Angular distribution of W bosons in hadron + hadron $\to W^{\pm}+\gamma+X$

K. O. Mikaelian
University of California, Lawrence Livermore National Laboratory, Livermore, California 94550
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We study the detailed kinematics of inclusive $W+\gamma$ production in hadron-hadron collisions and the phase-space integrations leading to the angular distribution of the W bosons. We also discuss the properties of the zero that appears in the subprocess $q_a\bar{q}_b\to W\gamma$.

I. INTRODUCTION

In this paper we consider

$$A+B\to W^{\pm}+\gamma+X\ , \tag{1}$$

where A and B are hadrons. The kinematics and dynamics of this process, based on the standard quark-parton picture, are shown in Figs. 1 and 2, respectively, and in Table I we give a list of definitions for ease of reference. We concentrate on the angular distributions of the W bosons since, as reported earlier,[1] it contains a peculiar zero whose physical origin remains to be explained.

The zero appears clearly in the center-of-mass frame of the W plus γ, hereafter referred to simply as the c.m. frame. This angular distribution, $d\sigma/d\cos\theta^{\text{c.m.}}$, is relatively easy to calculate but perhaps difficult to measure experimentally since it requires knowledge of the c.m. motion. In the laboratory frame, which we take to be the c.m. of the colliding hadrons A plus B, the opposite is true: measurement of $d\sigma/d\cos\theta^{\text{lab}}$ is relatively straightforward but the calculation is not.

The kinematics is of course quite general and applies to the inclusive production of a massive + a massless particle. We could not find such a treatment in the literature and therefore we present in Sec. II the kinematics and phase-space integration in some detail. The dynamics of the underlying quark-parton process is treated in Sec. III, and some comments are given in Sec. IV.

II. KINEMATICS

Referring to Fig. 1 we define the $\hat{z}$ axis by

$$\vec{P}_A^{\text{lab}}=-\vec{P}_B^{\text{lab}}=\hat{z}\sqrt{S}/2,\ \ S=(P_A+P_B)^2\ .$$

All angles are measured relative to this axis. The usual parameters x_A and x_B are defined by the four-vector relations $P_a^{\text{lab}}=x_AP_A^{\text{lab}}$ and $P_b^{\text{lab}}=x_BP_B^{\text{lab}}$, where a and b label the interacting quarks/partons. All masses, except the W mass M, are set to zero. The variables s, t, τ, etc., are defined in Table I.

The total cross section is straightforward to obtain:

$$\sigma(S,M^2)=\tfrac{1}{3}\sum_{a,b}\int_{\tau_0}^{1}d\tau\int_{\tau-1}^{1-\tau}dx(x^2+4\tau)^{-1/2}\mathscr{P}_a^A(x_A)\mathscr{P}_b^B(x_B)\int_{M^2-s}^{0}dt\frac{d\sigma^{ab}}{dt}(s,t,M^2)\ , \tag{2}$$

where the $\mathscr{P}$'s are probability functions, the $\frac{1}{3}$ factor is for color, and $d\sigma^{ab}/dt(s,t,M^2)$ is the differential cross section for $q_a+\bar{q}_b\to W^{\pm}+\gamma$ which we discuss in Sec. III.

From Eq. (2) and using $t=-\frac{1}{2}(s-M^2)(1-\cos\theta^{\text{c.m.}})$ we obtain

$$\frac{d\sigma}{d\cos\theta^{\text{c.m.}}}=\tfrac{1}{3}\sum_{a,b}\int_{\tau_0}^{1}d\tau\int_{\tau-1}^{1-\tau}dx(x^2+4\tau)^{-1/2}\mathscr{P}_a^A(x_A)\mathscr{P}_b^P(x_B)\tfrac{1}{2}(s-M^2)\frac{d\sigma^{ab}}{dt}\ . \tag{3}$$

As mentioned earlier $d\sigma/d\cos\theta^{\text{c.m.}}$ is easy to calculate but difficult to measure since one needs the angle $\theta^{\text{c.m.}}$ that the W makes in the $W+\gamma$ center-of-mass frame. Much of the rest of this paper is devoted to calculating $d\sigma/d\cos\theta^{\text{lab}}$ which is a directly measurable quantity.

We wish to write the total cross section, Eq. (2), in an alternative way:

$$\sigma(S,M^2)=\tfrac{1}{3}\sum_{a,b}\int_{-1}^{1}d\cos\theta^{\text{lab}}\int d\tau\int dx(J)\mathscr{P}_a^A(x_A)\mathscr{P}_b^B(x_B)\frac{d\sigma^{ab}}{dt} \tag{4}$$

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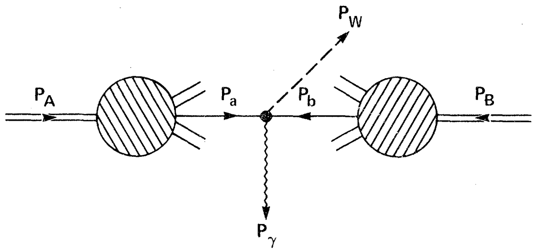

FIG. 1. The kinematics for $A+B\to W+\gamma+X$.

from which it is straightforward to get $d\sigma/d\cos\theta^{\text{lab}}$. The task therefore reduces to finding the Jacobian (J) and the physical region in the τ-x plane for fixed θ^{lab}.

Starting with the Jacobian,

$$(J)=\begin{vmatrix} \dfrac{\partial x_A}{\partial\cos\theta^{\text{lab}}} & \dfrac{\partial x_B}{\partial\cos\theta^{\text{lab}}} & \dfrac{\partial t}{\partial\cos\theta^{\text{lab}}} \\ \dfrac{\partial x_A}{\partial\tau} & \dfrac{\partial x_B}{\partial\tau} & \dfrac{\partial t}{\partial\tau} \\ \dfrac{\partial x_A}{\partial x} & \dfrac{\partial x_B}{\partial x} & \dfrac{\partial t}{\partial x} \end{vmatrix} = \frac{-1}{\sqrt{x^2+4\tau}}\frac{\partial t}{\partial\cos\theta^{\text{lab}}}\ . \tag{5}$$

To calculate

$$\frac{\partial t}{\partial\cos\theta^{\text{lab}}}=\tfrac{1}{2}(s-M^2)\frac{d\cos\theta^{\text{c.m.}}}{d\cos\theta^{\text{lab}}}$$

we need the relation

$$\cot\theta^{\text{lab}}=\gamma(\cos\theta^{\text{c.m.}}+c)/\sin\theta^{\text{c.m.}}\ , \tag{6}$$

where $\gamma(x_A,x_B)$ and $c(x_A,x_B)$ are given in Table I. Equations (5) and (6) give

$$(J)=-\frac{1}{(x^2+4\tau)^{1/2}}\frac{S}{2\gamma}(\tau-\tau_0)\times\frac{\sin^3\theta^{\text{c.m.}}}{\sin^3\theta^{\text{lab}}(1+c\cos\theta^{\text{c.m.}})}\ . \tag{7}$$

Note that (J) depends on $\theta^{\text{c.m.}}$ in addition to x_A, x_B, and θ^{lab}. We need $\theta^{\text{c.m.}}$ to evaluate both (J) and $d\sigma^{ab}/dt(s,t,M^2)$ since t involves $\theta^{\text{c.m.}}$.

From Eq. (6) we get

$$\cos\theta_{\pm}^{\text{c.m.}}=\frac{-c\pm[b^2(1+b^2-c^2)]^{1/2}}{1+b^2}\ , \tag{8}$$

where $b=(1/\gamma)\cot\theta^{\text{lab}}$. This means that for given x_A, x_B, and θ^{lab} there are two possible c.m. angles each of which, in general, must be substituted in the integrand of Eq. (4) and the two results must be added together. We find that the two possible roots are allowed only in a limited region of phase space which we now proceed to calculate.

The physical phase space must clearly lie within the "large" triangle defined by

$$\tau_0\le\tau\le1,\quad -(1-\tau)\le x\le 1-\tau\ ,$$

see Figs. 3 and 4. In addition, we require that [see Eq. (8)]

$$1+b^2-c^2\ge0\ , \tag{9}$$

which is an inequality involving x, τ, and θ^{lab}. After some algebra this condition reduces to

$$x^2\le\frac{(\tau-\tau_0)^2}{\tau_0{}^2\sin^2\theta^{\text{lab}}} \tag{10}$$

which eliminates the two symmetrically placed regions I in Figs. 3 and 4.

To find which root of Eq. (8) is allowed we cal-

TABLE I. Definition of variables used in this paper. See also Fig. 1.

$\vec{P}_A^{\text{lab}}+\vec{P}_B^{\text{lab}}=0$
$\vec{P}_a^{\text{c.m.}}+\vec{P}_b^{\text{c.m.}}=\vec{P}_W^{\text{c.m.}}+\vec{P}_\gamma^{\text{c.m.}}=0$
$P_a^{\text{lab}}=x_AP_A^{\text{lab}};\ P_b^{\text{lab}}=x_BP_B^{\text{lab}}$
All $P^2=0$ except $P_W{}^2=M^2$
$S=(P_A+P_B)^2$
$s=(P_a+P_b)^2=(P_W+P_\gamma)^2=x_Ax_BS$
$x=x_A-x_B$
$\tau=x_Ax_B;\ \tau_0=M^2/S<1$
$t=(P_W-P_a)^2=-\frac{1}{2}(s-M^2)(1-\cos\theta^{\text{c.m.}})$
$\beta=\dfrac{x_A-x_B}{x_A+x_B}=\dfrac{x}{(x^2+4\tau)^{1/2}}$
$\gamma=\dfrac{1}{(1-\beta^2)^{1/2}}$
$c=\beta(E_W^{\text{c.m.}}/\lvert\vec{P}_W^{\text{c.m.}}\rvert)$
$=\dfrac{x}{(x^2+4\tau)^{1/2}}(\tau+\tau_0)/(\tau-\tau_0)$

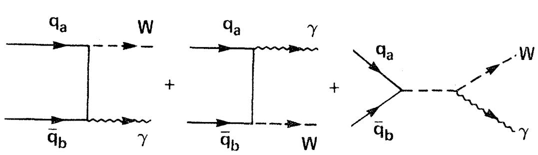

FIG. 2. Feynman diagrams for $q_a+\bar{q}_b\to W+\gamma$.

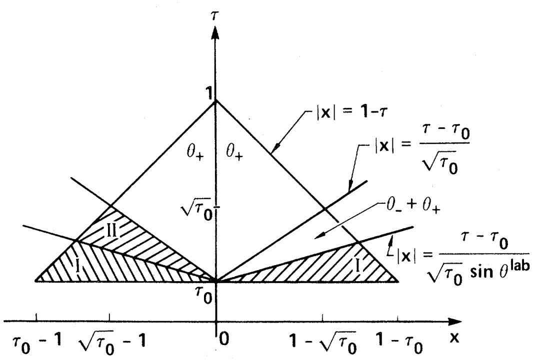

FIG. 3. Allowed phase space for events in the forward hemisphere, $0 \le \theta^{lab} \le \pi/2$. Regions I and II are forbidden. Regions labeled $\theta_\pm$ admit corresponding solutions $\cos\theta_\pm^{c.m.}$ given in Eq. (8). The lines $|x| = 1-\tau$ and $|x| = (\tau-\tau_0)/\sqrt{\tau_0}$ intersect at $\tau = \sqrt{\tau_0}$, $x = \pm(1-\sqrt{\tau_0})$ as indicated. The lines $|x| = 1-\tau$ and $|x| = (\tau-\tau_0)/\sqrt{\tau_0}\sin\theta^{lab}$ intersect at $\tau = \sqrt{\tau_0}(\sqrt{\tau_0}+\sin\theta^{lab})/(1+\sqrt{\tau_0}\sin\theta^{lab})$ and $x = \pm(1-\tau_0)/(1+\sqrt{\tau_0}\sin\theta^{lab}$ (these points are not indicated on the axes).

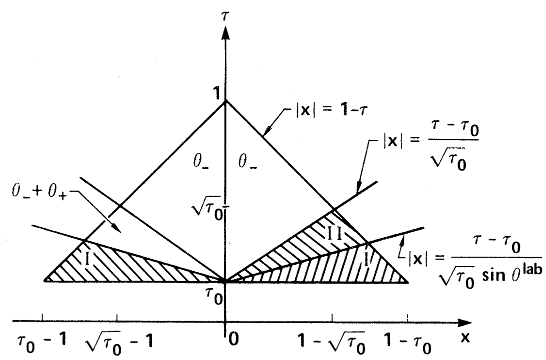

FIG. 4. Same as Fig. 3 but in the backward hemisphere $\pi/2 \le \theta^{lab} \le \pi$.

culate $|\vec{P}_W^{lab}|$ and require it to be positive or zero.[2] A convenient method is to evaluate the invariants $P_a \cdot P_W$ and $P_b \cdot P_W$ in both the c.m. and laboratory frames:

$$P_{a,b} \cdot P_W = E_{a,b}^{c.m.}(E_W^{c.m.} \mp |\vec{P}_W^{c.m.}| \cos\theta^{c.m.}) = E_{a,b}^{lab}(E_W^{lab} \mp \vec{P}_W^{lab} | \cos\theta^{lab}) \; . \tag{11}$$

Eliminating E_W^{lab} from these equations and after some algebra we find

$$|\vec{P}_W^{lab}| = \sqrt{S}\left[1+\frac{\tau_0}{\tau}\right](x_A - x_B) \times \left[1+\frac{\cos\theta^{c.m.}}{c}\right] \Big/ 4\cos\theta^{lab} \; . \tag{12}$$

We shall discuss the details only in the forward hemisphere, $\theta^{lab} < \pi/2$. The discussion of the backward hemisphere is similar.

From Eq. (12) and for $\cos\theta^{lab} > 0$, requiring $|\vec{P}_W^{lab}| \ge 0$ leads to

$$(x_A + x_B)\frac{|\vec{P}_W^{c.m.}|}{E_W^{c.m.}}\cos\theta^{c.m.} \ge x_B - x_A \; . \tag{13}$$

Using Eq. (8) we find that $\theta_-^{c.m.}$ is allowed only in a "small" region of phase space, i.e., in the region (see Fig. 3)

$$x_B + \frac{\tau-\tau_0}{\sqrt{\tau_0}} \le x_A \le x_B + \frac{\tau-\tau_0}{\sqrt{\tau_0}\sin\theta^{lab}} \; . \tag{14}$$

$\theta_+^{c.m.}$ is also allowed in this region. In addition, $\theta_+^{c.m.}$ is allowed in the somewhat "larger" region

$$|x| \le \frac{\tau-\tau_0}{\sqrt{\tau_0}} \tag{15}$$

which is symmetric around $x = 0$. Note that this is more restrictive than Eq. (10) which eliminated region I. Region II in Fig. 3 mirrors the above small region defined by Eq. (14). No solution is found in this region.

The proofs of the above statements are somewhat lengthy but straightforward. The results for the forward and backward hemispheres are shown in Figs. 3 and 4, respectively. The allowed regions are labeled by $\theta_\pm$ or $\theta_+ + \theta_-$. For example, in the forward hemisphere,

$$\frac{d\sigma}{d\cos\theta^{lab}} = \int_{\tau_0}^{\sqrt{\tau_0}} d\tau \int_{-(\tau-\tau_0)/\sqrt{\tau_0}}^{\tau-\tau_0/\sqrt{\tau_0}} dx[\theta_+] + \int_{\sqrt{\tau_0}}^{1} d\tau \int_{\tau-1}^{1-\tau} dx[\theta_+] + \int_{\tau_0}^{\sqrt{\tau_0}(\sqrt{\tau_0}+\sin\theta^{lab})/[1+\sqrt{\tau_0}\sin\theta^{lab}]} d\tau \int_{(\tau-\tau_0)/\sqrt{\tau_0}}^{(\tau-\tau_0)/\sqrt{\tau_0}\sin\theta^{lab}} dx[\theta_+ + \theta_-] + \int_{\sqrt{\tau_0}(\sqrt{\tau_0}+\sin\theta^{lab})/[1+\sqrt{\tau_0}\sin\theta^{lab}]}^{\sqrt{\tau_0}} d\tau \int_{(\tau-\tau_0)/\sqrt{\tau_0}}^{1-\tau} dx[\theta_+ + \theta_-] \; , \tag{16}$$

where

$$[\theta_\pm]=\frac{1}{3}\sum_{a,b}\mathscr{P}_a^A(x_A)\mathscr{P}_b^B(x_B)\,|\,J_\pm\,|\,\frac{d\sigma^{ab}}{dt}(s,\theta_\pm^{\text{c.m.}},M^2)\ . \tag{17}$$

A similar expression can be written down in the backward hemisphere, $\pi/2\le\theta^{\text{lab}}\le\pi$.

III. DYNAMICS

While the above kinematical analysis is general and as such covers the inclusive production of $W\gamma$, $Z^0\gamma$, $H\gamma$, etc., the dynamics depends on the particular interaction involved and appears in $d\sigma^{ab}/dt(s,t,M^2)$.

The amplitude for $q_a\bar{q}_b\to W^\pm\gamma$ is found, to lowest order in perturbation theory, by summing the three Feynman diagrams of Fig. 2. The triple-vector coupling at the W-γ-W vertex can be generalized[3] by introducing an "anomalous" magnetic moment κ whose value is one in the standard theory which uses Yang-Mills couplings among the gauge bosons. We shall omit the details of the calculation because $q_a\bar{q}_b\to W^\pm\gamma$ can be obtained by crossing from $\gamma q_a\to W^\pm q_b$, a process studied[4] in detail for arbitrary κ. The result for $\kappa=1$ is exceptionally simple:

$$\frac{d\sigma^{ab}}{dt}(s,t,M^2)=\frac{d\sigma}{dt}(q_a\bar{q}_b\to W^-\gamma)=\frac{\alpha}{s^2}\frac{M^2G_F}{\sqrt{2}}g_{ab}{}^2\left[Q_a+\frac{1}{1+t/u}\right]^2\left[\frac{t^2+u^2+2sM^2}{tu}\right], \tag{18}$$

where $g_{ab}=\cos\theta_C$ ($\sin\theta_C$) if $q_a\bar{q}_b=d\bar{u}$ or $s\bar{c}$ ($s\bar{u}$ or $d\bar{c}$), u is defined by $s+t+u=M^2$, and $Q_a\,|\,e\,|$ is the electric charge of q_a. The cross section for $W^+\gamma$ is obtained by interchanging u and t.

Since

$$Q_a+\frac{1}{1+t/u}=Q_a+\frac{1+\cos\theta^{\text{c.m.}}}{2} \tag{19}$$

the differential cross section $d\sigma/dt(q_a\bar{q}_b\to W^-\gamma)$ vanishes at $\cos\theta^{\text{c.m.}}=-\frac{1}{3}$ ($Q_a=-\frac{1}{3}$ in the standard model). Two properties make this zero interesting: (1) its location depends on the quark electric charge Q_a and (2) its physical origin is not clear.

The differential cross section in Eq. (18) is summed and averaged over final and initial spin directions. The factor $Q_a+1/(1+t/u)$, however, occurs in each helicity amplitude. It was shown[5] that for color SU(N) interactions between quarks and gluons a similar factorization takes place for four-body amplitudes $q\bar{q}\to gg$, etc., and a general proof that such a factorization always occurs was given recently.[6] It applies to any four-body process where at least one of the external particles is a massless gauge boson, always calculated to lowest order in perturbation theory.

While the masslessness of at least one of the gauge bosons is sufficient for this proof, one cannot say that it is necessary. For the case under study $q_a\bar{q}_b\to W^\pm\gamma$ an explicit calculation showed[7] that putting the photon off-mass-shell spoils the factorization property of the amplitude and the zero associated with it, suggesting that only real photon processes have a physically observable zero (to observe a zero in a gluon process one needs to distinguish among colors).

Indeed recently a systematic study[8] of the helicity amplitudes for $q_a\bar{q}_b\to W^+W^-$, Z^0Z^0, $W^\pm Z^0$, $W^\pm\gamma$, and $Z^0\gamma$ finds that only the $W^\pm\gamma$ channel has a nontrivial zero in all of its helicity amplitudes. There are, of course, "trivial" zeros for example, $W^\pm\gamma$ and $W^\pm Z^0$ production by right-handed quarks vanish identically because the W couples only to left-handed quarks.

The absence of a zero in W^+W^-, Z^0Z^0, and $W^\pm Z^0$ is presumably due to the finite mass of these bosons, while in $Z^0\gamma$ factor is trivial (factor $=1$) because it is essentially an Abelian interaction with no Z^0-γ-Z^0 coupling. These channels, however, have other interesting properties, e.g., dips at $\theta^{\text{c.m.}}=\pi/2$, as discussed in Ref. 8.

IV. COMMENTS

(i) Equation (16) describes the angular distribution of W bosons in the laboratory frame. The two-dimensional integration must be done numerically and is somewhat tricky because of the Jacobian peaks. An important test is to do a third integration over $\cos\theta^{\text{lab}}$ to obtain the total cross section which should agree with $\sigma(S,M^2)$ calculated in the much more straightforward manner of Eq.

(2). Numerical results for σ_{total} based on Eq. (2) have already been reported in Ref. 9 which discusses both $W^{\pm}Z^0$ and $W\gamma$ production.

(ii) The zero occurs only at a particular value of $\theta^{\text{c.m.}}$ and does not appear in $d\sigma/d\cos\theta^{\text{lab}}$. As stated earlier,[1] this is because the two-dimensional integration over x and τ, Eq. (16), washes out the zero since one is effectively integrating over $\theta^{\text{c.m.}}$ also. Clearly a double- or triple-differential cross section is needed to locate the zero.

(iii) If the factor in Eq. (19) had contained the ratio s/u or s/t in addition to, or perhaps in place of, the ratio t/u. then the position of the zero would change with s. As such, since

$$t/u=\frac{1-\cos\theta^{\text{c.m.}}}{1+\cos\theta^{\text{c.m.}}}\,,$$

the location of the zero is fixed in the c.m. frame, independently of the W or γ energy. We have no physical explanation as to why such a zero occurs in the first place and of course none as to why it remains fixed.

Relative to single-W production, the associated $W+\gamma$ production is suppressed by about α/π. However its strange behavior and its strong sensitivity to κ make it an attractive process to try and measure in a second round of experiments after the much awaited discovery of W bosons.

ACKNOWLEDGMENT

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