

Analysis of neutron-multiplicity frequencies observed in a double monitor

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The neutron-multiplicity distribution was measured in Turku, Finland (geomagnetic latitude 58° N, 40 m above sea level), in the range 1–380 by means of a double neutron monitor during the period 6 October 1980–22 February 1981. The analysis of the measured distribution was based on the computation of Monte Carlo predictions for neutron evaporation in hadron cascades initiated by cosmic-ray particles in the monitor. Results were obtained for the differential energy spectrum of cosmic-ray neutrons in the range 10–4000 MeV and of all hadrons in the range 4–1000 GeV at sea level. The results show changes in the slopes of the spectra at 270 MeV, 8.5 GeV, and 200 GeV. Interpretations for these changes are given.

INTRODUCTION

Experimental determination of the energy spectrum of hadrons at sea level involves several difficulties. Neutrons dominate in the low-energy region. At high energies they still are as abundant as protons, whereas muons are much more numerous than hadrons. The detection of both neutral and charged hadrons and their separation from the nuclear inactive muons set special demands for the measuring system.

The energy spectrum of cosmic-ray neutrons below 0.5 GeV has been measured by Hess *et al.*¹ by using a bismuth-fission ionization chamber. At comparable energies (80–300 MeV) Heidebreder *et al.*² used the double-elastic-scattering technique and at somewhat higher energies (0.4–1.2 GeV) Ashton *et al.*³ applied the charge-exchange reaction to determine the energy spectrum of neutrons. In the intermediate-energy range (300 MeV to a few GeV) the spectrum has been estimated from multiplicity distributions of neutrons in neutron monitors.^{4,5} At high energies (> 20 GeV), the spectrum has been obtained from the burst spectrum produced by neutral primary particles in a thick steel target.⁶

The energy spectrum of cosmic-ray protons has been measured at low energies (< 0.5 GeV) by various techniques.⁷ Above 0.5 GeV the measurements have been made by using a magnet spectrograph in connection with a neutron monitor.^{8,9}

At high energies there are several measurements of charged^{10–12} and all hadron^{13–15} spectra. These measurements are usually made with ionization calorimeters. Recently, a frequency spectrum of

detected neutron multiplicities in a neutron monitor in the range 5–200 was inverted into the energy spectrum of cosmic-ray hadrons at energies 0.5–500 GeV.¹⁶ The results showed reasonable agreement with the results obtained with ionization calorimeters at high energies and with other techniques at lower energies. Thus, it is evident that the neutron monitor provides a method to monitor the energy spectra of hadrons in a wide range, from 0.1 GeV up to 1000 GeV, by using only one apparatus. It is then easier to study the form and time variations of the hadron spectra.

In the following, we present the result of a measurement of the multiplicity distribution in our new double neutron monitor. We also give some Monte Carlo estimates of particle production due to cosmic-ray hadrons in the monitor and analyze the measurement on the basis of these estimates.

DOUBLE NEUTRON MONITOR

A drawback of standard neutron monitors, when applied to multiplicity studies of cosmic rays, is the very low efficiency for detecting evaporation neutrons. This results in a low counting rate of high-multiplicity events. Another disadvantage from the point of view of energy spectrum measurements is the poor energy resolution. To overcome these difficulties, a double neutron monitor was designed. The construction and characteristics of the apparatus was described earlier by Arvela *et al.*¹⁷

The cross section of the double neutron monitor is shown in Fig. 1. The essential parts of the ap-

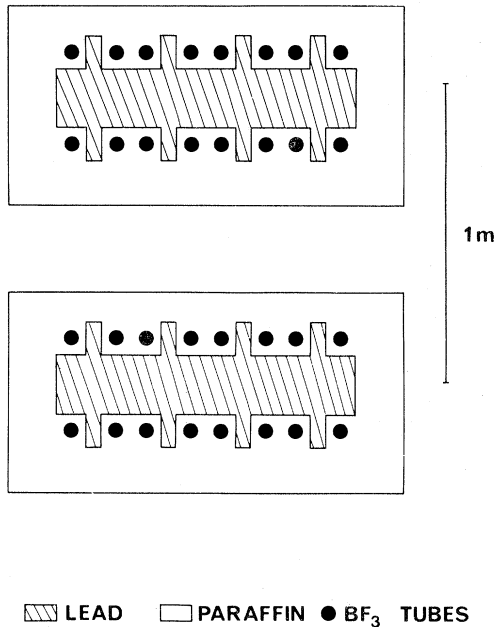


FIG. 1. Schematic representation of the double monitor.

paratus are two neutron monitors of special design, one on top of the other. The thickness of the lead producer in both monitors is 271 g/cm^2 , and the area is 1.0 m^2 . To achieve good efficiency, the neutron counters are mounted in two layers, above and under the lead target with eight BF_3 tubes in each layer. The efficiency was determined experimentally with a calibrated Ra-Be source and was found to be $\epsilon = (12.5 \pm 0.5)\%$ in both monitors.

An advantage of the double monitor is the relatively large total thickness of the production layer (about 3 mean free paths of high-energy hadrons), which provides a greater probability for inelastic interactions of the incident particle and of its secondaries in the monitor. Consequently, the average multiplicity of evaporation neutrons is also large.

Another important advantage of the double monitor is the possibility of obtaining two independent simultaneous measurements of the multiplicity associated with a cascade produced by a high-energy hadron. This is due to the thick layer of paraffin between the two monitors, which prevents the evaporation neutrons produced in one of the monitors from reaching the other. The two separate measurements of multiplicity increase the energy resolution of the monitor.

The numbers of neutrons detected in the upper and lower monitors are denoted by m_1 and m_2 ,

respectively. To determine the energy spectrum of cosmic-ray hadrons, the frequencies $f(m_1, m_2)$ of multiplicities m_1 and m_2 are measured during a certain period of time. A microprocessor is used for collecting and recording the data.

MEASUREMENT OF THE FREQUENCY SPECTRUM OF MULTIPLICITIES

The measurement with the double monitor was performed during the period 6 October 1980–22 February 1981. The effective duration of the experiment was 2021 h. The hourly counting rates were corrected to the standard air pressure of 1010 mbar. The barometric coefficients $\beta(m_1, m_2)$ were estimated for each multiplicity pair (m_1, m_2) . At high multiplicities $m_1 + m_2 \geq 10$, the values of β turned out to be nearly independent of m_1 and m_2 . In Fig. 2, the observed frequency spectrum is given as a function of the total multiplicity m ($=m_1 + m_2$).

MONTE CARLO METHOD FOR THE SIMULATION OF HADRON CASCADES

The inversion of the multiplicity distribution into the energy spectrum requires knowledge of hadron cascades in the monitor in a wide energy range from a few MeV up to several hundred or thousand GeV. We used the Monte Carlo method

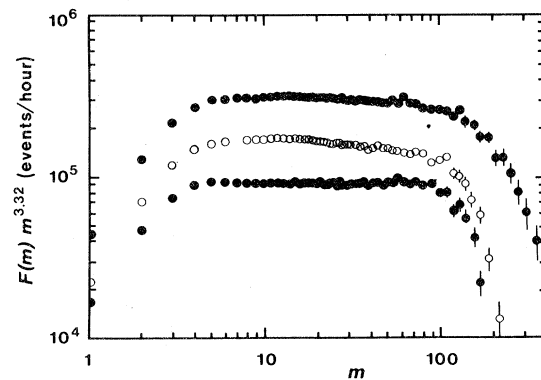


FIG. 2. Frequency spectrum of total multiplicities $F(m)$ observed in the Turku double neutron monitor (black points at the top), in the upper (open circles), and in the lower monitor (black points at the bottom).

of Nieminen and Torsti¹⁸ to calculate the neutron production in the monitor. In this method, the multiplicities of secondary neutrons, protons, pions, and light nuclei are first computed for an individual inelastic collision in three different energy intervals (I: $E \leq 30$ MeV; II: $30 \leq E \leq 380$ MeV; III: $E \geq 380$ MeV). The multiplicities of various particles are assumed to obey the Kobayashi-Nielsen-Olesen scaling hypothesis.^{19,20}

The energies and emission angles of the particles produced in the collision are generated using known empirical or theoretical relations. Especially, the scaling distributions make the cascade simulations possible at arbitrary high energies at least in principle. The propagation of secondary particles in the monitor is followed until the particles have reached the minimum energy of 5 MeV or leaked away from the monitor or decayed.

To analyze the measured multiplicity distribution, we calculated the probabilities $p(E, m_1^e, m_2^e)$ that a cosmic-ray hadron of energy E produces m_1^e and m_2^e evaporation neutrons in the upper and lower monitors, respectively.

The probability densities of evaporation neutron production for one incoming neutron or proton are shown in Table I. If the incoming particle is a neutron, the probabilities P_0 (no neutron production), P_1 (neutron production only in the upper monitor), P_2 (neutron production only in the lower monitor), and P_{12} (neutron production in both

monitors) dominate in the energy ranges $E \leq 80$ MeV, $80 \text{ MeV} \leq E \leq 5 \text{ GeV}$, $5 \text{ GeV} \leq E \leq 10 \text{ GeV}$, and $E \geq 10 \text{ GeV}$, respectively. In the case of protons, P_0 dominates up to 1 GeV. At energies $E > 3 \text{ GeV}$, no great differences exist in the low-energy neutron production between incident neutrons and protons. The "scaling" of the probabilities occurs at energies $E \geq 100 \text{ GeV}$, where $P_0 \simeq 0.25$, $P_2 \simeq 0.25$, $P_{12} \simeq 0.5$, and $P_1 \simeq 0$.

The average multiplicity of evaporation neutrons has the following energy dependence:

$$\langle m^e \rangle = \begin{cases} 34E^{0.64}, & \text{for } 0.05 < E \leq 5 \text{ GeV} , \\ 38E^{0.56}, & \text{for } 5 < E \leq 1000 \text{ GeV} , \end{cases}$$

if there is neutron production in both monitors. Thus, in the energy range where the observed multiplicity distribution is of power form, the energy spectrum of hadrons has also power form, if the multiplicity distribution of produced particles and their energy distribution show scaling behavior. Due to the rapid increase of the average multiplicity ($\langle m^e \rangle \simeq 1800$ at $E = 1000 \text{ GeV}$), the energy spectrum of cosmic-ray hadrons can be monitored by means of the double monitor, if more than a few percent of evaporation neutrons can be detected.

The counting rate of neutrons in the double monitor is

TABLE I. Probabilities of evaporation neutron production in the double monitor at various energies of neutrons and protons: P_0 = no neutron production, P_1 = neutron production only in the upper monitor, P_2 = neutron production only in the lower monitor, and P_{12} = neutron production in both monitors.

E (GeV)	P_0	P_1	P_2	P_{12}
Neutrons				
0.01	0.81	0.17	0.02	0.00
0.03	0.60	0.33	0.07	0.00
0.1	0.39	0.40	0.20	0.01
0.3	0.33	0.39	0.25	0.03
1.0	0.27	0.36	0.29	0.08
3	0.25	0.33	0.28	0.14
10	0.25	0.17	0.27	0.31
30	0.25	0.06	0.26	0.43
100–5000	0.25	0.02	0.26	0.47
Protons				
0.01–0.1	1.00	0.00	0.00	0.00
0.3	0.85	0.12	0.03	0.00
1	0.38	0.36	0.23	0.03
3	0.27	0.32	0.29	0.12

$$f(m_1, m_2) = \sum_{i=n, p, \pi, \dots} \int_0^\infty R_i(E, m_1, m_2) dE.$$

The integrand R_i is the energy response function for particles i ,

$$R_i(m_1, m_2) = AN_i(E)P_i(E, m_1, m_2).$$

Here, A is a constant (the acceptance), $N_i(E)$ is the differential vertical energy spectrum of particles i , and P_i is the probability distribution of detecting m_1 and m_2 neutrons in the upper and lower monitors, respectively, produced by an incident particle i of energy E . The probabilities P_i ($i=n, p$) were calculated from the distributions $p_i(E, m_1^e, m_2^e)$ by using the binomial distribution.

The average energies of cosmic-ray nucleons corresponding to multiplicity $m = m_1 + m_2$ are

$$\langle E_i(m) \rangle = \int_{E_{\min}}^\infty ER_i(E, m) dE / \int_{E_{\min}}^\infty R_i(E, m) dE.$$

By using the energy spectrum of protons given by Brooke and Wolfendale,⁸ it was found that the following relation is a good approximation to the relation between the observed multiplicity and the average energy of a cosmic-ray hadron incident on the double monitor:

$$\langle E_h \rangle = 0.04m^{1.8} \text{ GeV, for } m > 5.$$

INVERSION INTO THE ENERGY SPECTRUM

In order to invert the multiplicity spectrum into an energy spectrum of hadrons, $N_h(E)$, the following main assumptions and computations were made.

(1) The vertical energy spectrum of incoming hadrons is of power form in successive intervals:

$$f_i(m_1, m_2) = A \int_{E_{\min}}^\infty dE N_h(E) \sum_{m_1^e \geq m_1'} \sum_{m_2^e \geq m_2'} p_h(E, m_1^e, m_2^e) B(m_1^e, m_1') B(m_2^e, m_2') J(m_1', m_2' / m_1, m_2),$$

contain the acceptance factor of the monitor, A ($=1.21 \text{ m}^2\text{sr}$), and the binomial probabilities B which depend on the efficiency ϵ and on the real pulse rates m_1' and m_2' . The amplifier dead time ($8 \mu\text{s}$) and the gate time ($850 \mu\text{s}$) reduce the real pulse rates for $m_1, m_2 > 1$. According to the Monte Carlo calculations, a good approximation to the relation between the recorded and real pulse

$$N_h(E) = \begin{cases} A_1 E^{-\gamma_1}, & \text{for } E_1 \leq E \leq E_2, E_1 = 10 \text{ MeV} \\ A_2 E^{-\gamma_2}, & \text{for } E_2 < E \leq E_3 \\ \dots & \\ A_N E^{-\gamma_N}, & \text{for } E_N < E \end{cases}$$

There are two free parameters for each energy interval, $(A_1, \gamma_1), (E_2, \gamma_2), \dots, (E_N, \gamma_N)$. The number of intervals N is also a free parameter. Instead, parameters A_2, A_3, \dots depend on the other parameters because of the requirement of continuity.

(2) The intensity of protons deviates from that of neutrons in the low-energy region due to the energy losses in air. Therefore, the spectral indexes of neutrons and protons were allowed to be different in the region $E < 4 \text{ GeV}$.

(3) The contribution of muons to the observed frequencies in the multiplicity range $m \leq 9$ were taken into account. The estimates given by Arvela *et al.*¹⁷ were used for this correction.

(4) Monte Carlo predictions for the probability density of neutron evaporation, $p_i(E, m_1^e, m_2^e)$, were computed in the energy range from 10 MeV up to 5000 GeV of the incident particle. Multiplicities m_1^e and m_2^e represent the numbers of neutrons in the energy range $\leq 5 \text{ MeV}$. Probability densities were assumed to be independent of the particle species (n, p , or π^\pm) in the range $E > 5 \text{ GeV}$.

Numerically it was found that the probabilities p_i for evaporation particle production scale in the energy range $E \geq 10 \text{ GeV}$, and that a good scaling variable is $(m_1^e + m_2^e) / \langle m_1^e + m_2^e \rangle$. Earlier, a comparison was given¹⁸ between the predictions of the cascade model used here and that of Shen²¹ for the average multiplicity of evaporation neutrons in a 13-cm-thick lead target. The absolute agreement between the predictions was quite good in the range 0.05–20 GeV which indicates that the evaporation particle production is not so much model dependent as could be thought at first sight.

(5) The theoretical frequencies

rates m_i and m_i' is

$$m_i' = 1.045m_i(1 - 0.0015m_i)^{-1}, \quad i = 1, 2, \quad m_i \geq 5.$$

The function J is the Jacobian of the transformation between the recorded and real pulse rates.

(6) The values of the parameters in the hadron

spectrum were obtained by applying the usual minimization technique into the χ^2 function. This function represents the deviation of the observed from the theoretical frequencies. Each deviation was weighted by the inverse of its error consisting of both the statistical error of the measured frequency and of the error in the estimation of the theoretical frequency. The CERN-library program MINUIT was used in the numerical solving of the optimization problem.

The optimization gave the following values for χ^2/DF : 9.6 for $N=3$ and 1.5 for $N=4$. No further reduction was gained in χ^2/DF with the increase of N . Therefore, the value $N=4$ was accepted as the best fit to the spectrum. It follows that there are three changes in the spectrum in the range 10 MeV–1000 GeV.

In the energy region where neutrons dominate, the following result was obtained for the vertical differential energy spectrum:

$$N_n(E) = \begin{cases} (3.7 \pm 0.2)E^{-1.48 \pm 0.05}, & \text{for } 0.01 \leq E \leq 0.27 \text{ GeV}, \\ (0.98 \pm 0.05)E^{-2.51 \pm 0.05}, & \text{for } 0.27 < E \leq 4 \text{ GeV}. \end{cases}$$

Above this range, the vertical differential energy spectrum of all accompanied hadrons incident on the monitor was found to be of the form

$$N_h(E) = \begin{cases} (1.95 \pm 0.10)E^{-2.51 \pm 0.05}, & \text{for } 4 \leq E \leq 8.5 \text{ GeV} \\ (1.03 \pm 0.05)E^{-2.22 \pm 0.07}, & \text{for } 8.5 < E \leq 200 \text{ GeV} \\ (22 \pm 2)E^{-2.8 \pm 0.1}, & \text{for } E > 200 \text{ GeV}. \end{cases}$$

The unit of the intensities is $\text{m}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{GeV}^{-1}$.

DISCUSSION

Frequency spectra of total multiplicities

The three frequency spectra presented in Fig. 2 are of the form $F(m) \propto m^{-3.3}$ in a wide multiplicity region. However, at large multiplicities ($m \geq 100$), the frequency distributions are much steeper. The change in the distribution occurs approximately at multiplicities 100, 95, and 130 in the upper, lower, and double monitors, respectively. The frequencies corresponding to these multiplicities are approximately equal in all three cases, being 0.025–0.027 events/h. This agreement strengthens the result of the earlier measurement of

1752 h made with a single monitor (presently lower monitor) by Nieminen *et al.*¹⁶ In this case the change in the slope of the spectrum was at the multiplicity $m=105$. The corresponding counting rate was 0.025 events/h. The slopes before and after the change are also in good agreement with the new results.

The agreement obtained strengthens the physical interpretation of the change of the slope given earlier.¹⁶ According to this interpretation, there is a corresponding change in the energy spectrum of cosmic-ray hadrons. The possibility of some unknown fault in the experimental equipment as the cause of the change is diminished. We also hold the opinion that any breakdown in the scaling hypothesis, concerning the multiplicity distribution and particle production mechanism in hadron–lead-target interactions, is not the reason for the changes observed in $F(m)$.

Energy spectra

In Fig. 3, the results of various measurements and theoretical calculations of the differential neutron energy spectra are given. All the spectra have been adjusted to sea level assuming an attenuation length of 125 g/cm^2 . The neutron spectrum of Lal²² was determined at the height of 680 g/cm^2 . Therefore, the absolute intensities of Lal²² given in Fig. 3 may be somewhat uncertain due to the large influence of the value chosen for the attenuation length. The zenith angle dependence of neutrons was assumed to be of the form $f(\theta) \propto \cos^8 \theta$. This

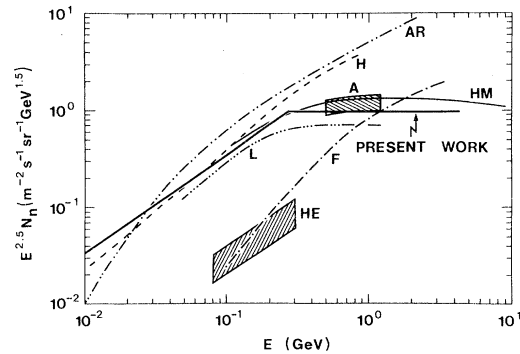


FIG. 3. Vertical energy spectrum of cosmic-ray neutrons at sea level according to various experiments and calculations. Thick solid curve, present work; A, Ashton *et al.* (Ref. 3); H, Hess *et al.* (Ref. 1); HM, Hughes and Marsden (Ref. 4); HE, Heidebreder *et al.* (Ref. 2); L, Lal (Ref. 22); F, Flückiger (Ref. 24); AR, Armstrong *et al.* (Ref. 23).

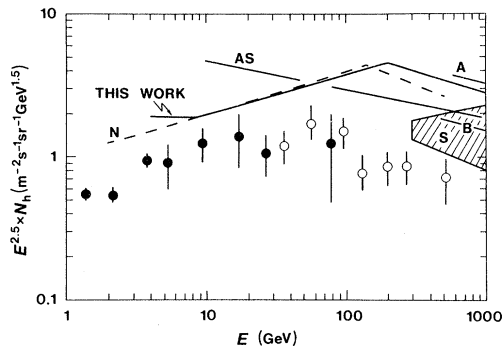


FIG. 4. Vertical energy spectra of cosmic-ray hadrons at sea level from various experiments. Thick solid curves, present work ($n + p + \pi$); \circ , Ashton and Coats (Ref. 6) (n); A, Ashton *et al.* (Ref. 14) ($n + p + \pi$); AS, Ashton and Saleh (Ref. 13) ($n + p + \pi$); B, Baruch *et al.* (Ref. 15) ($n + p + \pi$); \bullet , Brooke and Wolfendale (Ref. 8) (p); N, Nieminen *et al.* (Ref. 16) ($n + p + \pi$); S, Siohan *et al.* (Ref. 12) ($p + \pi$).

form was used to change the global spectra, as, for example, those of Hess *et al.*¹ and Hughes and Marsden,⁴ to vertical intensities.

In the theoretical spectrum of Armstrong *et al.*,²³ there appear quite large statistical fluctuations. These are smoothed out in Fig. 3. In the calculations of Armstrong *et al.*²³ and Flückiger,²⁴ primary α particles are taken into account. The theoretical spectrum of Flückiger²⁴ and the experimental results of Heidbreder *et al.*² differ considerably from the other results. The spectrum of Flückiger²⁴ given in Fig. 3 is very similar to that given later by Flückiger.²⁵ The former seems to have, however, a larger statistical weight.

According to Hess *et al.*¹ the spectrum falls off as $E^{-1.4}$ in the range from 1 MeV up to 800 MeV. Hughes and Marsden⁴ modified this result above 350 MeV, in which region they claimed the spectrum to be much steeper. This modification was based on neutron monitor data and on the assumption that the neutron spectrum should have the same form as the proton spectrum in the high-energy limit. The result of Ashton *et al.*³ based on the charge-exchange reaction confirmed the modified spectrum.

Our result agrees quite well with these results.

An explanation for the change in the spectrum at about 300 MeV could be the break in the fragmentation process of neutrons in neutron—air-nucleus collisions. This would manifest itself as a relative increase in the probability of neutron evaporation versus fragmentation at low energies.

The reason for the change in the slope of the hadron spectrum from -2.5 to -2.22 is that pions become gradually as numerous as neutrons and protons. In addition, our results indicate that the ratio of pions to nucleons reaches its limiting value at 200 GeV. Above this energy, the slope is steep (steeper than in the range $E < 8.5$ GeV where the pions are almost lacking). This reflects the logarithmic increase of the hadron—air-nucleus cross section with energy.

In Fig. 4 several results of differential hadron spectra are shown. The proton intensity of Brooke and Wolfendale⁸ amounts to roughly half of our intensity of all hadrons at energies 4–10 GeV. In the region of several hundred GeV our results are in reasonable agreement with the hadron spectra of Ashton and Saleh,¹³ Ashton *et al.*,¹⁴ and Baruch *et al.*¹⁵

For the sake of comparison, the energy spectrum of neutrons of Ashton and Coats⁶ and the spectrum of accompanied charged hadrons of Siohan *et al.*¹² are also given. These results are in qualitative agreement with our result in the region $E \gtrsim 300$ GeV, as can be seen by comparing our intensity with the sum of these comparison intensities.

The consistency between the present results and our previous results obtained by means of a single monitor¹⁶ is a demonstration of the reliability of the multiplicity distribution measurements and of the method used in the inversion of the multiplicity distribution of detected neutrons into the energy spectrum of hadrons.

ACKNOWLEDGMENTS

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