]. \tag{4.16}$$

For the two quartets the most general breaking pattern in the $E_{\alpha 1}$, $E_{\alpha 2}$ sector of interest is

$$\begin{aligned}\phi^a&=a(1,0,0,0)\,,\\ \phi^b&=b(\sin\gamma,\cos\gamma,0,0)\,.\end{aligned} \tag{4.17}$$

In terms of the parameters a, b, and γ we can determine the eigenvalues and eigenvectors of the gauge-boson mass matrix as

$$\widetilde{M}_1{}^2=\frac{g_H{}^2}{2}[(a^2+b^2)+\sqrt{A}\,]\,, \tag{4.18a}$$

$$\widetilde{M}_2{}^2=\frac{g_H{}^2}{2}[(a^2+b^2)-\sqrt{A}\,]\,, \tag{4.18b}$$

$$\tan\phi=\frac{\sqrt{A}-a^2+b^2\cos2\gamma}{b^2\sin2\gamma}\,, \tag{4.18c}$$

where $A=a^4+b^4-2a^2b^2\cos2\gamma$. From Eqs. (4.18) we can obtain the useful relation

$$\tan^2\phi=\frac{\widetilde{M}_2{}^2}{\widetilde{M}_1{}^2}(1+Y)\,, \tag{4.19}$$

where

$$Y=\frac{(a^2-b^2)[(a^2-b^2)-\sqrt{A}\,]}{2a^2b^2\sin^2\gamma}\,. \tag{4.20}$$

Finally, recalling that ϕ is also the d-s mixing angle ϕ_{ds} we find from Eq. (4.14) that

$$\tan^2\phi_{ds}=\frac{m_d}{m_s}(1+Y)\,. \tag{4.21}$$

The d-s mixing angle is thus expressed completely in terms of the parameters of the gauge-boson mass matrix.

With the parameter Y being completely unknown we cannot use Eq. (4.21) to extract out a numerical value for ϕ_{ds}. However we now note some unusual features of the structure of the mixing matrices. From Eq. (4.14) we observe that $\widetilde{M}_2{}^2/\widetilde{M}_1{}^2$ is a very small number since $m_e \ll m_\mu$. Hence from Eqs. (4.18) we see that

$$\frac{a^2b^2\cos^2\gamma}{(a^2+b^2)^2}\ll 1\,, \tag{4.22}$$

$$Y=\frac{b^2-a^2}{a^2\sin^2\gamma}\,. \tag{4.23}$$

Now there are four ways of satisfying the constraint of Eq. (4.22) and they lead to differing values of Y. Specifically they are

$$\begin{aligned}&\text{(i) } a\sim b\sin\gamma\gg b\cos\gamma;\ Y\sim 0\,,\\ &\text{(ii) } a\gg b\sin\gamma\sim b\cos\gamma;\ Y\sim 1\,,\\ &\text{(iii) } a\gg b\sin\gamma\gg b\cos\gamma;\ Y\sim 1\,,\\ &\text{(iv) } a\gg b\cos\gamma\gg b\sin\gamma;\ Y\gg 1\end{aligned} \tag{4.24}$$

(cases in which $b\gg a$ are equivalent). All of cases (i) through (iii) lead to

$$\tan^2\phi_{ds}=O\left[\frac{m_d}{m_s}\right]\ll 1 \tag{4.25}$$

while only case (iv) can make ϕ_{ds} large. Without further knowledge of the structure of the Higgs potential which produces the set of expectation values of Eq. (4.17) we cannot make any further statement. However, we note that case (iv) [and also case (iii)] requires that there be a hierarchy of two separate scales of breaking, while cases (i) and (ii) only require one such hierarchy. Since all of the Higgs expectation values are to arise from one single potential we regard it as perhaps unlikely that case (iv) could emerge. Hence, we can reasonably expect Y to be of order one, so that Eq. (4.25) is a realistic expectation for our model.

While Eq. (4.21) is an encouraging result we note that since the hyperfermions are generation singlets an analogous relation also exists in the u-c sector where the mixing angle satisfies

$$\tan^2\phi_{uc}=\frac{m_u}{m_c}(1+Y) \tag{4.26}$$

according to Eqs. (4.19) and (4.14). Thus from Eq. (4.14) we obtain

$$\phi_{ds}=\phi_{uc}\,. \tag{4.27}$$

Thus the extended-hypercolor contributions to the weak-interaction current mass matrix mixing are the same in the d-s and u-c sectors. Consequently an identification of ϕ_{ds}-ϕ_{uc} with the observed Cabibbo angle would lead us to conclude that there would be no Cabibbo mixing in our model. However as we noted in Ref. 9, $(\phi_{ds}-\phi_{uc})$ as determined from Eqs. (4.21) and (4.26) is not necessarily the observed Cabibbo angle since further mixing contributions can also come from strong-interaction chiral-symmetry breaking.

Specifically with four quarks the QCD Lagrangian possesses a global chiral flavor $SU(4)_L \times SU(4)_R$ symmetry. This symmetry is spontaneously broken by the color dynamics and gives Goldberger-Treiman masses m_u^0, m_d^0, m_s^0, and m_c^0 to the quarks to contribute an extra diagonal piece to the quark mass matrix. In the event that QCD spontaneously breaks $SU(4)_L \times SU(4)_R$ down beyond $SU(4)_{L+R}$ not all of these strong-interaction masses are degenerate, so that the actual observable quark mixing angles are only obtained after a simultaneous diagonalization of both the weak and strong contributions. In the d-s sector for instance we can express the full mass matrix $[m'_{ij}]$ in our initial f_i, f_j basis as

$$\sum_{ij} \bar{f}_i m'_{ij} f_j = \sum_{ij} \bar{f}_i m_{ij} f_j + \bar{f}_1 f_1 m_d^0 + \bar{f}_2 f_2 m_s^0 \tag{4.28}$$

according to Eq. (4.9). Expressing m_{ij} in terms of its eigenvectors and eigenvalues yields

$$[m'_{ij}] = \begin{bmatrix} m_d^0 + m_d\cos^2\phi + m_s\sin^2\phi & \sin\phi\cos\phi(m_d - m_s) \\ \sin\phi\cos\phi(m_d-m_s) & m_s^0 + m_d\sin^2\phi + m_s\cos^2\phi \end{bmatrix}, \tag{4.29}$$

where ϕ is still the extended-hypergluon mixing angle and m_d and m_s are still given by Eq. (4.13). The mixing angle associated with $[m'_{ij}]$ is readily calculated to be

$$\tan 2\theta_{ds} = \frac{\sin 2\phi}{(k_{ds}+\cos 2\phi)}, \tag{4.30}$$

where

$$k_{ds} = \frac{m_s^0 - m_d^0}{(m_s - m_d)}. \tag{4.31}$$

Similarly in the u-c sector we obtain

$$\tan 2\theta_{uc} = \frac{\sin 2\phi}{(k_{uc}+\cos 2\phi)}, \tag{4.32}$$

where

$$k_{uc} = \frac{m_c^0 - m_u^0}{(m_c - m_u)}. \tag{4.33}$$

Thus, finally, the observable Cabibbo angle is given as

$$\begin{aligned}\theta_C &= \theta_{ds} - \theta_{uc} \\ &= \tfrac{1}{2}\tan^{-1}\left[\frac{\sin 2\phi}{k_{ds}+\cos 2\phi}\right] \\ &\quad -\tfrac{1}{2}\tan^{-1}\left[\frac{\sin 2\phi}{k_{uc}+\cos 2\phi}\right],\end{aligned} \tag{4.34}$$

where ϕ is given in Eq. (4.19).

While the above discussion of the influence of QCD on θ_C is straightforward we note that such an analysis is not customarily made in the literature as it is always assumed that m_u^0, m_d^0, m_s^0, and m_c^0 are all equal; i.e., it is always assumed that QCD only breaks $SU(4)_L \times SU(4)_R$ down to $SU(4)_{L+R}$. As far as we are aware such an assumption has not been justified in the literature and in fact demands further study as a problem in its own right even independent of our work here, especially since it affects what we even mean by the Cabibbo angle. Further, the possible breaking of the strong-interaction chiral symmetry beyond $SU(4)_{L+R}$ is directly amenable to experimental testing. As we noted in Ref. 9 there would have to exist some observable scalar Goldstone bosons in addition to the familiar 15 pseudoscalar Goldstone bosons such as the pion and we advocate a vigorous search for such particles. On the phenomenological side we also take note of the detailed study of Das and Deshpande[15] using a partial conservation of the axial-vector current (PCAC) analysis. They found that there are good indications that $SU(4)_{L+R}$ is in fact broken spontaneously, and moreover quite a lot, although $SU(3)_{L+R}$ apparently survives intact as an unbroken symmetry. Hence we can effectively take m_s^0 equal to m_d^0 and m_c^0 very large. Then in the case where Y is zero we obtain finally from Eq. (4.34)

$$\tan^2\theta_C = \frac{m_d}{m_s}, \tag{4.35}$$

where we recall that m_d and m_s are the weak-interaction contributions to the quark masses. We are not of course asserting that Eq. (4.35) is strictly obeyed, by only that it constitutes a reasonable estimate of the smallness of the Cabibbo angle.

Since the quark mass pattern that we have used is somewhat different than the conventional one we would like to comment on the general determinations of the quark mass parameters. In the days before the discovery of charm the chiral symmetry of the strong interaction was taken to be $SU(3)_L \times SU(3)_R$ and the conventional PCAC analysis indicated that it was broken down spontaneously only to $SU(3)_{L+R}$ so that m_u^0, m_d^0, and m_s^0 are all equal, with a typical value of 300 MeV then usually being taken for these masses. The same PCAC analysis was also used to estimate the weak-interaction current masses m_u, m_d, and m_s with typical values of 5, 10, and 200 MeV, respectively, being found for them. The reliability of these numbers is still open to question since apart from anything else there is quite a big continuation to the kaon mass shell. The full constituent masses of the quarks which make up the hadrons are then given by combining m_i^0 and m_i $(i=u,d,s)$ perhaps geometrically. With the discovery of charm the above picture had to be extended to $SU(4)_L \times SU(4)_R$ and it became necessary to know how to distribute the 1.5 GeV constituent charm mass between m_c^0 and m_c. The conventional so-far-unjustified approach is to take m_c^0 simply equal to m_u^0, m_d^0, and m_s^0 [so that $SU(4)_{L+R}$ remains unbroken] while taking m_c very large. Instead we are suggesting that rather it is m_c^0 that is large and that m_c might be much smaller than previously anticipated. Further, if the QCD effects are not $SU(4)_{L+R}$ flavor invariant, then QCD itself will also make radiative contributions to m_u, m_d, m_s, and m_c. With m_c^0 large we might thus expect m_c to undergo a big renormalization so that the m_c as measured in PCAC for D mesons may no longer satisfy the relation $m_u/m_c = m_d/m_s$ of Eq. (4.14). Moreover, there are even further potential renormalizations of the weak masses in our model. It is possible that the fourth generation of fermions might have masses as heavy as the Weinberg-Salam gauge bosons or even some of the the six flavor-changing $SO(4)_H$ generators. They would then make radiative corrections to the self-energies of the light fermions to cause some further deviations from Eq. (4.14) (so that for instance m_d/m_s would no longer be equal to m_e/m_μ). Thus we should only take Eq. (4.14) as a guide to the light-fermion masses, and not as exact mass relations.

To conclude this section we remark again that by exploiting the group-theoretical correlation between hypercolor and the generation problem we have been able to construct a realistic theory of extended hypercolor which gives a reasonable estimation of the Cabibbo angle. We would also like to note that the pattern of masses and mixings we have discussed in detail is assuming that the extended-hypercolor mechanism is the only source. One might also use an appropriate Higgs system instead or in conjunction with the extended hypercolor within the framework of our model.

V. THE GOLDSTONE AND PSEUDO-GOLDSTONE BOSONS OF THE MODEL

We turn now to the classification of the various Goldstone and pseudo-Goldstone bosons associated with the hypercolor dynamics of our model. The Lagrangian of the coupling of the $(\underline{4},\underline{2},\underline{1};\underline{6},\underline{1},\underline{1})$ and $(\underline{4}^*,\underline{1},\underline{2};\underline{6},\underline{1},\underline{1})$ hyperfermions to the $(\underline{1},\underline{1},\underline{1};\underline{15},\underline{1},\underline{1})$ hypergluons which is effective in the TeV region possesses an $SU(8)_L \times SU(8)_R \times U(1)_{L+R}$ global chiral symmetry in the U_i, D_i, E, N space. [Technically there is also an axial $U(1)_{L-R}$ symmetry, but this is destroyed by instantons.] Each SU(8) contains seven $SU(3)_C$ color-singlet currents, four color octets, and eight color triplets. Thus $SU(8)_L \times SU(8)_R \times U(1)_{L+R}$ contains 15 color-singlet currents altogether. We shall not concern ourselves with the color-nonsinglet piece in the following since any associated Goldstone or pseudo-Goldstone bosons will be confined.

We break the global chiral symmetry with the Δ_R, Δ_L, and χ condensates of Eq. (2.9). The right-handed Δ_R breaks as $N_R C N_R$ and thus spontaneously breaks three right-handed currents, $\bar{N}_R \gamma_\lambda N_R$, $\bar{N}_R \gamma_\lambda E_R$, and $\bar{E}_R \gamma_\lambda N_R$ which are all color singlets. Similarly, the left-handed Δ_L breaks the three color-singlet currents $\bar{N}_L \gamma_\lambda N_L$, $\bar{N}_L \gamma_\lambda E_L$, and $\bar{E}_L \gamma_\lambda N_L$. The Dirac mass χ gives different masses M_U, M_D, M_E, and M_N to the hyperfermions. It thus breaks all 63 axial-vector currents and also breaks 44 vector currents leaving unbroken only the 20 vector currents $\bar{N}\gamma_\lambda N$, $\bar{E}\gamma_\lambda E$, $\bar{U}_i \gamma_\lambda U_j$, and $\bar{D}_i \gamma_\lambda D_j$ $(i,j=R,G,B)$. Of these last 20 there are 4 color singlets, $\bar{N}\gamma_\lambda N$, $\bar{E}\gamma_\lambda E$, $\sum \bar{U}_i \gamma_\lambda U_i$, and $\sum \bar{D}_i \gamma_\lambda D_i$. Thus χ breaks 11 singlet currents altogether, of which 4 are vector $(\bar{N}\gamma_\lambda E$, $\bar{E}\gamma_\lambda N$, $\sum \bar{U}_i \gamma_\lambda D_i$, and $\sum \bar{D}_i \gamma_\lambda U_i)$ and 7 are axial-vector [$\bar{N}\gamma_\lambda \gamma_5 N$, $\bar{E}\gamma_\lambda \gamma_5 E$, $\bar{N}\gamma_\lambda \gamma_5 E$,

$\bar{E}\gamma_\lambda\gamma_5 N$, $\sum \bar{U}_i\gamma_\lambda\gamma_5 U_i$, $\sum \bar{U}_i\gamma_\lambda\gamma_5 D_i$, $\sum \bar{D}_i\gamma_\lambda\gamma_5 U_i$, and $\sum \bar{D}_i\gamma_\lambda\gamma_5 D_i$, then minus the $\mathrm{U}(1)_{L-R}$ current]. Consequently Δ_R, Δ_L, and χ together break a grand total of 12 linearly independent color-singlet chiral currents. The dynamical potential thus produces 12 light spin-zero particles.

In our grand unified model we give a local extension to the 45 gauge bosons associated with the $\mathrm{SO}(10)_V$ subgroup of the chiral $\mathrm{SU}(8)_L \times \mathrm{SU}(8)_R \times \mathrm{U}(1)_{L+R}$. Within the $\mathrm{SO}(10)_V$ there are altogether 7 color-singlet currents which form the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{L+R}$ group studied in detail in Sec. III. Since one of its gauge bosons is to be the photon we see that only 6 color-singlet gauge bosons acquire masses by the Higgs mechanism. Hence, out of our initial 12 spin-zero particles above we see that 6 are removed by the Higgs mechanism while the other 6 remain in the spectrum. These latter 6 bosons can only acquire masses by radiative corrections involving the gauge bosons of the standard model and become relatively light pseudo-Goldstone particles with masses of order eM_W, i.e., of order perhaps 20 GeV or less. Such pseudo-Goldstone particles are typical of minimally gauged theories such as $\mathrm{SO}(10)_V$ where the gauge sector has a lower symmetry than that of the potential. In maximally gauged theories on the other hand the full $\mathrm{SU}(8)_L \times \mathrm{SU}(8)_R \times \mathrm{U}(1)_{L+R}$ group would be gauged with all twelve spin-zero particles then being removed by the Higgs mechanism to leave no observable pseudo-Goldstone particles at all. Thus we do not regard the pseudo-Goldstone bosons of our $\mathrm{SO}(10)_V \times \mathrm{SO}(10)_H$ model with too much concern, as they can all be removed if necessary by embedding the grand unified theory in an appropriately large-enough gauge group.

VI. CONSTRAINTS FROM RARE DECAYS

In this section we analyze the flavor-changing rare decay processes which occur in our model due to the generation mixing. The various possibilities we consider are tree-level single-$\mathrm{SO}(4)_H$-gauge-boson exchange, box-diagram extended-hypergluon exchange, pseudo-Goldstone-boson exchange, and tree-level Pati-Salam-$\mathrm{SU}(4)_V$-gauge-boson exchange. As we shall see a mass scale in the 1000-TeV region will suffice to provide sufficient suppression to keep all rare decay processes within current experimental bounds.

The single-gauge-boson exchanges yield tree graphs such as that of Fig. 2 with an effective coupling

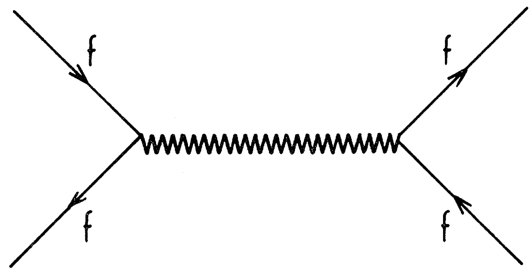

FIG. 2. A single-gauge-boson-exchange graph. The exchanged gauge bosons belong to the adjoint representation of $\mathrm{SO}(4)_H$.

$$V_T = \frac{g_H{}^2}{M^2} , \tag{6.1}$$

ignoring quantum numbers for the moment. The exchange of a pair of $(\underline{1},\underline{1},\underline{1};\underline{6},\underline{2},\underline{2})$ extended hypergluons gives the box graph of Fig. 3. Here the exchanged fermions are the same hyperfermions as those of Fig. 1. We evaluate the box graph by cutting off the internal momentum at M_{HF} as in Fig. 1 to obtain

$$V_{\mathrm{box}} \sim g_H{}^4 \frac{M_{\mathrm{HF}}{}^2}{M_{\mathrm{EHG}}{}^4} , \tag{6.2}$$

which we can reexpress as

$$V_{\mathrm{box}} \sim \frac{m^2}{M_{\mathrm{HF}}{}^4} , \tag{6.3}$$

using Eq. (4.2). Thus V_{box} is a number whose magnitude can be estimated independently of g_H and M_{EHG}. To evaluate the contributions due to the pseudo-Goldstone bosons, to be labeled P, we note that they only couple to the ordinary fermions via the hyperfermions and the extended hypergluons to give the effective Yukawa coupling of Fig. 4, viz.,

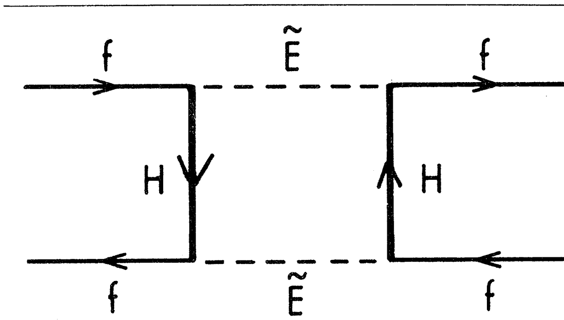

FIG. 3. Box diagram involving the exchange of hyperfermions and extended hypergluons.

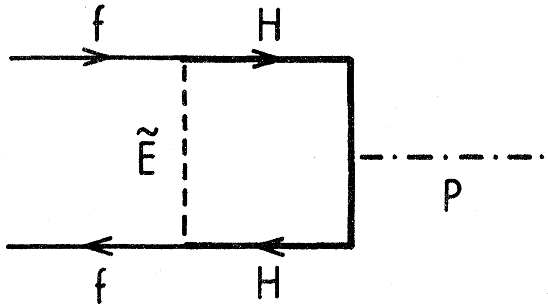

FIG. 4. Pseudo-Goldstone-boson effective coupling.

$$V_Y \sim \kappa g_H{}^2 \frac{M_{\rm HF}{}^2}{M_{\rm EHG}{}^2} , \tag{6.4}$$

where κ, the pseudo-Goldstone-boson—hyperfermion coupling constant, was introduced in Sec. IV. Using Eq. (4.2) and the hypercolor Goldberger-Treiman relation we can reexpress Eq. (6.4) as

$$V_Y \sim \frac{m}{F} , \tag{6.5}$$

where F is the hyperpion decay constant. Again we see that the strength of the interaction can be estimated. Finally pseudo-Goldstone-boson exchange leads to the graph of Fig. 5 with an effective strength

$$V_P \sim \frac{m^2}{F^2 M_P{}^2} , \tag{6.6}$$

where M_P is the pseudo-Goldstone-boson mass. To evaluate the effects due to all these processes we must now put in all the appropriate quantum numbers.

We discuss first the tree graphs due to the exchange of the $SO(4)_H$ generators, the $(\underline{1},\underline{1},\underline{1};\underline{1},\underline{3},\underline{1})$ and $(\underline{1},\underline{1},\underline{1};\underline{1},\underline{1},\underline{3})$ gauge bosons. To match our previous notation of Eq. (4.1) we shall label these gauge bosons E_{ij} $(i,j,=1,\ldots,4,\ i<j)$. These gauge bosons connect fermions of different generations to each other and are thus flavor changing even in the symmetry limit. In terms of our original $(\underline{4},\underline{2},\underline{1};\underline{1},\underline{2},\underline{2})$ fermion basis f_i we have couplings of the form

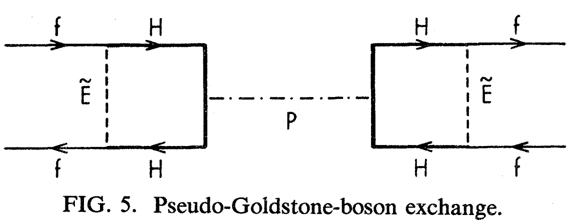

FIG. 5. Pseudo-Goldstone-boson exchange.

$$\mathscr{L} = g_H \sum_{i,j} E_{ij}^{\lambda}(\bar{f}_i \gamma_\lambda f_j - \bar{f}_j \gamma_\lambda f_i) . \tag{6.7}$$

We must now change the fermion basis to that of the eigenstates of the fermion mass matrix. We introduce the notation $d',s',u',c',\ldots$ to denote the exact mass eigenstates of the full $[m_{ij}']$ of Eq. (4.28) which includes both extended-hypergluon and QCD contributions. In terms of the convenient $d,s,\ldots$ basis which diagonalizes the extended-hypergluon-induced mass matrix $[m_{ij}]$ of Eq. (4.9) alone we can write

$$\begin{aligned} d' &= d \cos\alpha + s \sin\alpha , \\ s' &= -d \sin\alpha + s \cos\alpha \end{aligned} \tag{6.8}$$

and

$$\begin{aligned} u' &= u \cos\beta + c \sin\beta , \\ c' &= -u \sin\beta + c \cos\beta . \end{aligned} \tag{6.9}$$

Here

$$\begin{aligned} \tan 2\alpha &= \frac{\sin 2\phi}{(k_{ds}{}^{-1} + \cos 2\phi)} , \\ \tan 2\beta &= \frac{\sin 2\phi}{(k_{uc}{}^{-1} + \cos 2\phi)} , \end{aligned} \tag{6.10}$$

and k_{ds} and k_{uc} are defined in Eqs. (4.31) and (4.33), respectively. [Eq. (6.10) differs from Eq. (4.30) because we have expressed d' in terms of d, and not in terms of f_i as previously.] Thus we can express Eq. (6.7) in terms of mass eigenstates.

As far as the first two generations of fermions are concerned we note that the antisymmetric vertices of Eq. (6.7) are left invariant by the Cabibbo rotations used to diagonalize the fermion mass matrix. Thus Eq. (6.7) leads to flavor-changing vertices of interest of the form

$$\begin{aligned} \mathscr{L} = g_H E_{12}^{\lambda} [& (\bar{s}'\gamma_\lambda d' - \bar{d}'\gamma_\lambda s') + \bar{\mu}\gamma_\lambda e - \bar{e}\gamma_\lambda \mu \\ & + (\bar{c}'\gamma_\lambda u' - \bar{u}'\gamma_\lambda c') \\ & + \bar{\nu}_\mu \gamma_\lambda \nu_e - \bar{\nu}_e \gamma_\lambda \nu_\mu] \end{aligned} \tag{6.11}$$

[the lepton basis is that of Eq. (4.13)]. Thus Fig. 2 leads to typical effective interactions such as

$$\begin{aligned} \mathscr{L}_T = \frac{g_H{}^2}{M_{12}{}^2} & (\bar{s}'\gamma_\lambda d' - \bar{d}'\gamma_\lambda s')_{(1)} \\ & \times (\bar{s}'\gamma_\lambda d' - \bar{d}'\gamma_\lambda s')_{(2)} , \end{aligned} \tag{6.12}$$

where M_{12} is the mass of E_{12} and (1) and (2) denote the two vertices of Fig. 2. In the direct channel [where we construct a K meson by contracting $\bar{s}'$ in (1) with d' in (1)] the couplings are pure vector, and hence the pseudoscalar mesons cannot couple to the quarks at all. Thus there are no direct-channel contributions to both the K_S-K_L and D_S-D_L mass differences or to K and D decays into lepton pairs at all due to Fig. 2. There is, however, a vector $K\pi$ vertex so that $K \to \pi\mu e$ does occur in the direct channel. This process is then sufficiently suppressed if $M_{12} > 5$ TeV. Stronger constraints are found from the cross channel where we can construct a K meson by contracting $\bar{s}'$ in (1) with d' in (2). Unlike the direct channel this channel does possess pseudoscalar projections. The effective strangeness-changing vertices of interest are of the form

$$\mathscr{L}_T = \frac{g_H^2}{M_{12}^2}(\bar{s}'_{(1)}\gamma_5 d'_{(2)}\bar{s}'_{(2)}\gamma_5 d'_{(1)} + \bar{d}'_{(1)}\gamma_5 s'_{(2)}\bar{d}'_{(2)}\gamma_5 s'_{(1)}) \tag{6.13}$$

$$= \frac{g_H^2}{M_{12}^2}(K_0^2 + \bar{K}_0^2) = \frac{g_H^2}{M_{12}^2}(K_S^2 - K_L^2) \ .$$

This yields a K_S-K_L mass difference ΔM_K of the form

$$\frac{\Delta M_K}{M_K f_K^2} = \frac{g_H^2}{M_{12}^2} \ , \tag{6.14}$$

where f_K is the usual kaon decay constant. Experimentally the left-hand side of Eq. (6.14) is less than 10^{-6} TeV^{-2}. Thus with g_H of order 1 we require $M_{12} \geq 1000$ TeV. Similarly the tree graph contributes to the D_S-D_L mass difference. Though this has yet to be measured it has been estimated theoretically[16] to satisfy

$$\frac{\Delta M_D}{M_D f_D^2} = 10^{-4}\text{TeV}^{-2} \tag{6.15}$$

and is thus well suppressed by a 1000-TeV mass for E_{12}. Since there are no lepton-pair decays possible in the crossed channel we see that this one scale suppresses all flavor-changing processes due to the $SO(4)_H$ horizontal gauge bosons.

The value we have found for M_{12} is very encouraging since it is of the same order as $\tilde{M}_1$. It is thus very tempting to generate both these masses by the same Higgs mechanism. Indeed we can give all six $SO(4)_H$ generators a mass by breaking according to four $SO(4)_H$ quartets. At the same time this would give masses to all 24 of the $(\underline{1},\underline{1},\underline{1};\underline{6},\underline{2},\underline{2})$ extended hypergluons by breaking $SO(10)_H$ to $SO(6)_H$ in one step. Specifically with four quartets ϕ_i^a, ϕ_i^b, ϕ_i^c, and ϕ_i^d we obtain a mass term [analogous to Eq. (4.16)]

$$\mathscr{L}_{\text{mass}} = g_H^2 \sum_{a,\alpha}\left[\left|\sum_i E_{\alpha i}\phi_i^a\right|^2\right] + g_H^2\sum_{a,i}\left[\left|\sum_j E_{ij}\phi_j^a\right|^2\right] . \tag{6.16}$$

For simplicity we shall ignore mixings with the third and fourth generations by taking

$$\begin{aligned} \phi^a &= a(1,0,0,0) \ , \\ \phi^b &= b(\sin\gamma,\cos\gamma,0,0) \ , \\ \phi^c &= c(0,0,1,0) \ , \\ \phi^d &= d(0,0,0,1) \ . \end{aligned} \tag{6.17}$$

This yields [using Eqs. (4.18)]

$$\begin{aligned} \mathscr{L}_{\text{mass}} = &\sum_\alpha(\tilde{M}_1^2\tilde{E}_{\alpha1}^2 + \tilde{M}_2^2\tilde{E}_{\alpha2}^2 + g_H^2c^2E_{\alpha3}^2 + g_H^2d^2E_{\alpha4}^2) + g_H^2(a^2+b^2)E_{12}^2 + g_H^2(c^2+d^2)E_{34}^2 \\ &+ g_H^2[a^2E_{13}^2 + c^2(E_{13}^2+E_{23}^2) + b^2(E_{13}\sin\gamma + E_{23}\cos\gamma)^2] \\ &+ g_H^2[a^2E_{14}^2 + d^2(E_{14}^2+E_{24}^2) + b^2(E_{14}\sin\gamma + E_{24}\cos\gamma)^2] \ . \end{aligned} \tag{6.18}$$

Thus we find that

$$M_{12}^2 = g_H^2(a^2+b^2) = \tilde{M}_1^2 + \tilde{M}_2^2 \tag{6.19}$$

according to Eq. (4.18), so that indeed M_{12} can naturally be 1000 TeV in our model. Finally using Eq. (4.13) we can reexpress the K_S-K_L mass difference as

$$\frac{\Delta M_K}{M_K f_K^2} = \frac{m_e}{M_E^3} \tag{6.20}$$

to order m_e/m_μ, so we see that in our model it is m_e being small which suppresses the tree-graph contribution to ΔM_K. In Sec. VII we shall study further this interesting connection between the various mass scales.

The second type of rare decay processes that we need to consider are the extended-hypergluon box graphs of Fig. 3. Some simplification comes by noting that since $SO(10)_V \times SO(10)_H$ is an orthogonal group the coupling at each vertex in Fig. 3 is antisymmetric in the fermion indices. Thus after contracting the internal fermion lines the graphs will be symmetric in the external fermion lines. For instance contracting $(\bar{d}D - \bar{D}d)$ with $(\bar{s}D - \bar{D}s)$ yields $(\bar{s}d + \bar{d}s)$. Thus the box diagrams only couple to K_S in the direct channel. Analogous to our discussion of the crossed-channel structure of Fig. 2 we thus see that there are no extended-hypergluon box-diagram contributions to $K_L \to \mu e$ at all, although there are both direct- and crossed-channel contributions to the K_S-K_L mass difference.

To evaluate these contributions explicitly we find it convenient to first classify the graphs according to the eigenstates of $[m_{ij}]$ of Eq. (4.9) since this is the basis in which the extended-hypergluon mass matrix is diagonal, and then only at the end make the QCD-induced rotation to the final quark basis of $[m'_{ij}]$ of Eq. (4.28). Using Eq. (4.8) we find the box graphs yield an effective action of interest,

$$\mathscr{L}_{\text{box}} = \frac{4m_d^2}{M_D^4}(\bar{d}\gamma_5 d)^2 + \frac{4m_s^2}{M_D^4}(\bar{s}\gamma_5 s)^2 + \frac{m_d m_s}{M_D^4}(\bar{d}\gamma_5 s + \bar{s}\gamma_5 d)^2$$
$$+ \frac{4m_d m_e}{M_D^2 M_E^2}(\bar{d}\gamma_5 d)(\bar{e}\gamma_5 e) + \frac{4m_s m_\mu}{M_D^2 M_E^2}(\bar{s}\gamma_5 s)(\bar{\mu}\gamma_5\mu) + \frac{m_d m_\mu}{M_D^2 M_E^2}(\bar{d}\gamma_5 s + \bar{s}\gamma_5 d)(\bar{e}\gamma_5\mu + \bar{\mu}\gamma_5 e) \tag{6.21}$$

with an analogous expression in the charm sector. Changing now to the final d',s' basis of Eq. (6.8) yields

$$\mathscr{L}_{\text{box}} = \left[\frac{(m_d^2 + m_s^2)}{M_D^4}\sin^2 2\alpha + \frac{m_d m_s}{M_D^4}\cos^2 2\alpha\right](\bar{d}'\gamma_5 s' + \bar{s}'\gamma_5 d')^2$$
$$- \frac{2\sin 2\alpha}{M_D^2 M_E^2}(\bar{d}'\gamma_5 s' + \bar{s}'\gamma_5 d')(m_d m_e \bar{e}\gamma_5 e - m_s m_\mu \bar{\mu}\gamma_5\mu)$$
$$+ \frac{m_d m_\mu}{M_D^2 M_E^2}\cos 2\alpha(\bar{d}'\gamma_5 s' + \bar{s}'\gamma_5 d')(\bar{e}\gamma_5\mu + \bar{\mu}\gamma_5 e)\ . \tag{6.22}$$

We note that since the rotation of Eq. (6.8) is real the above interactions still only involve K_S in the direct channel. For the K_S-K_L mass difference Eq. (6.22) yields

$$\frac{\Delta M_K}{M_K f_K^2} = \frac{(m_d^2 + m_s^2)\sin^2 2\alpha + m_d m_s \cos^2 2\alpha}{M_D^4}\ . \tag{6.23}$$

With $m_s = 200$ MeV and $M_D = 1$ TeV the box-graph contribution is then at least an order of magnitude smaller than the experimental value. Further, Eq. (6.22) leads to a muon-number-violating $K_S \to \mu e$ decay. However, the branching ratio is found to be of order 10^{-22} and is thus essentially unobservable. The analogous charm-changing effective interactions are of the form

$$\mathscr{L}_{\text{box}} = \left[\frac{(m_u^2 + m_c^2)}{M_U^4}\sin^2 2\beta + \frac{m_u m_c}{M_U^4}\cos^2 2\beta\right](\bar{u}'\gamma_5 c' + \bar{c}'\gamma_5 u')^2$$
$$- \frac{2\sin 2\beta}{M_U^2 M_E^2}(\bar{u}'\gamma_5 c' + \bar{c}'\gamma_5 u')(m_u m_e \bar{e}\gamma_5 e - m_c m_\mu \bar{\mu}\gamma_5\mu)$$
$$+ \frac{m_c m_\mu}{M_U^2 M_E^2}\cos 2\beta(\bar{u}'\gamma_5 c' + \bar{c}'\gamma_5 u')(\bar{e}\gamma_5\mu + \bar{\mu}\gamma_5 e) \tag{6.24}$$

so that the D_S-D_L mass difference is well below the estimate of Ref. 16. Thus the box diagrams are also sufficiently suppressed.

The third type of decay processes that we need to consider are the pseudo-Goldstone-boson exchanges of

Fig. 5. The Yukawa couplings of Fig. 4 give an effective interaction of interest of the form

$$\mathscr{L}_Y=\frac{P}{F}(m_d\bar{d}\gamma_5 d+m_s\bar{s}\gamma_5 s+m_e\bar{e}\gamma_5 e+m_\mu\bar{\mu}\gamma_5\mu+m_u\bar{u}\gamma_5 u+m_c\bar{c}\gamma_5 c) \tag{6.25}$$

which is flavor diagonal in the $d,s,\ldots$ basis since the hyperfermion condensates are flavor singlets. Thus at this stage there are still no flavor-changing processes associated with Fig. 5. Rotating to the final $d',s',\ldots$ basis of Eqs. (6.8) and (6.9) however yields a flavor-changing interaction of interest,

$$\mathscr{L}_Y=\frac{P}{F}\{(m_s-m_d)\sin\alpha\cos\alpha(\bar{d}'\gamma_5 s'+\bar{s}'\gamma_5 d')+(m_c-m_u)\sin\beta\cos\beta(\bar{u}'\gamma_5 c'+\bar{c}'\gamma_5 u') + m_e\bar{e}\gamma_5 e+m_\mu\bar{\mu}\gamma_5\mu\} \ . \tag{6.26}$$

Thus we obtain

$$\frac{\Delta M_K}{M_K f_K{}^2}=\frac{(m_s-m_d)^2\sin^2\alpha\cos^2\alpha}{F^2M_P{}^2}\ , \qquad \frac{\Delta M_D}{M_D f_D{}^2}=\frac{(m_c-m_u)^2\sin^2\beta\cos^2\beta}{F^2M_P{}^2}\ . \tag{6.27}$$

The K_S-K_L mass difference will thus only be suppressed enough if

$$M_P>\sin2\alpha \tag{6.28}$$

in TeV mass units. Thus even with α as small as θ_C, M_P would have to be at least 500 GeV. However P is a pseudo-Goldstone boson and is thus perhaps as light as a few GeV. Thus the only way to suppress the P exchange enough is to take $\alpha=0$, i.e., to set $k_{ds}=0$. However we have exactly argued in Sec. IV that k_{ds} is in fact zero since $SU(3)_{L+R}$ is left unbroken after QCD breaks the global chiral $SU(4)_L\times SU(4)_R$ symmetry of the strong interactions. Thus the symmetry relation $m_s^0=m_d^0$ completely suppresses pseudo-Goldstone-boson-exchange contributions to the K_S-K_L mass difference. Then with $\alpha=0$ $\mathscr{L}_Y$ leads to no K_S leptonic decays either. Thus the global $SU(3)_{L+R}$ invariance of the strong interaction suppresses all strangeness-changing processes due to Fig. 5 in our model.

Since $SU(4)_{L+R}$ is broken by m_c^0 being unequal to m_u^0, m_d^0, and m_s^0, there will instead be charm-changing processes. Indeed with m_c^0 very large, $k_{uc}{}^{-1}=0$ so that $\beta=\phi$. According to Eq. (4.34) ϕ is θ_C in this limit, and so we take $\beta=\theta_C$. Equation (6.26) then yields

$$\frac{\Delta M_D}{M_D f_D{}^2}=\frac{(m_c-m_u)^2\sin^2\theta_C\cos^2\theta_C}{F^2M_P{}^2}\ . \tag{6.29}$$

Taking $m_c=m_u m_s/m_d\simeq 100$ MeV then requires M_P to be of order at least 20 GeV, which is just our previous estimate of Sec. V. Given the uncertainties in estimating ΔM_D and M_P theoretically the pseudo-Goldstone-boson exchanges do not appear to pose any difficulties at present. Of course experimental improvements in the charm sector (there are currently no bounds on leptonic decay modes) would considerably tighten the current freedom in the model.

The fourth and final class of exchanges that we must consider are the tree-level Pati-Salam-$SU(4)_V$-gauge-boson[10] exchanges. As we noted in Sec. II this $SU(4)_V$ group contains an $SU(3)_C\times(B-L)$ subgroup and six other generators (to be denoted generically as leptoquarks [LQ]) which transform as a $\underline{3}\oplus\underline{3}^*$ under $SU(3)_C$ and hence couple quarks to leptons. Additionally the $SU(4)_V$ Lagrangian also possesses a global $3B+L$ fermion-number invariance. Thus $SU(4)_V$ breaking can potentially lead to proton decay or lepton pair rare decays. With regard first to proton decay we note that in the symmetry limit the [LQ]-exchange diagrams lead only to elastic $\bar{q}+l\rightarrow\bar{q}+l$ processes and not to $\bar{q}+l\rightarrow q+q$. Thus there will be no proton decay unless baryon number is explicitly spontaneously broken at the $SU(4)_V$ level. Since breaking $SU(4)_V$ in the $\underline{10}$ (i.e., as $N_R C N_R$ as in Sec. III) breaks $SU(4)_V$ down to $SU(3)_C$ while only breaking $(B-L)$ and $(3B+L)$ in the right-handed neutrino sector, and since breaking $SU(4)_V$ in the $\underline{15}$ (i.e., as $\bar{N}N$) breaks $SU(4)_V$ down to $SU(3)_C\times(B-L)$ while not breaking the global $3B+L$ symmetry at all, we see that we can break down to the standard model without needing to break baryon number at the $SU(4)_V$ level at all. Hence we can, and will, give masses to the [LQ] gauge bosons which are much less than the 10^{15}-GeV grand unification mass scale.

With regard to flavor-changing processes we note that the $SU(4)_V$ generators are $SO(4)_H$ singlets and thus even in the symmetry limit connect quark-lepton combinations of different generations.

Specifically, in terms of our original $(\underline{4},\underline{2},\underline{1};\underline{1},\underline{2},\underline{2})$ fermion basis f_i we have couplings of the form

$$\mathscr{L}=[\mathrm{LQ}]\sum_i \bar{f}_i(q)f_i(l)\ , \tag{6.30}$$

where $f_i(q)$ denotes a quark in f_i, and $f_i(l)$ a lepton. When the $SO(4)_H$ symmetry is broken the fermions mix. As we noted in Eq. (4.7) the extended-hypercolor-induced mixings are the same for quarks and leptons. Hence in terms of the basis of Eq. (4.13) the interaction of Eq. (6.30) yields flavor-changing vertices of the form

$$\mathscr{L}=[\mathrm{LQ}](\bar{d}e+\bar{s}\mu+\bar{b}\tau+\bar{l}\sigma+\bar{u}\nu_e+\bar{c}\nu_\mu+\bar{t}\nu_\tau+\bar{h}\nu_\sigma)\ . \tag{6.31}$$

Thus in the crossed channel [LQ] exchange leads to both K_S and K_L decays into μe. The current bound of 2×10^{-9} on the $K_L\to\mu e$ branching ratio then requires that the [LQ] masses be at least of order 50 TeV. Thus a mass scale of only 50 TeV suppresses all $SU(4)_V$-induced rare decays, enabling $SU(4)_V$ to be a relatively low-lying symmetry. Also, and most intriguingly, we see that this leptoquark mass scale is again typical of the horizontal mass scales we have just found. Consequently, we can anticipate that all of these gauge bosons acquire their masses from a common origin. As we shall show in detail in Sec. VII, the leptoquark gauge bosons can also get their masses from the $SO(4)_H$ quartets of Higgs fields of Eq. (6.17). In such a situation we find that the [LQ] mass is at most of order of $\tilde{M}_1$. Then we obtain a *lower* bound on the muon-number-violating $K_L\to\mu e$ decay

$$\frac{\Gamma(K_L\to\mu e)}{\Gamma(K_L\to \mathrm{all})}>10^{-18}\ . \tag{6.32}$$

While this number is too small to be practical its significance (and also that of the box-diagram rate for $K_S\to\mu e$ presented earlier) lies in the fact that it is a lower rather than an upper bound on a rare decay. Our ability to bound $K_L\to\mu e$ from below is due to the fact that the extended hypergluon masses are bounded from above by the observed lepton masses. It is amusing to note that the present lower bound on $K_L\to\mu e$ obtained here is of the same order of magnitude as that obtained in an $SU(4)_L\times U(1)$ weak interaction model where the gauge-boson masses were bounded from above by the observed rate for *CP* violation.[17]

Finally, to conclude this section we remark again that a single 1000-TeV mass scale is sufficient to suppress all the rare decay processes which occur in our model, so that these decays do not present us with any problems at all.

VII. THE MASS SCALES OF THE MODEL

Out model possesses a variety of scales and in this section we classify all of them. With regard first to $SO(10)_V$ we recall that, like its SU(5) subgroup, this group is baryon-number violating in the Lagrangian, i.e., in the symmetry limit, through the exchange of the $(\underline{6},\underline{2},\underline{2};\underline{1},\underline{1},\underline{1})$ gauge bosons. Consequently they must acquire masses of the order of 10^{15} GeV to keep proton decay within experimental bounds. This value for the grand unification mass scale also leads to an acceptable renormalization of the Weinberg angle. Hence we first break $SO(10)_V$ by some fundamental Higgs field at 10^{15} GeV. This Higgs breaking is required to leave $SU(4)_V\times SU(2)_L\times SU(2)_R$ unbroken [or possibly $SU(4)_V\times SU(2)_L\times T_R^3$ only since the charged gauge bosons of $SU(2)_R$ play no role at all in the subsequent breaking to the standard-model phenomenology given in Sec. III]. The easiest way to break $SO(10)_V$ down to $SU(4)_V\times SU(2)_L\times SU(2)_R$ is with a single $\underline{54}$ representation of SO(10), as it contains exactly one singlet under the subgroup. Moreover, since

$$\begin{aligned}
&\underline{10}\times\underline{10}=\underline{1}+\underline{45}+\underline{54}\ ,\\
&\underline{16}\times\underline{10}=\underline{16}+\underline{144}\ ,\\
&\underline{16}\times\underline{16}=\underline{10}+\underline{120}+\underline{126}\ ,\\
&\underline{16}\times\underline{16}^*=\underline{1}+\underline{45}+\underline{210}\ ,
\end{aligned} \tag{7.1}$$

we see that breaking in the $(\underline{54},\underline{1})\oplus(\underline{1},\underline{54})$ representation of $SO(10)_V\times SO(10)_H$ will not give a mass to any of the fermions of our model, since they are only allowed to have Majorana masses at the $SO(10)_V\times SO(10)_H$ level. On the $SO(10)_H$ side we note that no gauge boson can get a 10^{15} GeV mass according to our analysis of Sec. IV and thus the discrete symmetry between $SO(10)_V\times SO(10)_H$ is broken spontaneously at the 10^{15}-GeV level. Thus in the primary stage of breaking we break $SO(10)_V\times SO(10)_H$ according to the $(\underline{54},\underline{1})\oplus(\underline{1},\underline{54})$ representation to reduce the symmetry to $SU(4)_V\times SU(2)_L\times SU(2)_R\times SO(10)_H$.

Before we discuss the second stage of breaking at 1000 TeV we take note of some constraints on the model due to asymptotic freedom requirements. The standard renormalization-group analysis gives a lowest-order β function of the form

$$\beta=\frac{-g^3}{12\pi^2}\left[\frac{11C_2}{4}-\frac{T(R)}{2}-\frac{T(R)}{8}\right], \tag{7.2}$$

where C_2 is the contribution of the adjoint, $T(R)/2$ that of each irreducible representation of two-component spinors, and $T(R)/8$ that of each real irreducible representation of scalars. For the adjoint representation of SU(N) the value of C_2 is N, for the fundamental $T(R)$ is $\frac{1}{2}$, and for the symmetric and antisymmetric second-rank tensors $T(R)$ is $(N+2)/2$ and $(N-2)/2$, respectively. We calculate first the value of β for $SO(10)_V \times SO(10)_H$. An easy way to do this is to decompose SO(10) according to SU(5), viz.,

$$\underline{10}=\underline{5}+\underline{5}^*,\quad \underline{45}=\underline{1}+\underline{24}+\underline{10}+\underline{10}^*,$$
$$\underline{16}=\underline{1}+\underline{5}^*+\underline{10},\quad \underline{54}=\underline{15}+\underline{15}^*+\underline{24}, \tag{7.3}$$

to give $C_2(45)=8$, $T(16)=2$, $T(10)=1$, and $T(54)=12$. Thus for the adjoint gauge bosons, the $(\underline{16},\underline{10})$ and the $(\underline{10},\underline{16})$ fermions, and the $(\underline{54},\underline{1})\oplus(\underline{1},\underline{54})$ scalars we obtain for either $SO(10)_V$ or $SO(10)_H$,

$$\beta_{\rm GUT}=\frac{-g^3}{12\pi^2}(22-10-8-\tfrac{3}{2})<0\ . \tag{7.4}$$

Thus as we noted in Ref. 8,[18] our $SO(10)_V \times SO(10)_H$ model is asymptotically free despite its large fermionic content.

We now study the renormalization for the subgroups. For $SO(6)_H$ the same analysis gives [ignoring now the superheavy $(\underline{54},\underline{1})\oplus(\underline{1},\underline{54})$ scalars as they play no role at low energies]

$$\beta_{\rm hyp}=\frac{-g^3}{12\pi^2}(11-18) \tag{7.5}$$

which is positive. The reason for this change of sign is that we have reduced the number of gauge bosons but not the number of fermions compared with Eq. (7.4). Thus the only way to retain asymptotic freedom for the hypercolor group is to freeze out some of the fermionic degrees of freedom. We thus give the $(\underline{10},\underline{16})$ fermions (which have so far played no role in this paper) large masses so that after they condense out only the $(\underline{16},\underline{10})$ fermions survive. These latter fermions alone then yield

$$\beta_{\rm hyp}=\frac{-g^3}{12\pi^2}(11-8) \tag{7.6}$$

which is now nicely asymptotically free.

In order to give the $(\underline{10},\underline{16})$ fermions a mass we note that the product of a pair of $(\underline{10},\underline{16})$ representations contains the following hypercolor singlets:

$$\begin{gathered}(\underline{1},\underline{1},\underline{1};\underline{1},\underline{2},\underline{2})\ ,\ (\underline{1},\underline{1},\underline{1};\underline{1},\underline{2},\underline{2})\ ,\\ (\underline{15},\underline{1},\underline{1};\underline{1},\underline{2},\underline{2})\ ,\ (\underline{1},\underline{3},\underline{1};\underline{1},\underline{2},\underline{2})\ ,\\ (\underline{20},\underline{1},\underline{1};\underline{1},\underline{2},\underline{2})\ ,\ (\underline{1},\underline{1},\underline{3};\underline{1},\underline{2},\underline{2})\ ,\\ (\underline{6},\underline{2},\underline{2};\underline{1},\underline{2},\underline{2})\ ,\ (\underline{1},\underline{3},\underline{3};\underline{1},\underline{2},\underline{2})\ ,\\ (\underline{6},\underline{2},\underline{2};\underline{1},\underline{2},\underline{2})\ .\end{gathered} \tag{7.7}$$

Out of these a set of four is of particular interest, viz.,

$$\begin{gathered}(\underline{1},\underline{1},\underline{1};\underline{1},\underline{2},\underline{2})\ ,\ (\underline{1},\underline{1},\underline{1};\underline{1},\underline{2},\underline{2})\ ,\\ (\underline{15},\underline{1},\underline{1};\underline{1},\underline{2},\underline{2})\ ,\ (\underline{1},\underline{1},\underline{3};\underline{1},\underline{2},\underline{2})\ .\end{gathered} \tag{7.8}$$

This set of four is just sufficient to break $SO(10)_H$ all the way down to $SO(6)_H$ on the horizontal side, while breaking $SU(4)_V \times SU(2)_L \times SU(2)_R$ down to $SU(3)_C \times (B-L) \times SU(2)_L \times T_R^3$ on the vertical side as required in Sec. VI. Further with these same four representations we can also give masses to all of the $(\underline{10},\underline{16})$ fermions. While the fourth representation in Eq. (7.8) gives masses to the charged gauge bosons of $SU(2)_R$ it leaves the neutral sector alone, which, as we already noted, is sufficient for subsequently obtaining the usual Weinberg-Salam phenomenology. It is thus remarkable that the set of representations of Eq. (7.7) contains just the right set of fields with just the right quantum numbers to yield the set of Eq. (6.17), which is precisely the set we used to be able to control all the rare decays with one common scale. Thus with four fields which transform as in Eq. (7.8) we can simultaneously give common 1000-TeV region masses to the 160 $(\underline{10},\underline{16})$ fermions, the 24 extended hypergluons, the 6 $SO(4)_H$ generators, the 2 charged gauge bosons of $SU(2)_R$ and the 6 leptoquark [LQ] gauge bosons of the Pati-Salam $SU(4)_V$ group. While we shall use Higgs fields to explicitly do this it is extremely tempting to speculate that the $(\underline{10},\underline{16})$ fermions undergo dynamical symmetry breaking through some new 1000-TeV dynamics—or perhaps, more economically, through hypercolor exchange itself. [This is not so unreasonable since we have seen that some of the extended hypergluons associated with the third and fourth generations have masses in the 1-to-10-TeV region. Also the group-theory factors associated with hypercolor exchanges are quite different for the $\underline{6}$ and $\underline{4}$ hypercolor representations so that the $(\underline{10},\underline{16})$ condensates could anyway have different scales than the $(\underline{16},\underline{10})$ condensates even while both are bound by the same hypergluons.] We feel that the mechanism for giving the (10,16) fermions their masses needs to be explored in detail in the future, as we appear to have uncovered a new scale at 1000 TeV characteristic of unified theories. Thus the desert is beginning to bloom.

With the (10,16) fermions now frozen out at 1000 TeV the hypercolor group is asymptotically free at lower energies so that the (16,10) fermions can condense and dynamically break $SU(3)_C \times (B-L) \times SU(2)_L \times T_R^3 \times SO(6)_H$ down to $SU(3)_C \times U(1)_{EM} \times SO(6)_H$ as we discussed previously in Sec. III. As we continue down in energy from 1000 TeV there would at first appear to be a possibility that QCD could break its global symmetry before $SO(6)_H$ does. This is nicely avoided on our model however, since we note that the complete (16,10) contribution to the $SU(3)_C$ renormalization is

$$\beta_C = \frac{-g^3}{12\pi^2}\left(\tfrac{33}{4} - 10\right) \tag{7.9}$$

which is positive, not negative. Thus as we continue down below 1000 TeV only β_{hyp} is negative so that $SO(6)_H$ causes dynamical symmetry breaking first in the TeV region. After this happens the hyperfermions freeze out leaving only the four generations of ordinary fermions. Their contribution to QCD alone gives

$$\beta_C = \frac{-g^3}{12\pi^2}\left(\tfrac{33}{4} - 4\right) \tag{7.10}$$

which is now negative again, so that below the hypercolor threshold only the ordinary fermions are operative allowing QCD to finally spontaneously break the strong-interaction chiral symmetry in the GeV region.

Having now identified the various breaking scales we can now calculate the observable low-energy parameters of our model. Unlike the usual $SO(10)_V$ extrapolation from the grand unified mass scale M down to an ordinary mass scale m we note that in our theory we have to go through various dynamical thresholds to get to the low-energy region. At these thresholds highly nonperturbative effects are taking place which we do not know how to calculate. Thus without a detailed theory of dynamical symmetry breaking we are actually unable to do anything. We note that the usual renormalization-group analysis actually provides two different types of information, namely, explicit low-energy values, $g_3(m)$, $g_2(m)$, and $g_1(m)$ for the $SU(3)_C \times SU(2)_L \times U(1)_{WS}$ coupling constants, and second, a low-energy value for $\sin^2\theta_W(m)$. Because we go through all the dynamical thresholds and freeze out fermions at different energies we do not believe that the standard renormalization-group extrapolation for $g_3(m)$, $g_2(m)$, and $g_1(m)$ is reliable in our model and hence we shall make no attempt to evaluate these quantities. However we recall that $\sin^2\theta_W(m)$ only depends on the difference $[g_1(m)^{-2} - g_2(m)^{-2}]$. In this difference all fermionic contributions to the appropriate β functions drop out and so this difference is presumably not sensitive to threshold effects. Consequently the standard renormalization-group analysis should hold in our model for $\sin^2\theta_W(m)$.

In explicitly evaluating the renormalization of the Weinberg angle we note first that the (16,10) and (10,16) fermions contribute in the standard manner to yield $\sin^2\theta_W(M) = \frac{3}{8}$ at the grand unified mass scale. The low-energy value for $\sin^2\theta_W(m)$ depends on which subgroup of $SO(10)_V$ is light. If only $SU(2)_L \times U(1)_{WS}$ is light we obtain the standard relation

$$\sin^2\theta_W(m) = \tfrac{3}{8} - \frac{55e^2(m)}{96\pi^2}\, ln\frac{M}{m} \; . \tag{7.11}$$

Now , it was noted in the third reference cited in Ref. 7 that this same relation is obtained if $SU(2)_L \times T_R^3 \times U(1)_{L+R}$ is light, with the relation

$$\sin^2\theta_W(m) = \tfrac{3}{8} - \frac{11e^2(m)}{48\pi^2}\, ln\frac{M}{m} \tag{7.12}$$

obtaining if the full chiral $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ is light. With the standard low-energy values for $\sin^2\theta_W(m)$ and $e^2(m)$ we find that Eq. (7.11) requires $M \sim 10^{15}$ GeV while Eq. (7.12) leads to a completely unacceptable value for M. Now we recall that in our analysis of the mass scales of our model given above we precisely found that $SU(2)_R$ should not be a light subgroup but rather only T_R^3. Thus that analysis dovetails nicely with the analysis of $\sin^2\theta_W(m)$ leading us to Eq. (7.11) and grand unification in the 10^{15}-GeV region. Finally, we note that this value for M leads to a proton lifetime in the standard 10^{30} yr region. However, because we are unable to extrapolate $g_3(m)$, $g_2(m)$, and $g_1(m)$ back to the grand unified scale we cannot give an exact value for the lifetime, but only an order-of-magnitude estimate. Hence in our model even though $\sin^2\theta_W(m)$ is not sensitive to the details of the dynamical symmetry breaking, the proton lifetime is, and thus remains a little uncertain.

Thus we identify the three main scales of our model, a primary superheavy breaking at 10^{15} GeV and then two sequential low-lying breakings at

1000 TeV and then at 1 TeV. For convenience we display the complete pattern in Fig. 6. We note that there emerges a very attractive approximate low-energy $SU(4)_V \times SU(4)_H$ symmetry which is broken in the 1000-TeV region only and is otherwise good up to the grand unification mass scale. Thus the hypercolor group emerges as an approximate horizontal counterpart of the Pati-Salam color group.

VIII. CONCLUDING REMARKS

With quarks and leptons as the fundamental fermionic entities, the occurrence of repeated fermionic generations poses a serious and difficult problem for the program of grand unification of the strong, weak, and electromagnetic interactions. Attempts in the literature[19] to construct a multigenerational grand unified theory based on a simple group and satisfying some physically reasonable constraints have proved quite unsatisfactory. The problem is made more acute because of a recent demonstration by Tosa and Okubo[20] who show under very general assumptions that SU(5) and SO(10) are almost unique as unitary and orthogonal group candidates for a grand unified theory. But both of these lead to a single generation of the basic fermions. Accommodating more than one generation in such theories is accompanied by a large number of free, adjustable parameters. If simple groups cannot provide the needed framework, the next simplest alternative is a semisimple group structure with a discrete symmetry imposed so as to ensure a single coupling constant. From this point of view we have studied in this paper a grand unified theory based on $SO(10)_V \times SO(10)_H$.

The semisimple group structure is forced upon us for another reason. The problems in theories in which fundamental scalars bring about the spontaneous symmetry breaking have been discussed so often in the literature that it is not necessary to repeat them here. The alternative to the Higgs mechanism is the so-called dynamical symmetry breaking which requires forces that become strong in the TeV region. Since the QCD forces described by SU(3) color symmetry become strong in the GeV region, according to the prevailing ideas on how renormalization-group equations govern the strength of the coupling constant, the required superstrong group must be larger than or equal to SU(4). This requirement along with some others which have become standard rules of the game cannot be met within the framework of a simple group.[21] As we have seen they are met in our semisimple grand unified model, however.

The model discussed in this paper has indeed some very attractive features from a group-theoretic point of view. The "vertical" $SO(10)_V$ describes each family of ordinary fermions and hence incorporates automatically all the good features of the grand unified theory based on $SO(10)_V$.[2] The "horizontal" $SO(10)_H$ contains hypercolor, extended hypercolor, and horizontal interactions. The irreducible representation—an unusual combination of spinor and vector representations, namely $(\underline{16},\underline{10}) \oplus (\underline{10},\underline{16})$—is almost uniquely selected out. The ensuing particle spectrum and quantum numbers allow the existence of all the needed interactions, unlike other hypercolor models where one has to introduce such interactions from the outside. Further, the model satisfies the criteria for renormalizability and asymptotic freedom. It has a nontrivial generation structure of four conventional fermionic families.

The important relation $M_W = M_Z \cos\theta$ can be satisfied to any desired degree of accuracy provided the hypercolor dynamics satisfies certain criteria. These criteria are not unique to our model; they are shared by other theories. The masses of the ordinary quarks and their mixings come out satisfactorily, although much more work is necessary to establish them on a quantitative basis. In this regard we take an unconventional view concerning the mass matrix of the usual fermions in the low-energy region. It appears that in the literature one either neglects or implicitly assumes that strong forces preserve an $SU(N)_{L+R}$ symmetry while breaking the $SU(N)_L \times SU(N)_R$ global chiral flavor symmetry when considering the mass matrix. The entire mass matrix is then the current quark mass matrix generated either through Higgs or an extended hypercolor mechanism. We propose that this is not necessarily the correct procedure. The QCD forces could break part of the vector flavor

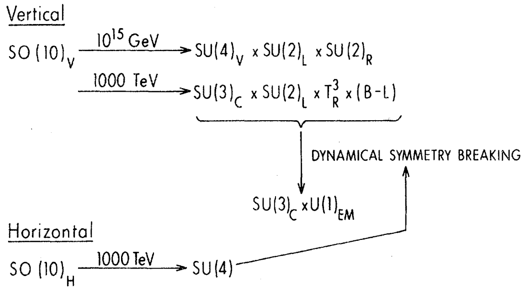

FIG. 6. Symmetry breaking; mass scales.

symmetry as well. On that basis, our results concerning masses and mixing angles are encouraging. Also our analysis of the rare decay modes shows that no serious difficulties are encountered there. This is due to a fortuitous combination of group-theoretic features of the model, which constrain the effective exchanges of single bosons in their couplings to K and D mesons. There are definite predictions concerning the rare decays of the charmed particles. When experimental information concerning these decays becomes available the model can be further tightened.

Perhaps the most unsatisfactory feature of the model is the use of more than one primary Higgs mechanism to cause the initial symmetry breaking. The discrete symmetry which fixes the group structure and the fermionic representation has to be broken right away. Nonetheless, an attractive feature emerges, namely that there is a surviving correspondence between Pati-Salam SU(4) color in the vertical sector and the SU(4) hypercolor in the horizontal sector. In the TeV region the dynamical symmetry breaking takes over with no further need for fundamental scalars. Clearly a great deal of further work is necessary to clarify and establish many features of the model. It does however contain many realistic features and has a rich structure which merits further study.

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APPENDIX

In this paper we have considered in great detail the decomposition $SO(10)_H \to SO(6)\times SO(4)$, identifying the $SO(6)\sim SU(4)$ subgroup as the candidate for the hypercolor group. In this appendix we briefly consider other possible subgroups which could serve as candidates for hypercolor and analyze the hypercolor content of the $(\underline{16},\underline{10}) \oplus (\underline{10},\underline{16})$ and $(\underline{16},\underline{16})$ representations for each case.

(1) $SO(10)_H \to SO(8)\times SO(2)$. Under this decomposition

$$\begin{aligned}\underline{16}&=(\underline{8},\underline{1})+(\underline{8}',\underline{1})\ ,\\ \underline{10}&=(\underline{8},\underline{1})+(\underline{1},\underline{2})\ .\end{aligned}\tag{A1}$$

If we identify SO(8) itself as the hypercolor group, it would imply that the $(\underline{16},\underline{10}) \oplus (\underline{10},\underline{16})$ representation contains two ordinary generations, while the $(\underline{16},\underline{16})$ representation contains no ordinary fermions at all. This choice is thus excluded phenomenologically.

(2) $SO(10)_H \to SO(7)\times SO(3)$. Here

$$\begin{aligned}\underline{16}&=(\underline{8},\underline{2})\ ,\\ \underline{10}&=(\underline{7},\underline{1})+(\underline{1},\underline{3})\ .\end{aligned}\tag{A2}$$

With SO(7) as the hypercolor subgroup, we see that the $(\underline{16},\underline{16})$ representation contains no hypercolor singlets. The $(\underline{16},\underline{10}) \oplus (\underline{10},\underline{16})$ representation contains *three hypercolor-singlet generations* which transform like the vector representation of SU(2). This is then the horizontal flavor-chiral version of the Wilczek and Zee model.[22] The hypercolor β function contribution due to the $(\underline{16};\underline{7},\underline{1})$ fermions is

$$\beta=-\frac{g^3}{12\pi^2}\left(\tfrac{55}{4}-8\right)\ .\tag{A3}$$

[In SO(N), $C_2=N-2$ for the adjoint, and $T(R)=1$ for the vector representation.] The decomposition is thus asymptotically free and therefore provides a viable phenomenological alternative, and is worthy of further study. We did not pursue it here only because the hypercolor group is orthogonal rather than unitary. Though there is no compelling reason to use a unitary hypercolor group, we note that only unitary groups have a nontrivial topological structure, which could provide an origin for quantum number confinement.

(3) $SO(10)_H \to SO(5)\times SO(5)$. Here

$$\begin{aligned}\underline{16}&=(\underline{4},\underline{4})\ ,\\ \underline{10}&=(\underline{5},\underline{1})+(\underline{1},\underline{5})\ .\end{aligned}\tag{A4}$$

With SO(5) as the hypercolor group, we see that the $(\underline{16},\underline{16})$ representation contains no ordinary fermions; the $(\underline{16},\underline{10}) \oplus (\underline{10},\underline{16})$ representation contains five generations. The hypercolor β function due to the $(\underline{16};\underline{5},\underline{1})$ fermions is given by

$$\beta=-\frac{g^3}{12\pi^2}\left(\tfrac{33}{4}-8\right)\ .\tag{A5}$$

Though β is negative, it is so small that it is unlikely to lead to dynamical symmetry breaking in the TeV region.

(4) $SO(10)_H \to SU(4)\times SO(2)$. Here we identify SU(4) as an SO(8) subgroup using the standard connection between SO($2N$) and SU(N) groups. Then

$$\underline{16}=(\underline{4}+\underline{4}^*,\underline{1})+(\underline{1}+\underline{6}+\underline{1},\underline{1})\ ,$$
$$\underline{10}=(\underline{4}+\underline{4}^*,\underline{1})+(\underline{1},\underline{2})\ . \tag{A6}$$

The $(\underline{16},\underline{16})$ representation then provides only two generations of ordinary fermions and hence can be set aside phenomenologically. The $(\underline{16},\underline{10})$ representation also contains only two generations; however, the $(\underline{10},\underline{16})$ representation contains some hypercolor singlets. Thus the representation $(\underline{16},\underline{10})\oplus(\underline{10},\underline{16})$ contains altogether two ordinary [$SO(10)_V$ spinor] and two extraordinary [$SO(10)_V$ vector] fermionic generations. It leads to the topless model of Georgi and Glashow.[23] If this decomposition is to be viable experimentally (to accommodate the b quark), both the $(\underline{16},\underline{10})$ and $(\underline{10},\underline{16})$ representations of the fermions must be light. But then, however, the hypercolor β function is positive [see Eq. (7.5)]. Further, currently available experimental information favors b-quark assignment to an ordinary family. On these grounds one may reject this possibility.

(5) $SO(10)_H \rightarrow SU(3)\times SO(4)$. As in the previous case, here we identify SU(3) through its connection with SO(6). Then

$$\underline{16}=(\underline{3}+\underline{1},\underline{2})+(\underline{3}^*+\underline{1},\underline{2}')\ ,$$
$$\underline{10}=(\underline{3}+\underline{3}^*,\underline{1})+(\underline{1},\underline{4})\ . \tag{A7}$$

Now the $(\underline{16},\underline{16})$ representation contains four generations as in the preferred case discussed in the paper. However, the hypercolor β function due to the $(\underline{16},\underline{16})$ fermions is

$$\beta=-\frac{g^3}{12\pi^2}\left(\tfrac{33}{4}-16\right)\ , \tag{A8}$$

which is not asymptotically free and so must be rejected. The $(\underline{16},\underline{10})$ representation contains four generations, while the $(\underline{10},\underline{16})$ representation this time also contains four generations. With altogether eight generations of light fermions, the hypercolor group is not asymptotically free as previously note in Sec. VII. With only the $(\underline{16},\underline{10})$ fermions light,

$$\beta=-\frac{g^3}{12\pi^2}\left(\tfrac{33}{4}-8\right)\ . \tag{A9}$$

This is again unrealistically small and so must be rejected.

Thus we see that there are only two choices allowed if we take into account the requirements of asymptotic freedom and consider it as established that more than two generations exist. These are case (2) and the case considered in the main body of the paper. The latter is the only case which gives *a unitary hypercolor group.*

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