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Quark and diquark fragmentation into mesons and baryons

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Quark and diquark fragmentation into mesons and baryons is treated in a cascade-type model based on six coupled integral equations. An analytic solution including flavor dependence is found. Comparison with experimental data is given. Our results indicate a probability of about 50% for the diquark to break up.

I. INTRODUCTION

It is commonly accepted that quarks are confined and fragment into jets of hadrons which are observed in high-energy reactions. The properties of these jets are usually described in terms of so-called fragmentation functions. In the phenomenological picture which has been developed to understand this process of hadronization (see, e.g., Refs. 1–3) the fragmentation of a highly energetic quark into mesons proceeds by creation of quark-antiquark pairs in the color field of the original quark and recursive pairing of quarks with antiquarks starting with the original quark.

In quark jets also baryons are produced[4] at a rate of 5–10%. This can be incorporated into the phenomenological description by assuming that also diquark-antidiquark pairs are created and a quark together with a diquark forms a baryon.[5] With regard to color the diquark behaves like an antiquark, but since it has a higher effective mass than a quark, diquark-antidiquark pairs are produced less frequently. When a baryon is produced in a quark jet an antidiquark continues the chain. If the antidiquark acts as a single unit then the next hadron produced in the chain is an antibaryon (local conservation of baryon number[6]) and strong correlations between the emitted baryon and antibaryon have to be expected. On the other hand, since the diquark is a bound object it may also break up. Then mesons are emitted before the antibaryon is formed. In this case the strong correlation between the baryon and the antibaryon is lost.

Besides quark fragmentation into baryons there are further phenomena where the concept of a diquark plays an important role. In deep-inelastic lepton scattering when a quark is knocked out from a nucleon, the diquark is the main component of the target remnant responsible for target fragmentation. Diquark fragmentation will occur in large-p_T hadron-hadron collisions, too, but there the reaction mechanism is certainly more complicated. Some of the diquark fragmentation functions have already been determined phenomenologically[7]; a recent calculation is given in Ref. 8. The concept of the diquark also appears in the calculation of higher-twist contributions[9] and, furthermore, it might be useful in baryon spectroscopy.[10]

In a proper treatment of quark and diquark fragmentation it is necessary to simultaneously take into account emission of mesons, baryons, and antibaryons from the quark as well as from the diquark. This leads to a system of six coupled integral equations for the quark and diquark fragmentation functions.[5] It is the aim of this paper to investigate this system of integral equations and to look also for other solutions besides a Monte Carlo one. In order to include resonance decays, masses, etc., a Monte Carlo simulation of the emission process is certainly best suited for analyzing experimental data, but it is also important to know about exact analytic solutions or other approximate ones. From a comparison with experiment one will then learn about the role of the diquark in the process of hadronization and whether it behaves as a colored entity or not.

In Ref. 11 a mechanism for baryon production in quark jets based on the chromoelectric-flux-tube

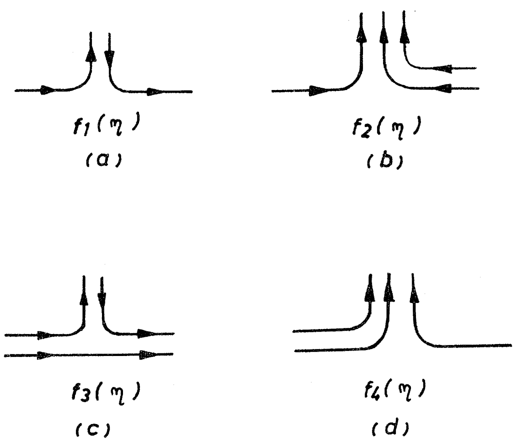


FIG. 1. Elementary processes for emission of primary particles: (a) meson from quark, (b) baryon from quark, (c) meson from diquark, (d) baryon from diquark.

model has been discussed. It relies on the SU(3) nature of the color quantum number and does not make use of the notion of the diquark. If the ideas presented there turn out to be right, then our model involving a diquark which can break up could be considered a phenomenological description of baryon production although the diquark would then be a less fundamental object.

In Sec. II we describe our basic system of equations for the fragmentation functions and draw some general conclusions. In Sec. III we show how to include flavor. An exact analytic solution and an approximate one for the fragmentation functions are presented in Sec. IV. In Sec. V we compare with experimental data using a Monte Carlo procedure and discuss our results.

II. THE MODEL EQUATIONS

The elementary processes for emitting a single (first-rank) meson or baryon from a quark or diquark are shown in Fig. 1. We shall denote by $f_1(\eta)$ the probability for emission of a meson from a quark jet [Fig. 1(a)] regardless of flavor when the meson carries away the fraction $1-\eta$ of the longitudinal momentum of the quark and leaves longitudinal-momentum fraction η to the rest of the chain. Similarly $f_2(\eta)$, $f_3(\eta)$, and $f_4(\eta)$ are the probability functions for emission of a baryon from a quark, a meson from a diquark, and a baryon from a diquark, respectively [Figs. 1(b)–1(d)]. In particular $f_3(\eta)\neq 0$ means that a diquark can split and the two quarks become part of different hadrons. If we then call $M_q(z)$, $B_q(z)$, and $\bar{B}_q(z)$ the probabilities of finding in a quark jet a primary meson, baryon, or antibaryon, respectively, independent of rank and with momentum fraction z, and analogously $M_d(z)$, $B_d(z)$, and $\bar{B}_d(z)$ the probabilities to find a primary meson, baryon, or antibaryon, respectively, in a diquark jet, the following set of equations can be written[5]:

$$M_q(z)=f_1(1-z)+\int_z^1 \frac{d\eta}{\eta} f_1(\eta) M_q\left[\frac{z}{\eta}\right] + \int_z^1 \frac{d\eta}{\eta} f_2(\eta) M_d\left[\frac{z}{\eta}\right], \tag{1a}$$

$$B_q(z)=f_2(1-z)+\int_z^1 \frac{d\eta}{\eta} f_1(\eta) B_q\left[\frac{z}{\eta}\right] + \int_z^1 \frac{d\eta}{\eta} f_2(\eta) \bar{B}_d\left[\frac{z}{\eta}\right], \tag{1b}$$

$$\bar{B}_q(z)=\int_z^1 \frac{d\eta}{\eta} f_1(\eta) \bar{B}_q\left[\frac{z}{\eta}\right] + \int_z^1 \frac{d\eta}{\eta} f_2(\eta) B_d\left[\frac{z}{\eta}\right], \tag{1c}$$

$$M_d(z)=f_3(1-z)+\int_z^1 \frac{d\eta}{\eta} f_3(\eta) M_d\left[\frac{z}{\eta}\right] + \int_z^1 \frac{d\eta}{\eta} f_4(\eta) M_q\left[\frac{z}{\eta}\right], \tag{1d}$$

$$B_d(z)=f_4(1-z)+\int_z^1 \frac{d\eta}{\eta} f_3(\eta) B_d\left[\frac{z}{\eta}\right] + \int_z^1 \frac{d\eta}{\eta} f_4(\eta) \bar{B}_q\left[\frac{z}{\eta}\right], \tag{1e}$$

$$\bar{B}_d(z)=\int_z^1 \frac{d\eta}{\eta} f_3(\eta) \bar{B}_d\left[\frac{z}{\eta}\right] + \int_z^1 \frac{d\eta}{\eta} f_4(\eta) B_q\left[\frac{z}{\eta}\right]. \tag{1f}$$

Here we have made use of charge-conjugation invariance, i.e., $\bar{B}_{\bar{d}}(z)=B_d(z)$, $\bar{B}_q(z)=B_{\bar{q}}(z)$, etc. The right-hand side of Eq. (1a) follows because the emitted meson can be of rank one, or it can be of higher rank, the first emitted particle being a meson or being a baryon, giving rise to the first, second, and third terms, respectively. In an analogous way also Eqs. (1b) to (1f) can be understood. Here transverse momentum of the emitted particles is not taken into account, as we discuss longitudinal-momentum distributions only. Equation (1a) without the last term is just the equation of Feynman and Field.[1] Equation (1b) without the last term has been used in Ref. 6. Equations similar to (1d) and (1e) without the last term were used in Ref. 8 as a model for diquark fragmentation. This means that in Refs. 6 and 8 multiple-baryon production has been neglected. Conservation of longitudinal momentum gives the normalization conditions

$$\int_0^1 z[M_q(z)+B_q(z)+\bar{B}_q(z)]dz=1\ , \tag{2a}$$

$$\int_0^1 z[M_d(z)+B_d(z)+\bar{B}_d(z)]dz=1 \tag{2b}$$

and these lead to

$$\int_0^1 [f_1(z)+f_2(z)]dz=1\ , \tag{3a}$$

$$\int_0^1 [f_3(z)+f_4(z)]dz=1\ . \tag{3b}$$

One way to obtain a solution of Eqs. (1) is by performing a Mellin transformation. Defining moments as

$$\widetilde{M}_q(r)=\int_0^1 M_q(z)z^r dz \tag{4}$$

and in an analogous way for $\widetilde{B}_q(r)$, $\widetilde{\bar{B}}_q(r)$, $\widetilde{M}_d(r)$, $\widetilde{B}_d(r)$, and $\widetilde{\bar{B}}_d(r)$ as well as

$$g_i(r)=\int_0^1 f_i(z)z^r dz\ , \tag{5a}$$

$$h_i(r)=\int_0^1 f_i(1-z)z^r dz\ , \tag{5b}$$

Eqs. (1) are transformed into

$$\widetilde{M}_q(r)=h_1(r)+g_1(r)\widetilde{M}_q(r)+g_2(r)\widetilde{M}_d(r)\ , \tag{6a}$$

$$\widetilde{B}_q(r)=h_2(r)+g_1(r)\widetilde{B}_q(r)+g_2(r)\widetilde{\bar{B}}_d(r)\ , \tag{6b}$$

$$\widetilde{\bar{B}}_q(r)=g_1(r)\widetilde{\bar{B}}_q(r)+g_2(r)\widetilde{B}_d(r)\ , \tag{6c}$$

$$\widetilde{M}_d(r)=h_3(r)+g_3(r)\widetilde{M}_d(r)+g_4(r)\widetilde{M}_q(r)\ , \tag{6d}$$

$$\widetilde{B}_d(r)=h_4(r)+g_3(r)\widetilde{B}_d(r)+g_4(r)\widetilde{\bar{B}}_q(r)\ , \tag{6e}$$

$$\widetilde{\bar{B}}_d(r)=g_3(r)\widetilde{\bar{B}}_d(r)+g_4(r)\widetilde{B}_q(r)\ . \tag{6f}$$

The solution of Eqs. (6) is easily obtained as

$$\widetilde{M}_q(r)=\frac{1}{\Delta(r)}\{h_1(r)[1-g_3(r)] +g_2(r)h_3(r)\}\ , \tag{7a}$$

$$\widetilde{B}_q(r)=\frac{1}{\Delta(r)}h_2(r)[1-g_3(r)]\ , \tag{7b}$$

$$\widetilde{\bar{B}}_q(r)=\frac{1}{\Delta(r)}h_4(r)g_2(r)\ , \tag{7c}$$

$$\widetilde{M}_d(r)=\frac{1}{\Delta(r)}\{h_3(r)[1-g_1(r)] +h_1(r)g_4(r)\}\ , \tag{7d}$$

$$\widetilde{B}_d(r)=\frac{1}{\Delta(r)}h_4(r)[1-g_1(r)]\ , \tag{7e}$$

$$\widetilde{\bar{B}}(r)=\frac{1}{\Delta(r)}h_2(r)g_4(r)\ , \tag{7f}$$

with

$$\Delta(r)=[1-g_1(r)][1-g_3(r)]-g_2(r)g_4(r)\ . \tag{8}$$

The inverse of the Mellin transformation of Eqs. (7)

$$M_q(z)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dr\, z^{-r-1}\widetilde{M}_q(r)\ , \tag{9}$$

etc., are in general not obtainable in closed form. In Sec. IV we shall discuss a special case where this is possible. But some general remarks can already be made here. From Eq. (8) one can see that $\Delta(r)$ vanishes when $r\to 0$. Consequently, the functions $\widetilde{M}_q(r)$, $\widetilde{B}_q(r)$, etc., in Eqs. (7) have poles for $r\to 0$. These poles then completely control the $z\to 0$ behavior of the functions $M_q(z)$, $B_q(z)$, etc. To see this more clearly we write

$$p_i=g_i(0)=\int_0^1 f_i(z)dz\ , \tag{10}$$

where from Eqs. (3), $p_1+p_2=1$, $p_3+p_4=1$. p_1 (p_2) and p_3 (p_4) are the z-integrated probabilities that a meson (baryon) is the first emitted particle from a quark or diquark, respectively. Then one can show that

$$\lim_{z\to 0} M_q(z)=\lim_{z\to 0} M_d(z)=\frac{1}{z}\frac{p_1p_4+p_2p_3}{A}\ , \tag{11a}$$

$$\lim_{z\to 0} B_q(z)=\lim_{z\to 0} \bar{B}_q(z)=\lim_{z\to 0} B_d(z) =\lim_{z\to 0} \bar{B}_d(z)=\frac{1}{z}\frac{p_2p_4}{A}\ , \tag{11b}$$

with

$$A=-p_4\int_0^1 [f_1(z)+f_2(z)]\ln z\, dz -p_2\int_0^1 [f_3(z)+f_4(z)]\ln z\, dz\ . \tag{12}$$

This means that the height of the rapidity plateau for a given particle type is the same for quark and diquark fragments. The ratio of the plateau heights for mesons and baryons is given by

$$\lim_{z\to 0}\frac{M_q(z)}{B_q(z)}=\lim_{z\to 0}\frac{M_d(z)}{B_d(z)}=\frac{p_1}{p_2}+\frac{p_3}{p_4} . \tag{13}$$

It depends only on the parameters p_i and not on the shape of the momentum-sharing functions $f_i(z)$. Furthermore, in the scaling limit where Eqs. (1) are supposed to hold, antibaryons develop a plateau with the same height as baryons. These results also hold if flavor is taken into account [see Eqs. (19), (23), and (25) to (29) below]. The quantities p_i are the basic parameters of the model.

It is interesting to note that neglecting multiple-baryon production, this means omitting the last term in Eqs. (1b) and (1e) (as was done in Refs. 6 and 8) does not lead to a $1/z$ behavior of $B_q(z)$ and $B_d(z)$ or, equivalently, not to a rapidity plateau for baryons. Clearly, for z large enough multiple-baryon production can be safely neglected. More specifically, taking the parameters as obtained in our phenomenological analysis in Sec. V, we have found that for $z>0.2$ omission of the last term in Eqs. (1b) and (1e) changes $B_q(z)$ and $B_d(z)$ by less than 20%. For $z<0.1$, however, multiple-baryon production is important.

III. INCLUSION OF FLAVOR

This can be achieved in a way analogous to that of Ref. 1. Let us call γ_a the probability of creating a quark-antiquark pair with flavor a and γ_{ab} the probability of creating a diquark-antidiquark pair with flavor content ab. With the normalizations (2) and (3) we have

$$\sum_a \gamma_a=\sum_{ab}\gamma_{ab}=1 . \tag{14}$$

We call $M_q^{a\bar b}(z)$ and $B_q^{abc}(z)$ the probabilities that in the jet of the quark with flavor q a primary meson with quark content $(a\bar b)$ or a primary baryon with quark content (abc), respectively, is produced with longitudinal-momentum fraction z. $M_{q_1q_2}^{a\bar b}(z)$ and $B_{q_1q_2}^{abc}(z)$ are the analogously defined functions for a diquark jet of flavor q_1q_2. An analogous notation for the antibaryons is used. We can then rewrite Eqs. (1) as

$$M_q^{a\bar b}(z)=\delta_{qa}\gamma_b f_1(1-z)+\int_z^1\frac{d\eta}{\eta}f_1(\eta)\sum_e\gamma_e M_e^{a\bar b}\left[\frac{z}{\eta}\right]+\int_z^1\frac{d\eta}{\eta}f_2(\eta)\sum_{ef}\gamma_{ef}M_{ef}^{a\bar b}\left[\frac{z}{\eta}\right] , \tag{15a}$$

$$B_q^{abc}(z)=\sum_{\{abc\}}\delta_{qa}\gamma_{bc}f_2(1-z)+\int_z^1\frac{d\eta}{\eta}f_1(\eta)\sum_e\gamma_e B_e^{abc}\left[\frac{z}{\eta}\right]+\int_z^1\frac{d\eta}{\eta}f_2(\eta)\sum_{ef}\gamma_{ef}\bar B_{ef}^{\bar a\bar b\bar c}\left[\frac{z}{\eta}\right] , \tag{15b}$$

$$\bar B_q^{\bar a\bar b\bar c}(z)=\int_z^1\frac{d\eta}{\eta}f_1(\eta)\sum_e\gamma_e\bar B_e^{\bar a\bar b\bar c}\left[\frac{z}{\eta}\right]+\int_z^1\frac{d\eta}{\eta}f_2(\eta)\sum_{ef}\gamma_{ef}B_{ef}^{abc}\left[\frac{z}{\eta}\right] , \tag{15c}$$

$$\begin{aligned}M_{q_1q_2}^{a\bar b}(z)=&\tfrac{1}{2}(\delta_{q_1a}+\delta_{q_2a})\gamma_b f_3(1-z)+\int_z^1\frac{d\eta}{\eta}f_3(\eta)\tfrac{1}{2}\sum_e\left[\gamma_e M_{eq_2}^{a\bar b}\left[\frac{z}{\eta}\right]+\gamma_e M_{q_1e}^{a\bar b}\left[\frac{z}{\eta}\right]\right]\\&+\int_z^1\frac{d\eta}{\eta}f_4(\eta)\sum_e\gamma_e M_e^{a\bar b}\left[\frac{z}{\eta}\right] ,\end{aligned} \tag{15d}$$

$$\begin{aligned}B_{q_1q_2}^{abc}(z)=&\sum_{\{abc\}}\delta_{q_1a}\delta_{q_2b}\gamma_c f_4(1-z)+\int_z^1\frac{d\eta}{\eta}f_3(\eta)\tfrac{1}{2}\sum_e\left[\gamma_e B_{eq_2}^{abc}\left[\frac{z}{\eta}\right]+\gamma_e B_{q_1e}^{abc}\left[\frac{z}{\eta}\right]\right]\\&+\int_z^1\frac{d\eta}{\eta}f_4(\eta)\sum_e\gamma_e\bar B_e^{\bar a\bar b\bar c}\left[\frac{z}{\eta}\right] ,\end{aligned} \tag{15e}$$

$$\bar B_{q_1q_2}^{\bar a\bar b\bar c}(z)=\int_z^1\frac{d\eta}{\eta}f_3(\eta)\tfrac{1}{2}\sum_e\left[\gamma_e\bar B_{eq_2}^{\bar a\bar b\bar c}\left[\frac{z}{\eta}\right]+\gamma_e\bar B_{q_1e}^{\bar a\bar b\bar c}\left[\frac{z}{\eta}\right]\right]+\int_z^1\frac{d\eta}{\eta}f_4(\eta)\sum_e\gamma_e B_e^{abc}\left[\frac{z}{\eta}\right] . \tag{15f}$$

These are the basic equations for our study of quark and diquark fragmentation. Here and in the following $\sum_{\{abc\}}$ means the sum over all *different* permutations of the quarks a,b,c (notice that in the convention used the order of q_1 and q_2 has to be kept fixed). In the following it will also be more convenient to work with the Mellin transforms of Eqs. (15). $\tilde M_q^{a\bar b}(r)$, $\tilde B_q^{abc}(r)$ etc., then denote the Mellin transforms of $M_q^{a\bar b}(z)$,

$B_q^{abc}(z)$, etc., defined analogously to Eq. (4). Then, e.g., from the Mellin transformations of Eqs. (15a) and (15d) we get

$$\sum_e \gamma_e \widetilde{M}_e^{a\bar{b}}(r)=\gamma_a\gamma_b h_1(r)+g_1(r)\sum_e \gamma_e \widetilde{M}_e^{a\bar{b}}(r)+g_2(r)\sum_{ef}\gamma_{ef}\widetilde{M}_{ef}^{a\bar{b}}(r)\ , \tag{16a}$$

$$\sum_{ef}\gamma_e\gamma_f M_{ef}^{a\bar{b}}(r)=\gamma_a\gamma_b h_3(r)+g_3(r)\sum_{ef}\gamma_e\gamma_f\widetilde{M}_{ef}^{a\bar{b}}(r)+g_4(r)\sum_e \gamma_e M_e^{a\bar{b}}(r)\ . \tag{16b}$$

Equations (16a) and (16b) together with Eqs. (6a) and (6d) lead to

$$\sum_e \gamma_e \widetilde{M}_e^{a\bar{b}}(r)=\gamma_a\gamma_b\widetilde{M}_q(r)\ , \tag{17}$$

$$\sum_{ef}\gamma_{ef}\widetilde{M}_{ef}^{a\bar{b}}(r)=\sum_{ef}\gamma_e\gamma_f\widetilde{M}_{ef}^{a\bar{b}}(r)=\gamma_a\gamma_b\widetilde{M}_d(r)\ . \tag{18}$$

From Eqs. (6a), (15a), (17), and (18) one then obtains

$$M_q^{a\bar{b}}(z)=\delta_{qa}\gamma_b f_1(1-z)+\gamma_a\gamma_b[M_q(z)-f_1(1-z)]\ . \tag{19}$$

From Eqs. (14) and (15d) one furthermore gets

$$\sum_e \gamma_e[\widetilde{M}_{eq_2}^{a\bar{b}}(r)+\widetilde{M}_{q_1e}^{a\bar{b}}(r)]=\frac{1}{1-\frac{1}{2}g_3(r)}\left\{[\tfrac{1}{2}(\delta_{q_1a}+\delta_{q_2a})\gamma_b+\gamma_a\gamma_b]h_3(r)+g_3(r)\sum_{ef}\gamma_e\gamma_f\widetilde{M}_{ef}^{a\bar{b}}(r)+2g_4(r)\sum_e\gamma_e\widetilde{M}_e^{a\bar{b}}(r)\right\}\ . \tag{20}$$

Inserting this and Eqs. (17) and (18) again in Eq. (15d) and using Eqs. (6) gives

$$\widetilde{M}_{q_1q_2}^{a\bar{b}}(r)=\tfrac{1}{2}(\delta_{q_1a}+\delta_{q_2a})\gamma_b\frac{1}{1-\frac{1}{2}g_3(r)}h_3(r)+\gamma_a\gamma_b\left[\widetilde{M}_d(r)-\frac{1}{1-\frac{1}{2}g_3(r)}h_3(r)\right]\ . \tag{21}$$

Defining a function $\kappa(z)$ by

$$\int_0^1 \kappa(z)z^r dz=\frac{\frac{1}{2}g_3(r)}{1-\frac{1}{2}g_3(r)} \tag{22}$$

the inverse Mellin transform of Eq. (21) can be performed leading to

$$M_{q_1q_2}^{a\bar{b}}(z)=\tfrac{1}{2}(\delta_{q_1a}+\delta_{q_2a})\gamma_b[f_3(1-z)+J_M(z)]+\gamma_a\gamma_b[M_d(z)-f_3(1-z)-J_M(z)] \tag{23}$$

with

$$J_M(z)=\int_z^1\frac{d\eta}{\eta}\kappa(\eta)f_3\left[1-\frac{z}{\eta}\right]\ . \tag{24}$$

In a similar way one obtains

$$B_q^{abc}(z)=\sum_{\{abc\}}\{\delta_{qa}\gamma_{bc}f_2(1-z)+\gamma_a\gamma_{bc}[B_q(z)-f_2(1-z)]\}\ , \tag{25}$$

$$B_{q_1q_2}^{abc}(z)=\sum_{\{abc\}}\{\delta_{q_1a}\delta_{q_2b}\gamma_c f_4(1-z)+\tfrac{1}{2}(\delta_{q_1a}\gamma_b+\delta_{q_2b}\gamma_a)\gamma_c J_B(z)+\gamma_a\gamma_{bc}[B_d(z)-f_4(1-z)-J_B(z)]\} \tag{26}$$

with

$$J_B(z)=2\int_z^1\frac{d\eta}{\eta}\kappa(\eta)f_4\left[1-\frac{z}{\eta}\right] \tag{27}$$

as well as

$$\bar{B}_q^{\bar{a}\bar{b}\bar{c}}(z)=\sum_{\{abc\}}\gamma_a\gamma_{bc}\bar{B}_q(z)\ , \tag{28}$$

$$\bar{B}_{q_1q_2}^{\bar{a}\bar{b}\bar{c}}(z)=\sum_{\{abc\}}\gamma_a\gamma_{bc}\bar{B}_d(z)\ . \tag{29}$$

In analogy to Eqs. (17) and (18) the following equalities must hold:

$$\sum_e \gamma_e B_e^{abc}(z)=\sum_{\{abc\}}\gamma_a\gamma_{bc}B_q(z)\ , \tag{30}$$

$$\sum_{ef}\gamma_{ef}B_{ef}^{abc}(z)=\sum_{ef}\gamma_e\gamma_f B_{ef}^{abc}(z)=\sum_{\{abc\}}\gamma_a\gamma_{bc}B_d(z)\ . \tag{31}$$

By expanding Eqs. (21) into powers of $\frac{1}{2}g_3(r)$ it can be seen that the terms $J_M(z)$ and $J_B(z)$ in Eqs. (23) and (26), respectively, come from such contributions where one of the quarks in the original diquark runs as a spectator through the whole chain until the last emitted particle. Denoting the meson $a\bar{b}$ by its symbol M and the baryon abc by its symbol B, the results for the fragmentation functions, Eqs. (19), (23), (25), and (26) can be cast into the more convenient form

$$D_q^M(z)=\alpha_M A_q^M f_1(1-z)+\alpha_M B^M[M_q(z)-f_1(1-z)]\ , \tag{32a}$$

$$D_q^B(z)=\alpha_B A_q^B f_2(1-z)+\alpha_B B^B[B_q(z)-f_2(1-z)]\ , \tag{32b}$$

$$D_{q_1q_2}^M(z)=\alpha_M C_{q_1q_2}^M[f_3(1-z)+J_M(z)]+\alpha_M B^M[M_d(z)-f_3(1-z)-J_M(z)]\ , \tag{32c}$$

$$D_{q_1q_2}^B(z)=\alpha_B C_{q_1q_2}^B f_4(1-z)+\alpha_B E_{q_1q_2}^B J_B(z)+\alpha_B B^B[B_d(z)-f_4(1-z)-J_B(z)]\ , \tag{32d}$$

where the coefficients $A_q^M=\delta_{qa}\gamma_b$ and $B^M=\gamma_a\gamma_b$ are already listed, e.g., in Table I of Ref. 1. Setting $\gamma_u=\gamma_d=\gamma$, $\gamma_s=1-2\gamma$, and $\gamma_q=0$, for the heavy quarks the quantities

$$C_{q_1q_2}^M=\tfrac{1}{2}(\delta_{q_1a}+\delta_{q_2a})\gamma_b$$

as well as

$$C_{q_1q_2}^B=\sum_{\{abc\}}\delta_{q_1a}\delta_{q_2b}\gamma_c$$

and

$$E_{q_1q_2}^B=\tfrac{1}{2}\sum_{\{abc\}}(\delta_{q_1a}\gamma_b+\delta_{q_2b}\gamma_a)\gamma_c$$

can be calculated. They are listed in Tables I and II (e.g., $\gamma=0.4$ was taken in Ref. 1, $\gamma=0.44$ in Ref. 8). In order to compute

$$A_q^B=\sum_{\{abc\}}\delta_{qa}\gamma_{bc}$$

TABLE I. Flavor coefficients for mesons in diquark jets. θ is the mixing angle of isoscalar mesons.

	$C_{q_1q_2}^M$			B^M
	uu	ud	dd	
π^+,ρ^+	γ	$\frac{1}{2}\gamma$	0	γ^2
π^-,ρ^-	0	$\frac{1}{2}\gamma$	γ	γ^2
π^0,ρ^0	$\frac{1}{2}\gamma$	$\frac{1}{2}\gamma$	$\frac{1}{2}\gamma$	γ^2
K^+,K^{*+}	$1-2\gamma$	$\frac{1}{2}(1-2\gamma)$	0	$\gamma(1-2\gamma)$
K^0,K^{*0}	0	$\frac{1}{2}(1-2\gamma)$	$1-2\gamma$	$\gamma(1-2\gamma)$
$\bar{K}^0,\bar{K}^{*0}$	0	0	0	$\gamma(1-2\gamma)$
K^-,K^{*-}	0	0	0	$\gamma(1-2\gamma)$
η,ω	$\frac{1}{2}\gamma\sin^2\theta$	$\frac{1}{2}\gamma\sin^2\theta$	$\frac{1}{2}\gamma\sin^2\theta$	$\gamma^2\sin^2\theta+(1-2\gamma)^2\cos^2\theta$
η',ϕ	$\frac{1}{2}\gamma\cos^2\theta$	$\frac{1}{2}\gamma\cos^2\theta$	$\frac{1}{2}\gamma\cos^2\theta$	$\gamma^2\cos^2\theta+(1-2\gamma)^2\sin^2\theta$

TABLE II. Flavor coefficients for baryons in diquark jets.

	$C^B_{q_1q_2}$ uu	$C^B_{q_1q_2}$ ud	$C^B_{q_1q_2}$ dd	$E^B_{q_1q_2}$ uu	$E^B_{q_1q_2}$ ud	$E^B_{q_1q_2}$ dd	B^B
p,Δ^+	γ	γ	0	$2\gamma^2$	$\frac{3}{2}\gamma^2$	γ^2	$3\gamma^3$
n,Δ^0	0	γ	γ	γ^2	$\frac{3}{2}\gamma^2$	$2\gamma^2$	$3\gamma^3$
$\Lambda,\Sigma^0,\frac{1}{2}\Sigma^{*+}$	0	$\frac{1}{2}(1-2\gamma)$	0	$\gamma(1-2\gamma)$	$\gamma(1-2\gamma)$	$\gamma(1-2\gamma)$	$3\gamma^2(1-2\gamma)$
Σ^+,Σ^{*+}	$1-2\gamma$	0	0	$2\gamma(1-2\gamma)$	$\gamma(1-2\gamma)$	0	$3\gamma^2(1-2\gamma)$
Σ^-,Σ^{*-}	0	0	$1-2\gamma$	0	$\gamma(1-2\gamma)$	$2\gamma(1-2\gamma)$	$3\gamma^2(1-2\gamma)$
Ξ^0,Ξ^{*0}	0	0	0	$(1-2\gamma)^2$	$\frac{1}{2}(1-2\gamma)^2$	0	$3\gamma(1-2\gamma)^2$
Ξ^-,Ξ^{*-}	0	0	0	0	$\frac{1}{2}(1-2\gamma)^2$	$(1-2\gamma)^2$	$3\gamma(1-2\gamma)^2$
Δ^{++}	γ	0	0	γ^2	$\frac{1}{2}\gamma^2$	0	γ^3
Δ^-	0	0	γ	0	$\frac{1}{2}\gamma^2$	γ^2	γ^3
Ω^-	0	0	0	0	0	0	$(1-2\gamma)^3$

and

$$B^B=\sum_{\{abc\}}\gamma_a\gamma_{bc}$$

an assumption about the probabilities γ_{bc} has to be made. For the sake of simplicity we shall put $\gamma_{bc}=\gamma_b\gamma_c$. The resulting coefficients A_q^B and B^B are given in Table III. Finally, the coefficient α_M takes into account the relative probability that a pseudoscalar or a vector meson is made from the quark pair $(a\bar{b})$. We shall assume $\alpha_p=\alpha_V=0.5$. Similarly, α_B measures the relative production rate of spin-$\frac{1}{2}$ to spin-$\frac{3}{2}$ baryons from the quarks *abc*. Again we shall assume $\alpha_{1/2}=\alpha_{3/2}=0.5$ if spin-$\frac{1}{2}$ baryon production is possible, $\alpha_{1/2}=0$ and $\alpha_{3/2}=1$ otherwise. Production of higher resonances is neglected.

The treatment of flavor as presented here is exact and the results of Eqs. (32) are obtained in closed form whereas in Ref. 8 the inclusion of flavor is done only in an iterative way up to third order.

IV. SOLUTIONS OF THE MODEL EQUATIONS

From Eqs. (32) one can see that also in the case of including flavor it suffices to know the solution of Eqs. (1). Although Eqs. (1) look rather complicated there are special cases which allow exact analytic solutions in a closed form being not only simple but also realistic and useful for practical purposes. We shall first present such an example and then discuss a way of obtaining approximate solutions for more general cases.

A. Exact analytic solution

The Mellin transforms of the solutions of Eqs. (1) are already given in Eqs. (7) and the problem would be solved if one could do the inverse Mellin transformation [Eqs. (9)] analytically. For arbi-

TABLE III. Flavor coefficients for baryons in quark jets (notice that a slightly different convention is used in Ref. 6).

	A_q^B u	A_q^B d	A_q^B s	B^B
p,Δ^+	$2\gamma^2$	γ^2	0	$3\gamma^3$
n,Δ^0	γ^2	$2\gamma^2$	0	$3\gamma^3$
$\Lambda,\Sigma^0,\frac{1}{2}\Sigma^{*0}$	$\gamma(1-2\gamma)$	$\gamma(1-2\gamma)$	γ^2	$3\gamma^2(1-2\gamma)$
Σ^+,Σ^{*+}	$2\gamma(1-2\gamma)$	0	γ^2	$3\gamma^2(1-2\gamma)$
Σ^-,Σ^{*-}	0	$2\gamma(1-2\gamma)$	γ^2	$3\gamma^2(1-2\gamma)$
Ξ^0,Ξ^{*0}	$(1-2\gamma)^2$	0	$2\gamma(1-2\gamma)$	$3\gamma(1-2\gamma)^2$
Ξ^-,Ξ^{*-}	0	$(1-2\gamma)^2$	$2\gamma(1-2\gamma)$	$3\gamma(1-2\gamma)^2$
Δ^{++}	γ^2	0	0	γ^3
Δ^-	0	γ^2	0	γ^3
Ω^-	0	0	$(1-2\gamma)^2$	$(1-2\gamma)^3$

trary input functions $f_i(\eta)$ this is in general not possible. However, for the special case of a power-law ansatz

$$f_i(\eta)=p_i(d_i+1)\eta^{d_i}\ , \tag{33}$$

we have

$$g_i(r)=p_i\frac{d_i+1}{r+d_i+1}\ , \tag{34}$$

$$h_i(r)=p_i\frac{\Gamma(d_i+2)\Gamma(r+1)}{\Gamma(r+d_i+2)}\ , \tag{35}$$

and the problem is reduced to solving an algebraic equation for finding all zeros of $\Delta(r)$ in Eq. (8) (besides that one at $r=0$). Specializing further we put $d_1=d_4=1$, $d_2=d_3=3$, i.e.,

$$f_1(\eta)=p_1 2\eta\ , \tag{36a}$$

$$f_2(\eta)=p_2 4\eta^3\ , \tag{36b}$$

$$f_3(\eta)=p_3 4\eta^3\ , \tag{36c}$$

$$f_4(\eta)=p_4 2\eta\ , \tag{36d}$$

which is suggested by counting rules[12] and we obtain from Eq. (18)

$$\Delta(r)=\frac{r(r+2p_2+4p_4)}{(r+2)(r+4)}\ . \tag{37}$$

Inserting into Eqs. (7) the inverse Mellin transform is obtainable in a straightforward way, giving

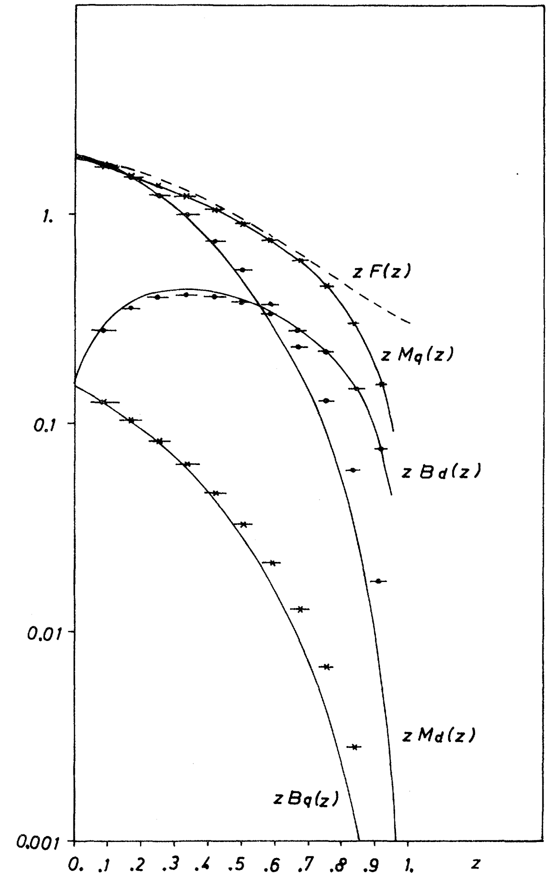


FIG. 2. Solid curves, exact solution for the fragmentation functions $M_q(z)$, $B_q(z)$, $M_d(z)$, and $B_d(z)$ of Eqs. (38) with $f_i(\eta)$ given in Eq. (36). Dashed curve for comparison fragmentation function of Ref. 1. x ● fragmentation functions obtained by the method of approximate inverse Mellin transformation, the length of the averaged z interval as indicated.

$$M_q(z)=\frac{8(p_1p_4+p_2p_3)}{2p_2+4p_4}\frac{1}{z}+\frac{2p_1-8p_1p_4-16p_2p_3}{2p_2+4p_4-1}+\frac{16p_2p_3}{2p_2+4p_4-3}z^2$$
$$-\frac{8p_2p_3}{2p_2+4p_4-4}z^3-(2p_2+4p_4-2)Q(z)\ , \tag{38a}$$

$$B_q(z)=4p_2\left[\frac{2p_4}{2p_2+4p_4}\frac{1}{z}-\frac{4p_4-1}{2p_2+4p_4-1}+\frac{4p_4-3}{2p_2+4p_4-3}z^2+\frac{2p_3}{2p_2+4p_4-4}z^3\right.$$
$$\left.-\frac{12p_2}{(2p_2+4p_4)(2p_2+4p_4-1)(2p_2+4p_4-3)(2p_2+4p_4-4)}z^{2p_2+4p_4-1}\right]\ , \tag{38b}$$

$$\bar{B}_q(z)=8p_2p_4\left[\frac{1}{2p_2+4p_4}\frac{1}{z}-\frac{1}{2p_2+4p_4-1}+\frac{z^{2p_2+4p_4-1}}{(2p_2+4p_4)(2p_2+4p_4-1)}\right]\ , \tag{38c}$$

$$M_d(z)=\frac{8(p_1p_4+p_2p_3)}{2p_2+4p_4}\frac{1}{z}+\frac{12(p_3-2p_2p_3-p_1p_4)}{2p_2+4p_4-1}+\frac{4p_1(p_4-6p_3)}{2p_2+4p_4-2}z$$
$$+\frac{4p_3(1+2p_1)}{2p_2+4p_4-3}z^2+(2p_2+4p_4-4)Q(z)\ , \tag{38d}$$

$$B_d(z)=2p_4\left[\frac{4p_2}{2p_2+4p_4}\frac{1}{z}+\frac{3-6p_2}{2p_2+4p_4-1}-\frac{2p_1}{2p_2+4p_4-2}z\right.$$
$$\left.-\frac{4p_4(2p_2+4p_4-4)z^{2p_2+4p_4-1}}{(2p_2+4p_4)(2p_2+4p_4-1)(2p_2+4p_4-2)}\right]\ , \tag{38e}$$

$$\bar{B}_d(z)=8p_2p_4\left[\frac{1}{2p_2+4p_4}\frac{1}{z}-\frac{3}{2p_2+4p_4-1}+\frac{3}{2p_2+4p_4-2}z-\frac{1}{2p_2+4p_4-3}z^2\right.$$
$$\left.+\frac{6z^{2p_2+4p_4-1}}{(2p_2+4p_4)(2p_2+4p_4-1)(2p_2+4p_4-2)(2p_2+4p_4-3)}\right]\ , \tag{38f}$$

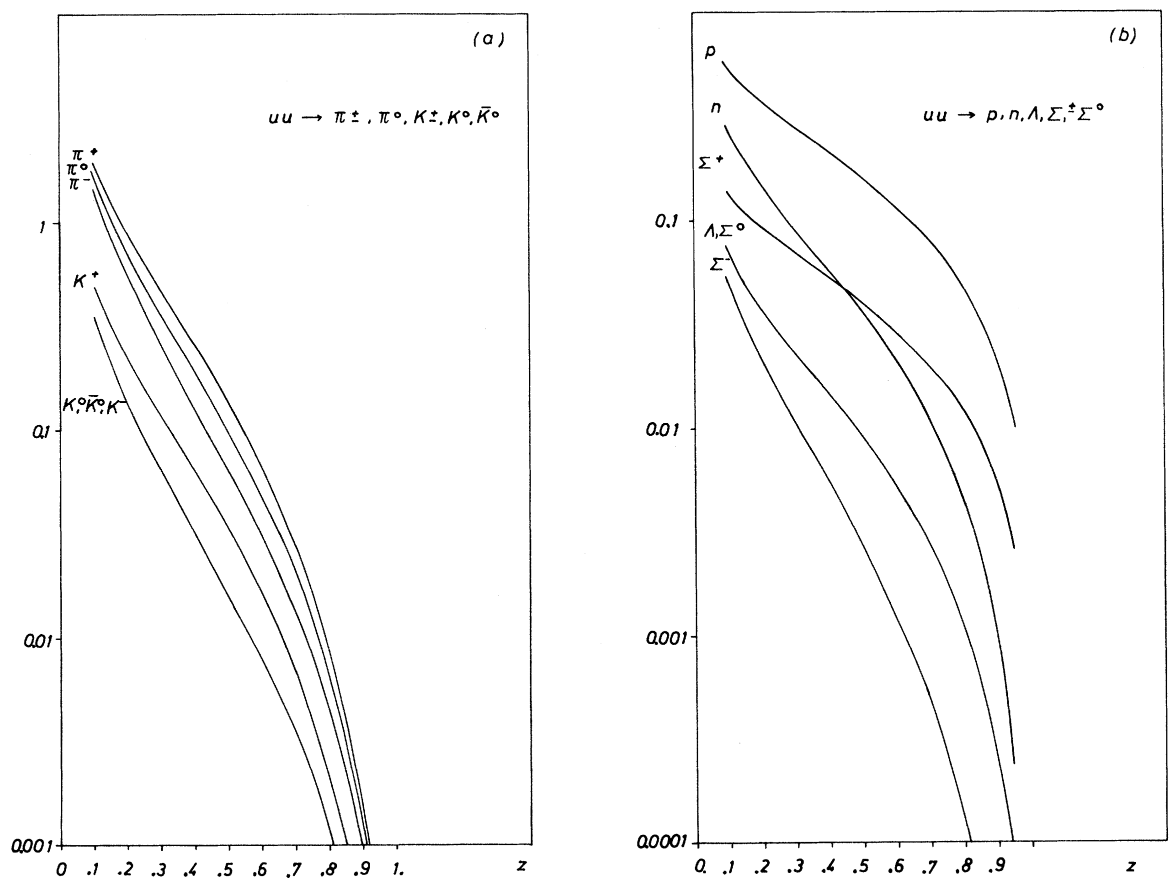


FIG. 3. Diquark fragmentation functions following from Eqs. (32) and (38) with the flavor coefficients as given in Tables I and II ($\gamma_s=0.1$, $\alpha_P=\alpha_V=\alpha_{1/2}=\alpha_{3/2}=0.5$), without resonance contributions. (a) $uu\to$mesons, (b) $uu\to$baryons, (c) $ud\to$mesons, (d) $ud\to$baryons.

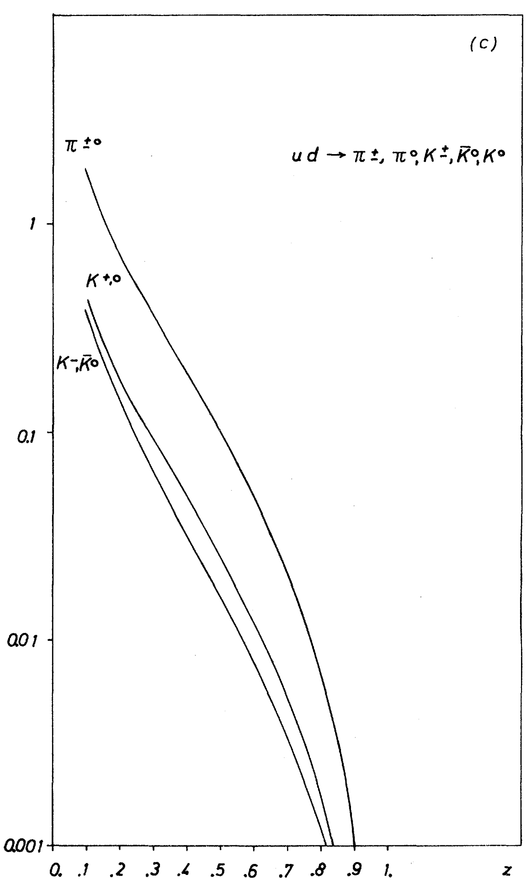
with

$$Q(z)=\left[\frac{4p_1p_3}{(2p_2+4p_4)(2p_2+4p_4-1)(2p_2+4p_4-2)}\right.$$
$$\left.-\frac{96p_2p_3}{(2p_2+4p_4)(2p_2+4p_4-1)(2p_2+4p_4-2)(2p_2+4p_4-3)(2p_2+4p_4-4)}\right]z^{2p_2+4p_4-1}\,. \tag{39}$$

Note that by Eqs. (3) only p_1 and p_3 (or p_2 and p_4) are independent parameters. As one would expect $B_q(z)$ $[B_d(z)]$ vanishes when $p_2 \to 0$ $[p_4 \to 0]$, but $M_q(z)$ and $M_d(z)$ vanish only if both p_1 and p_3 go to zero. We have plotted $zM_q(z)$, $zM_d(z)$, $zB_q(z)$, and $zB_d(z)$ in Fig. 2. The parameters were chosen as $p_1=0.92$, $p_3=0.5$, as this corresponds also to the realistic situation when one compares with experimental data. For comparison we have shown in Fig. 2 also $zF(z)$ from Ref. 1 which is obtained with an input function $f_1(\eta)=1-a+3a\eta^2$, with $a=0.7$. As one can see, the difference between $zF(z)$ and $zM_q(z)$ is less than 20%, apart from the range $z>0.9$ although rather different functions $f_i(z)$ are used.

Usually, the analysis of meson spectra in quark jets is done with a momentum-sharing function $f_1(z)=p_1(1-a+3az^2)$ as in Ref. 1. One can find an exact analytic solution to Eqs. (1) also in this case. The resulting formulas are somewhat lengthy and, therefore, are not explicitly given here. For $a=0.7$ the numerical difference to the solution (38) is less than 10%.

In Figs. 3(a)–3(d) we plotted the fragmentation functions for uu and ud diquarks into *primary* pseudoscalar mesons and spin-$\frac{1}{2}$ baryons, following from Eqs. (32c), (32d), (38d), and (38e). From Eqs. (22), (24), and (27) one can also calculate in this case



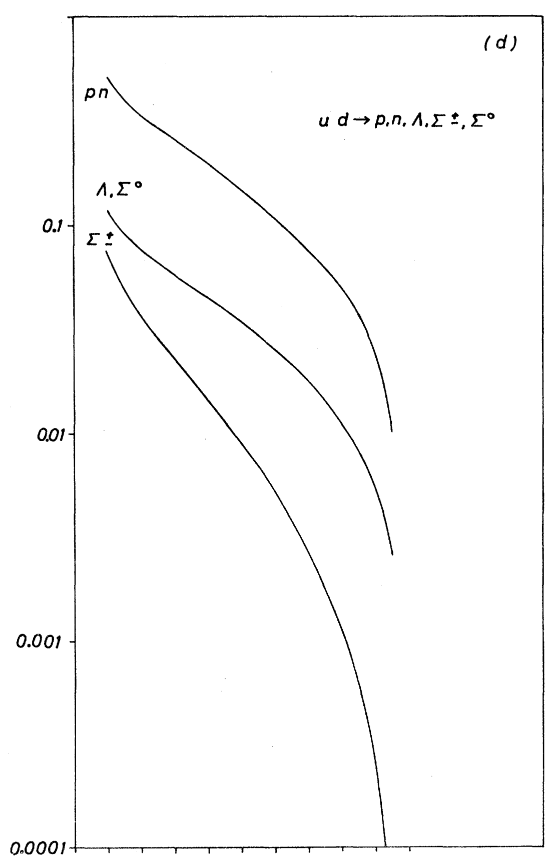


FIG. 3. (*Continued.*)

$$\kappa(z)=2p_3 z^{3-2p_3} , \tag{40}$$

$$J_M(z)=8p_3{}^2\left[\frac{1}{3-2p_3}-\frac{3z}{2-2p_3}+\frac{3z^2}{1-2p_3}+\frac{z^3}{2p_3}-\frac{3z^{3-2p_3}}{p_3(3-2p_3)(2-2p_3)(1-2p_3)}\right], \tag{41}$$

$$J_B(z)=8p_3p_4\left[\frac{1}{3-2p_3}-\frac{z}{2-2p_3}+\frac{z^{3-2p_3}}{(3-2p_3)(2-2p_3)}\right]. \tag{42}$$

B. Solution by approximate inverse Mellin transformation

In the analysis of deep-inelastic scattering several methods were developed to approximately reconstruct the structure functions of the nucleon from their moments. For arbitrary input functions $f_i(z)$ the moments of the fragmentation functions are known from Eqs. (7). Thus, if an exact solution cannot be obtained one can try such a method of approximate inversion here, too. We shall do this by following a procedure proposed in Ref. 13 which uses the normalized Bernstein polynomials:

$$b^{(N,k)}(z)=\frac{(N+1)!}{k!}\sum_{l=0}^{N-k}\frac{(-1)^l}{l!(N-k-l)!}z^{k+l}, \quad k=0,1,\ldots,N . \tag{43}$$

At each of the points

$$z_{N,k}=\int_0^1 dz\, b^{(N,k)}(z)z=\frac{k+1}{N+2} , \tag{44}$$

the average over the function $M_q(z)$, for example, is calculated by

$$\hat{M}_q(z_{N,k})=\int_0^1 dz\, b^{(N,k)}(z)M_q(z) ,$$

which by Eqs. (43) is given as

$$\hat{M}_q(z_{N,k})=\frac{(N+1)!}{k!}\sum_{l=0}^{N-k}\frac{(-1)^l}{l!(N-k-l)!}\tilde{M}_q(k+l) . \tag{45}$$

Analogous expressions hold for the other fragmentation functions. The average values of the fragmentation functions $\hat{M}_q(z_{N,k})$, etc., are directly obtainable from their moments [Eq. (7)]. The z interval $\Delta_{N,k}$ over which the functions are averaged, is given by

$$\Delta_{N,k}{}^2=\frac{N-k+1}{(N+2)^2(N+3)} . \tag{46}$$

For $k=2,3,\ldots N$ also a correction term to Eq. (45) can be calculated which is

$$\delta M_q(z_{N,k})=\frac{1}{2}\frac{(N+1)!(N-k+1)}{k!(N+2)^2(N+3)}\sum_{l=0}^{N-k}\frac{(-1)^l(k+l)(k+l-1)}{l!(N-k-l)!}\tilde{M}_q(k+l-2) . \tag{47}$$

For further details we refer to Ref. 13. With the same input functions and parameters as for the exact solution we calculated the fragmentation functions according to Eqs. (45) and (47) for $N=12$. The results for $zM_q(z)$, $zM_d(z)$, $zB_q(z)$, and $zB_d(z)$ are also plotted in Fig. 2, where the length of the averaging interval, $\Delta_{N,k}$, is also shown. As one can see, the method works well for $M_q(z)$ and $B_d(z)$. For $M_d(z)$ and $B_q(z)$ which vary more strongly, deviations from the exact solution up to 40% occur. The corrections according to Eq. (47) were less than 2%. It turned out that for $N\geq 15$ the results become numerically unstable. As we can compare with an exact solution, we have provided in this way a test of this method of approximate moment inversion.

V. COMPARISON WITH EXPERIMENT AND DISCUSSION

An essential amount of the particles observed in a jet is not directly produced but comes from resonance decays. For a comparison with experimental data it is therefore necessary to also include the

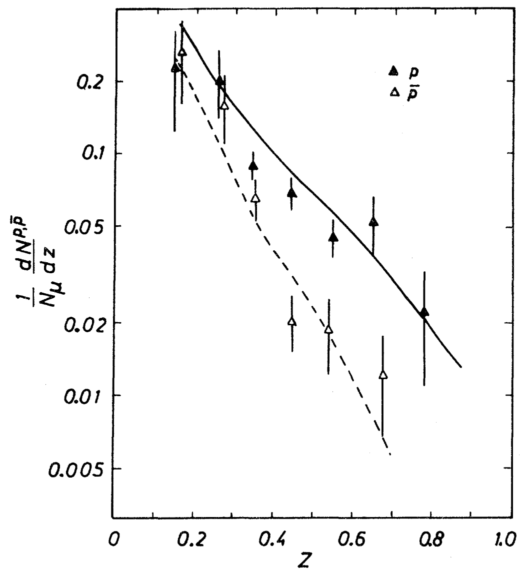


FIG. 4. Comparison of our model with p and $\bar{p}$ data of the European Muon Collaboration (Ref. 14). Theoretical curves obtained with $f_2=2p_2\eta$, $f_4=2p_4\eta$; $p_2=0.08$, $p_4=0.5$. Solid curve, p production. Dashed curve, $\bar{p}$ production. Here $z=E_h/\nu$, and jet energy corresponding to $\langle W\rangle=13$ GeV, $\langle Q^2\rangle=15$ GeV.

production and decay of resonances. This is most conveniently done by simulating the generation of a quark or a diquark jet by the Monte Carlo method. Our procedure is an extension of that of

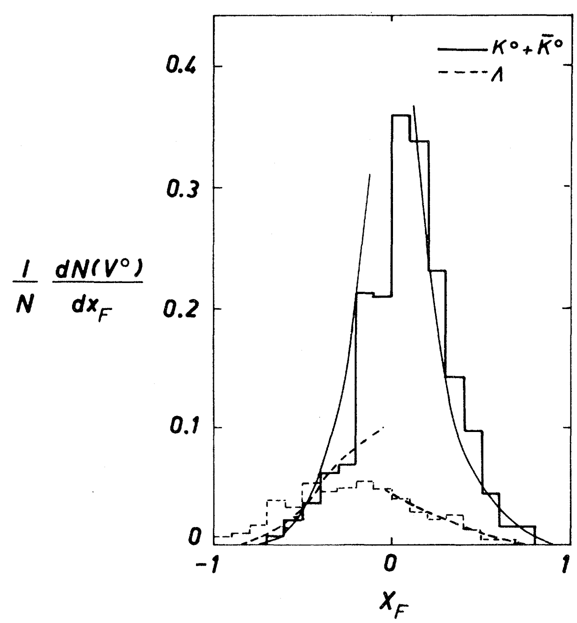


FIG. 5. Comparison with Λ and $K^0+\bar{K}^0$ neutrino-production data of BEBC (Ref. 15). Theoretical curves obtained with a quark or diquark jet energy of 3 GeV, corresponding to $\langle W\rangle=6$ GeV.

Field and Feynman by including particle production involving diquarks according to Figs. 1(b)–1(d) in addition to Fig. 1(a). Without inclusion of resonance decay this is of course equivalent to the analytic solution of the integral equations (15), as discussed previously. As usual the primary mesons are pseudoscalars or vectors in the rato 1:1. For baryon production only the $l=0$ states with spin $\frac{1}{2}$ and $\frac{3}{2}$ according to the ratio 1:1 (except of course for $\Omega^-,\Delta^{++},\Delta^-$) are taken into account. The two-particle decays are assumed to be isotropic in their rest system and the three-particle decays are supposed to go via two-particle decays. For the created quarks and diquarks we have included transverse momentum according to a Gaussian distribution with $\sigma_q=\sigma_{qq}=330$ MeV. The actual value taken for σ_{qq} has practically no influence on the longitudinal-momentum distributions.

It is not our intention to present detailed fits to all available experimental data. Instead we shall rather concentrate on characteristic examples to discuss the main features and determine the parameters. For this purpose we shall deal here only with jets in neutrino and muon production because in this case one quark flavor dominates in the interaction. These processes are therefore better suited for the study of fragmentation of light quarks and diquarks. At present we shall not treat fragmentation of heavy quarks ($c,b,\ldots$) as the weak decays play an important role here. Therefore, we shall for the moment not compare with e^+e^- data.

Choosing

$$f_1(\eta)=p_1(1-a+3a\eta^2)\ ,$$

$$a=0.7\ ,$$

and

$$f_i=p_i(d_i+1)\eta^{d_i}$$

for $i=2,3,4$ the parameters to be determined are p_1, p_3, and the powers d_i. The parameter $p_2=1-p_1$ is given by the rate of baryon production in quark jets. With the choice $p_2=0.08$ we got a satisfactory description of proton production by high-energy muons[14] as can be seen in Fig. 4. As for the power d_2 it turned out that the form $f_2(\eta)=2p_2\eta$ gave better agreement with data than the counting-rule ansatz $f_2(\eta)=4p_2\eta^3$. In particular with $d_2=3$ the rate of antiproton production at large z would be higher than for protons, which is not seen in the data. (Notice that in Fig. 4 and in Ref. 14 the variable $z=E_h/\nu$ and is therefore dif-

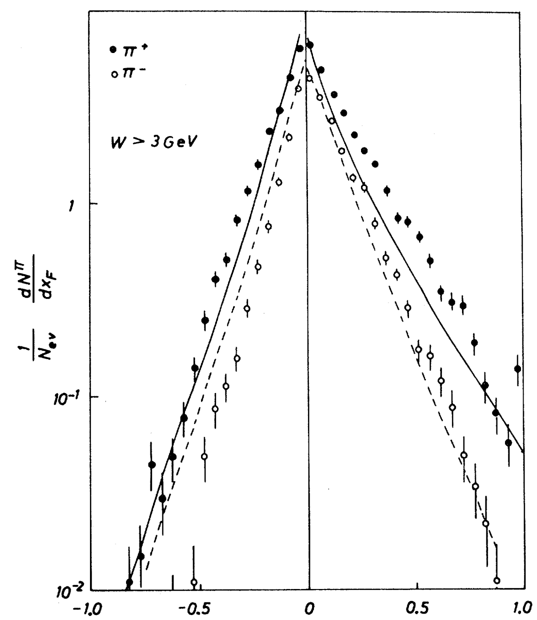


FIG. 6. π^+ and π^- neutrino-production data of BEBC (Ref. 17) compared to our model predictions. Solid curve π^+; dashed curve π^-. Quark or diquark jet energy taken to be 3 GeV. $f_1=p_1\,(1-a+3a\eta^2)$, $a=0.7$, $p_1=0.92$, $f_3=4p_3\eta^3$, $p_3=0.5$. For $x_F>0$ the result corresponds within 10% to the Feynman-Field model (Ref. 1).

ferent from the variable z used in our paper.) With $f_2=2p_2\eta$ and $p_2=0.08$ also a good description of Λ production by neutrinos[15] for $x_F>0$ (see Fig. 5) was obtained. The flavor dependence is essentially

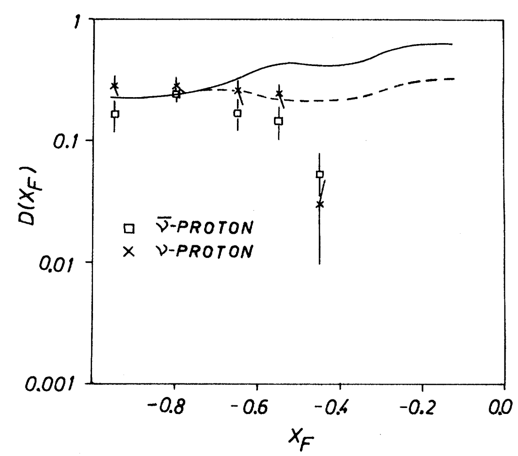


FIG. 7. Comparison with proton (anti) neutrino-production data of Ref. 18 in the target fragmentation region. Solid curve, νp; dashed curve, $\bar{\nu}p$. Model calculation including the leading-particle effect, with diquark jet energy of 3 GeV.

described by the parameter $\gamma_s=1-2\gamma$. A comparison with $K^0+\bar{K}^0$ production by neutrinos (Fig. 5) suggests a value $\gamma_s=0.1$, smaller than that originally used in Ref. 1. Such a value for γ_s was also found experimentally in Ref. 16 and used in Ref. 8. Moreover, it can be seen that the x_F dependence of $K^0+\bar{K}^0$ for $x_F>0$ is reasonably well reproduced. For the sake of completeness also a comparison with the forward π^+ and π^- neutrino-production data[17] is shown in Fig. 6. Owing to the value $p_1=0.92$ this reproduces within 10% the prediction of the Feynman-Field model.

In describing diquark fragmentation, the momentum-sharing functions $f_3(\eta)=4p_3\eta^3$ and $f_4(\eta)=2p_4\eta$ reproduce satisfactorily the observed z behavior. The remaining parameter is p_3. With the choice of $p_3=0.5$ we obtain fair agreement with π^+, π^-, $K^0+\bar{K}^0$, and Λ neutrino production for $x_F<0$ as can be seen in Figs. 5 and 6. The rate of antibaryons produced in quark jets is also directly proportional to $p_4=1-p_3$ [see Eq. (38c)]. $p_3=0.5$ also describes the $\bar{p}$ production by muons (see Fig. 4) quite well.

We tried to determine more accurately the parameter p_3 which is a measure of the breakup of the diquark. On the basis of the present experimental situation we can only say that p_3 has to be in the range between 0.4 and 0.8. The case $p_3=0$ (no splitting of the diquark) seems to be excluded because then the rates of π^+ and π^- in neutrino production would practically become equal for $x_F<-0.5$ contrary to the data (see Fig. 6). The following quantities are sensitive to the value of p_3: the ratio of π^+ to π^- neutrino production for $x_F<0$, baryon production for $x_F<0$, in particular the ratio $D_{uu}^{\Lambda}(z)/D_{ud}^{\Lambda}(z)$, and antibaryon production in muon and neutrino scattering. More accurate data on these processes are therefore necessary for a better determination of p_3.

With $f_4(z)$ as given above we could not reproduce the proton production by (anti)neutrinos of Ref. 18 for $x_F<-0.6$. It seems (see Fig. 7) that the spectrum does not fall off as strongly as one would expect from the fragmentation functions shown in Fig. 2. A possible explanation for this behavior could be that the backward protons show the so-called leading-particle effect. A further indication for this effect could also be seen in the difference between forward and backward multiplicities in neutrino production.[17] We simulated such a behavior of the proton in the following simple way which is most easily implemented into the Monte Carlo procedure: if the final proton is produced as a rank-one particle, its momentum depen-

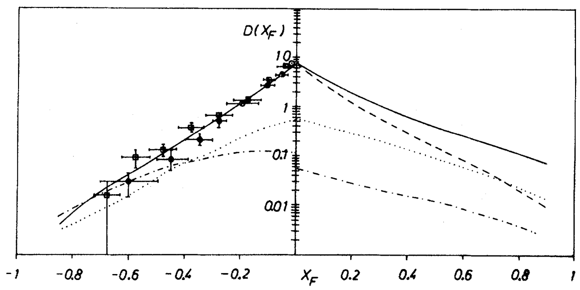


FIG. 8. Model predictions for antineutrino production of π^+, π^-, $K^0+\bar{K}^0$, and Λ for quark and diquark fragmentation with quark or diquark jet energy of 3 GeV. Data for π^+ and π^- production from Ref. 18. Solid curve, π^- (for $x_F>0$), and $\pi^\pm$ (for $x_F<0$). Dashed curve, π^+. Dotted curve, $K^0+\bar{K}^0$. Dashed-dotted curve, Λ. □ negative mesons, data from Ref. 18, ● positive mesons, data from Ref. 18.

dence is not given by $f_4(1-z)=2p_4(1-z)$ but by a function $\tilde{f}(z)$ which for simplicity we take as $\tilde{f}(z)=2p_4\theta(z-0.5)$. The higher-rank protons are distributed according to $f_4(\eta)$. With this simple method one gets reasonable agreement for $x_F<-0.5$ (Fig. 7).

In Fig. 8 we show the predictions of the model for antineutrino production of $\pi^+,\pi^-,K^0+\bar{K}^0$. We compared them with the data of Ref. 18. Whereas we got good agreement for the π^+ and π^- distributions, for $K^0+\bar{K}^0$ and Λ production our results are too high. As our model agrees with the neutrino-production data[15] of BEBC and one expects practically equal fragmentation functions D_u^Λ and D_d^Λ, we have no explanation for this discrepancy. Further data are needed to draw final conclusions.

Summarizing we can say that the treatment of quark and diquark fragmentation within the system of integral equations as discussed here provides a satisfactory description of meson, baryon, and antibaryon distributions in the current and target fragmentation regions in leptoproduction. We found an analytic solution including the flavor dependence which shows the characteristic features of the fragmentation functions and clearly exhibits the dependence on the main parameters. The experimental baryon spectra suggest that in a quark jet the first-rank particle is a baryon with about 8% probability. Moreover, we get better agreement with the momentum-sharing function $f_2(\eta)\sim\eta$ than with the counting-rule behavior $f_2(\eta)\sim\eta^3$. By comparing with present data our results also indicate that the diquark has an approximately 50% probability to break up. So far it seems that the diquark forming a baryon in a quark jet behaves in the same way as the diquark responsible for target fragmentation. The only difference could be the leading-particle effect of the target proton which, however, can be incorporated quite easily.

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