Fits of the baryon magnetic moments to the quark model and spectrum-generating SU(3)

A. Bohm
Center for Particle Theory, University of Texas, Austin, Texas 78712
and Physikalisches Institut der Universität Würzburg, 8700 Würzburg, Germany

R. B. Teese
Institut für Physik, Johannes-Gutenberg-Universität, 6500 Mainz, Germany
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We show that for theoretical as well as phenomenological reasons the baryon magnetic moments that fulfill simple group transformation properties should be taken in intrinsic rather than nuclear magnetons. A fit of the recent experimental data to the reduced matrix elements of the usual octet electromagnetic current is still not good, and in order to obtain acceptable agreement, one has to add correction terms to the octet current. We have tested two kinds of corrections: U-spin-scalar terms, which are singled out by the model-independent algebraic properties of the hadron electromagnetic current, and octet U-spin vectors, which could come from quark-mass breaking in a nonrelativistic quark model. We find that the U-spin-scalar terms are more important than the U-spin vectors for various levels of demanded theoretical accuracy.

I. INTRODUCTION

Hyperon magnetic moments have been measured recently to such a precision[1–3] that they can be used for testing theoretical ideas that describe fine-structure effects for the hadrons. Such fine-structure effects come from (1) the mass differences in the multiplets and (2) the admixture of other multiplets with the usual octet currents.

The assumption that the magnetic moments in nuclear magnetons behave like a U-spin-scalar SU(3)-octet operator was rather *ad hoc* and the resulting Coleman-Glashow[4] relations were already shown by earlier experiments to be too inaccurate. Symmetry-breaking effects were then called upon in two ways: (1) by taking into consideration quark-mass scale factors[5] and (2) by taking into consideration hadron-mass scale factors.[6] The former, whose predictions are identical to those of the bag model,[7] had to be generalized[8] by taking not only $\xi = m_u/m_s$ but also $\lambda = m_u/m_d$ as a free parameter which is determined by the fit to the experimental values for the hyperon magnetic moments. Such a fit in terms of the quark masses m_u, m_d, and m_s can also be viewed as a fit in terms of the reduced matrix elements of an SU(3) magnetic-moment operator that transforms like the sum of a U-spin-scalar SU(3)-octet operator, a U-spin-vector SU(3)-octet operator, and an SU(3)-scalar operator.[8] Expressed in terms of currents, the quark model therefore shows that the electromagnetic current $V_\mu^{\text{el}}(0)$ contains in addition to the Gell-Mann—Nishijima current

$$V_\mu^{\text{GMN}}(0) = V_\mu^{\pi_0}(0) + \frac{1}{\sqrt{3}} V_\mu^{\eta}(0) \tag{1.1}$$

other SU(3)-tensor operators $V_\mu^{U=1} + V_\mu^S$. This means that quark-mass scale factors for the baryon magnetic moments do not describe symmetry-breaking effects that are expressible by the differences in the hadron masses, but indicate the existence of unusual operators in the electromagnetic current. Fits using these three parameters were acceptable,[8,9] especially if in addition a phenomenological hadron-mass scale factor was admitted.[10] Fits using only the usual octet and an SU(3) scalar had not given satisfactory agreement, even with a hadron-mass scale factor,[6] thus at least one more term was needed. However U-spin vectors in the electromagnetic current are theoretically disfavored, as we explain in Sec. IV. Now the new experimental value[3] for μ_{Ξ^0} shows that they are also phenomenologically disfavored.

We will, in Sec. III, consider in addition to the U-spin-vector SU(3)-octet and SU(3)-scalar operators other U-spin scalars and U-spin vectors.[11,12]

We will see that phenomenologically all these operators are equally important [(5−10)% of the usual octet operator]. We will then for theoretical reasons—explained in Sec. IV—restrict ourselves to U-spin scalars only and give in Sec. VI various fits with different hadron-mass scale factors.

Hadron-mass scale factors are to describe deviations from SU(3) symmetry. They arise naturally in the framework in which SU(3) is considered as a spectrum-generating group.[13] The most prominent problem connected with the hadron-mass scale factors is the question of whether the magnetic moments should be taken in nuclear magnetons or in intrinsic magnetons.[11] We will show in Sec. V that the SU(3) property of the electromagnetic current operator favors the intrinsic magnetons, and that in the spectrum-generating group (SG) approach it follows unambiguously from the assumed transformation property of the electromagnetic current that $m_\alpha \mu_\alpha$ (the magnetic moments in intrinsic magnetons) and not μ_α (the magnetic moments in nuclear magnetons) is given by the group transformation property. This has already been pointed out for the SG approach in Ref. 6. It has recently also been suggested in a different context by Oneda *et al.*[14] and by Lipkin[15] and Tomozawa[8] for the quark model. Although in terms of currents intrinsic magnetons appear as the natural units, for the nonrelativistic quark model nuclear magnetons appear more natural. We will see from fits of Sec. V that phenomenology with the new experimental data definitely favors the choice of intrinsic magnetons.

II. QUARK MODEL

In the additive quark model,[16] the magnetic moment of baryon α is

$$\mu_\alpha = \left\langle \alpha, s_3 = \tfrac{1}{2} \,\middle|\, \sum_q \mu_q \sigma_3^q \,\middle|\, \alpha, s_3 = \tfrac{1}{2} \right\rangle , \qquad (2.1)$$

where $|\alpha, s_3\rangle$ is the SU(6) baryon state vector, σ_3^q is the Pauli matrix for quark q, and μ_q is the quark magnetic moment, which is taken to be

$$\mu_q = g_q e_q / 4 m_q , \qquad (2.2)$$

where g_q, e_q, and m_q are the gyromagnetic ratio, charge, and mass parameter for the quark q. If a fit to the data is made using (2.1) and (2.2), the fit parameters are g_q/m_q. As long as the quark masses are not compared to the masses obtained from other calculations, one cannot determine the gyromagnetic ratios. Nevertheless, it is commonly assumed that quarks are Dirac particles,[5] so we will set $g_q = 2$. The quark magnetic moment can also be written as an operator in terms of λ matrices,[8]

$$\tilde{\mu} = a\lambda_3 + b\lambda_8 + c\lambda_0 . \qquad (2.3)$$

It is clearly the sum of an SU(3) scalar, an octet U-spin scalar, and an octet U-spin vector. The three parameters a, b, and c can be related to the quark masses m_u, m_d, and m_s [cf. Eq. (3.4)].

Although (2.1) appears unambiguous, it is still an open question whether the quark-mass parameters define a unique mass scale, or whether they somehow depend upon the baryon containing them. It has therefore been suggested that the formulas resulting from (2.1) should be multiplied by m_p/m_α, where m_α is the mass of baryon α, in order to take the baryon mass scale into account.[8] That is, one interprets the quark-model predictions as being in intrinsic magnetons rather than nuclear magnetons. On the other hand, the baryon masses are supposed to be derivable from the quark masses and their interactions,[5] so this prescription amounts to assuming a more complicated form for the quark magnetic-moment operator. Rather than obscurely using two mass scales and a simple moment operator, we prefer to use an unambiguous mass scale and investigate the complications to the magnetic-moment operator. This is what we do in the following section.

The formulas (2.1) and (2.2) have been tested in a χ^2 fit to the data with the three parameters m_u, m_d, and m_s.[9] The fits were found to be rather poor. The use of the new Ξ^0 and Ξ^- data only changes the fitted values slightly, but it does increase the χ^2 considerably.

The quark-model predictions can undoubtedly be improved by taking various corrections into account, such as radiative corrections, relativistic effects or configuration mixing. Although a lot of work has been done on this problem, the results have been only modest improvements in the fits.[17]

III. EXPANSION OF THE MAGNETIC MOMENT INTO SU(3) TENSORS

In this section we will assume that the magnetic moment is a sum of terms which have definite transformation properties with respect to SU(3). We will see in the next section that these properties can be related to the properties of the electromagnetic current operator. In the previous section we already explained that the quark model singles out two terms in addition to the usual octet. To check whether the experimental data would also single out a particular set of tensors, we expand the moments in terms of a complete set of SU(3) tensors having $Y=I_3=0$ ($U_3=Q=0$) and well-defined U-spin properties.[11,12] We assume that the baryons correspond to pure SU(3)-octet states, so the magnetic-moment operator can be expanded in terms of the Clebsch-Gordan series for $\underline{8}\times\underline{8}$:

$$\mu=\mu^{\underline{1}}+\mu_1^{\underline{8}}+\mu_2^{\underline{8}}+\mu^{\underline{10}}+\mu^{\overline{\underline{10}}}+\mu^{\underline{27}}\,. \tag{3.1}$$

If μ is to have a definite Hermiticity property, $\mu^{\underline{10}}$ and $\mu^{\overline{\underline{10}}}$ cannot be independent. We therefore only consider the combination $\underline{10}+\overline{\underline{10}}$. The Clebsch-Gordan coefficients $C_U^{\underline{\lambda}}(\alpha',\alpha)$ of these operators are shown in Table I. They are labeled according to the representation $\underline{\lambda}$ (including symmetry, e.g., $\underline{8}F$ or $\underline{8}D$), total U spin U ($U_3=Q=0$) and baryon states α, α'. They have been given a consistent normalization because we want to be able to judge the relative importance of the various parameters.[12] The octet $U=0$ coefficients in Table I agree with those in Ref. 6 up to a multiplicative factor of $-1/\sqrt{6}$. The parameters of the expansion

$$\mu_\alpha^{\text{nuc mag}}=\left[\frac{m_p}{m_\alpha}\right]^\beta \sum_{\underline{\lambda},U} a_U^{\underline{\lambda}} C_U^{\underline{\lambda}}(\alpha,\alpha) \tag{3.2}$$

are the scalar $a_0^{\underline{1}}$, the octet $a_0^{\underline{8}F}$, $a_0^{\underline{8}D}$, $a_1^{\underline{8}F}$, and $a_1^{\underline{8}D}$, the decuplet $a_1^{\underline{10}+}=(1/\sqrt{2})(a_1^{\underline{10}}+a_1^{\overline{\underline{10}}})$ and the 27-plet $a_0^{\underline{27}}$, $a_1^{\underline{27}}$, and $a_2^{\underline{27}}$. The mass factor in (3.2) has been inserted to fix the scale. By definition, μ_α is in nuclear magnetons, so $\beta=0$ if the SU(3) predictions are in nuclear magnetons, and $\beta=1$ if the magnetic moments in intrinsic magnetons are given

TABLE I. Clebsch-Gordan coefficients $C_U^{\underline{\lambda}}(\alpha,\alpha')$.

$\alpha\alpha'$ \ $\underline{\lambda}$	$\underline{1}$	$\underline{8}F$	$\underline{8}D$	$\underline{8}F$	$\underline{8}D$	$\underline{10}+\overline{\underline{10}}$	$\underline{27}$	$\underline{27}$	$\underline{27}$
U	0	0	0	1	1	1	0	1	2
p	1	$-\sqrt{2}$	$\frac{\sqrt{10}}{5}$	$-\left[\frac{2}{3}\right]^{1/2}$	$-\left[\frac{6}{5}\right]^{1/2}$	$\frac{2}{\sqrt{3}}$	$\left[\frac{3}{5}\right]^{1/2}$	$\frac{2}{\sqrt{5}}$	0
n	1	0	$-\frac{2\sqrt{10}}{5}$	$-2\left[\frac{2}{3}\right]^{1/2}$	0	$\frac{-2}{\sqrt{3}}$	$-\frac{1}{\sqrt{15}}$	0	$\frac{2}{\sqrt{3}}$
Λ	1	0	$-\frac{\sqrt{10}}{5}$	0	$-\left[\frac{6}{5}\right]^{1/2}$	0	$-\left[\frac{3}{5}\right]^{1/2}$	$\frac{-3}{\sqrt{5}}$	$-\sqrt{3}$
Σ^+	1	$-\sqrt{2}$	$\frac{\sqrt{10}}{5}$	$\left[\frac{2}{3}\right]^{1/2}$	$\left[\frac{6}{5}\right]^{1/2}$	$-\frac{2}{\sqrt{3}}$	$\left[\frac{3}{5}\right]^{1/2}$	$\frac{-2}{\sqrt{5}}$	0
Σ^0	1	0	$\frac{\sqrt{10}}{5}$	0	$\left[\frac{6}{5}\right]^{1/2}$	0	$\frac{-7}{\sqrt{15}}$	$\frac{3}{\sqrt{5}}$	$-\frac{1}{\sqrt{3}}$
Σ^-	1	$\sqrt{2}$	$\frac{\sqrt{10}}{5}$	$-\left[\frac{2}{3}\right]^{1/2}$	$\left[\frac{6}{5}\right]^{1/2}$	$\frac{2}{\sqrt{3}}$	$\left[\frac{3}{5}\right]^{1/2}$	$-\frac{2}{\sqrt{5}}$	0
Ξ^0	1	0	$-\frac{2\sqrt{10}}{5}$	$2\left[\frac{2}{3}\right]^{1/2}$	0	$\frac{2}{\sqrt{3}}$	$-\frac{1}{\sqrt{15}}$	0	$\frac{2}{\sqrt{3}}$
Ξ^-	1	$\sqrt{2}$	$\frac{\sqrt{10}}{5}$	$\left[\frac{2}{3}\right]^{1/2}$	$-\left[\frac{6}{5}\right]^{1/2}$	$-\frac{2}{\sqrt{3}}$	$\left[\frac{3}{5}\right]^{1/2}$	$\frac{2}{\sqrt{5}}$	0
$\Sigma^0\Lambda$	0	0	$\left[\frac{6}{5}\right]^{1/2}$	0	$-\left[\frac{2}{5}\right]^{1/2}$	0	$\frac{-2}{\sqrt{5}}$	$-\left[\frac{3}{5}\right]^{1/2}$	1

by the group transformation property.

The broken quark model[16] is expressed in terms of formula (3.2) by the conditions $\beta=0$,

$$a_1^{10+}=a_0^{27}=a_1^{27}=a_2^{27}=0 \quad (3.3a)$$

and

$$\frac{a_0^{8D}}{a_0^{8F}}=\frac{a_1^{8D}}{a_1^{8F}}=-\frac{\sqrt{5}}{2}\ . \quad (3.3b)$$

This is a special case of what Dothan calls a linear symmetric model.[12] The three parameters a_0^1, a_0^{8F}, and a_1^{8F} can be related to the quark magnetic moments μ_u, μ_d, and μ_s as follows:

$$\begin{aligned}\mu_u&=a_0^1-\sqrt{2}a_0^{8F}\ ,\\ \mu_d&=a_0^1+\frac{1}{\sqrt{2}}a_0^{8F}-\frac{\sqrt{6}}{2}a_1^{8F}\ ,\\ \mu_s&=a_0^1+\frac{1}{\sqrt{2}}a_0^{8F}+\frac{\sqrt{6}}{2}a_1^{8F}\ .\end{aligned} \quad (3.4)$$

We have inverted Eq. (3.2), using the Clebsch-Gordan coefficients of Table I, the experimental values from Table III, and the assumption

$$\left[\frac{m_{\Sigma^0}}{m_p}\right]^\beta \mu_{\Sigma^0}=\frac{1}{2}\left[\left[\frac{m_{\Sigma^+}}{m_p}\right]^\beta \mu_{\Sigma^+}+\left[\frac{m_{\Sigma^-}}{m_p}\right]^\beta \mu_{\Sigma^-}\right] \quad (3.5)$$

to estimate the value for μ_{Σ^0}. The results are shown in Table II. We see that the octet $U=0$ components dominate, as expected, but among the others no one is dominant. To see if certain coefficients could be forced to vanish by varying the experimental values within reasonable limits, we have also made plots of the coefficients as a function of a given moment. Typical such plots with $\beta=1$ are shown in Fig. 1 for $\mu_{\Sigma^0\Lambda}$ and in Fig. 2 for μ_{Σ^-} because their experimental values have the largest error and are most likely to change. From Fig. 1 we see that we could make the 27-plet terms small by shifting $\mu_{\Sigma^0\Lambda}$ about two standard deviations lower, but that other contributions would still be different from zero. From Fig. 2 we see that a lower absolute value for Σ^- would also be theoretically preferred, a conclusion which we will also reach from our χ^2 fits in Sec. VI. Owing to lack of space, we cannot reprint all of the plots here. It is clear, though, that the experimental values do not select any particular one or two of the additional tensor operators, say e.g., the scalar or $U=0$ 27-plet or the $U=1$ octet. The plots do not show whether any parameters vanish when several experimental values are varied. The χ^2 fits described in Sec. VI are better for this purpose.

TABLE II. The expansion coefficients of Eq. (3.2) obtained from the experimental values of μ_α of Table III. The nuclear-magneton values are obtained with $\beta=0$ and the intrinsic-magneton values with $\beta=1$.

	Nuclear magnetons	Intrinsic magnetons
a_0^1	−0.04	−0.12
a_0^{8F}	−1.29	−1.52
a_0^{8D}	1.32	1.53
a_1^{8F}	0.16	0.12
a_1^{8D}	−0.29	−0.25
a_1^{10+}	0.07	−0.11
a_0^{27}	−0.06	−0.16
a_1^{27}	−0.05	−0.15
a_2^{27}	0.10	0.15

We note also that the values in Table II are in disagreement with a constant D/F ratio [see Eq. (3.3)] which we would expect to hold in a quark model.[8,12]

IV. THE SU(3)-TENSOR PROPERTY OF THE ELECTROMAGNETIC CURRENT

Since the present experimental values do not favor any one or two of the additional SU(3) tensors and the usual $U=0$ octet term is not sufficient to fit the experimental moments, we have to rely

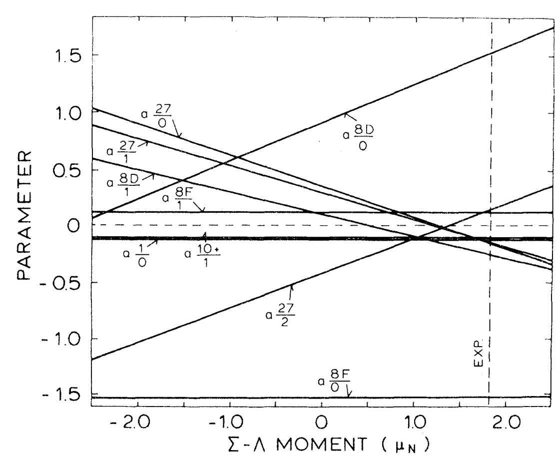

FIG. 1. Plots of the expansion parameters of Eq. (3.2) as functions of the Σ-Λ moment for $\beta=1$. The Σ-Λ moment is in nuclear magnetons (μ_N) and the parameters are in intrinsic magnetons.

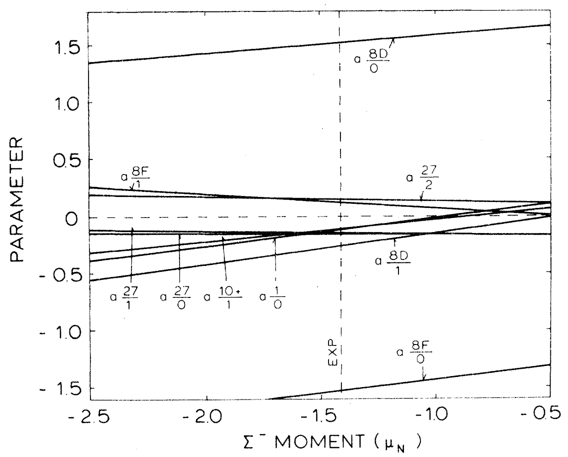

FIG. 2. Plots of the expansion parameters of Eq. (3.2) as functions of the Σ^- moment for $\beta=1$. The Σ^- moment is in nuclear magnetons (μ_N) and the parameters in intrinsic magnetons.

on theoretical arguments to eliminate some of these seven SU(3) tensors. Basing our arguments on the properties of the electromagnetic current operator, we will see that the U-spin scalars are the theoretically preferred SU(3) tensors and that the electromagnetic current is given by

$$V_\mu^{\rm el}(0)=V_\mu^{\rm GMN}(0)+V_\mu^s(0)+V_\mu^{(27)}(0)\ , \qquad (4.1)$$

where $V_\mu^{\rm GMN}$ is given by (1.1), V_μ^s is the SU(3) scalar, and $V_\mu^{(27)}$ is a U-spin-scalar SU(3)-27-plet current operator:

$$V_\mu^{(27)}=\frac{5}{\sqrt{15}}V_\mu^{(27)I=2,I_3=0,Y=0}+V_\mu^{(27)I=0,I_3=0,Y=0}-\frac{1}{\sqrt{3}}V_\mu^{(27)I=0,I_3=0,Y=0}\ . \qquad (4.2)$$

The charges are already given by $V_\mu^{\rm GMN}$; therefore, any term in addition to $V_\mu^{\rm GMN}$ in $V_\mu^{\rm el}$ must be a purely magnetic operator. Thus its electric charge matrix elements (i.e., the diagonal matrix elements between meson states and the form factors multiplying γ_μ for the baryon states) must be zero. A purely magnetic SU(3)-scalar operator V_μ^s of this kind has already been introduced.[18,6] By the same arguments as in Ref. 18 we will now see that also the diagonal matrix elements of $V_\mu^{(27)}$ between the meson states, which contribute to the charges, are zero:

$$\langle M\mid V_\mu^{(27)}\mid M\rangle=0\ . \qquad (4.3)$$

If $V_\mu^{\rm el}$ is to have a definite charge-conjugation property, $U_c^\dagger V_\mu^{\rm el}U_c=-V_\mu^{\rm el}$, $V_\mu^{(27)}$ must also fulfill

$$U_c^\dagger V_\mu^{(27)}U_c=-V_\mu^{(27)}\ . \qquad (4.4)$$

Taking the diagonal matrix elements of this between meson states with charge-conjugation parity C_M, we have

$$\langle M\mid V_\mu^{(27)}\mid M\rangle=-\langle \bar M\mid V_\mu^{(27)}\mid \bar M\rangle \bar C_M C_M\ , \qquad (4.5)$$

where M and $\bar M$ represent the quantum numbers of the meson and its antiparticle, respectively. On the other hand, from the SU(3) transformation property of $V_\mu^{(27)}$ it follows that

$$\langle M\mid V_\mu^{(27)}\mid M\rangle=\langle \bar M\mid V_\mu^{(27)}\mid \bar M\rangle \qquad (4.6)$$

so that (4.3) follows.

Suppose we try to introduce any U-spin-vector operator $V_\mu^{U=1,U_3=0}$. It is easy to see that this could contribute to the meson charges, by considering K^0 and $\bar K^0$. These are not only charge conjugates of each other, but also the $U_3=1$ and $U_3=-1$ members of a U-spin triplet, respectively. It then follows from the Wigner-Eckart theorem and Clebsch-Gordan coefficients for U-spin SU(2) that

$$\langle K^0\mid V_\mu^{U=1,U_3=0}\mid K^0\rangle=-\langle \bar K^0\mid V_\mu^{U=1,U_3=0}\mid \bar K^0\rangle\ , \qquad (4.7)$$

so that Eq. (4.6) does not hold and therefore its charge matrix elements need not be zero. It could also not contribute because of (4.7) to the magnetic transitions of vector mesons.

Of course, we could introduce a U-spin vector anyway, and just force its meson reduced matrix elements to vanish. For an SU(3)-octet U-spin vector, this would mean introducing a second octet of operators. To see this, assume that the U-spin scalar and U-spin-vector SU(3)-octet operators are in the same octet. Then the current operator would be

$$\begin{aligned}V_\mu^{\rm el}&=V_\mu^{\pi^0}+\frac{1}{\sqrt{3}}V_\mu^\eta\\&\quad+a'\left[V_\mu^\eta-\frac{1}{\sqrt{3}}V_\mu^{\pi^0}\right]+V_\mu^{\rm add}\\&=aV_\mu^{\pi^0}+b\frac{1}{\sqrt{3}}V_\mu^\eta+V_\mu^{\rm add}\ ,\end{aligned}$$

where $V_\mu^{\rm add}$ contains the additional SU(3)-tensor operators. One can show that the matrix elements of $a'[V_\mu^\eta-(1/\sqrt{3})V_\mu^{\pi^0}]+V_\mu^{\rm add}$ can in general not be zero if the matrix elements of $V_\mu^{\pi^0}+(1/\sqrt{3})V_\mu^\eta$ are to be proportional to the charges. Thus a U-spin-vector SU(3)-octet operator is only possible if one introduces a second octet of currents, which is an unattractive theoretical feature. (We remark that in the mass-broken quark model[8,16] the magnetic-moment operator contains a U-spin vector, so the corresponding electromagnetic current must contain two different octets of operators. In the quark model this is not surprising, since the quark moments are built up out of the quark-charge operator and the quark-mass operator.)

Summarizing, we have seen that the $U=1$ current operators have unattractive features, so that we exclude them. The $U=2$ currents cannot be excluded by these arguments, but we have made fits including them and found that their contributions are still smaller than those of the $U=0$ 27-plet and scalar operators and that they do not significantly improve the fits. We shall therefore take for the electromagnetic current the SU(3) tensor with $U=0$ given by (4.1).

V. HADRON-MASS SCALE FACTORS FOR μ_α AND THE SG APPROACH

In this section we will discuss the origin of hadron-mass scale factors, in particular those of the form $(m_p/m_\alpha)^\beta$ and of these in particular the one for $\beta=1$, which is the one required when the magnetic moments in intrinsic magnetons are given by the group transformation property. We will justify the factor m_p/m_α in two ways: the first uses the conventional formalism for the electromagnetic current and is, therefore, suggestive but not unambiguous[11]; the second uses the SG approach and is totally unambiguous.

The matrix element of the electromagnetic current operator is usually written as

$$\langle p'\alpha \mid V_\mu^{\text{el}}(0)\mid \alpha p\rangle = \bar{u}_\alpha(p')\left[\tilde{f}_1^\alpha\gamma_\mu+\frac{\tilde{f}_2^\alpha}{2m_\alpha}q^\nu i\sigma_{\mu\nu}\right]u_\alpha(p), \quad (5.1)$$

where α stands for $(II_3Y\cdots)$ and labels the baryon. The magnetic moment of the baryon α is usually written as $(e/2m_p)\mu_\alpha$ where μ_α is the value of the magnetic moment in units of proton magnetons and m_p is the proton mass. Expressed in terms of the conventional form factors the magnetic moment is given in terms of the charge $e\tilde{f}_1^\alpha$ and anomalous magnetic moment $e\tilde{f}_2^\alpha/2m_\alpha$ by

$$\frac{e}{2m_p}\mu_\alpha=\frac{e}{2m_\alpha}\left[\tilde{f}_1^\alpha+\tilde{f}_2^\alpha\right]. \quad (5.2)$$

From the SU(3) transformation property (4.1) of V_μ^{el} it follows by the use of the Wigner-Eckart theorem that

$$\langle p'\alpha' \mid V_\mu^{\text{el}}(0)\mid \alpha p\rangle=\sum_\gamma C(\gamma;\alpha,\alpha')\langle p'\|V_\mu^{(\gamma)}(0)\|p\rangle=\sum_\gamma C(\gamma;\alpha,\alpha')\bar{u}_{\alpha'}\left[\tilde{f}_1^{(\gamma)}\gamma_\mu+\frac{\tilde{f}_2^{(\gamma)}q^\nu}{2m_\alpha}i\sigma_{\mu\nu}\right]u_\alpha, \quad (5.3)$$

where $\langle p'\|V_\mu^{(\gamma)}(0)\|p\rangle$ are the reduced matrix elements and $\tilde{f}_i^{(\gamma)}$ are their form factors. The Clebsch-Gordan (CG) coefficients $C(\gamma;\alpha,\alpha')\equiv C_U^{\lambda}(\alpha,\alpha')$ and the $\tilde{f}_i^{(\gamma)}$ are summed over all γ that occur in V_μ^{el}. We abbreviate $\gamma=(\underline{1},U=0)$ by S, $\gamma=(\underline{8}F,U=0)$ by F, $\gamma=(\underline{8}D,U=0)$ by D, and $\gamma=(\underline{27},U=0)$ by T. Comparison of (5.1) and (5.3) shows that

$$\tilde{f}_i^\alpha=\sum_\gamma C(\gamma;\alpha,\alpha)\tilde{f}_i^{(\gamma)},\quad i=1,2\,, \quad (5.4)$$

so that we obtain for the magnetic moment (5.2)

$$\frac{e}{2m_p}\mu_\alpha=\frac{e}{2m_\alpha}\sum_\gamma C(\gamma;\alpha,\alpha)\left[\tilde{f}_1^{(\gamma)}+\tilde{f}_2^{(\gamma)}\right]. \quad (5.5)$$

Thus $(m_\alpha\mu_\alpha)$ is given by the group transformation property, i.e., expressed in terms of products of CG coefficients and reduced matrix elements. The above arguments, however, do not constitute a derivation of (5.5) because there is no reason to distinguish the form factors $\tilde{f}_i^\alpha$ or $\tilde{f}_i^{(\gamma)}$. Instead of the dimensionless $\tilde{f}_2^\alpha$ one could, e.g., have used $f_2^\alpha=(\tilde{f}_2^\alpha/2m_\alpha)$ and compared the factors of γ_μ and $q^\nu\sigma_{\mu\nu}$ in (5.1) and (5.3) instead of the factors of γ_μ and $\hat{q}^\nu\sigma_{\mu\nu}=(q^\nu/m)\sigma_{\mu\nu}$. This ambiguity is removed in the SG approach, from which it follows indeed that $(m_\alpha\mu_\alpha)$ is given in terms of products of CG coefficients and reduced matrix elements, as we shall show now.

The SG approach[13] uses as one of its basic assumptions the Werle relation

$$[\hat{P}_\mu,\text{SU(3)}_E]=0,\quad \hat{P}_\mu=P_\mu M^{-1}\,. \quad (5.6)$$

Therefore, the four-velocities $\hat{p}_\mu = p_\mu^{(\alpha)}/m_\alpha$ but not the momenta are invariants of the group $SU(3)_E$, which is taken as the group that classifies the hadrons. With this assumption the effects of the mass differences due to the "SU(3)-symmetry breaking" can be precisely taken into account. Therefore, instead of the usual vector (and axial-vector) currents $V_\mu^\beta(0)$ which have the dimension of $(\text{length})^{-3}$ or $(\text{mass})^3$, the SG approach uses the dimensionless transition operators V_μ^β ("generalized currents") which transform like components of $SU(3)_E$-tensor operators.[19] The physical current or physical transition operator J_μ is then constructed from these $SU(3)_E$-tensor operators and the mass operator. An ansatz of this kind for the weak hadronic current has recently been tested for the semileptonic decay of baryons.[20] For the electromagnetic transition operator (and also for the electromagnetic current) this ansatz reads

$$J_\mu^{\text{el}} = V_\mu^{\text{el}} + A_1[M,[M,V_\mu^{\text{el}}]] + A_2[M,[M,[M,[M,V_\mu^{\text{el}}]]]] , \tag{5.7}$$

where — in analogy to (4.1) —

$$V_\mu^{\text{el}} = V_\mu^{\text{GMN}} + V_\mu^s + V_\mu^{(27)} \tag{5.8}$$

is the electromagnetic component of a dimensionless $SU(3)_E$-tensor operator. A_1 and A_2 are SU(3)-invariant phenomenological parameters.[20]

Under assumption (5.6) one considers the matrix elements of the transition operators V_μ^β between generalized velocity eigenvectors $|\,\alpha\hat{p}\rangle$ and obtains from the Wigner-Eckart theorem

$$\langle \hat{p}'\alpha' \,|\, V_\mu^\beta \,|\, \alpha\hat{p}\rangle = \sum_{\gamma\epsilon\beta} c(\gamma;\alpha,\alpha')\langle \hat{p}' \,||\, V_\mu^{(\gamma)} \,||\, \hat{p}\rangle = \sum_{\gamma\epsilon\beta} c(\gamma;\alpha,\alpha')\bar{u}\left[F_1^{(\gamma)}\gamma_\mu + F_2^{(\gamma)}\hat{q}^\nu i\sigma_{\mu\nu}\right]u \ , \tag{5.9}$$

where $\hat{q}^\nu = p'^\nu/m_{\alpha'} - p^\nu/m_\alpha$ is the $SU(3)_E$-invariant velocity transfer. Both sides of the above equations contain only dimensionless quantities; $\langle \hat{p}' \,||\, V_\mu^{(\gamma)} \,||\, \hat{p}\rangle$ are the proper [SU(3)-invariant] reduced matrix elements and $F_1^{(\gamma)}(\hat{q}^2)$, $F_2^{(\gamma)}(\hat{q}^2)$ are the dimensionless form factors which are proper SU(3) invariants in contrast to the form factors $\tilde{f}_i^{(\gamma)}$ in (5.3).

The connection between the SU(3)-invariant form factors $F_i^{(\gamma)}$ and the conventional form factors can be obtained by calculating physical quantities like cross sections or decay rates in terms of the new $F_i^{(\gamma)}$ using the dimensionless velocity basis vectors and dimensionless transition operators, and comparing them with the same quantities in terms of the conventional form factors. The result of such a procedure is given, e.g., in Eq. (18) of Ref. 20. Specialized to our $\tilde{f}^\alpha = f_1^{\alpha,\text{el},\alpha}$ and $\tilde{f}_2^\alpha/2m = f_2^{\alpha,\text{el},\alpha}$ Eq. (18) of Ref. 20 reads [the $\sqrt{2}$ comes from the normalization $F_1^{pn}(0)=1$]

$$\sqrt{2}\tilde{f}_1^\alpha = \sum_{\gamma=F,D,S,T} C(\gamma;\alpha,\alpha)F_1^{(\gamma)} ,$$
$$\sqrt{2}\frac{\tilde{f}_2^\alpha}{2m_\alpha} = \sum_{\gamma=F,D,S,T} C(\gamma;\alpha,\alpha)F_2^{(\gamma)}\frac{1}{m_\alpha} . \tag{5.10}$$

which are valid for

$$J_\mu^{\text{el}} = V_\mu^{\text{el}} = V_\mu^{\text{GMN}} + V_\mu^s + V_\mu^{(27)} \tag{5.11}$$

and also for J_μ^{el} given by (5.7).

Inserting (5.10) into (5.2) we obtain for the magnetic moments

$$\frac{e}{2m_p}\mu_\alpha = \frac{e}{2m_\alpha}\sum_\gamma C(\gamma;\alpha,\alpha)\left[F_1^{(\gamma)} + 2F_2^{(\gamma)}\right]\frac{1}{\sqrt{2}} . \tag{5.12}$$

In contrast to (5.5), which was obtained by suggestive arguments but not by a proper derivation,[11] (5.12) is an unambiguous consequence of the basic assumptions of the SG approach. The $F_i^{(\gamma)}(\hat{q}^2)$, in contrast to the $\tilde{f}_i^{(\gamma)}(q^2)$, are proper [i.e., $SU(3)_E$-invariant] reduced matrix elements which depend upon the $SU(3)_E$-invariant velocity transfer squared. Equation (5.12) says that the magnetic moments in intrinsic magnetons, $m_\alpha\mu_\alpha$, are given by the SU(3) properties. Thus the assumption of a well-defined tensor character for the electromagnetic current $V_\mu^{\text{el}}(0)$ or the electromagnetic transition operator V_μ^{el} leads in the SG approach to a well defined tensor character of $m_\alpha\mu_\alpha$. If one considers the currents as the fundamental quantities, then there is no reason [except for historical reasons from the time when the mass differences were ignored, in which case μ_α and $m_\alpha\mu_\alpha = m\mu_\alpha$ have the same SU(3) property] for the belief that μ_α should have a definite SU(3)-tensor character. This is still the case if the electromagnetic current $J_\mu^{\text{el}}(0)$ (or J_μ^{el}) is given by expressions like (5.7), only the off-diagonal matrix elements, i.e., the transition magnetic moment, will be given by more complicated expressions (see below). It can of course be that the physical transition operator $J^{\rm el}_\mu$ is given by a still more complicated function of $V^{\rm el}_\mu$ and M than (5.7), in which case the $\mu_\alpha \sim (1/m_\alpha)\times$CG coefficient need not hold.

The relations between the usual dimensional currents $J^{\rm el}_\mu(0)$, $V^{\rm el}_\mu(0)$ and the dimensionless transition operators $J^{\rm el}_\mu$, $V^{\rm el}_\mu$ are given by[21]

$$J^{\rm el}_\mu(0) \Longleftrightarrow M^{3/2} J^{\rm el}_\mu M^{3/2}\ ,\qquad V^{\rm el}_\mu(0) \Longleftrightarrow M^{3/2} V^{\rm el}_\mu M^{3/2}\ . \tag{5.13}$$

The relations between the conventional momentum eigenvectors normalized by

$$\langle p' \,|\, p\rangle = \frac{2E}{m}\delta^3(\vec{p}-\vec{p}\,')$$

and the velocity eigenvectors normalized $SU(3)_E$ invariantly,

$$\langle \hat{p}' \,|\, \hat{p}\rangle = 2\frac{E}{m}\delta^3\left[\frac{\vec{p}}{m}-\frac{\vec{p}\,'}{m'}\right],$$

are given by

$$|\,\alpha,p\,\rangle \Longleftrightarrow \frac{1}{m_\alpha{}^{3/2}}\,|\,\alpha,\hat{p}\,\rangle\ . \tag{5.14}$$

Using SU(3) as a spectrum-generating group thus means to work with quantities of dimensionality mass^0 (automodelity principle).[22] Whereas $V^{\rm el}_\mu$ and $|\,\alpha\hat{p}\,\rangle$ are well-defined quantities in the SG approach, the $V^{\rm el}_\mu(0)$ and $|\,\alpha p\,\rangle$ may not be well defined. As $V^{\rm el}_\mu$ have well-defined $SU(3)_E$ transformation properties, $V^{\rm el}(0)$, having dimension (mass)^3, cannot be a simple $SU(3)_E$-tensor operator if M is not taken to be an SU(3) scalar. Using (5.13) and (5.14), (5.10) is immediately obtained by comparing (5.1) and (5.9).

As $\tilde{f}^{\,\alpha}_1$ must be proportional to the electric charge, i.e., proportional to $C(F;\alpha,\alpha)$, one must have

$$F_1^{(D)}=0,\quad F_1^{(S)}=0,\quad F_1^{(T)}=0 \tag{5.15}$$

(V^s_μ and $V^{(27)}_\mu$ are purely magnetic). We therefore write (5.12) as

$$\mu_\alpha = \left[\frac{m_p}{m_\alpha}\right]^\beta [C(F;\alpha,\alpha)f + C(D;\alpha,\alpha)d + s + C(T;\alpha,\alpha)t]\ , \tag{5.16}$$

where $\beta=1$ under the above-described conditions [i.e., if the electromagnetic current is given by an expression like (5.7)] and where f,d,s,t,

$$\sqrt{2}f = F_1^{(F)} + 2F_2^{(F)},\quad d=\sqrt{2}F_2^{(D)}\ ,\qquad s=2F_2^{(S)},\quad t=2F_2^{(T)}\ , \tag{5.17}$$

are the reduced matrix elements to be fitted from the experimental data. In some of the fits β will be treated as a free parameter to check for possible deviations from the form (5.7) for the electromagnetic current operator. $\beta=0$ corresponds to the case—which had erroneously been considered the case of SU(3) symmetry—wherein the magnetic moments in nuclear magnetons are given by the group transformation property.

For the transition magnetic moment $\mu_{\Sigma^0\Lambda}$, off-diagonal matrix elements of the electromagnetic current enter and the mass differences give rise to more complicated expressions. Using Eq. (18) of Ref. 18 and the well-known connection between the transition magnetic moment in nuclear magnetons $\mu^{\rm nuc\,mag}_{\Sigma^0\Lambda}$ and the usual form factors $\tilde{f}_2^{\,\Sigma^0\Lambda}$ (Ref. 23) one obtains

$$\left|\mu^{\rm nuc\,mag}_{\Sigma^0\Lambda}\right| = \phi_{\Sigma^0\Lambda}\frac{(m_{\Sigma^0}+m_\Lambda)^2}{2m_{\Sigma^0}m_\Lambda}\frac{1}{\sqrt{2}}\left[-\frac{1}{\sqrt{5}}d+\frac{2}{\sqrt{3}}\frac{1}{\sqrt{15}}t\right]\frac{2m_p}{m_{\Sigma^0}+m_\Lambda} \tag{5.18}$$

(in the normalization of Ref. 18) where, for the current (5.7), $\phi_{\Sigma^0\Lambda}$ is given by [cf. Eq. (16) of Ref. 20]

$$\phi_{\Sigma^0\Lambda} = 1 - A_1(m_{\Sigma^0}-m_\Lambda)^2 + A_2(m_{\Sigma^0}-m_\Lambda)^4\ . \tag{5.19}$$

$\phi_{\Sigma^0\Lambda}=1$ if the phsical current is given by the $SU(3)_E$-tensor current $J^{\rm el}_\mu(0)=V^{\rm el}_\mu(0)$; $\phi_{\Sigma^0\Lambda}=0.838$ if one uses the parameters A_1 and A_2 obtained from the fit of the semileptonic decay data.[20] Fortunately the experimental errors for $\mu_{\Sigma^0\Lambda}$ are much larger than for the magnetic moments so that our uncertainty about the value for $\phi_{\Sigma^0\Lambda}$ is not crucial for the fit. The second factor on the right-hand side of (5.18) comes from Eq. (18) of Ref. 20 and the last factor on the right-hand side of (5.18) is due to the conversion from $\mu_{\Sigma^0\Lambda}$ (in intrinsic magnetons) to $\mu_{\Sigma^0\Lambda}^{\text{nuc mag}}$ (in proton magnetons).

VI. DISCUSSION OF THE FITS

We have made several fits to the data, as shown in Tables III–V. The data are the same as in Ref. 9, except that $\mu(\Xi^0)$ is the final value[2] and $\mu(\Xi^-)$ a preliminary value[3] from the ongoing Fermilab experiment.

The baryon magnetic moments have now been measured to high accuracy. In particular, the proton and neutron moments have experimental errors that are orders of magnitude smaller than $\alpha/2\pi$, the radiative correction (electrodynamic anomalous magnetic moment) for a charged point particle. Since the baryons are extended particles, the radiative corrections could be much higher, perhaps several percent. If the contributions from higher-order corrections had the same transformation properties as the hadron electromagnetic current itself, then these corrections would not affect a fit to the data. However, it is not clear that this is the case, so we should allow for some minimum uncertainty in the fits.

On the other hand, this is not just a question of radiative corrections. Every theory is only an approximate description, and we need to ask what level of accuracy can be expected of a particular theory. It is almost certain that the proton and neutron moments are measured to much higher accuracy than we could expect to predict with present theories of hadronic structure (strong interaction). In our fits we have therefore included a "theoretical error" $x\mu$ which is a fraction (percentage error $p=100x$) of the measured moment μ. That is, we have made χ^2 fits using the program MINUIT,[24] but instead of the actual experimental errors $\Delta\mu$ we used adjusted errors $\Delta\mu^{\text{adj}}$. We define these quadratically, as suggested by Dothan[12]:

$$\Delta\mu^{\text{adj}}=[(\Delta\mu)^2+(x\mu)^2]^{1/2} . \qquad (6.1)$$

The χ^2 resulting from these fits does not have the usual statistical interpretation (because theoretical corrections are not random variables), but it is useful for comparing fits of different theories made with the same adjusted errors. We use the notation $\chi^2(p\%)$ as a reminder that different χ^2's should only be compared at the same theoretical error $p\%$.

The fits in Table III were made with a theoretical error of 0.04%. This is a little smaller than $\alpha/2\pi$ but large enough to avoid any computational problems (we used a single-precision version of MINUIT). Since this adjustment only affects the proton and neutron errors significantly (they become ± 0.001), these fits can be considered essentially exact. Fits (a) and (b) are to the $U=0$ hypothesis [Eq. (5.16)] for $\beta=0$ (nuclear magnetons) and $\beta=1$ (intrinsic magnetons), respectively. The results $\chi^2(0.04\%)=2000$ for $\beta=0$ and $\chi^2(0.04\%)=76$ for $\beta=1$ show an overwhelming phenomenological preference for $\beta=1$. This is reinforced by fit (c), in which β is a free parameter. The fitted value $\beta=1.26$ is close to unity, and the $\chi^2(0.04\%)=8.6$ shows that it is a fairly good fit. We also notice that the d/f ratio in fit (b) is close to the SU(6) value[25] [eq. (3.3b)]. In fit (d) we have fixed the d/f ratio at the SU(6) value, so the parameters are s, f, and t. The resulting $\chi^2(0.04\%)=79$ is only slightly higher than in fit (b), but is for $n_D=5$ instead of $n_D=4$.

Fits (e) and (f) are quark-model fits for $\beta=0$ and $\beta=1$, made with conditions (3.3 a,b). Assuming that the quarks have Dirac moments, the parameters correspond to quark masses of $m_\mu=338$ MeV, $m_d=322$ MeV, and $m_s=530$ MeV for $\beta=0$, and $m_u=338$ Mev, $m_d=321$ MeV, and $m_s=417$ MeV for $\beta=1$. The results $\chi^2(0.04\%)=175$ and $\chi^2(0.04\%)=87$, respectively, show that these fits are somewhat worse than those for the $U=0$ hypothesis.

The main contribution to χ^2 in the fits (b) and (d) is the value of $\mu(\Xi^0)$ which was recently measured at Fermilab.[2] These fits would be excellent if this experimental value were about 10% larger in magnitude. The value of $\mu(\Xi^0)$ depends only on the $U=0$ property of the electromagnetic current, which predicts (for $\beta=1$)

$$\frac{\mu_{\Xi^0}}{\mu_n}=\frac{m_n}{m_{\Xi^0}} . \qquad (6.2)$$

An explanation of the current value for $\mu(\Xi^0)$ therefore requires either a $U\neq 0$ term in the electromagnetic current or a more complicated symmetry-breaking mechanism than that expressed by an integral value of β, both of which are theoretically unattractive. From fit (e) we also see that the U-spin vector resulting from quark-mass breaking is insufficient for explaining $\mu(\Xi^0)$. We therefore hope that a remeasurement of this moment will yield a value closer to $-1.4\,\mu_N$ (μ_N = nuclear magneton). A similar conclusion was also reached by Glashow.[26]

Table IV contains fits made at the 4% level of

TABLE III. The data with their measured errors, and fits made with 0.04% artificial error. These errors are about ±0.001 for p and n, and the measured errors for the other moments. Fitted values of the free parameters are shown with the errors calculated by MINUIT. Parameters shown without errors were either held constant (β) or calculated from the fixed d/f ratio (d, a_1^{8D}). Equivalent quark masses, where applicable, are given in the text. The data are from Ref. 27, except for Ξ^0 (Ref. 2), Ξ^- (preliminary; see Ref. 3) and $\Sigma\Lambda$ (Ref. 28). Apparent errors of addition are due to rounding off.

	Data	Fit (a)	$\chi^2(0.04\%)$	Fit (b)	$\chi^2(0.04\%)$	Fit (c)	$\chi^2(0.04\%)$	Fit (d)	$\chi^2(0.04\%)$	Fit (e)	$\chi^2(0.04\%)$	Fit (f)	$\chi^2(0.04\%)$
p	2.793	2.793	0.0	2.793	0.0	2.793	0.0	2.793	0.0	2.79	0.0	2.79	0.0
n	−1.913	−1.909	11.6	−1.913	0.2	−1.913	0.0	−1.913	0.2	−1.91	0.0	−1.91	0.0
Λ	−0.614±0.005	−0.615	0.0	−0.614	0.0	−0.614	0.0	−0.615	0.0	−0.59	23.0	−0.63	10.3
Σ^+	2.33 ±0.13	2.79	12.7	2.20	0.9	2.07	3.9	2.20	0.9	2.67	6.7	2.14	2.0
Σ^0		1.97		1.29		1.14		1.29		0.78		0.66	
Σ^-	−1.41 ±0.25	−0.79	6.2	−0.87	4.7	−0.89	4.4	−0.98	3.0	−1.10	1.5	−0.82	5.5
Ξ^0	−1.250±0.014	−1.91	2220.0	−1.37	69.4	−1.25	0.0	−1.37	69.0	−1.40	120.7	−1.15	47.7
Ξ^-	−0.75 ±0.06	−0.79	0.4	−0.78	0.3	−0.78	0.3	−0.89	5.3	−0.46	22.9	−0.48	20.4
$(\Sigma\Lambda)$	$1.82^{+0.25}_{-0.18}$	2.67	16.3	1.99	0.6	1.83	0.0	1.98	0.6	1.94	0.3	1.58	1.3
	total χ^2		2267.2		76.2		8.6		79.4		175.3		87.3

Parameters	Fit (a)	Fit (b)	Fit (c)	Fit (d)	Fit (e)	Fit (f)
$a_0^1=s$	0.194±0.001	0.06±0.02	0.01±0.02	0.019±0.001	0.0965±0.0007	0.043±0.002
$a_0^{8F}=f$	−1.265±0.001	−1.38±0.03	−1.41±0.03	−1.429±0.002	−1.2408±0.004	−1.279±0.001
$a_0^{8D}=d$	1.739±0.001	1.62±0.01	1.58±0.02	1.597	1.3864	1.429
$a_0^{27}=t$	−0.38 ±0.01	−0.31±0.02	−0.29±0.02	−0.335±0.008		
a_1^{8F}					0.1560±0.006	0.092±0.002
a_1^{8D}					−0.1743	−0.103
β	0	1	1.26±0.03	1	0	1

TABLE IV. Fits made with 4% artificial errors. See caption to Table III.

	Fit (g)	$\chi^2(4\%)$	Fit (h)	$\chi^2(4\%)$	Fit (i)	$\chi^2(4\%)$	Fit (j)	$\chi^2(4\%)$	Fit (k)	$\chi^2(4\%)$
p	2.81	0.0	2.61	2.9	2.82	0.1	2.59	3.4	2.89	0.8
n	−1.68	9.2	−1.49	30.0	−1.83	1.1	−1.88	0.1	−1.91	0.0
Λ	−0.71	13.7	−0.61	0.1	−0.61	0.0	−0.61	0.0	−0.66	3.1
Σ^+	2.22	0.5	2.61	3.0	2.23	0.4	2.47	0.7	2.24	0.3
Σ^0	0.66		1.16		1.16		0.68		0.72	
Σ^-	−0.89	4.0	−0.75	6.5	−0.86	4.4	−1.11	1.3	−0.79	5.8
Ξ^0	−1.20	0.9	−1.49	21.5	−1.31	1.3	−1.38	6.7	−1.20	0.8
Ξ^-	−0.80	0.6	−0.75	0.0	−0.78	0.2	−0.49	13.8	−0.52	11.1
$\Sigma\Lambda$	1.41	3.8	1.82	0.0	1.85	0.0	1.84	0.0	1.61	1.0
total χ^2		32.9		64.0		7.5		26.1		22.9
Parameters	Fit (g)		Fit (h)		Fit (i)		Fit (j)		Fit (k)	
s			0.16±0.03		0.07±0.03		0.03±0.04		0.07±0.04	
f	−1.39±0.05		−1.19±0.03		−1.39±0.03		−1.18±0.03		−1.32±0.03	
d	1.33±0.03		1.32		1.55		1.32		1.47	
t			−0.09±0.04		−0.24±0.05					
a_1^{8F}							0.15±0.02		0.07±0.02	
a_1^{8D}							−0.17		−0.08	
β	1		0		1		0		1	

TABLE V. Mixed fits. Fit (l) is equivalent to fit (i) with quark-mass corrections. Fit (m) is equivalent to fit (e) with decuplet corrections (see text).

	Fit (l)	$\chi^2(4\%)$	Fit (m)	$\chi^2(0.04\%)$
p	2.81	0.0	2.79	0.0
n	−1.94	0.1	−1.91	0.0
Λ	−0.61	0.1	−0.61	0.0
Σ^+	2.19	0.8	2.39	0.2
Σ^0	1.06		0.79	
Σ^-	−0.95	3.2	−0.81	5.8
Ξ^0	−1.28	0.4	−1.25	0.0
Ξ^-	−0.73	0.1	−0.69	1.2
$\Sigma\Lambda$	1.86	0.0	1.81	0.0
total χ^2		4.6		7.3
Parameters	Fit (l)		Fit (m)	
s	0.03±0.04		0.088±0.002	
f	−1.38±0.03		−1.180±0.001	
d	1.54		1.318	
t	−0.22±0.05			
a_1^{8F}	0.04±0.03		0.107±0.002	
a_1^{8D}	−0.05		−0.120	
$a^{10}_{-}+$			0.137±0.001	
β	1		0	

theoretical accuracy. Fit (g) tests the usual Gell-Mann—Nishijima form of the electromagnetic current and is equivalent to the Coleman-Glashow relation[4] in intrinsic magnetons. The high value $\chi^2(4\%)=33$ shows that this is not a good theory at the 4% level [the Coleman-Glashow fit in nuclear magnetons, which is not shown in the table, is still worse and gives $\chi^2(4\%)=112$]. Again we see that the d/f ratio is not far from the SU(6) value, and that $\beta=1$ is preferred. Fits (h) and (i) include all $U=0$ contributions, with the SU(6) d/f ratio, for both values of β. The results $\chi^2(4\%)=64$ ($n_D=5$) for $\beta=0$ and $\chi^2(4\%)=7.5$ ($n_D=5$) for $\beta=1$ again show a strong preference for $\beta=1$. In addition, fit (i) with 5 degrees of freedom, is a very good fit at the 4% level. The two quark-model fits, (j) and (k), also with 5 degrees of freedom, have $\chi^2(4\%)=26$ for $\beta=0$ and $\chi^2(4\%)=23$ for $\beta=1$, so they are much poorer fits at this level of accuracy. It is interesting to note that the quark-model fits in nuclear magnetons and in intrinsic magnetons are about equally bad at the 4% level.

These same trends continue as the theoretical errors increase, except that the quark-model fits seem to improve faster for $\beta=0$ than for $\beta=1$. At the 15% level, the quark-model fits (not shown in the tables) give $\chi^2(15\%)=5$ ($n_D=5$) for $\beta=0$ and $\chi^2(15\%)=7$ ($n_D=5$) for $\beta=1$, so $\beta=0$ gives a good fit at the 15% level.

Finally, in Table V we show some "mixed" fits. These do not fit either in the $U=0$ category or the quark-model category, but they are good fits to the present data. Perhaps they will provide clues about how the theories will have to be modified if the data do not change substantially. Fit (l) is equivalent to a quark-model fit *with* $\beta=1$ and 27-plet corrections. Actually, since a_1^{8F} turns out to be small, this could be viewed as a fit to the $U=0$ hypothesis, with small corrections coming from quark-mass breaking. With $\chi^2(4\%)=4.6$, it is a good fit at the 4% level. The parameters correspond to quark masses of $m_u=316$ MeV, $m_d=314$ MeV, and $m_s=349$ MeV. Fit (m) is equivalent to a quark-model fit with $\beta=0$ and decuplet correction. With $\chi^2(0.04\%)=8.4$ for 4 degrees of freedom, it is a good fit to the actual experimental data. The up and down masses corresponding to these parameters are almost exactly equal ($m_u \simeq m_d=355$ MeV), and $m_s=509$ MeV is essentially fixed by $\mu(\Lambda)$.

VII. CONCLUSIONS

Our phenomenological analysis strongly indicates that the magnetic moments which have simple SU(3) transformation properties must be taken in intrinsic magnetons ($\beta=1$) rather than nuclear magnetons ($\beta=0$). This also follows from the theoretical assumptions of the spectrum-generating SU(3) approach.

The largest contribution to the magnetic moments comes from the $U=0$ octet operator, so the Gell-Mann—Nishijima formula for the electromagnetic current [even with d/f fixed by SU(6)] is a good first approximation. If we do not demand more than 20% accuracy from the theory, then this ansatz, in intrinsic magnetons, would be sufficient. To get better accuracy, we need to add more parameters. We have tried two general approaches for doing this. Based on the transformation properties of the phenomenological hadron electromagnetic current, we have included all $U=0$ correction terms. The other approach is to include U-spin-vector contributions, as expected in the nonrelativistic quark model. Our findings and conclusions can best be summarized as follows:

(1) The baryon magnetic moments should be taken in intrinsic rather than nuclear magnetons, for both theoretical and phenomenological reasons.

(2) In intrinsic magnetons they are described roughly by the old SU(3) assumption with the Gell-Mann—Nishijima form for the electromagnetic current.

(3) Even for a demanded "theoretical accuracy" of only 4%, one or two additional terms are needed. Of these, the $U=0$ SU(3)-27-plet-plus-singlet combination gives a better fit than the U-spin-vector SU(3)-octet-plus-singlet that is equivalent to the broken quark model. The corrections are about 1/10 as big as the octet terms.

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[1](a) L. Schachinger *et al.*, Phys. Rev. Lett. 41, 1348 (1978); (b) R. Settles *et al.*, Phys. Rev. D 20, 2154 (1979).

[2]P. T. Cox *et al.*, Phys. Rev. Lett. 46, 877 (1981).

[3]O. E. Overseth, in *Baryon 1980, proceedings of the IVth International Conference on Baryon Resonances, Toronto,* edited by N. Isgur (Univ. of Toronto, Toronto, 1981).

[4]S. Coleman and S. Glashow, Phys. Rev. Lett. 6, 423 (1961).

[5]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[6]A. Bohm, Phys. Rev. D 18, 2547 (1978).

[7]R. H. Hackman, N. G. Deshpande, D. A. Dicus, and V. L. Teplitz, Phys. Rev. D 18, 2537 (1978); T. Barnes, Nucl. Phys. B96, 353 (1975); E. Allen, Phys. Lett. 57B, 263 (1975).

[8]Y. Tomozawa, Phys. Rev. D 19, 1626 (1979).

[9]R. B. Teese and R. Settles, Phys. Lett. 87B, 111 (1979).

[10]The contributions of the *U*-spin vector and SU(3) scalar were approximately 10% of the usual octet contribution [Ref. (8)]. See also F. Scheck, Acta Phys. Austria. Suppl. 18, 629 (1977).

[11]T. P. Cheng and H. Pagels, Phys. Rev. 172, 1635 (1968).

[12]Y. Dothan, Univ. of Minnesota report, 1981 (unpublished).

[13]A. Bohm, Lett. Nuovo Cimento 11, 13 (1974); A. Bohm and J. Werle, Nucl. Phys. B106, 165 (1976); A. Bohm, Phys. Rev. D 13, 2110 (1976). For a review see A. Bohm and R. B. Teese, in *Lecture Notes in Physics 94, Group Theoretical Methods in Physics,* proceedings of the Austin Conference, 1978, edited by W. Beiglböck, A. Bohm, and E. Takasugi (Springer, Berlin, 1979), pp. 301-324.

[14]S. Oneda, T. Tanuma, T. Toiya and M. D. Slaughter, Phys. Rev. D 23, 787 (1981).

[15]H. J. Lipkin, Phys. Lett. 89B, 358 (1980).

[16]J. Franklin, Phys. Rev. 172, 1807 (1968).

[17]J. L. Rosner, *in High Energy Physics—1980,* proceedings of the XXth International Conference, Madison, Wisconsin, edited by L. Durand and L. G. Pondrom (AIP, New York, 1981). M. Böhm, R. Huerta, and A. Zepeda, Phys. Rev. D 25, 223 (1982); N. Isgur and G. Karl, *ibid.* 21, 3175 (1980).

[18]A. Bohm, Phys. Rev. D 17, 3127 (1978).

[19]Such a distinction between dimensionless and dimensional quantities is unnecessary if the mass differences are only taken into account in the phase space.

[20]A. Bohm and P. Kielanowski, University of Texas Report No. DOE-ER 0392-430, 1980 (unpublished).

[21]Note that the matrix elements in Ref. 20 are formulated in terms of the dimensional currents $J_\mu(0)$ and $V_\mu^\beta(0)$ (which are denoted there by J_μ and V_μ^β). Strictly speaking only the quantities on the right-hand side of $\Longleftrightarrow$ are defined in the SG approach and $\Longleftrightarrow$ means that they are equal to the quantities on the left-hand side in an approximate description in which the masses in a multiplet are considered equal, i.e., in the limit when $[M, \mathrm{SU}(3)]=0$.

[22]V. A. Matveev, in Proceedings of the 1973 CERN-JINR School of Physics, Report No. CERN-73-12 (unpublished).

[23]H. Pilkuhn, *Zahlenwerte and Funktionen aus Naturwissenschaft und Technik,* edited by Landolt-Börnstein (Springer, Berlin, 1972), Vol. I-16.

[24]F. James and M. Roos, CERN Internal Report No. DD/75/20, 1975 (unpublished).

[25]M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Lett. 13, 514 (1964).

[26]S. L. Glashow, Harvard Report No. HUTP-80/A089, 1980 (unpublished).

[27]Particle Data Group, Rev. Mod. Phys. 52, S1 (1980).

[28]F. Dydak *et al.*, Nucl. Phys. B118, 1 (1977).