S-matrix approach to the theory of hadron structure functions

N. G. Antoniou
Department of Theoretical Physics, University of Athens, Athens 621, Greece

S. D. P. Vlassopulos
Physics Laboratory II, National Technical University, Zographou, Athens 624, Greece
(Received 20 January 1982)

We argue that the factorized long-distance part of the hadron cross section can be constrained by S-matrix techniques. Specifically, assuming (i) that the low-p_T multiplicity, being a by-product of a multijet production mechanism generated by iteration of the standard hard-scattering cross section in the QCD framework, increases indefinitely with energy, and (ii) that the hard hadron cross section is a logarithmic power of the energy asymptotically, we get a unique prediction for the hadron structure function.

Although the study of hadronic processes involving hard excitation of partons is not the most appropriate method to test quantum chromodynamics (QCD), due to the complexity of hadron-hadron collisions, it may lead to a strong constraint for the hadron structure functions if we employ the properties of the hadronic S matrix. In this work we show that, in fact, the study of a particular mechanism for multijet production in hadron-hadron collisions may lead, under some fairly general assumptions, to a solution for the hadron structure function $G(x,Q^2)$ which is consistent with perturbative QCD.

In a hard process involving parton subcollisions with large subprocess momentum transfers one is usually interested in studying inclusive hadrons emerging from the hadronization of a hard parton (jet) in order to clarify the fundamental properties of parton interactions within the framework of large-p_T physics. On the other hand, there exists a whole class of multijet production diagrams, given by iteration of the standard hard-parton-scattering mechanism[1] [Fig. 1(a)], in the framework of QCD, which generate inclusive hadrons with low transverse momenta [Fig. 1(b)]. These inclusive processes represent a well-defined component of low-p_T physics, which is characterized by long-rapidity-range correlations, due to the presence of hard partons in the final state. These processes are distinct from those involving the production of only low-p_T hadrons, which are analyzed within the framework of Reggeon field theory.

The basic quantity needed to build up the above production mechanism is the contribution of the hard parton scattering (one-gluon exchange) to the total hadronic cross section[1] [Fig. 1(a)],

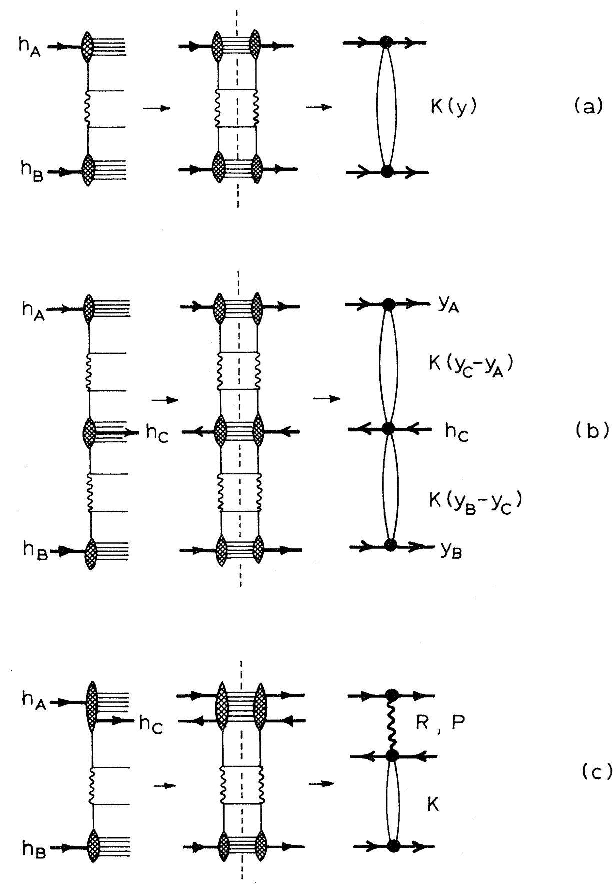

FIG. 1. Inclusive production of low-p_T hadrons from a class of tree graphs involving hard quark scattering and high-p_T jet production. (a) Standard (lowest-order) graph: Two-jet production with any low-p_T hadrons. (b) The lowest-order graph is iterated once to give rise to four high-p_T jets plus inclusive production of one central low-p_T hadron. (c) The standard hard-scattering mechanism coupled to Reggeon exchange.

$$\sigma_h(s)=\int_{s_0/s}^{1}dx_a\int_{s_0/sx_a}^{1}dx_b\int_{-\hat{s}/2}^{\hat{t}_{\min}}d\hat{t}\,G(x_a,-\hat{t})G(x_b,-\hat{t})\frac{d\hat{\sigma}}{d\hat{t}}\,, \tag{1}$$

where $G(x,Q^2)$ is the hadron structure function and $d\hat{\sigma}/d\hat{t}$ the differential cross section of the subprocess. The hard cross section (1) may play the role of a Mueller propagator in the inclusive diagrams [Fig. 1(b)] if we assume that the joint probability $G(x_b,x_c;-\hat{t}_{cd})$ appearing in the one-particle-inclusive cross section

$$\frac{d\sigma^{\text{in}}}{dy}=\int dx_a dx_b d\hat{t}_{ab} dx_c dx_d d\hat{t}_{cd} G(x_a,-\hat{t}_{ab})\frac{d\hat{\sigma}_{ab}}{d\hat{t}_{ab}}G(x_b,x_c;-\hat{t}_{ab},-\hat{t}_{cd})\frac{d\hat{\sigma}_{cd}}{d\hat{t}_{cd}}G(x_d,-\hat{t}_{cd}) \tag{2}$$

is factorized as follows:

$$G(x_b,x_c;-\hat{t}_{ab},-\hat{t}_{cd})=G(x_b,-\hat{t}_{ab})G(x_c,-\hat{t}_{cd})\,.$$

This assumption is motivated by the general properties of the parton model and the fact that the probing momentum transfers $\hat{t}_{ab},\hat{t}_{cd}$ for the two partons b and c are independent.

Hence, the semi-inclusive cross section for the production of q hadrons with rapidities $y_1,y_2,\ldots,y_q$ and small momentum transfers together with a hadronic missing mass of $2(q+1)$ hard jets and any number of soft hadrons is given by the factorized form

$$\frac{d^q\sigma^{\text{in}}}{dy_1dy_2\cdots dy_q}=K(y_A-y_q)K(y_q-y_{q-1})\cdots K(y_1-y_B)\,. \tag{3}$$

The Mueller propagator in (3) is identified with the hard cross section (1), i.e., $K(y_i-y_{i-1})\equiv\sigma_h(s_{i,i-1})$.

From the densities (3) we may extract the semiexclusive propagator $K_0(y_i-y_{i-1})$ which generates the factorized semiexclusive distributions, i.e.,

$$\frac{d^q\sigma^{\text{ex}}}{dy_1dy_2\cdots dy_q}=K_0(y_A-y_q)K_0(y_q-y_{q-1})\cdots K_0(y_1-y_B)\,. \tag{4}$$

Equation (4) gives the cross section for producing q soft hadrons with rapidities $y_1,y_2,\ldots,y_q$ plus any large-p_T hadrons. If we introduce the Laplace transform

$$\tilde{K}(p)=\int_0^\infty e^{-py}K(y)dy\,, \tag{5}$$

we obtain the representation

$$K_0(y)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}e^{py}\tilde{K}_0(p)dp\,,$$

with

$$\tilde{K}_0(p)=\frac{1}{\tilde{K}^{-1}(p)+1}\,, \tag{6}$$

for the semiexclusive propagator $K_0(y)$, in terms of the semi-inclusive one $K(y)$.

We may now use QCD in order to factorize the large- and short-distance parts of the propagator $\tilde{K}(p)$. From Eq. (1) we obtain

$$\tilde{K}(p)=\int_{-\infty}^{\hat{t}_{\min}}d\hat{t}\,\tilde{G}^2(p,Q^2)C(p,Q^2),\quad Q^2\equiv-\hat{t}\,, \tag{7}$$

where $\tilde{G}(p,Q^2)$ is the usual moment of order p of the structure function

$$\tilde{G}(p,Q^2)=\int_0^1 x^pG(x,Q^2)dx\,,$$

and $C(p,Q^2)$ is defined by

$$C(p,Q^2)=\int_0^\infty e^{-pz}\theta\left[\frac{\hat{s}}{2}+\hat{t}\right]\frac{d\hat{\sigma}}{d\hat{t}}dz,\quad z\equiv\ln\frac{\hat{s}}{s_0}\,. \tag{8}$$

Ignoring, for simplicity, the flavor content of the structure function and employing the renormalization-group equation for the moment $G(p,Q^2)$, we may write

$$\tilde{G}(p,Q^2)=\tilde{G}(p,Q_0{}^2)e^{\gamma(p)\xi(Q^2,Q_0{}^2)}\,, \tag{9}$$

where $\gamma(p)$ is calculable in perturbative QCD and

$$\xi=\ln\left[\ln\frac{Q_0{}^2}{\Lambda^2}\Big/\ln\frac{Q^2}{\Lambda^2}\right]$$

is the usual evolution variable.[2] From (7) and (9) we obtain

$$\tilde{K}(p)=\tilde{G}^2(p,Q_0{}^2)R(p,Q_0{}^2)\,, \tag{10}$$

where the factor

$$R(p,Q_0{}^2)\equiv\int_{-\infty}^{-Q_0{}^2}d\hat{t}\,e^{2\gamma(p)\xi(Q^2,Q_0{}^2)}C(p,Q^2) \tag{10a}$$

depends on the short-distance properties of the quark-gluon interaction and it is calculable in perturbation theory, whereas the moments of the structure function $\widetilde{G}(p,Q_0{}^2)$, at the initial value Q_0, belong to the unknown sector of the theory and they cannot be calculated in perturbative QCD.

Equation (10) shows that any attempt to constrain the Mueller propagator $\widetilde{K}(p)$, by studying the component (3) of hadronic production within the framework of S-matrix theory, offers at the same time a method to constrain the structure function $G(x,Q_0{}^2)$ at the initial point Q_0 without referring to the controversial space-time properties of QCD at large distances (confinement). Motivated by these last remarks we focus our discussion to the one-dimensional classical fluid in rapidity space[3] defined by the densities (3). One can prove the following theorem:

If in the fluid defined by Eqs. (3), $K(y) \sim y^{-\eta}$ for $y \to \infty$ and the average multiplicity $\langle n \rangle \to \infty$ for $Y \to \infty$ $(Y \equiv y_A - y_B)$, then $0 < \eta < 1$ and $\widetilde{K}_0(p) = \exp(-bp^{1-\eta})$ with $b > 0$.

The assumptions of this theorem simply express the general belief that for any hadron-production mechanism, the cross section behaves asymptotically as a logarithmic power of the energy and the average multiplicity grows indefinitely with energy.

First, we show that the exponent η is restricted in the range $0 < \eta < 1$. The wide range of values $\eta \geq -2$, allowed by the Froissart bound, is reduced to the range of positive values, $\eta > 0$, since otherwise the densities (3) with $-2 \leq \eta \leq 0$ violate positivity $(\sigma_n > 0)$.[4] On the other hand, it can be easily shown that if $\eta > 1$ the average multiplicity $\langle n \rangle$ of the production mechanism is constant for $s \to \infty$, contrary to our assumption. Indeed, the factorized form (3) leads to the following representation for the average multiplicity:

$$K(Y)\langle n \rangle = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} [\widetilde{K}(p)]^2 e^{pY} dp, \quad Y = \ln \frac{s}{s_0} . \tag{11}$$

The discontinuity of $[\widetilde{K}(p)]^2$ near the branch point $p \to 0_-$ has the form

$$\mathrm{Im}[\widetilde{K}(p)]^2 \underset{p \to 0_-}{\simeq} 2\widetilde{K}(0)\mathrm{Im}\widetilde{K}(p) , \tag{12}$$

where

$$\widetilde{K}(0) = \int_0^{\infty} K(y) dy$$

is finite for $\eta > 1$. Hence, from Eqs. (11) and (12) we find

$$\lim_{Y \to \infty} \langle n \rangle = 2\widetilde{K}(0) .$$

For completeness, we note that the marginal value $\eta = 1$ is also excluded since the corresponding propagator, $K(y) \sim y^{-1}$, violates unitarity when coupled to ordinary Reggeons in the Mueller graphs[5] [Fig. 1(c)].

To complete the proof of the theorem, we note that our previous work[6] on the properties of the Feynman-Wilson fluid renders the following statements obvious:

(i) The production mechanism, defined by the densities (3) with

$$K(y) \sim y^{-\eta} \quad (y \to \infty,\ 0 < \eta < 1) ,$$

satisfies Koba-Nielsen-Olesen scaling,

$$\langle n \rangle \sigma_n / \sigma_t = \psi(n / \langle n \rangle) ,$$

and the scaling function behaves asymptotically like

$$\psi(x) \sim e^{-\alpha x^{1/\eta}} \quad (x \to \infty,\ \alpha > 0) .$$

(ii) The generating function (grand partition function) of the system,

$$Q(z,Y) = \sum_{n=0}^{\infty} z^n \sigma_n(Y)/\sigma_t(Y) , \tag{13}$$

has a normal thermodynamic (Regge) behavior $Q(z,Y) \sim e^{Yp(z)}$ for $Y \to \infty$ and the pressure-fugacity relation is

$$p(z) = \left[\frac{1}{b} \ln z \right]^{1/(1-\eta)} ,$$

$$b \equiv \frac{1}{(1-\eta)c} \left[\frac{1-\eta}{\eta} \alpha \right]^{\eta} . \tag{14}$$

The constant c is related to the rise of the average multiplicity with energy, i.e., $\langle n \rangle \sim c Y^{1-\eta}$.

The factorized form (4), interpreted in the context of the hadronic fluid dynamics, defines the nearest-neighbor interaction potential $U(y)$ through the Boltzmann factor as follows: $e^{-U(y)} = K_0(y)$. On the other hand, in one-dimensional classical systems with nearest-neighbor interactions, the Laplace transform of the Boltzmann factor is simply $z^{-1}(p)$ if the pressure p is identified with the Laplace variable conjugate to y.[7] We thus obtain

$$\widetilde{K}_0(p) = e^{-bp^{1-\eta}} , \tag{15}$$

which completes the proof of our theorem.

Using now Eq. (6) we determine the inclusive propagator $\widetilde{K}(p)$, which is directly related to the hard processes with multijet production, shown in Fig. 1, namely

$$\widetilde{K}(p)=(e^{bp^{1-\eta}}-1)^{-1} . \tag{16}$$

We may now determine the hadron structure function $G(x,Q_0^2)$ using the propagator (16), which was found from the study of the hadronic S matrix, in the framework of the Feynman-Wilson fluid dynamics. From Eqs. (10) and (16) we obtain

$$\widetilde{G}(p,Q_0^2)=\{R(p,Q_0^2)[-1+\exp(bp^{1-\eta})]\}^{-1/2} . \tag{17}$$

Using the results of perturbative QCD for the evaluation of the quantity $R(p,Q_0^2)$,[2,8] with a steepest-descent expansion of the inverse transform

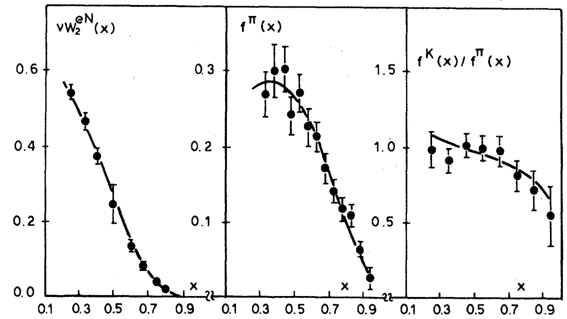

FIG. 2. Fit to the hadron-structure-function data (from Ref. 10) with the model described in the text.

$$G(x,Q_0^2)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} x^{-p}\{R(p,Q_0^2)[-1+\exp(bp^{1-\eta})]\}^{-1/2}dp , \tag{18}$$

we find that the behavior of $G(x,Q_0^2)$ near the end point $x=1$ is (see also Ref. 9)

$$G(x,Q_0^2)\simeq g(Q_0^2,\eta,b,\Lambda)\left[\ln\frac{1}{x}\right]^{-(3\kappa+1)/2}\exp\left\{-\frac{[(1-\eta)b]^{\kappa}}{2^{\kappa}(\kappa-1)(\ln 1/x)^{\kappa-1}}\right\} \quad (x\sim 1,\ \kappa=1/\eta) . \tag{19}$$

The constant $g(Q_0^2,\Lambda,\eta,b)$ in the solution (19) depends on the parameters of the model. We note that κ and b have their origin in the hadronic fluid dynamics (S matrix), whereas Q_0^2 and Λ are the basic parameters of the quark-gluon dynamics (perturbative QCD).

The form (19) shows that the threshold behavior near the point $x=1$ is controlled by an essential singularity, contrary to the popular power behavior $G(x,Q_0^2)\sim(1-x)^{\beta}$ used in phenomenological studies.

Although the expression (19) is exact only in the limit $x\to 1$, we have checked that including more terms in the steepest-descent expansion of the integral (18), we can have a good approximation for smaller values of x (e.g., $x\gtrsim 0.3$). The model fails near $x=0$, since the perturbation expansion for the anomalous dimension $\gamma(p)$ breaks down near the point $p=0$ and one does not have a reliable behavior of the quantity $R(p,Q_0^2)$ in Eq. (17) near this point. We have applied this approach to a phenomenological study of the hadron structure functions and in Fig. 2 a simultaneous fit to the data[10] for nucleon, pion, and kaon structure functions is illustrated. The Q^2 dependence of these structure functions required by perturbative QCD can easily be incorporated in the model.[9]

Finally, we note that although the hadronic propagator $\widetilde{K}(p)$, which plays a central role in our model, manifests itself in the longitudinal region, it bares strong memory of its hard origin. Its most fundamental property is that it implies a characteristic energy threshold, beyond which a new contribution to hadron scattering switches on. Indeed, inverting Eq. (16) we find a rising component to the hadronic cross section[6] which is the solution for the hard cross section (1), i.e.,

$$\sigma_h(s)\simeq g_h^2\left[\ln\frac{s}{s_0}\right]^{-(\kappa+1)/2}\times\exp\left\{-\frac{[(1-\eta)b]^{\kappa}}{(\kappa-1)(\ln s/s_0)^{\kappa-1}}\right\} . \tag{20}$$

The expression (20) is exact near the threshold of the mechanism $s\simeq s_0$, which is related with the transverse-momentum cutoff $-t_{\min}=Q_0^2=s_0/2$, beyond which perturbative QCD is applicable.

The coupling of the propagator $\widetilde{K}(p)$ with ordinary Reggeons in the Mueller graphs [Fig. 1(c)] gives rise to long-range effects in the hadron multiplicities and correlations. These phenomena have already been studied[11] in the framework of the critical Feynman-Wilson fluid and it has been shown that the data support the relevance of the propagator $\widetilde{K}(p)$ to hadron physics.

We are grateful to E. Zevgolatakos for invaluable help with computer facilities. This work was supported in part by Alexander S. Onassis public benefit Foundation.

[1]S. M. Berman, J. D. Bjorken, and J. B. Kogut, Phys. Rev. D 4, 3388 (1971); J. Kogut and L. Susskind, Phys. Rep. 8C, 75 (1973); S. D. Ellis and M. B. Kislinger, Phys. Rev. D 9, 2027 (1974); R. K. Ellis, H. Georgi, M. Machacek, H. D. Politzer, and G. G. Ross, Phys. Lett. 78B, 281 (1978); Nucl. Phys. B152, 285 (1979).

[2]See, e.g., A. J. Buras, Rev. Mod. Phys. 52, 199 (1980).

[3]R. P. Feynman, talk presented at Argonne Symposium on High Energy Interactions and Multiparticle Productions, 1970 (unpublished); K. Wilson, Cornell Report No. CLNS-131, 1970 (unpublished).

[4]N. G. Antoniou, C. B. Kouris, and G. M. Papaioannou, Phys. Lett. 52B, 207 (1974).

[5]N. G. Antoniou, C. B. Kouris, and G. M. Papaioannou, Phys. Rev. D 14, 264 (1976).

[6]N. G. Antoniou, P. N. Poulopoulos, C. B. Kouris, and S. D. P. Vlassopulos, Phys. Rev. D 14, 3578 (1976); N. G. Antoniou and S. D. P. Vlassopulos, Lett. Nuovo Cimento 20, 285 (1977); Phys. Rev. D 18, 4320 (1978).

[7]*Mathematical Physics in One Dimension,* edited by E. H. Lieb and D. C. Mattis (Academic, London, 1966); T. D. Lee, Phys. Rev. D 6, 3617 (1972).

[8]B. L. Combridge, J. Kripfganz, and J. Ranft, Phys. Lett. 70B, 234 (1977); R. Cutler and D. Sivers, Phys. Rev. D 16, 679 (1977); 17, 196 (1978).

[9]N. G. Antoniou, C. Chiou-Lahanas, S. D. P. Vlassopulos, and E. Zevgolatakos, Phys. Lett. 93B, 472 (1980).

[10]A. Bodek *et al.,* Phys. Rev. D 20, 1471 (1979); C. B. Newman *et al.,* Phys. Rev. Lett. 42, 951 (1979); J. Badier *et al.,* work presented at XXth International Conference on High Energy Physics, Madison, Wisconsin, 1980, Report No. CERN/EP 80-148 (unpublished).

[11]N. G. Antoniou, C. Chiou-Lahanas, X. N. Maintas, and S. D. P. Vlassopulos, Lett. Nuovo Cimento 24, 339 (1979); Nuovo Cimento 56A, 97 (1980).