

## Cabibbo theory and hyperon semileptonic decays

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A detailed comparison of the Cabibbo theory and hyperon semileptonic decays is performed. Radiative corrections and the  $q^2$  dependence of the leading vector and axial-vector form factors are considered. Radiative corrections significantly reduce the deviations between theory and experiment and they are the most important out of all the corrections. The overall agreement between theory and experiment is not satisfactory. It is shown that the deviations come mainly from polarization asymmetries in  $\Lambda \rightarrow p e \nu$  and  $\Sigma^- \rightarrow n e \nu$ ; otherwise the agreement with the Cabibbo theory is good. The current situation can be described as a twofold situation. Either the tendency shown by the polarization data is substantially changed or the Cabibbo theory would lose its simplicity.

## INTRODUCTION

The experimental understanding of hyperon semileptonic decays (HSD),

$$A \rightarrow B l \nu,$$

evolves very slowly, much more slowly than is commonly believed. One cannot even tell in some of these decays whether vector ( $V$ ) and axial-vector ( $A$ ) currents are both present; even less can one tell whether scalar or other interactions are absent. For example, until 1975,  $\Sigma^- \rightarrow n e \nu$  decay could have been described with either  $V$  or  $A$  current alone,<sup>1</sup> and today one cannot yet eliminate the possibility of some other interaction form. Nevertheless, progress has been achieved and—what is more important—in the next couple of years substantial improvement will materialize, once current experiments are concluded. The reason behind this is the availability of the so-called hyperon beams in several laboratories.

In contrast, on the theoretical side the situation is quite different. The  $V-A$  theory was introduced<sup>2</sup> 23 years ago and the Cabibbo theory (CT) was formulated<sup>3</sup> almost two decades ago. With these two theories being some of the key ingredients theoretical progress has been steady, leading eventually to gauge theories<sup>4</sup> and opening high expectations for grand unification schemes.<sup>5</sup> There is thus a clear mismatch between the progress in experiment and theory in HSD.

The main consequence of this mismatch is that, although it has been confirmed<sup>6,7</sup> that there is an overall consistency between the CT and HSD, very important detailed questions have not yet been

thoroughly verified. The CT assumes that  $SU_3$ -symmetry breaking (other than mass differences between hyperons) is very small and can be safely neglected in practice. In this respect, the CT—not committing itself to any symmetry-breaking scheme—is intended to be only approximate and eventually deviations from experiment will appear. As a matter of fact, in a recent paper<sup>7</sup> a deviation between the CT prediction for the rate of  $\Sigma^- \rightarrow \Lambda e \nu$  and its experimental value has been interpreted as a symmetry-breaking effect. Also, experiments in  $\Lambda \rightarrow p e \nu$  have reported<sup>8</sup> values for the corresponding axial-vector-to-vector form-factor ratio ( $g_1/f_1$ ) that are either in good agreement or show a not small deviation from the CT prediction. Therefore, the question of how accurate is the CT does not have a definite answer yet.

Establishing agreement or differences between the CT and experimental evidence in HSD requires great care. There are radiative corrections to be accounted for and the momentum transfer<sup>9</sup>  $q \equiv p_2 - p_1$  is not always negligible. In addition, some pieces of experimental data may not have been firmly determined yet. When the emitted charged lepton is a muon one needs to go beyond the CT and use partial conservation of the  $A$  current<sup>10</sup> (PCAC) to predict the induced pseudo-scalar form factor.

Our aim is to provide a thorough careful analysis of the status of the CT in HSD including all the data that have been published up to the present. In a previous Rapid Communication<sup>11</sup> we reported our conclusions. The purpose of this paper is to give all the details and the full discussion that were not included in Ref. 11. We shall not in-

roduce symmetry-breaking effects, except for the hyperon mass differences (see discussions in Secs. IV, V, and IX); but we shall consider all those contributions that are required by the precision of the data. We shall also study in detail the formulation of universality in the CT. In the past the CT has been confronted<sup>6,7</sup> to experimental values for the rates and  $g_1/f_1$  ratios. One must do better than this and use the available angular correlation and asymmetry coefficients. The reason is that the  $g_1/f_1$  ratios are not directly measurable and their experimental values already assume a lot about the required corrections, either by ignoring them (i.e., assuming they are negligible) or by giving them a certain theoretical value. Also, it is desirable that only information from free baryon decays is used.

We shall discuss the experimental evidence on HSD in Sec. I. In order to set a starting point, we shall make a rough comparison of the CT and the data in Sec. II. The radiative corrections will be discussed in Sec. III. The  $q^2$  dependence of the  $V$  and  $A$  form factors will be incorporated in Secs. IV and V, respectively. In Sec. VI, we shall take these three contributions simultaneously. The induced pseudoscalar form factor and the effects of the model dependence in the radiative corrections will be discussed in this section. In Sec. VII, the data is partitioned in two subsets in order to determine which pieces show significant deviations from CT. In Sec. VIII we shall allow for some modification of the Cabibbo universality that may be required by certain unification schemes. Finally, in Sec. IX, we shall discuss our findings and draw our conclusions.

## I. EXPERIMENTAL EVIDENCE ON HSD

The available<sup>1,8,12-15</sup> experimental evidence on HSD consists of transition rates, electron-neutrino angular distributions (or equivalently, energy spectrum of the emitted hyperon), and, in some cases, angular distribution of an emitted  $i$ th particle with respect to the polarization direction of the decaying hyperon. The angular information can be best put into the form of electron-neutrino angular-correlation coefficients and asymmetry coefficients,  $\alpha_{ev}^{AB}$  and  $\alpha_i^{AB}$ , respectively. In addition, it is customary to quote "experimental values" for the  $g_1/f_1$  ratios. We have collected all this information in Table I.

The comparison of the CT with the  $g_1/f_1$  ratios can lead to inconsistencies when the  $q^2$  dependence of these form factors is considered. The problem

TABLE I. Experimental evidence on HSD, from Refs. 1, 8, 12, and 13.  $R_{AB}$  is the transition rate of  $A \rightarrow B l \nu$  when  $l$  is an electron and  $R_{AB\mu}$  is the transition rate when  $l$  is a muon; the units are  $10^6 \text{ sec}^{-1}$ , except for  $R_{np}$  which has units  $10^{-3} \text{ sec}^{-1}$ . The upper indices in the angular coefficients refer to the decay process and the lower indices refer to the type of angular correlation. For example,  $ev$  means angular correlation of electron and neutrino directions,  $e$  alone means angular correlation of the spin of the decaying hyperon and the electron direction, etc.

Observable	Experiment
$R_{np}$	$1.091 \pm 0.017$
$R_{\Sigma^+ \Lambda}$	$0.253 \pm 0.059$
$R_{\Sigma^- \Lambda}$	$0.412 \pm 0.034$
$R_{\Lambda p}$	$3.165 \pm 0.058$
$R_{\Sigma^- n}$	$7.29 \pm 0.28$
$R_{\Xi^- \Lambda}$	$1.71 \pm 0.73$
$R_{\Xi^- (\Lambda, \Sigma^0)}$	$4.14 \pm 1.3$
$R_{\Lambda p \mu}$	$0.60 \pm 0.13$
$R_{\Sigma^- n \mu}$	$3.04 \pm 0.27$
$R_{\Xi^- \Lambda \mu}$	$1.58 \pm 1.58$
$\alpha_{ev}^{np}$	$-0.074 \pm 0.004$
$\alpha_e^{np}$	$-0.084 \pm 0.003$
$\alpha_v^{np}$	$1.001 \pm 0.038$
$\alpha_{ev}^{\Sigma^\pm \Lambda}$	$-0.35 \pm 0.05$
$\alpha_{ev}^{\Sigma^- n}$	$0.28 \pm 0.05$
$\alpha_e^{\Sigma^- n}$	$0.04 \pm 0.27$
$\alpha_e^{\Lambda p}$	$0.125 \pm 0.066$
$\alpha_{ev}^{\Lambda p}$	$-0.01 \pm 0.02$
$\alpha_v^{\Lambda p}$	$0.821 \pm 0.060$
$\alpha_p^{\Lambda p}$	$-0.508 \pm 0.065$
$(g_1/f_1)_{np}$	$1.254 \pm 0.007$
$(g_1/f_1)_{\Lambda p}$	$0.702 \pm 0.026$
$ g_1/f_1 _{\Sigma^- n}$	$0.435 \pm 0.035$
$(f_1/g_1)_{\Sigma^- \Lambda}$	$0.10 \pm 0.22$

is quite subtle. The world averages for  $(g_1/f_1)_{\Lambda p}$  and  $(g_1/f_1)_{\Sigma^- n}$  quoted in Table I were obtained averaging experimental values obtained under different assumptions; in some case no  $q^2$  dependence for  $g_1(q^2)$  and  $f_1(q^2)$  was taken into account, and in other cases a particular choice for their  $q^2$  dependence was made. This should not present any problem as long as the experimental error bars on these ratios are substantially larger than the uncertainty introduced by the  $q^2$  dependence contributions, as is the case in neutron decay. But in  $\Lambda \rightarrow pe\nu$  and  $\Sigma^- \rightarrow ne\nu$  the error bars are comparable and even smaller than such an uncertainty.

This consistency problem can be avoided by using correlation and asymmetry coefficients, since their values do not involve any assumption about the  $q^2$  dependence. A similar discussion applies when the contributions of the induced form factors are considered.

There are nevertheless several points about the angular coefficients that must be mentioned. In some experiments only values for the absolute value of the  $g_1/f_1$  ratios are given and the values of the  $\alpha_{e\nu}$  coefficients are not quoted. The corresponding  $\alpha_{e\nu}$  values can be obtained by substituting the  $|g_1/f_1|$  into the general expressions for the  $\alpha_{e\nu}$  under exactly the same assumptions that were used in obtaining the  $|g_1/f_1|$ . This is the case of  $\alpha_{e\nu}^{\Sigma^- \Lambda}$  and  $\alpha_{e\nu}^{\Sigma^- n}$ . In addition, for  $\alpha_{e\nu}^{\Sigma^- n}$  there are two experimental results available,<sup>12</sup> but they differ too much and we have chosen the result of Refs. 1 and 13 only. Our choice is based on a very recent preliminary result<sup>14</sup> that seems to confirm the measurement of Refs. 1 and 13, although we have not incorporated this preliminary value into Table I. For all angular coefficients experimental numbers are quoted directly.

An objection may be raised to using  $\alpha_e^{\Lambda p}$ ,  $\alpha_\nu^{\Lambda p}$ , and  $\alpha_p^{\Lambda p}$ , since their experimental values are not statistically independent and there is a risk of introducing some bias. We can appreciate the importance of this bias by fitting  $(g_1/f_1)_{\Lambda p}$  to these asymmetries alone and comparing the result to the value of  $(g_1/f_1)_{\Lambda p}$  obtained from the statistically independent combinations  $\alpha_e^{\Lambda p} + \alpha_\nu^{\Lambda p}$ ,  $\alpha_e^{\Lambda p} + \alpha_p^{\Lambda p}$ , and  $\alpha_\nu^{\Lambda p} + \alpha_p^{\Lambda p}$ , using in both cases the same assumptions. The value from our fit is  $0.32 \pm 0.10$  and from Ref. 8 it is  $0.33 \pm_{0.09}^{0.14}$ . There is, indeed, some bias, but it is much smaller than the corre-

sponding error bars. So, for the meantime we can safely use each asymmetry coefficient in  $\Lambda \rightarrow pe\nu$ . Nevertheless, this point should be kept in mind and it is recommended that in future experiments values for the statistically independent combinations should also be given.

As a final point concerning the data, we shall use the available<sup>12</sup> evidence on free neutron decay. Often the vector coupling constant obtained in superallowed nuclear decays<sup>10</sup> is used. This is of course quite all right. But given the recent improvements in the free neutron decay measurements, we find it more attractive to limit ourselves to using data on elementary-particle decays. As a matter of fact, it is most desirable that no information from other fields be used. This way the value of the vector coupling constant coming from particle physics can be compared to the one coming from nuclear physics, thus providing some independent check on the accuracy of the nuclear-structure and isospin-breaking calculations used in studying superallowed decays.

## II. ROUGH COMPARISON OF THE CT AND HSD DATA

In this section we shall make a first confrontation of the CT to the experimental evidence on HSD. This will serve to set a point of reference that will allow us to appreciate, on the one hand, the relevance of several corrections that will be introduced in the following sections and, on the other hand, the precision and consistency of different pieces of data.

The transition amplitude of a HSD is<sup>9</sup>

$$M_0 = G_V \bar{u}_B \{ f_1^{AB}(q^2) \gamma_\mu + f_2^{AB}(q^2) i \sigma_{\mu\nu} q^\nu + f_3^{AB}(q^2) q_\mu + [g_1^{AB}(q^2) \gamma_\mu + g_2^{AB}(q^2) i \sigma_{\mu\nu} q^\nu + g_3^{AB}(q^2) q_\mu] \gamma_5 \} u_A \bar{u}_1 \gamma^\mu (1 - \gamma_5) v_\nu .$$

As we mentioned in the introduction, we shall restrict ourselves to the original assumptions of the CT. Therefore  $f_3^{AB} = g_2^{AB} = 0$ , because second-class currents are presumed to be absent.  $f_1^{AB}(0)$  and  $f_2^{AB}(0)$  will be related to the electromagnetic form factors of the neutron and the proton through the conserved-vector-current (CVC) hypothesis; namely,

$$f_i^{AB}(0) = C_F^{AB} F_i + C_D^{AB} D_i, \quad i = 1, 2 .$$

$C_F^{AB}$  and  $C_D^{AB}$  are Clebsch-Gordan coefficients<sup>9</sup>; they are given in Table II. The reduced form fac-

tors  $F_i$  and  $D_i$  are given by

$$F_1 = \sqrt{6} ,$$

$$D_1 = 0 ,$$

$$F_2 = \left[ \frac{\mu_p}{2} + \frac{\mu_n}{4} \right] \frac{\sqrt{6}}{M_p} ,$$

and

$$D_2 = \frac{\mu_n}{4M_p} \sqrt{30} .$$

$\mu_p$  and  $\mu_n$  are the anomalous magnetic moments of

TABLE II. Clebsch-Gordan coefficients relevant to HSD.

	$C_F^{AB}$	$C_D^{AB}$
$n \rightarrow pev$	$\frac{1}{\sqrt{6}}$	$-\left(\frac{3}{10}\right)^{1/2}$
$\Sigma^\pm \rightarrow \Lambda ev$	0	$-\frac{1}{\sqrt{5}}$
$\Lambda \rightarrow pl\nu$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{5}}$
$\Sigma^- \rightarrow ne\nu$	$-\frac{1}{\sqrt{6}}$	$-\left(\frac{3}{10}\right)^{1/2}$
$\Xi^- \rightarrow \Lambda l\nu$	$\frac{1}{2}$	$\frac{1}{2\sqrt{5}}$
$\Xi^- \rightarrow \Sigma^0 l\nu$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{2}\left(\frac{3}{5}\right)^{1/2}$

the proton and neutron, respectively.<sup>10,12</sup>

The leading  $A$  form factor is given by

$$g_1^{AB}(0) = C_F^{AB}F + C_D^{AB}D,$$

where  $F$  and  $D$  are unknown  $SU_3$  reduced form factors. The pseudoscalar form factor  $g_3^{AB}(0)$  would have a form similar to  $g_1^{AB}$  but in terms of two other reduced form factors, which remain unspecified in the CT. For HSD where the emitted charged lepton is an electron or positron the contribution of  $g_3^{AB}$  is suppressed because the mass of the electron appears as a factor. Therefore, in these electron modes  $g_3^{AB}$  can be ignored. But, from muon modes its contribution may be noticeable.

The formulation of universality of weak decays in the CT is

$$G_V = G_\mu \times \begin{cases} \cos\theta, & \Delta S = 0, \\ \sin\theta, & |\Delta S| = 1, \end{cases}$$

where<sup>10</sup>

$$\begin{aligned} G_\mu &= 1.02678 \times 10^{-5} / M_p^2 \\ &= 1.43581 \times 10^{-49} \text{ erg cm}^3 \end{aligned}$$

is the  $\mu$ -decaying coupling constant after applying radiative corrections to the  $\mu$  lifetime.

A very important assumption in the CT is the hypothesis that, other than mass differences between the hyperons,  $SU_3$ -symmetry breaking is small and can be neglected. This assumption we shall keep throughout this paper. It is something

more than a working hypothesis, because the usefulness of an internal symmetry is intimately related to the extent to which it is approximately exact. When the CT was originally introduced it was assumed also that contributions coming from radiative corrections, the  $q^2$  dependence of the form factors and  $g_3^{AB}$  could be neglected. These approximations are, indeed, working hypotheses and are intended to be valid only as long as the precision of the data allows.

As a starting point<sup>16</sup> we have performed a rough comparison of the CT to the data of Table I ignoring all the corrections mentioned in the last paragraph. The result is displayed in Table III(a). In the last column we give the contribution of each prediction to the total  $\chi^2$ . We include also the predictions for the  $g_1/f_1$  ratios. These ratios have not been fitted, but we find it illustrative to give the  $\Delta\chi^2$  the predictions for them may be contributing.

The free parameters are  $F$ ,  $D$ , and  $\theta$ , i.e., three, and therefore the number of degrees of freedom  $n_D$  is 17. From the  $\chi^2$  point of view the fit is poor, with  $\chi^2/n_D \sim 2.4$ . The large value of  $\chi^2$  is built up essentially by six quantities:  $R_{np}$ ,  $R_{\Sigma^- \Lambda}$ ,  $R_{\Xi^- \Lambda}$ ,  $\alpha_e^{\Sigma^- n}$ ,  $\alpha_e^{\Lambda p}$ , and  $\alpha_v^p$ . The first three, being rates and therefore easier to measure, are worrisome.

It is interesting to remark that if  $\chi^2$  were built up from the contributions of the rates and the  $g_1/f_1$  ratios it would amount to  $\sim 21.54$  and  $\chi^2/n_D \sim 2.15$ . It is clear then that using the angular coefficients instead of the  $g_1/f_1$  ratios not only avoids inconsistencies but provides a more sensitive test.

### III. RADIATIVE CORRECTIONS

The radiative corrections to processes where hadrons are involved are a problem by themselves. The interference of strong interactions makes them model dependent and therefore they cannot be rigorously computed. They are also affected by details of weak interactions, e.g., the intermediate vector boson. Nevertheless, an approach originally introduced<sup>17</sup> by Sirlin to deal with the radiative corrections to the electron energy spectrum in neutron decay can be extended<sup>18</sup> readily to the observables in HSD, within certain reasonable approximations. We can assert that it solves the problem of the experimental analysis, but it does not solve the theoretical problem.

The results of this approach can be briefly stated. The radiative corrections to HSD have three

TABLE III. Comparison of the CT with HSD data. (a) No radiative corrections included, and no  $q^2$  dependence of the leading  $V$  and  $A$  form factors accounted for. The last column gives the contribution to  $\chi^2$  from each prediction. The asterisk on some quantities means that such quantities were not fitted. (b) Effect of radiative corrections. The model-dependent part of radiative corrections has been neglected. There is no contribution from the  $q^2$  dependence of the leading  $V$  and  $A$  form factors. Other conventions are as in (a). (c) Effect of  $V$  slopes. Radiative corrections and the  $q^2$  dependence of the leading  $A$  form factors are not accounted for. Other conventions are as in (a).

	(a)		(b)		(c)	
	Prediction	$\Delta\chi^2$	Prediction	$\Delta\chi^2$	Prediction	$\Delta\chi^2$
$R_{np}$	1.047	6.72	1.081	0.34	1.048	6.45
$R_{\Sigma^+\Lambda}$	0.290	0.39	0.286	0.32	0.281	0.23
$R_{\Sigma^-\Lambda}$	0.480	3.99	0.474	3.38	0.466	2.49
$R_{\Lambda p}$	3.171	0.01	3.178	0.05	3.166	0.00
$R_{\Sigma^-n}$	7.084	0.55	7.083	0.55	7.040	0.80
$R_{\Xi^-\Lambda}$	2.811	2.28	2.787	2.19	2.914	2.73
$R_{\Xi^-(\Lambda,\Sigma^0)}$	3.352	0.35	3.318	0.38	3.439	0.28
$R_{\Lambda p\mu}$	0.509	0.44	0.524	0.30	0.520	0.33
$R_{\Sigma^-n\mu}$	3.142	0.15	3.142	0.15	3.294	0.91
$R_{\Xi^-\Lambda\mu}$	0.761	0.27	0.754	0.27	0.837	0.22
$\alpha_{e\nu}^{np}$	-0.079	1.38	-0.076	0.22	-0.079	1.27
$\alpha_e^{np}$	-0.087	1.49	-0.084	0.00	-0.087	1.30
$\alpha_\nu^{np}$	0.988	0.13	0.988	0.11	0.988	0.12
$\alpha_{e\nu}^{\Sigma^\pm\Lambda}$	-0.400	0.99	-0.400	0.99	-0.400	0.99
$\alpha_{e\nu}^{\Sigma^-n}$	0.340	1.44	0.337	1.28	0.326	0.87
$\alpha_e^{\Sigma^-n}$	-0.687	7.26	-0.691	7.32	-0.614	5.86
$\alpha_e^{\Lambda p}$	0.017	2.68	0.021	2.48	0.014	2.81
$\alpha_{e\nu}^{\Lambda p}$	0.017	1.60	0.024	2.63	0.002	0.35
$\alpha_\nu^{\Lambda p}$	0.977	6.77	0.975	6.61	0.976	6.71
$\alpha_p^{\Lambda p}$	-0.579	1.19	-0.580	1.24	-0.576	1.08
$(g_1/f_1)_{np}$	1.269*	4.82	1.258*	0.25	1.269*	4.32
$(g_1/f_1)_{\Lambda p}$	0.715*	0.24	0.706*	0.02	0.723*	0.64
$(g_1/f_1)_{\Sigma^-n}$	-0.395*	1.33	-0.397*	1.18	-0.369*	3.57
$(f_1/g_1)_{\Sigma^-\Lambda}$	0.000*	0.21	0.000*	0.21	0.000*	0.21
$F_{(a)}=1.071$		$D_{(a)}=-1.519$		$\sin\theta_{(a)}=0.233$		$\chi^2_{(a)}=40.08$
$F_{(b)}=1.054$		$D_{(b)}=-1.510$		$\sin\theta_{(b)}=0.232$		$\chi^2_{(b)}=30.81$
$F_{(c)}=1.102$		$D_{(c)}=-1.495$		$\sin\theta_{(c)}=0.229$		$\chi^2_{(c)}=35.80$

effects.

(i) The leading  $V$  and  $A$  form factors are modified by additive constants containing all the model dependence,

$$f'_1(0) = f_1(0) + \frac{\alpha}{\pi} c \quad (1)$$

and

$$g'_1(0) = g_1(0) + \frac{\alpha}{\pi} d \quad (2)$$

(ii) There is a model-independent correction that affects the rates,

$$R' = R'_0 \left[ 1 + \frac{\alpha}{\pi} \Phi \right].$$

(iii) The angular coefficients are not affected by  $\Phi$ , i.e.,

$$\alpha'_{e\nu} = \alpha'_{0e\nu},$$

$$\alpha'_e = \alpha'_{0e}, \text{ etc.},$$

except for  $\alpha_p^{\Lambda p}$ . But it turns out that for the CT there is a cancellation of the model-independent contribution to  $\alpha_p^{\Lambda p}$ , and one ends with

$$\alpha_p'^{\Lambda p} = \alpha_{0p}'^{\Lambda p} .$$

The subscript zero means that the same general uncorrected expressions for the observables<sup>16</sup> must be used. The prime indicates that the primed form factors, Eqs. (1) and (2), replace the unprimed ones. We omitted the indices  $A$  and  $B$ , but they should be understood. The explicit form of  $\Phi$  can be found in Ref. 18, along with its numerical values.

Although  $c$  and  $d$  are difficult to compute one can obtain some estimates for them that may be reliable. We shall not go into this problem now. Instead we shall proceed in two steps. In this section we shall only incorporate the model-independent radiative correction to the CT, assuming that  $c$  and  $d$  can be ignored. Later, in Sec. VI we shall focus our attention on the possible consequences of nonzero  $c$  and  $d$ .

The effects of the different  $\Phi_i$  on the comparison of the CT to the HSD data can be seen in Table III(b).  $\chi^2$  is reduced by 9.3 compared to its value in Table III(a). The main change comes from  $R_{np}$ . There are several small changes all over and the prediction for  $R_{\Sigma^-\Lambda}$  is somewhat improved. Radiative corrections are certainly helpful.

#### IV. $V$ SLOPES

Only the  $q^2$  dependence of the leading-vector form factor is numerically relevant. It can be parametrized as a linear function of  $q^2$ . The corresponding parameters we shall call the  $V$  slopes, for short. For this parametrization it is important not to put  $f_1^{AB}(0)$  as a common factor, since in the case it is zero then the  $q^2$  dependence vanishes as well. This would happen for  $\Sigma^\pm \rightarrow \Lambda e \nu$ . Therefore, we shall use

$$f_1^{AB}(q^2) = f_1^{AB}(0) + q^2 \lambda_f^{AB} .$$

In terms of  $SU_3$  reduced form factors, we get

$$\begin{aligned} f_1^{AB}(q^2) &= C_F^{AB} F_1(q^2) + C_D^{AB} D_1(q^2) \\ &= C_F^{AB} [F_1(0) + \lambda_{F_1} q^2] + C_D^{AB} [D_1(0) + \lambda_{D_1} q^2] . \end{aligned} \quad (3)$$

There are actually two slope parameters. They can be determined using the experimental information<sup>10</sup> on the  $q^2$  dependence of the electromagnetic form factors of the neutron and the proton and CVC. The result is

$$\lambda_{F_1} = 6.13 \text{ GeV}^{-2} \quad (4)$$

and

$$\lambda_{D_1} = 0.12 \text{ GeV}^{-2} . \quad (5)$$

There is an ambiguity in the above procedure. The experimental numbers from the neutron and proton electromagnetic form factors should include already symmetry-breaking effects. Therefore, in order to use  $SU_3$  symmetry, one should first correct those numbers and extract the symmetry limit values and then perform the rotations to get the symmetric values of  $\lambda_{F_1}$  and  $\lambda_{D_1}$ . But since the contributions of the  $V$  slopes to the observables amount to a few percent, because they are of the order of  $q^2$  contributions, the bias introduced in the observables is very small (around 20% of a few percent). We can take, then, the values of Eqs. (4) and (5) as good estimates for our purposes and leave further refinements for a future more precise experimental situation.

The incorporation of the  $V$  slopes into the fit of Sec. II is given in Table III(c). They do make an improvement, especially in  $R_{\Sigma^-\Lambda}$ , whose  $\Delta\chi^2$  is reduced by 1.5.

#### V. $A$ SLOPES

For the octet  $A$ -current matrix elements, it is again the  $q^2$  dependence of the leading form factor that is numerically relevant. An expansion similar to Eq. (3) can be proposed,

$$\begin{aligned} g_1^{AB}(q^2) &= C_F^{AB} [F(0) + q^2 \lambda_F] \\ &\quad + C_D^{AB} [D(0) + q^2 \lambda_D] , \end{aligned} \quad (6)$$

introducing two slope parameters, which we shall call  $A$  slopes, for short. Unfortunately, there does not exist enough experimental results that can determine the  $A$  slopes. Only the slope of the neutron-decay leading  $A$  form factor has been measured through the reaction  $\nu_\mu + p \rightarrow n + \mu$ .

Using a dipole parametrization,

$$g_1^{np}(q^2) = \frac{g_1^{np}(0)}{(1 - q^2/M_A^2)^2} . \quad (7)$$

The measured slope is the inverse of the square of a certain mass  $M_A$ . The most recent value for  $M_A$  is<sup>10</sup>

$$M_A = 0.96 \pm 0.03 \text{ GeV} .$$

Expanding Eq. (7) to first order in  $q^2$  and comparing to Eq. (6) with the appropriate Clebsch-Gordan coefficients, we get one relation between the  $A$  slopes; namely,

$$\lambda_D = \left[ \frac{\lambda_F}{\sqrt{6}} - \frac{2 \times 1.26}{(0.96)^2} \right] / \sqrt{0.3}. \quad (8)$$

We have put  $g_1(0) = 1.26$ . In deriving this relation, we have assumed, in analogy to the case of the  $V$  slopes, that  $SU_3$ -symmetry-breaking effects are not important enough at this level and the neutron-proton measured  $A$  slope need not be corrected for them. Equation (8) is good enough for our purposes.

We are left with one unknown parameter. The best would be to leave it free and to determine it, along with the CT parameters, from HSD data. Or else, we could try some pole-dominance approach to make an estimate for a fixed value for it. Or, as a third possibility, we can try some dipole approach as indicated by the experimental evidence on  $g_1^{np}(q^2)$ . We shall try these three options.

Pole-dominance gives

$$g_1^{AB}(q^2) = g_1^{AB}(0) \left[ 1 + \frac{q^2}{M_A'^2} \right], \quad (9)$$

to first order in  $q^2$ . For  $\Delta S = 0$  decays,  $M_A' \sim 1.1$  GeV and for  $\Delta S = 1$  decays,  $M_A' \sim 1.3$  GeV. There is some symmetry breaking here, but it is numerically quite irrelevant. Equations (6), (8), and (9) together fix  $\lambda_F$ . For the dipole approach we extend expression (7) to other HSD with the same  $M_A$ ,

$$g_1^{AB}(q^2) = g_1^{AB}(0) (1 + q^2/2M_A^2),$$

keeping the first order in  $q^2$  only.

We have performed three fits to test the effect of  $A$  slopes alone, i.e., excluding the radiative corrections and the  $V$  slopes. In the first one we used  $\lambda_F$  as a free parameter, in the second one we put the  $A$  slopes as given by pole dominance, and in the third one we put the  $A$  slopes as given by a dipole approach. The corresponding  $\chi^2$  were 40.0, 38.6, and 40.1. In none of the three cases is there a significant improvement with respect to the fit of Sec. II, and, so, we shall not reproduce detailed tables for these three fits. Leaving  $\lambda_F$  free gives an unacceptable negative slope for  $\Xi^- \rightarrow \Lambda_e \nu$ . The changes in  $\chi^2$  for pole dominance and dipole leave  $F$ ,  $D$ , and  $\theta$  practically at their values of Sec. II. It is clear that the experimental data on HSD do not seem to require large  $A$  slopes.

## VI. COMBINED EFFECT

Let us now incorporate in the CT the three contributions of Secs. III–V simultaneously. The re-

sults are shown in Table IV. We can see that the  $A$  slopes, which by themselves did not improve the agreement between theory and experiment, now do provide an improvement. This can best be appreciated in  $R_{\Sigma^- \Lambda}$ . In Sec. II, its predicted value exceeds its measured value and the  $A$ -slope contribution to it, being of positive sign, tends to increase this deviation, as can be seen in Table III(c). But when radiative corrections and  $V$  slopes are also incorporated the corresponding  $\Delta\chi^2$  drops from 4 to 2.2 or 2.3. With variable  $\lambda_F$  it drops to 1.8; but this fit should be rejected because again the  $A$  slope for  $\Xi^- \rightarrow \Lambda_e \nu$  comes out negative; for this reason we have not displayed this fit in Table IV. The reason for the improvement is that the three contributions together allow for some extra freedom for the CT parameters. In particular, the value of  $D$  is allowed to decrease. This decrease, apparently very small, is enough to give a net reduction of  $R_{\Sigma^- \Lambda}$ . Comparing Tables III(a) and IV we can find many other improvements all over, except with the polarization data of  $\Lambda \rightarrow p e \nu$  and  $\Sigma^- \rightarrow n e \nu$ .

So far we have left out the  $g_3$  form factors. These should be incorporated to the  $\mu$ -mode decays. We can either leave them free through their corresponding  $F$  and  $D$  reduced form factors or use<sup>10</sup> the partial conservation of the axial-vector current (PCAC) to put them in terms of the  $g_1$ . In this latter approach, we can use

$$g_3(0) = 2M_N g_1(0) \left[ \frac{1}{m_i^2} - \frac{g_1'(0)}{g_1(0)} \right],$$

where  $M_N$  is the nucleon mass,  $m_i$  is the pion or the kaon mass if  $\Delta S = 0$  or  $\Delta S = 1$ , respectively, and  $g_1'(0)$  is the derivative of  $g_1(q^2)$  at  $q^2 = 0$ . It turns out that the effect of  $g_3$ , as determined from PCAC, is almost unnoticeable, as if the  $g_3$  had been left out. And, if the  $g_3$  are left free only very large values—some ten times larger than the PCAC values—make some contributions, which are however still very small. The  $\mu$ -mode data are still far from showing any sensitivity to  $g_3$ . The role of the  $\mu$ -mode rates is then just to check on the  $e$ - $\mu$  universality hypothesis which is adopted into the CT. In our opinion these pieces of data must still be kept along with the rest of the HSD data. They keep playing some role. In what follows we shall simply ignore  $g_3$ .

We can now come back to the radiative corrections. So far we have assumed that their model-dependent parts  $c$  and  $d$  could be ignored. This is not really justified. We shall try to extract them

TABLE IV. Combined effect of radiative corrections,  $V$  slopes, and  $A$  slopes. (a) corresponds to pole-dominance  $A$  slopes, (b) corresponds to dipole  $A$  slopes, and (c) corresponds to fitted model-dependent radiative correction parameter with pole-dominance  $A$  slopes. Other conventions are as in Table III.

	(a)		(b)		(c)	
	Prediction	$\Delta\chi^2$	Prediction	$\Delta\chi^2$	Prediction	$\Delta\chi^2$
$R_{np}$	1.081	0.34	1.079	0.52	1.085	0.13
$R_{\Sigma^+\Lambda}$	0.279	0.19	0.279	0.20	0.279	0.20
$R_{\Sigma^-\Lambda}$	0.462	2.18	0.464	2.34	0.463	2.28
$R_{\Lambda p}$	3.173	0.02	3.174	0.02	3.174	0.03
$R_{\Sigma^-n}$	7.037	0.83	7.024	0.92	7.037	0.83
$R_{\Xi^-\Lambda}$	2.865	2.51	2.810	2.28	2.869	2.53
$R_{\Xi^-(\Lambda,\Sigma^0)}$	3.378	0.33	3.319	0.38	3.38	0.32
$R_{\Lambda p\mu}$	0.539	0.19	0.546	0.15	0.54	0.19
$R_{\Sigma^-n\mu}$	3.301	0.95	3.315	1.06	3.301	0.95
$R_{\Xi^-\Lambda\mu}$	0.823	0.23	0.809	0.24	0.824	0.23
$\alpha_{ev}^{np}$	-0.076	0.12	-0.075	0.04	-0.075	0.07
$\alpha_e^{np}$	-0.083	0.02	-0.083	0.15	-0.083	0.08
$\alpha_v^{np}$	0.989	0.11	0.989	0.10	0.989	0.10
$\alpha_{ev}^{\Sigma^\pm\Lambda}$	-0.404	1.16	0.410	1.46	-0.404	1.16
$\alpha_{ev}^{\Sigma^-n}$	0.306	0.28	0.266	0.08	0.307	0.30
$\alpha_e^{\Sigma^-n}$	-0.624	6.06	-0.640	6.35	-0.623	6.03
$\alpha_e^{\Lambda p}$	0.012	2.92	-0.003	3.75	0.012	2.90
$\alpha_{ev}^{\Lambda p}$	-0.004	0.08	-0.038	1.79	-0.004	0.09
$\alpha_v^{\Lambda p}$	0.975	6.60	0.977	6.76	0.975	6.59
$\alpha_p^{\Lambda p}$	-0.574	1.03	-0.567	0.83	-0.574	1.03
$(g_1/f_1)_{np}$	1.256*	0.05	1.253*	0.02	1.254*	0.00
$(g_1/f_1)_{\Lambda p}$	0.713*	0.19	0.712*	0.16	0.713*	0.17
$(g_1/f_1)_{\Sigma^-n}$	-0.371*	3.36	-0.369*	3.59	0.370*	3.46
$(f_1/g_1)_{\Sigma^-\Lambda}$	0.000	0.21	0.000*	0.21	0.000*	0.21
$F_{(a)}=1.084$	$D_{(a)}=-1.485$		$\sin\theta_{(a)}=0.277$			$\chi^2_{(a)}=26.15$
$F_{(b)}=1.083$	$D_{(b)}=-1.480$		$\sin\theta_{(b)}=0.224$			$\chi^2_{(b)}=29.42$
$F_{(c)}=1.083\pm 0.021$	$D_{(c)}=-1.483\pm 0.019$		$\sin\theta_{(c)}=0.227\pm 0.0025$	$C=0.003\pm 0.009$		$\chi^2_{(c)}=26.04$

from the data. Since their contributions are suppressed by a factor  $\alpha$ , we can assume that the difference between them, which is in any case not expected to be large, can be ignored and we may put  $c \simeq d$ ; i.e., we assume that we have just one model-dependent constant in each decay. In addition, the change in it from one decay to another is also not expected to be large and we may as well neglect those changes. In all, it seems reasonable for our purposes to take one unknown model-dependent constant common to all HSD. The constant can be incorporated into the overall vector coupling constant,

$$G'_V = G_V(1+C),$$

where  $C \equiv (\alpha/\pi)c$ . This constant  $C$  can be treated

as a free parameter together with  $F$ ,  $D$ , and  $\theta$ . In Table IV(c) we produce the results when pole-dominance  $A$  slopes are used. It is interesting to compare it with Table IV(a). The changes are really minor despite the fact that a new parameter was introduced and that it is determined to be nonzero, i.e.,  $C=0.003\pm 0.009$ . In this sense we might say that  $C$  can be measured from HSD. The fit of Table IV(c) amounts to our best fit of the CT and we have, therefore, computed<sup>19</sup> for it the corresponding error bars of the different parameters. If dipole  $A$  slopes are used practically the same results are obtained, so there is no need to reproduce them.

A very careful analysis by Sirlin, using the Weinberg-Salam model and a current-algebra approach, gives<sup>20</sup> in our notation,



$$C = \frac{\alpha}{4\pi} \left[ 3 \ln \left[ \frac{m_z}{M_p} \right] + 6\bar{Q} \ln \left[ \frac{m_z}{M} \right] + 2C' + A\bar{g} \right], \quad (10)$$

where the first term is a universal photonic contribution that arises from the  $V$  current, the second and third terms correspond to photonic corrections induced by the  $A$  current and  $A\bar{g}$  is induced by strong interactions in the asymptotic domain.  $\bar{Q}$  is the average  $u$  and  $d$  quark charges,  $m_z$  is the intermediate-neutral-boson mass,  $M_p$  is the proton mass and  $M$  is a hadronic mass of the order of  $M_{A_1}$ . The estimates for  $C'$  and  $A\bar{g}$  make it plausible that their contributions are negligible, some 30–40 times smaller than other contributions in Eq. (10). In the simplest version of the Weinberg-Salam model with standard  $SU_3^c$ ,  $\bar{Q} = \frac{1}{6}$ . Putting  $m_z = 91.5$  GeV which comes from assuming the weak interaction angle at  $\sin^2\theta_W = 0.23$  and using  $M_p \simeq M \simeq 1$  GeV, we get

$$C \simeq 0.0105 = 1.05\% .$$

The values for  $C$  from the fits stay short of this prediction, but they do have the same sign and thus they are in the right direction.

## VII. PARTITION OF THE DATA

All in all, the agreement between the CT and HSD data is not satisfactory. Looking through Tables III and IV, one remarks that there is a well separated subset of the data that carries most of the weight of the deviations from the theory. This subset is composed mainly of the polarization asymmetries  $\Lambda \rightarrow pev$  and  $\Sigma^- \rightarrow nev$ . None of the corrections introduced in Secs. III–V, nor their combined effect in Sec. VI, gave any substantial change in the predictions of the CT for them. Their contribution to the total  $\chi^2$  has remained almost unchanged, despite the fact that in Table IV  $\chi^2$  was reduced to close to  $\frac{2}{3}$  its value in Table III(a).

We shall now compare the CT, including all the corrections and a variable  $C$  to (i) the transition alone, (ii) the transition rates and the angular-correlation coefficients, excluding the polarization asymmetries, and, for comparison's sake, (iii) the transition and the  $(g_1/f_1)$  ratios. The results are shown in Table V. They are very illustrative. When only rates are used important changes of the CT parameters are allowed and even an unaccept-

ably large value for  $C$  is obtained. When rates and  $g_1/f_1$  ratios are used  $C$  comes out even smaller than in Sec. VI and  $R_{\Sigma^- \Lambda}$  shows once more some deviation. In contrast, when rates and angular-correlation coefficients are used  $C$  comes out very close to half of its gauge-theory prediction and  $R_{\Sigma^- \Lambda}$  remains at as good agreement as before. We have reproduced in Table V the predictions for those quantities that were left out in each one of the fits along with their potential contribution to  $\chi^2$ . We have used pole-dominance  $A$  slopes; except for the  $g_1/f_1$  ratios, for which we did not attempt to make any corrections for  $A$  slopes because of the inconsistencies mentioned in the Introduction. If dipole  $A$  slopes are used, the same pattern is obtained, but with the  $\chi^2$  some two units larger. We shall not reproduce the corresponding results here. Fits with variable  $\lambda_F$  give negative  $A$  slope for  $\Xi^- \rightarrow \Lambda ev$ ; for this reason we do not reproduce them here.

Clearly, the experimental information on the transition rates alone is insufficient, while the information on the rates and angular-correlation coefficients is in quite acceptably good agreement with the CT. Contrastingly, the fit with rates and  $(g_1/f_1)$  ratios is satisfactory, which is misleading because it hides the deviations in the polarization data. It must be remarked that the predictions for the polarization data of  $\Lambda \rightarrow pev$  and  $\Sigma^- \rightarrow nev$  are the same as before. Evidently, the CT parameters and the different corrections we have considered are completely constrained by the experimental values of the transition rates and angular-correlation coefficients. The deviations between the polarization data and theory must be attributed to some other cause.<sup>19</sup>

## VIII. UNIVERSALITY

Before we conclude, we would like to consider one possibility. The form given to universality in the CT may have to be revised in the light of the existence of new quantum numbers. In a model with more than four quarks it is conceivable that the universality of weak interactions is modified, as illustrated by the model of Kobayashi and Maskawa.<sup>21</sup> This, of course, would be a change that should not be attributed to symmetry-breaking effects. Following a notation similar to Ref. 20, we put  $V_{11} = \cos\theta$  and we replace  $\sin\theta$  by another parameter  $V_{12} = \sin\theta \cos\theta_1$ , where  $\theta_1$  is another Cabibbo-type angle independent of  $\theta$ . A modification of the CT universality could be detected by

TABLE V. Partition of HSD data. (a) corresponds to fitting transition rates only, (b) corresponds to fitting transition rates and  $(g_1/f_1)$  ratios, and (c) corresponds to fitting transition rates and angular-correlation coefficients only. In all cases radiative corrections were included and pole-dominance  $A$  slopes were used. Other conventions are as in Table III.

	(a)		(b)		(c)	
	Prediction	$\Delta\chi^2$	Prediction	$\Delta\chi^2$	Prediction	$\Delta\chi^2$
$R_{np}$	1.089	0.02	1.084	0.17	1.085	0.13
$R_{\Sigma^+\Lambda}$	0.261	0.02	0.284	0.28	0.280	0.21
$R_{\Sigma^-\Lambda}$	0.434	0.41	0.471	2.99	0.464	2.33
$R_{\Lambda p}$	3.165	0.00	3.155	0.03	3.171	0.01
$R_{\Sigma^-n}$	7.083	0.55	7.196	0.11	7.061	0.68
$R_{\Xi^-\Lambda}$	3.148	3.89	2.849	2.45	2.872	2.54
$R_{\Xi^-(\Lambda,\Sigma^0)}$	3.630	0.15	3.363	0.34	3.385	0.32
$R_{\Lambda p\mu}$	0.538	0.20	0.536	0.21	0.538	0.19
$R_{\Sigma^-n\mu}$	3.335	1.21	3.372	1.54	3.312	1.04
$R_{\Xi^-\Lambda\mu}$	0.905	0.18	0.819	0.23	0.825	0.23
$\alpha_{e\nu}^{np}$	-0.046*	57.02	-0.075*	0.08	-0.075	0.01
$\alpha_e^{np}$	-0.049*	156.59	-0.083*	0.06	-0.082*	0.28
$\alpha_\nu^{np}$	0.996*	0.02	0.989*	0.11	0.989*	0.10
$\alpha_{e\nu}^{\Sigma^\pm\Lambda}$	-0.404*	1.17	-0.404*	1.16	-0.404	1.16
$\alpha_{e\nu}^{\Sigma^-n}$	0.387*	4.57	0.289*	0.03	0.306	0.28
$\alpha_e^{\Sigma^-n}$	-0.526*	4.40	-0.644*	6.41	-0.624*	6.05
$\alpha_e^{\Lambda p}$	0.038*	1.73	0.015*	2.80	0.013*	2.86
$\alpha_{c\nu}^{\Lambda p}$	0.047*	7.31	0.000*	0.23	-0.002	0.15
$\alpha_\nu^{\Lambda p}$	0.960*	5.37	0.974*	6.51	0.975*	6.56
$\alpha_p^{\Lambda p}$	-0.580*	1.24	-0.575*	1.05	-0.574*	1.04
$(g_1/f_1)_{np}$	1.138*	272.52	1.254	0.00	1.252*	0.12
$(g_1/f_1)_{\Lambda p}$	0.655*	3.22	0.708	0.06	0.711*	0.12
$(g_1/f_1)_{\Sigma^-n}$	-0.311*	12.59	-0.384	2.14	-0.371*	3.38
$(f_1/g_1)_{\Sigma^-\Lambda}$	0.000*	0.21	0.000	0.21	0.000*	0.21
$F_{(a)}=1.014$	$D_{(a)}=-1.323$		$\sin\theta_{(a)}=0.220$	$C_{(a)}=0.0845$	$\chi^2_{(a)}=6.63$	
$F_{(b)}=1.066$	$D_{(b)}=-1.495$		$\sin\theta_{(b)}=0.227$	$C_{(b)}=0.002$	$\chi^2_{(b)}=10.55$	
$F_{(c)}=1.079\pm 0.023$	$D_{(c)}=-1.481\pm 0.025$		$\sin\theta_{(c)}=0.227\pm 0.0026$	$C_{(c)}=0.004\pm 0.013$	$\chi^2_{(c)}=9.28$	

handling  $V_{11}$  and  $V_{12}$  as two independent parameters.

There is no need to reproduce the predictions for all the observables because there is no apparent change. Thus, we have listed in Table VI only the values obtained for  $V_{11}$ ,  $V_{12}$ ,  $F$ , and  $D$  for fits where the data are subdivided into transition rates and angular-correlation coefficients only and when all the data are used. We have performed fits under different assumptions as explained in the caption to Table VI. To see an effect attributable to the presence of  $\cos\theta_1$  it is required that  $V_{11}^2 + V_{12}^2 < 1$ . Going through Table VI it can be seen that when  $C=0$  there is no such effect even after the incorporation of the  $V$  and  $A$  slopes.

(Variable  $\lambda_F$  still gives negative  $A$  slope for  $\Xi^- \rightarrow \Lambda e \nu$ .) Only when  $C=1\%$  and when pole-dominance  $A$  slopes are used an effect in the correct direction appears, fits (G) and (H). But when dipole  $A$  slopes are used the situation is less clear, fits (I) and (J); in fit (I) the effect disappears. Apparently, if only the subset of data that better agrees with the CT is used one cannot see any significant deviation from the Cabibbo universality. One might say that in order to see, in such a sector, the effect of  $\cos\theta_1$ , further symmetry-breaking must be introduced. The deviation from Cabibbo universality seen when all the data are used cannot be taken too seriously, because the polarization data in  $\Lambda \rightarrow p e \nu$  and  $\Sigma^- \rightarrow n e \nu$ , which have been

TABLE VI. Fitted values of  $V_{11}$  and  $V_{12}$ . (A) and (B) correspond to using rates and angular-correlation coefficients only and all of the data, respectively, assuming no  $V$  or  $A$  slopes present and  $C=0$ . In all the following fits CVC  $V$  slopes are incorporated. In (C) and (D) the data are partitioned as in (A) and (B), respectively, assuming  $C=0$  and pole-dominance  $A$  slopes. (E) and (F) are as (C) and (D), but with dipole  $A$  slopes. The last four fits assume  $C=1\%$ . Otherwise, (G) and (H) are as (C) and (D) with pole-dominance  $A$  slopes; and (I) and (J) are as (E) and (F) with dipole  $A$  slopes, respectively. The quantities in parentheses are the percentage change from 1 in  $V_{11}^2+V_{12}^2$ .

	$F$	$D$	$V_{11}$	$V_{12}$	$V_{11}^2+V_{12}^2$	$\chi^2$
(A)	1.054	-1.515	0.972	0.232	0.999 (-0.14%)	12.99
(B)	1.054	-1.509	0.974	0.232	1.002 (+0.25%)	30.78
(C)	1.079	-1.481	0.978	0.228	1.008 (+0.85%)	9.27
(D)	1.083	-1.483	0.977	0.227	1.006 (+0.60%)	26.06
(E)	1.073	-1.466	0.985	0.226	1.021 (+2.1%)	10.71
(F)	1.082	-1.477	0.979	0.225	1.009 (+0.91%)	29.23
(G)	1.079	-1.481	0.968	0.225	0.988 (-1.23%)	9.27
(H)	1.083	-1.483	0.967	0.225	0.986 (-1.43%)	26.06
(I)	1.073	-1.466	0.975	0.223	1.000 (0.04%)	10.71
(J)	1.082	-1.477	0.969	0.222	0.988 (-1.18%)	29.23

showing deviations from the CT all along, were not better fit and remain just as before.

Nevertheless, there is room enough for the effect of  $\cos\theta_1$  to be present. This is only due to the current precision of the data. The error bars<sup>19</sup> on  $V_{11}$  and  $V_{12}$  are 1.3% and 1.4%, respectively, if only rates and correlation coefficients are used, and 1% and 1.4% if all the data are used. Therefore we cannot conclude that  $\cos\theta_1$  is excluded.<sup>22</sup>

To close this section, let us remark that in HSD only the combinations  $G_V'\cos\theta$  and  $G_V'\sin\theta\cos\theta_1$  can be determined experimentally. Hence, the appearance of  $\cos\theta_1$  is intimately related to the value of  $C$ . So, within HSD the existence of  $\cos\theta_1$  can only be solved theoretically by requiring a very reliable estimation of  $C$ .

## IX. DISCUSSION AND CONCLUSIONS

We have confronted the CT to HSD data allowing for several corrections, but assuming that symmetry-breaking effects—other than hyperon mass differences—are absent. This last statement must be clarified. The precision of the data requires that radiative corrections and  $V$  and  $A$  slopes (see Secs. III and IV) be accounted for. Strictly speaking, we are allowing for some symmetry breaking through the weak-magnetism form factor and the  $V$  slopes, when their corresponding parameters are fixed in terms of measured electromagnetic form factors of the neutron and proton, because the relevant experimental numbers

should already contain symmetry-breaking effects. The same applies to the  $A$  slopes. Nevertheless, the bulk of these symmetry-breaking effects is much suppressed. What we really mean is that  $f_1(0)$  and  $g_1(0)$  are taken at their symmetric values and, also, that no pseudotensor form factor  $g_2$  is induced.

There is one important lesson to be learned. All three corrections must be incorporated simultaneously before any conclusion is drawn. The  $A$  slopes seem, by themselves, to be a correction that would introduce some divergence between theory and experiment. The prediction for the rate  $R_{\Sigma-\Lambda}$  exceeds the experimental value and the  $A$  slope increases the predicted value even more. But when the three corrections are considered simultaneously the net effect is that the prediction for  $R_{\Sigma-\Lambda}$  is decreased and better agreement with experiment is obtained. Another important point is that, once radiative corrections are taken into account, the neutron decay rate and angular coefficients can be used instead of the vector coupling constant measured in superallowed nuclear  $\beta$  decays. This allows for a meaningful comparison of the CT with elementary-particle data alone, relaxing the necessity of using information from other fields.

All in all, the current situation in HSD is not satisfactory. The comparison between the CT and experiment shows several deviations which manifest themselves through a  $\chi^2 \sim 26$  (see Sec. VI), even after the introduction of relevant corrections. We cannot, at this stage, assert the origin of such deviations. They might be attributed to the pres-

ence of symmetry breaking or they may be due to some, possibly fortuitous, temporary experimental situation.

We have compared the CT, with all corrections included, to HSD data split into two subsets. These subsets are the transition rates alone and the transition rates and the electron-neutrino angular-correlation coefficients, excluding the polarization-asymmetry coefficients. When only the rates are used the CT parameters are not yet well determined and important changes can occur, as illustrated by the unacceptably large value of  $C$  (see Sec. VII). If only rates and angular-correlation coefficients are used, the agreement with the CT is very good. Even the fitted value of  $C$  comes out close to its estimated value from the Weinberg-Salam model. The CT parameters are quite tightly fixed. If these were the only pieces of data available, everything would be right in its place and we could conclude that  $SU_3$  symmetry is remarkably exact.<sup>23</sup> All of the important symmetry breaking would come just from hyperon mass differences. Clearly, it is only the polarization asymmetries in  $\Lambda \rightarrow pev$  and  $\Sigma^- \rightarrow nev$  that are the source of the deviations with the CT.

These asymmetries come from several experiments. It must be remarked that it is only the statistical averages for these asymmetries that show the deviations. None of the single experiments shows by itself a deviation of great significance. Nevertheless, one must admit that the statistical averages show a rather strong tendency to diverge from the CT. As a matter of fact, if the present central values were confirmed, the deviation would be important enough to require an important revision of the hypothesis in the CT. Probably, very large symmetry-breaking corrections would be implied, diminishing greatly the usefulness of the  $SU_3$ -symmetry limit. We are thus facing a twofold situation. Either the polarization asymmetries will be substantially changed in current and future experiments, or the CT will lose its simplicity.

There are two ways to approach this twofold situation. The first and more obvious one is to have better measurements of the asymmetries, but the polarization experiments are difficult and time-consuming. The second way which provides a cross check is to look for a second solution, different from the CT solution, in the rates and angular-coefficients sector.<sup>24</sup> If the polarization asymmetries tendency is true, then there must exist another solution for the several form factors with a nonzero pseudotensor form factor  $g_2$ , since it is only this form factor that remains available to ex-

plain the polarization asymmetries. In contrast, if a solution with nonzero  $g_2$  is proved to be absent in the rates and angular coefficients sector,<sup>23</sup> then we would know for sure that the tendency shown by the polarization asymmetries is only fortuitous and will be reversed. Let us emphasize that it is a general practice in experimental analysis of HSD data to put  $g_2$  equal to zero. Both ways should be tried, always keeping track of the several corrections simultaneously, whose absence would certainly lead to misinterpretations. In this respect, it would be most useful that experimental estimates and, if possible, determinations of the  $V$  and  $A$  slope parameters be performed, before any theoretical estimates be used.

All along in our analysis we have kept the contribution to  $\chi^2$  from the predictions for the  $g_1/f_1$  ratios and their experimental counterparts. It is clear that they do provide a consistency check on the CT, but it is also clear that they provide less information. Also their use makes it easier to be misled. For example, in Ref. 7,  $R_{\Sigma-\Lambda}$  contributed  $\Delta\chi^2 \sim 7$ , while we have obtained  $\Delta\chi^2 \leq 3$  [see Table IV(c) and Table V(c)], which would be a significantly smaller symmetry-breaking effect if so interpreted; but it can very likely be just a statistical fluctuation. It is, therefore, not idle at all to encourage experimentalists to quote only genuine experimentally measurable quantities such as angular-correlation and asymmetry coefficients.

Finally, let us mention that there seems to be no evidence for a modification of the CT universality that may be suggested in higher unification schemes, although the error bars still leave room for it. Curiously, such modification may require the incorporation of symmetry-breaking corrections that are not too small. It is conceivable that at the end there will indeed be symmetry breaking that is not small but that is not large enough to spoil the features of the CT and that gives room for higher-symmetry schemes. Evidently, much work remains to be done to bring about the required precision to fully exploit HSD.

*Note added.* Recent measurements<sup>25</sup> of  $R_{\Sigma-\Lambda}$  seem to indicate distinctly the presence of  $SU_3$ -symmetry breaking which could be interpreted as only a first-order correction. Detailed discussions are given in Ref. 26.

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