On the measurability of electroweak asymmetries in hadronic jets in e^+e^- annihilation

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Using the standard model of electroweak interactions and the Field-Feynman model for quark fragmentation we estimate the observable asymmetries of the jet-axis angular distribution in e^+e^- annihilation in the PETRA-PEP energy region. To do this, rules for the selection of quark versus antiquark jets according to final hadronic channel are proposed. These rules are characterized by a calculable parameter that gives a quantitative measure of relative merit of the various channels. A particularly promising method of identifying jet flavor involves prompt leptons from heavy-quark decays. We also discuss the dependence of the observable effects on the fragmentation-model parameters and some experimental cuts. The jet asymmetries are compared with muonic asymmetries for energies of 30, 50, and 70 GeV.

I. INTRODUCTION

The tests of the standard model of electroweak interactions are generally believed to be one of the main goals of the experiments on PETRA and PEP storage rings. The most easily measurable experimental quantities in this class seem to be the angular asymmetries of either leptonic or hadronic final states. The latter are not as clearly defined, both theoretically and experimentally, as the former, but provide an independent test of quark couplings to a neutral heavy boson and are physically equally interesting.

In a recent paper[1] we have proposed a simple formalism connecting the electroweak production asymmetries of quarks in high-energy e^+e^- annihilation with the observable asymmetries of hadronic jets. All information obtained from the quark-fragmentation model was represented by a set of easily calculable parameters, which for the jet of type k and flavor i were called ω_i^k. The physical interpretation of ω_i^k is straightforward: it is the probability that the quark of flavor i will fragment into a jet of type k. It is the aim of this paper to give the estimates of the magnitudes of the ω_i^k probabilities and hence the experimentally observable jet asymmetries for various types of jets.

Since the largest existing e^+e^- storage rings, PETRA and PEP, suffer from serious luminosity problems and the experiments anticipated there may have quite low statistics, it is crucial that a universal criterion be established to compare the practical measurability of various types of jets. Such a criterion should take into account the magnitude of measurable asymmetry simultaneously with the statistical efficiency and experimental acceptance of all final-state channels. We introduce a useful parameter directly calculable from the model and incorporating all aforementioned requirements. This parameter is used later in the search for the hadronic channel most suitable to observe the electroweak asymmetries.

The idea of measuring the weak couplings of quarks through asymmetries of hadronic jets is not new[2]; in particular, several papers have been recently devoted to the investigation of its experimental feasibility.[3–5] When compared with the experimentally clean channel $e^+e^-\rightarrow\mu^+\mu^-$, hadronic jets have two obvious advantages: (i) The quark production asymmetries, inversely proportional to quark charges, are, at PETRA-PEP energies, 1.5 (u,c) or 3.0 (d,s,b) times larger than muonic asymmetry. (ii) Hadronic events are more copiously produced than $\mu^+\mu^-$ by a factor $R=3.67$ [for five flavors without QCD corrections]. Unfortunately, the advantages of these two helpful factors are in general overshadowed by the enormous smearing effects of several experimental difficulties. The worst ones are (a) the lack of means to identify unambiguously the quark and antiquark jets, e.g., from particle distribution, on an event-by-event basis; (b) low experimental acceptance of detectors and problems with particle identification of leptons, pions, and kaons at high energies; (c) uncertainties in measured jet-axis direction, partially due to unobserved neutral particles and partially due to three-jet events from hard-gluon emission, etc. Most of the simple estimates of the magnitude of observable jet asymmetries are of the order of $1-4\%$ at the center-of-mass energy $\sqrt{s}=30$ GeV. This might be classified as unmeasurably small in view of present poor experimental statistics. On the other hand, it is possible to find a clean sample of jets with much higher asymmetry of the order of $10-20\%$. Unfortunately, such events are extremely scarcely produced and it may take years to collect a statistically significant sample. It is the purpose of this paper to investigate the interplay of these various factors to find the most convenient and suitable hadronic channels for the purpose of measuring the electroweak asymmetries.

Despite the fact that the subject of jet asymmetries has been covered quite extensively in the literature,[2–5] we feel that the present approach introduces at least three significant features. First, we establish in a quantitative way a statistical criterion which allows for critical reevaluation of previous proposals. It shows that sometimes the apparent advantage of large asymmetries in certain channels may be misleading due to their low statistical efficiency. Second, unlike most of the other authors, we concentrate on asymmetries of the thrust jet axis (not particular hadrons), using the hadronization products for flavor identification only. In particular, we give a general discussion of determination of the asymmetries of one- and two-leading-particles jets. Finally, having found that the unique identification of heavy flavors may turn out to be crucial for our purposes, we investigate the application of our recent observation[6] that the prompt electrons with a large momentum component perpendicular to the jet axis give a very clean signal for bottom-quark production. Actually, this seems to be the most promising channel to measure the quark asymmetries in the future.

Although in our analysis of various hadronic channels we tried to cover all the interesting final states, our paper is not intended to be a general review of jet asymmetries. We would like to apologize to the authors whose work has been omitted from our reference list, which could be called at best incomplete.

The paper is organized as follows. In Sec. II we introduce our notation, define various types of jets, and derive the statistical criterion of measurability of asymmetries. The formulas for one- and two-leading-particle jet asymmetries are derived in Sec. III, based on the notion of the rank-violation probabilities, discussed in the Appendix. Section IV is devoted to the discussion of the dependence of our results on various fragmentation parameters. The search for optimal types of jets using our criterion from Sec. II is performed in Sec. V. Finally, in Sec. VI, we give the summary of our paper and some general conclusions.

II. STATISTICAL CRITERION OF MEASURABILITY

As our considerations in this paper require the introduction of large numbers of parameters and symbols, we first give a review of our notation. In Table I we present a catalog of jets. In Sec. II B we derive our criterion.

A. Notation

We use the following notation.

$\sigma_h=$ total hadronic cross section for $e^+e^-\to$ hadrons.

$L=$ integrated luminosity.

$N=$ total number of hadronic events at a given center-of-mass energy $\sqrt{s}$. Obviously $N=\sigma_h L$.

k is an index enumerating various types of jets. The definition of "jet type" is quite general; it may involve any quantum numbers or kinematical characteristics of one or more particles in a jet, e.g., "jets with leading π^+ of momentum $p_{\parallel}>0.3\sqrt{s}$", where $p_{\parallel}$ is the momentum component along the jet axis. In particular, we will use $k=\mu$ to denote the reaction $e^+e^-\to\mu^+\mu^-$.

i,j,l,p,r are flavor indices ($=u,d,s,c,b$); for simplicity we will also denote $j=\mu$ in case of the reaction $e^+e^-\to\mu^+\mu^-$.

$N_k=$ total number of jets of type k (note that it is possible to have two type-k jets in a single event).

obs is a superscript meaning "observed".

B and F are subscripts meaning "backward" and "forward" with respect to the electron beam, e.g., $N_{k,F}^{\rm obs}$ denotes the number of observed jets of type k in the forward hemisphere.

$\eta_k=N_k/N=$ theoretical efficiency of jets of type k (depending on the production and fragmentation only).

$e_k=N_k^{\rm obs}/N_k=$ experimental acceptance of jets of type k (depending on detector only, includes identification of particles, geometric acceptance of

TABLE I. Catalog of jets.

Name	Abbreviation	Symbol k	Definition
One-leading-particle jet	1LP	a	Jet with leading particle a
Two-leading-particles jet	2LP	a,b	Jet with leading particle a and second leading b
Nonleading heavy-flavor jet	NLH	NL,a	Jet with a heavy meson a (D or B) produced with arbitrary momentum
Alternating jet	ALT	a/b	Jet with either a or b leading particles (may be also used with nonleading)
Correlated jets	COR	a-b	Event with two jets with a particle leading in one jet and b particle leading in the opposite one
Large-$k_\perp^{e^-}$ jets	$k_\perp^{e^-}$	$k_\perp^{e^-}$	Jet with an electron of $k_\perp > k_\perp^{e^-}$

apparatus, etc.).

$\widetilde{A}_k^{\rm obs}=(N_{k,F}^{\rm obs}-N_{k,B}^{\rm obs})/(N_{k,F}^{\rm obs}+N_{k,B}^{\rm obs})=$ experimentally observed global jet asymmetry for type-k jets, as defined in Ref. 1.

$\Delta\widetilde{A}_k^{\rm obs}=$ experimental error in asymmetry measurement.

$m=\widetilde{A}_k^{\rm obs}/\Delta\widetilde{A}_k^{\rm obs}$ is a parameter defining the precision of measurement in terms of standard deviations.

n_k^m (L_k^m)$=$ total number of hadronic events (integrated luminosity) required to observe the asymmetry in channel k at the level of m standard deviations.

ω_i^k ($\overline{\omega}_i^k$)$=$ probability that quark (antiquark) of flavor i will fragment into a jet of type k.

$\sigma_i=$ total production cross section of quarks of flavor i.

$\gamma_i=\sigma_i/\sigma_h=$ production rate of flavor i.

$\xi_i=$ probability of exciting $q_i\overline{q}_i$ pair out of the sea in fragmentation model (note that this parameter is denoted γ_i in Ref. 7).

α,β are indices enumerating ranks of particles in Field-Feynman (FF) fragmentation model[7] (see Fig. 5).

$P_{\alpha\beta}=$ probability that a particle of rank α is leading in a jet, and a particle of rank β is second leading.

$P_\alpha=\sum_{\beta\neq\alpha}P_{\alpha\beta}=$ probability that a particle of rank α is leading in a jet.

$\widetilde{A}_i=$ production asymmetry of quark q_i (see Ref. 1).

$k_\perp=$ lepton momentum component perpendicular to jet axis in the laboratory frame.

B. Derivation of the statistical parameter P_k

In the present situation when we have to investigate small effects by means of low-statistics experiments, it is crucial to find a way to make comparisons of the various hadronic channels which may differ from each other by orders of magnitude in asymmetry and statistics. To derive a reasonably general criterion we make the following assumptions.

1. In all channels (enumerated by k) we ask for the same relative size of the measured effect in terms of standard deviations:

$$\frac{\widetilde{A}_k^{\rm obs}}{\Delta\widetilde{A}_k^{\rm obs}}=m\ . \tag{2.1}$$

(For example, if $m=5$ we ask for a 2% precision when measuring a 10% effect and a 4% precision will be required for a 20% effect.)

2. As a basic quantity allowing for direct com-

parison of feasibility of various channels, we take n_k^m to be the total number of produced hadronic events required to observe the asymmetry at the precision level defined by m. Obviously, the smaller n_k^m is, the more suitable the channel k will be.

3. Only statistical errors are taken into account.
4. Only an effective global detector acceptance (e_k) is included without considering any detailed angular dependence.

The last two assumptions, adopted for the sake of generality, may be appropriately modified by the interested experimental groups.

A straightforward calculation shows that from these assumptions follows the formula

$$\Delta\widetilde{A}_k^{\rm obs} \sim \frac{[1-(\widetilde{A}_k^{\rm obs})^2]^{1/2}}{(N_k^{\rm obs})^{1/2}} \tag{2.2a}$$

from which we get a conservative upper limit of statistical experimental error

$$\Delta\widetilde{A}_k^{\rm obs} \le \left[\frac{2}{N_k^{\rm obs}}\right]^{1/2} . \tag{2.2b}$$

The constant factor [2 in (2.2b)] depends strongly on the technical details of statistical hypothesis one assumes in this estimate but it will be of no consequence in what follows. Assuming an equality in Eq. (2.2b) and using symbols defined in Sec. II A, we get from Eq. (2.1)

$$N_k^{\rm obs} = \frac{2m^2}{(\widetilde{A}_k^{\rm obs})} = e_k\eta_k n_k^m . \tag{2.3}$$

Hence

$$n_k^m = \frac{2m^2}{e_k\eta_k(\widetilde{A}_k^{\rm obs})^2} \tag{2.4}$$

and

$$L_k^m = \frac{2m^2}{\sigma_h e_k\eta_k(\widetilde{A}_k^{\rm obs})^2} . \tag{2.5}$$

The last two quantities depend on m, which can change from experiment to experiment. To find a more objective measure and to represent all theoretical information as simply as possible, e.g., in terms of one (model-dependent) parameter, we introduce a statistical quantity P_k defined by means of the ratio of integrated luminosities

$$\frac{L_{e^+e^-\to {\rm jet}k}}{L_{e^+e^-\to\mu^+\mu^-}} ,$$

$$P_k \equiv \frac{e_k L_k^m}{e_\mu L_\mu^m} , \tag{2.6}$$

where e_μ is the experimental acceptance of muons. The quantity P_k is a measure of the relative merit of the hadronic channel and the μ channel. For best measurability it should be minimized. Clearly, we can write

$$P_k = \frac{\eta_\mu(\widetilde{A}_\mu^{\rm obs})^2}{\eta_k(\widetilde{A}_k^{\rm obs})^2} . \tag{2.7}$$

(From the definitions of Sec. II A, it follows that $\eta_\mu = 1/R$, where $R = \sigma_h/\sigma_\mu$.) In the following, when investigating a given type of jets, e.g., k, we always calculate three relevant quantities: $\widetilde{A}_k^{\rm obs}$, describing the size of observed asymmetry, η_k, reflecting the frequency of occurrence of this type of event, and most important, P_k, telling us directly how much accelerator time is needed to observe the effect under consideration with the required precision.

To complete this section, let us quote after Ref. 1 the formulas for η_k and $\widetilde{A}_k^{\rm obs}$ expressed in terms of quark production rates (γ_i), asymmetries $(\widetilde{A}_i)$, and fragmentation probabilities (ω_i^k)

$$\eta_k = \sum_j \gamma_j(\omega_j^k + \overline{\omega}_j^k) , \tag{2.8}$$

$$\widetilde{A}_k^{\rm obs} = \frac{1}{\eta_k}\sum_i \gamma_i\widetilde{A}_i(\omega_i^k - \overline{\omega}_i^k) . \tag{2.9}$$

Note that for simplicity we assumed in (2.8) that the events with k-type jets on both sides are accepted. The change in the formula given in Ref. 1 is minor and numerically negligible. From these two equations it follows that

$$P_k = \gamma_\mu\widetilde{A}_\mu{}^2 \frac{\sum_j \gamma_j(\omega_j^k + \overline{\omega}_j^k)}{\left[\sum_i \gamma_i\widetilde{A}_i(\omega_i^k - \overline{\omega}_i^k)\right]^2} \tag{2.10}$$

(for definitions of symbols see Sec. II A).

III. ASYMMETRIES FOR LEADING-PARTICLE JETS

In order to calculate the quantities given by Eqs. (2.8)–(2.10), we have to know three sets of parameters $(\widetilde{A}_i, \gamma_i, \text{and } \omega_i^k)$ for all flavors (u,d,s,c,b). Following Ref. 1 we assume factorization of the production and the fragmentation processes and use γ_i's and $\widetilde{A}_i$'s calculated there in lowest order of perturbation theory of the standard electroweak model, including γ and Z^0 exchange diagrams. Corrections to these values will be briefly discussed in Sec. VI. We devote this section to the derivation of probabilities ω_i^k in the general case of jets defined by one leading particle (1LP) and two leading particles (2LP).

So far no satisfactory fundamental theory of quark-fragmentation processes has been found; however, the FF model[7] has been quite successful in reproducing the main trends in the data and, with some modifications, is commonly accepted as the best tool to simulate quark fragmentation by means of the Monte Carlo method. Most of the experimental groups working on jets have at their disposal a detailed FF Monte Carlo program and can readily calculate the ω_i^k's. Here we shall concern ourselves with the investigation of the sensitivity of ω_i^k's to the variation of FF parameters; therefore, a full Monte Carlo simulation is unnecessary. Below, we list the ingredients of the FF model which are irrelevant for our purposes and hence may be omitted from our simplified version of this model.

1. If we define the leading particle as the particle with the largest momentum component along the jet axis, all distributions in the plane perpendicular to the jet axis (primordial $p_\perp$ distribution) become unimportant for our results.

2. Since the light neutral particles, such as π^0, η^0, K_S^0, consist of an equal admixture of quarks and antiquarks of the same flavor, the following symmetry relation is satisfied:

$$\omega_i^{\pi^0,\eta^0,K_S^0,\dots}=\bar{\omega}_i^{\pi^0,\eta^0,K_S^0,\dots} \tag{3.1}$$

and it follows immediately from (2.9) that

$$\tilde{A}^{\text{obs}}_{\pi^0,\eta^0,K_S^0,\dots}=0 \ . \tag{3.2}$$

(Note that this is not true for D^0 or B^0, where D^0-$\bar{D}^0$ and B^0-$\bar{B}^0$ mixing is presumably small.) Hence, we exclude from our considerations the light neutral particles, avoiding all flavor mixing problems.

3. To get rid of the unnecessary complications connected with the production and the subsequent decay of vector mesons, we introduce a cut in the variable $z=2p_{\|}^{\text{hadron}}/\sqrt{s}$. As the data seem to indicate,[8] a cut which restricts data to $z>0.3$ eliminates most of the vector-meson contributions. Since the production cross section for vector mesons and pseudoscalar mesons are about the same,[7,9] the above cut means that a common factor 0.5 is to multiply all the ω's for the light quarks. This does not change asymmetries but doubles the value of the P_k parameter.

With all these simplifications, the FF model is simplified to contain two unknown parameters. The first one governs the flavor dependence reflected in the values of ξ_i, the probabilities of $q\bar{q}$ pairs of various flavors being excited out of the sea (denoted γ_i in Ref. 7). Due to the symmetry of u and d quarks and negligible production of heavy flavors,[8] just one parameter ξ_s defines the whole set,

$$\xi_u=\xi_d=\tfrac{1}{2}(1-\xi_s), \quad \xi_c\approx\xi_b\approx 0 \ . \tag{3.3}$$

The other parameter a controls the dynamics of the fragmentation process via the FF function

$$f(x)=1-a+3a(1-x)^2 \ , \tag{3.4}$$

where $x=p_{\|}^{\text{hadron}}/p_{\text{quark}}$ and $f(x)$ represents the probability density that the quark emits a meson with momentum xp_{quark} along the jet axis. In principle, one could take different f functions for light and heavy quarks, especially in view of strong theoretical arguments that the heavier the quark the harder its fragmentation function.[10] However, as will be shown in Sec. V, for heavy flavors the assumption that the particle is "leading" does not play any role. The observation of a heavy meson (B or D) gives sufficient evidence for heavy-flavor quark production in primary process, independently of kinematical characteristics of this meson. Hence, we keep the flavor-independent fragmentation function for illustration purposes.

To gain some physical insight into the fragmentation process, instead of using parameter a directly in the calculation of the ω_i^k's, we introduce a concept of rank-violation probabilities P_α and $P_{\alpha\beta}$. The P_α is for 1LP jets and the $P_{\alpha\beta}$ for 2LP jets, where α and β enumerate the ranks[7] of particles emitted in the cascade (see Sec. II A for definitions). Obviously, $P_\alpha\to 0$ for large values of α. The harder the $f(x)$ is (smaller a), the faster P_α decreases with α and the more pronounced the effect of leading particles would be. In principle, one may calculate P_α and $P_{\alpha\beta}$ parameters analytically but in practice this would be too complicated to be useful. We have calculated them by means of a Monte Carlo program and the results, together with rather complicated formulas for ω_i^k's are presented in the Appendix, Eqs. (A3)–(A7′).

IV. PARAMETER DEPENDENCE OF ASYMMETRIES AND STATISTICS

As follows from the previous section, we have three unknown fragmentation parameters for 1LP jets, ξ_s, a, and the cut in variable z which we will call z_1^0. Their values may be found from one-particle inclusive distributions in e^+e^- or neutrino data. Unfortunately, different experimental groups seem to adopt different approaches and a good fit of these parameters is still lacking. Originally, Field and Feynman used values[7]

$$a_{\rm FF}=0.88,\quad \xi_s^{\rm FF}=0.20\ . \tag{4.1}$$

Recent fits of PETRA data lead to a somewhat harder spectrum of hadrons[9]:

$$a_{e^+e^-}=0.55\ , \tag{4.2}$$

but the other parameter was kept fixed at the FF value. On the other hand, from the ratio of strange to nonstrange particles in neutrino data[8] the value of

$$\xi_s^{\nu}=0.12 \tag{4.3}$$

has been inferred but the fits were obtained by keeping the parameter a fixed at the FF value. As for the third parameter z_1^0, a value of $\frac{1}{3}$ will be used. This value should be sufficient to get rid of resonance effect.[8] Recently, however, much larger values were suggested[5] as a means to increase the asymmetries (the rank violation drops with the increasing momentum and hence the identification of quarks improves).

In view of these large uncertainties, rather than predict some specific magnitudes of the observable asymmetries, we will first determine how sensitive the jet asymmetries are to the variation of these parameters.

A. The parameter ξ_s

Since the values of ω_i^k and $\overline{\omega}_i^k$ are in general comparable, it is clear from Eq. (2.9) that the large cancellations may occur in $\widetilde{A}_k^{\rm obs}$. The final result depends strongly on two competing mechanisms. If the large rank violation occurs (see the Appendix) it enhances the ω's for nonvalence quarks of the leading particle with respect to those of its valence quarks. However, this effect may be wiped out by the small value of ξ_i. For instance, no rank violation may lead to substantial production of D mesons from a quark because ξ_c is negligible. On the other hand, the large value of ξ_u and ξ_d make pions easy to produce from any quark.

The best illustration of an intermediate situation is provided by K production, where the outcome of the cancellations depends in a crucial way on parameter ξ_s.

In Fig. 1 we present the $\widetilde{A}_{K^+}^{\rm obs}$ and P_{K^+} dependence on ξ_s (solid lines; the meaning of the dashed lines will be given in Sec. V). It turns out that for $\xi_s=0.2$ the asymmetry is almost exactly zero, whereas for $\xi_s=0.12$ it increases substantially to 2%. The conclusion is quite obvious: the results are very sensitive to this parameter and it is crucial to measure its value with high accuracy. In particular, if $\xi_s=0.2$ is the correct value, kaons are totally useless for measuring the electroweak effect and this conclusion may hold for a wide range of energies under the Z^0 mass. On the other hand, if ξ_s is closer to 0.12, kaons will give a much larger asymmetry than pions which may turn out to be relevant for the study of Z^0 resonance. (Note that the pion asymmetry is not sensitive to ξ_s variation: for $0.1<\xi_s<0.2$, $\widetilde{A}_{\pi^+}^{\rm obs}$ varies from 1.4 to 1.5% only.)

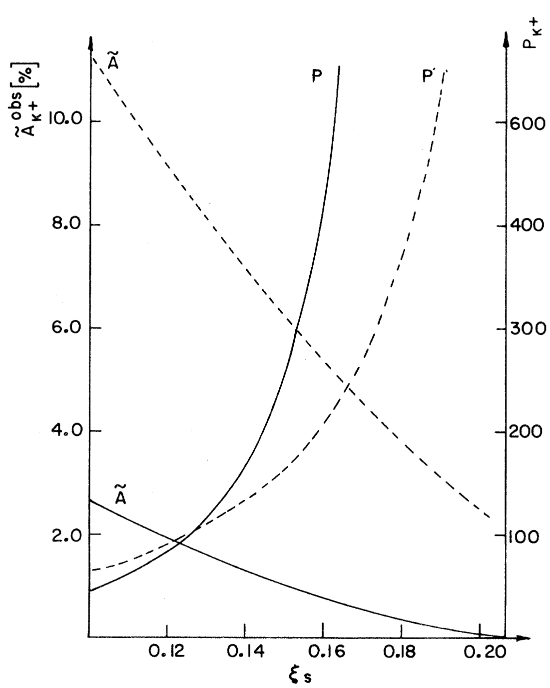

FIG. 1. The dependence of the observable asymmetry of leading K^+ jets and of the statistical parameter P_{K^+} on the parameter ξ_s. The left-hand scale refers to asymmetries and the right-hand scale to statistical parameters. Solid lines correspond to the case of K^+ 1LP jets, the dashed ones to COR K^+-K^- jets. Other parameters assumed $a_{e^+e^-}=0.55$ and $z_1^0=\frac{1}{3}$.

B. The parameter a, the hardness of the particle spectra

As it could be qualitatively expected, this parameter does not play a dominant role in our results. In the Appendix we illustrate the influence of a on the values of rank-violation probabilities, in Tables VI and VII. Although this influence is quite visible, it does not change P_a very much. In Fig. 2 we present the $\tilde{A}^{obs}_{\pi^+}$ and P_{π^+} dependence on a. Clearly, the harder the spectrum is (the smaller the value a), the more pronounced the asymmetries are, but no dramatic dependence is expected.

C. The momentum cut z_1^0

It is a general rule that the larger the momentum cut z_1^0, the faster the rank-violation probabilities tend to zero with growing rank index. Hence, in the case of large z_1^0 the probability ω_i^k for valence quarks should dominate, the amount of cancellation should be reduced, and one could expect to obtain a growing value of observable asymmetries. This fact has been used recently[5] to propose the measurements of electroweak jet asymmetries using jets with very fast leading particles on both sides. However, some caution seems advisable for experimentalists here. Although asymmetries clearly increase (sometimes by a factor of 3), there is a tremendous drop in statistics which, unfortunately, more than compensates the gain in the value of the asymmetry. This situation is illustrated in Figs. 3(a)—3(c) (solid lines). In Fig. 3(a) we show the asymmetry of π^+ 1LP jets corresponding to FF parameters (4.1). The same quantity for fitted values (4.2) and (4.3) is presented in Fig. 3(b). Analogous results for K^+ 1LP jets are given in Fig. 3(c). It follows consistently from all these cases that the values of P_k show fast increases for $z_1^0 > 0.4$. In other words, in trying to increase the size of asymmetries, we have to increase the time of data sampling. Although attractive, this method does not seem to be very practical for the next generation of experiments.[11]

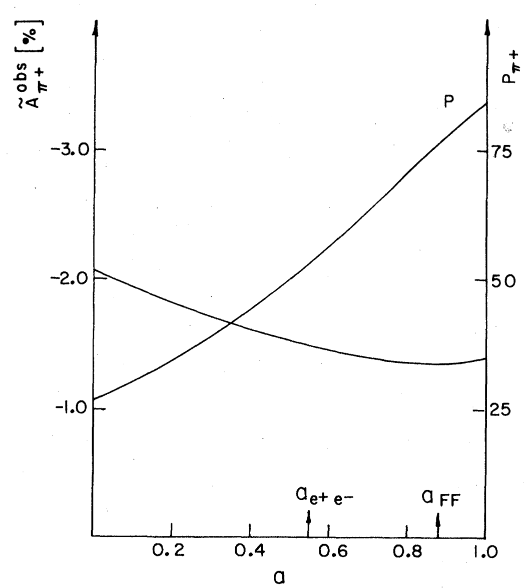

FIG. 2. The dependence of asymmetry and statistical parameter of leading π^+ jets on parameter a. Scales as in Fig. 1. Other parameters assumed fixed $\xi_s = 0.2$, $z_1^0 = \frac{1}{3}$.

D. Angular acceptance

Although this is not an unknown dynamical parameter, the technical problem of limited angular acceptance will occur in every experiment. On the one hand, the quark production cross section shows[1] that the asymmetry is more pronounced in a small region around forward and backward direction than at large angles and one may be tempted to optimize global asymmetry by limiting the range of integration to these regions. On the other hand, the higher-order QED effects,[12] substantial in the forward and the backward directions,[5] may give an argument for leaving out exactly those regions and integrating over the rest of the phase space. Below, we present the discussion of both possibilities.

Let us denote by $A_i'(\theta_{max})$ and $A_i''(\theta_{min})$ the following quark production asymmetries:

$$A_i'(\theta_{max}) = \frac{\sigma_i(0,\theta_{max}) - \sigma_i(\pi-\theta_{max},\pi)}{\sigma_i(0,\theta_{max}) + \sigma_i(\pi-\theta_{max},\pi)} , \quad (4.4)$$

$$A_i''(\theta_{min}) = \frac{\sigma_i(\theta_{min},\pi/2) - \sigma_i(\pi/2,\pi-\theta_{min})}{\sigma_i(\theta_{min},\pi/2) + \sigma_i(\pi/2,\pi-\theta_{min})} , \quad (4.5)$$

where the integrated cross section for the production of quark of flavor i in a limited angular range $\theta_1 \le \theta \le \theta_2$ is

$$\sigma_i(\theta_1,\theta_2) \equiv \int_{\cos\theta_1}^{\cos\theta_2} \frac{d\sigma_i}{d(\cos\theta)} d(\cos\theta) . \quad (4.6)$$

The expressions for $d\sigma_i / d(\cos\theta)$ including the quark mass effects may be found in Refs. 1 and 6.

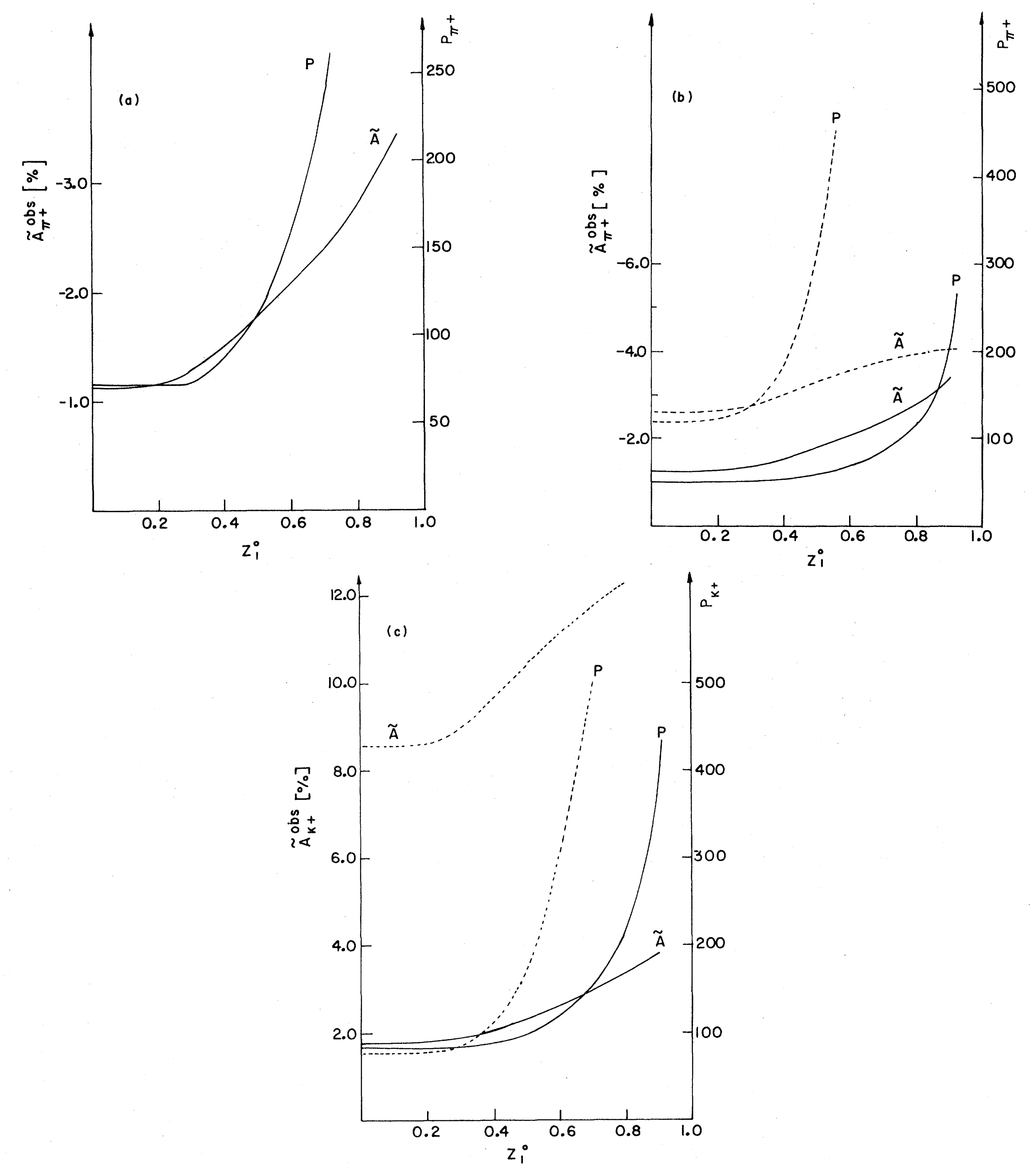

FIG. 3. The dependence of asymmetry and statistical parameter on the momentum cut of leading particle, z_1^0, for: (a) π^+ 1LP jets with $\xi_s=0.2$, $a_{FF}=0.88$; (b) π^+ 1LP jets with $\xi_s=0.12$, $a_{e^+e^-}=0.55$; (c) K^+ 1LP jets with $\xi_s=0.12$, $a_{e^+e^-}=0.55$. Notation is the same as in Fig. 1.

In what follows we will neglect the small mass effects. The integration region in the case of Eq. (4.4) is depicted in Fig. 4(a) and for Eq. (4.5) in Fig. 4(b) (shaded area in the inserts). It follows immediately from the definitions[1] that in the case of full angular acceptance they all coincide:

$$\tilde{A}_i = A'(\pi/2) = A''(0) . \tag{4.7}$$

To separate this effect from all other dynamical effects, instead of using directly the values of A' and A'' or corresponding parameters $P_k'(\theta_{max})$ and $P_k''(\theta_{min})$, we introduce the (flavor-independent) ratios

$$g'(\theta_{max}) \equiv \frac{A_i'(\theta_{max})}{\tilde{A}_i} = \frac{1+\cos\theta_{max}}{1+\frac{1}{4}\cos\theta_{max}+\frac{1}{4}\cos^2\theta_{max}} , \tag{4.8}$$

$$g''(\theta_{min}) \equiv \frac{A_i''(\theta_{min})}{\tilde{A}_i} = \frac{4\cos\theta_{min}}{3+\cos^2\theta_{min}} , \tag{4.9}$$

$$r'(\theta_{max}) \equiv \frac{P_k'(\theta_{max})}{P_k} = \frac{1+\frac{1}{4}\cos\theta_{max}+\frac{1}{4}\cos^2\theta_{max}}{(1-\cos^2\theta_{max})(1+\cos\theta_{max})} , \tag{4.10}$$

$$r''(\theta_{min}) \equiv \frac{P_k''(\theta_{min})}{P_k} = \frac{3+\cos^2\theta_{min}}{4\cos^3\theta_{min}} . \tag{4.11}$$

Since functions (4.8) and (4.9) are flavor independent (except for negligible mass effects), it follows from linearity of relations (2.8)–(2.10) that they also represent the relative observable asymmetry, e.g.,

$$f'(\theta_{max}) = \frac{A_k^{obs}(\theta_{max})}{\tilde{A}_k^{obs}} . \tag{4.12}$$

The shapes of the functions g', g'', r', and r'' are presented in Figs. 4(a) and 4(b). It is interesting, although with no practical consequences, that $r'(\theta_{max})$ has a minimum at $\theta_{max}=77°$. Unfortunately, by choosing this optimal value we can only gain about 8% in total luminosity. The asymmetry may be enhanced at most by a factor of $\frac{4}{3}$ but this causes a drop in efficiency and worsens the statistical parameter P_k. As for the complementary approach, it follows from Fig. 4(b) that the statistical loss due to cutting out the QED-dominated regions should not be too serious. For $\theta_{min}=20°$ (Ref. 5) we get a loss in luminosity of 17% and for 30° of 44%.

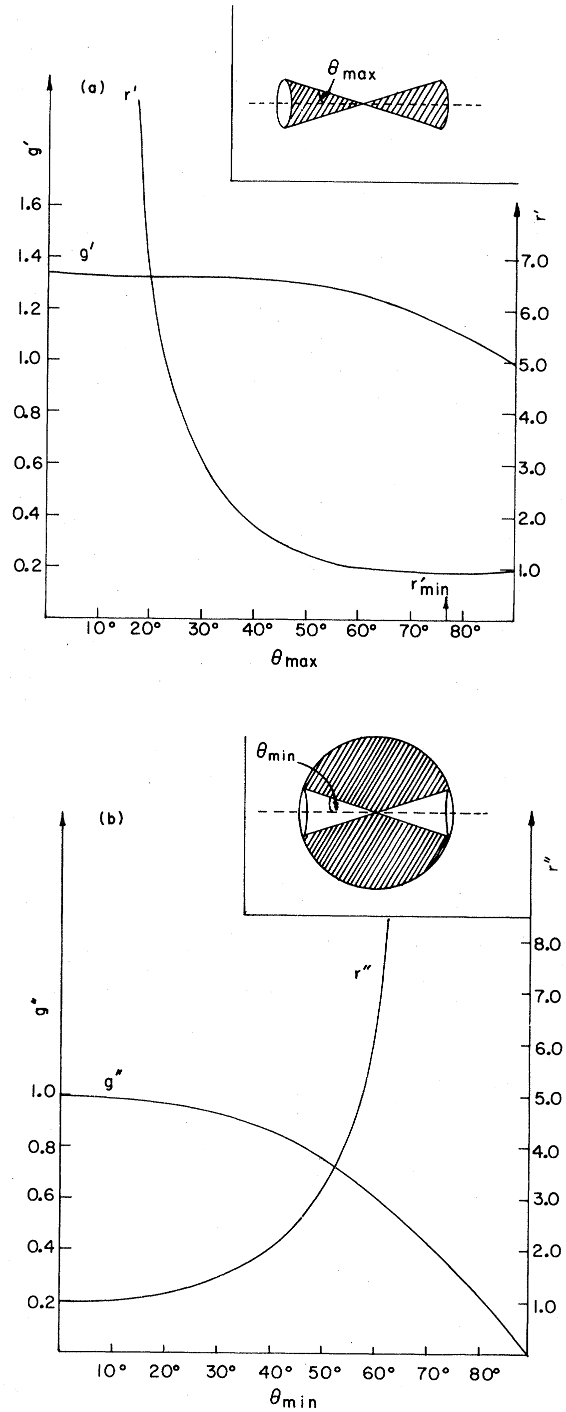

FIG. 4. The dependence of relative asymmetries and relative statistical parameters on the angular cuts (a) in the case when jets are accepted only in the forward and backward cones; (b) in the case when jets are accepted everywhere but in the forward and backward cones. The inserts illustrate the geometric acceptance of detectors.

V. OPTIMIZATION OF ASYMMETRIES AND STATISTICS

The solid curves in Figs. 1–3 show that the 1LP jets in the case of light flavors (π,K) lead to very small observable effects, of the order of 1–3 % at $\sqrt{s}=30$ GeV. This is in strong contrast to the large quark production asymmetries,[1] 10% (u,c) and 20% (d,s,b) at this energy. Obviously, using simple criteria such as 1LP, we allow for too much smearing. In this section we will review some of the more obvious selections of events that may lead to an improvement of quark identification on an event-by-event basis and, hence, increase the observable asymmetry.

In analyzing various particles in the final state it is necessary to treat the cases of light (u,d,s) and heavy (c,b) quarks separately. There are two basic differences between them. First, the values of ξ_c and ξ_b are negligible in comparison with ξ_u, ξ_d, and ξ_s. For our calculations we took $\xi_c \approx \xi_b = 10^{-4}$ to account for a very low charm production in neutrino data[8] but the actual values have no practical importance for our results as long as they are more than an order of magnitude smaller than those of the light quarks.[13] This means that unlike the case of light flavors where pions and kaons may be fragmentation product of any quark as a result of materialization of color vacuum polarization, an observation of heavy-flavored meson is a clear indication of the production of its heavy valence quark. Unfortunately, under ordinary circumstances the heavy mesons are extremely difficult to identify on an event-by-event basis.[14] It is interesting to check whether some improvement in heavy-flavor identification may allow it to compete with a much easier detectable, but also much more symmetrically distributed light flavors. Below we consider separately the cases of light and heavy quarks.

A. Mesons and quarks of light flavors

In our search for a mechanism to increase the jet asymmetries we shall follow the approach suggested in Ref. 1—reduce all the ω_i^k's but one to minimalize the cancellations. An unavoidable byproduct of this procedure is some reduction in the number of events in a data sample. It is then the role of P_k parameter to tell us whether our method is practical or not. We will not try to give here an overview of all possible jet-selection criteria. For the sake of illustration, we shall use three such criteria: 2LP jets, alternating 1LP (ALT) jets, and correlated (COR) jets (for definitions see Table I).

(a) Two-leading-particle jets. This seems to be an obvious choice for the purpose of increasing the contribution of a given quark in comparison with all the others. If the rank violation were negligible we would have the inequalities

$$\omega_u^{\pi^+},\bar{\omega}_d^{\pi^+} >> \omega_{i\neq u}^{\pi^+},\bar{\omega}_{i\neq d}^{\pi^+} \tag{5.1}$$

or

$$\omega_u^{K^+},\bar{\omega}_s^{K^+} >> \omega_{i\neq u}^{K^+},\bar{\omega}_{i\neq s}^{K^+}, \tag{5.2}$$

so that the leading particles would tend to include the parent quarks. Then, by tagging the second leading particle, we would be able to distinguish between the two otherwise comparable probabilities. For instance, the following inequalities might be naively expected to hold:

$$\bar{\omega}_d^{\pi^+,K^-} >> \omega_u^{\pi^+,K^-} \tag{5.3}$$

or

$$\bar{\omega}_s^{K^+,\pi^-} >> \omega_u^{K^+,\pi^-} \tag{5.4}$$

since the ω's in the left-hand side involve no rank violation and those in the right-hand side do, as illustrated in Fig. 5. As it turns out, however, the contribution of the rank-violation effect is substantial (see the Appendix) and this effect above cannot compensate for the suppression due to the presence of additional factors of ξ_s (e.g., contrary to naive expectations $\omega_s^{\pi^+,K^-} > \bar{\omega}_d^{\pi^+,K^-}$) or production rate γ_i which adds weight to u quarks. In many cases approximate estimates of asymmetries using rank violation only tend to be misleading.

In Table II we give the comparison of 1LP and 2LP jets in the case of parameters values $a_{e^+e^-}=0.55$, $\xi_s=0.12$, $z_1^0=0.33$, $z_2^0=0.20$. Although the detailed results depend on the parameters, two conclusions are general: (i) In all cases the addition of another particle increases the asymmetry (sometimes by a factor of 3). (ii) The gain in asymmetry is not large enough to compensate for the loss in statistics, as is clearly seen from the values of P_k. This second fact implies that this method of isolating pure samples by putting more restrictions on a jet is definitely impractical for most situations.

(b) Alternating 1LP jets. The spirit of this method is opposite to the previous one, it should smear asymmetries and increase statistics. The

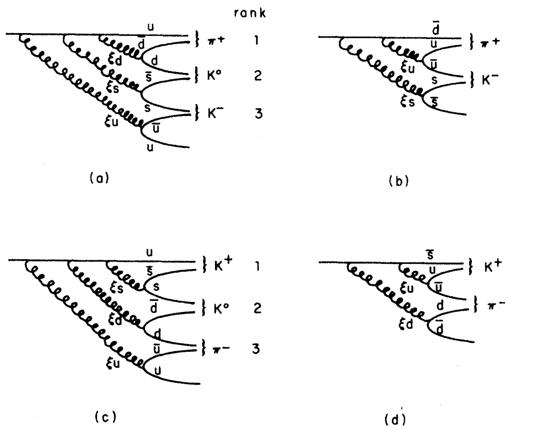

FIG. 5. Leading fragmentation diagrams for (a) $\omega_u^{\pi^+,K^-}$, (b) $\overline{\omega}_d^{\pi^+,K^-}$, (c) $\omega_u^{K^+,\pi^-}$, (d) $\overline{\omega}_s^{K^+,\pi^-}$. The contributions from these diagrams are (a) $\frac{1}{2}P_{13}\xi_d\xi_s\xi_u$, (b) $\frac{1}{2}P_{12}\xi_u\xi_s$, (c) $\frac{1}{2}P_{13}\xi_s\xi_d\xi_u$, (d) $\frac{1}{2}P_{12}\xi_u\xi_d$, respectively [see Eq. (A6)].

ω_i^k's in the formulas (2.8)–(2.10) should be modified in an obvious way, e.g., for π^+/K^+ ALT jets we get

TABLE II. The comparison of light-flavored-meson jet asymmetries at $\sqrt{s}=30$ GeV. The parameters used for this calculation are $a_{e^+e^-}=0.55$, $\xi_s=0.12$, $z_1^0=0.33$, and (for 2LP jets only) $z_2^0=0.2$.

Particles identified	Jet type	$\tilde{A}_k^{\rm obs}$ (%)	η_k (%)	P_k
π^+	1LP	-1.41	12.34	51.72
K^+		$+1.94$	3.99	84.10
π^+,π^+	2LP	-2.44	0.45	473.8
π^+,π^-		-0.27	1.88	917.8
π^+,K^+		-0.05	0.14	3.2×10^6
π^+,K^-		-4.11	0.32	231.4
K^+,K^-		-0.13	0.37	2.0×10^5
K^+,π^-		$+5.04$	0.34	146.0
π^+/K^+	1LP	-0.59	16.33	222.6
π^+/K^-	ALT	-1.54	16.33	32.7
π^+-π^-	1LP	-2.83	1.07	147.4
K^+-K^-	COR	$+9.16$	0.17	90.2
π^+/K^+-π^-/K^-	1LP	-1.65	1.82	127.6
π^+/K^--π^-/K^+	ALT	-3.37	1.65	67.4
	COR			

$$\omega_i^{\pi^+/K^+}=\omega_i^{\pi^+}+\omega_i^{K^+}, \quad (5.5)$$

$$\overline{\omega}_i^{\pi^+/K^+}=\overline{\omega}_i^{\pi^+}+\overline{\omega}_i^{K^+}. \quad (5.6)$$

From an experimental point of view, it would be very desirable to accept pions and kaons of the same charge together without having to distinguish between them. Hence, for π^+/K^+ ALT jet experimental acceptance e_k should increase substantially. Unfortunately, since π^+ and K^+ have opposite signs of asymmetries, their mixture involves a large amount of smearing. Apart from the experimental-acceptance effect, the choice of π^+/K^- ALT jet would seem more suitable, as follows from our results quoted in Table II. Incidentally, this seems to be the best result of Table II but one should keep in mind that it requires very good identification of pions and kaons.

(c) Correlated 1LP jets. The only modification in Eqs. (2.8)–(2.10) consists of replacing each ω_i^k by $(\omega_i^k)^2$. The results are also listed in Table II and plotted in Figs. 1, 3(b), and 3(c) (dashed lines). Clearly, in spite of a large gain in asymmetries (in the case of K^+-K^- COR jets almost a factor of 5), this method is not advisable, as the P_k values show. Of course, the feasibility of the various methods will ultimately depend on experimental details such as detector acceptance and particle identification.

Finally, one can combine the last two ideas and investigate 1LP alternating and correlated jets. We give the results for this case in the last two rows in Table II. They are rather discouraging too.

B. Heavy-flavored mesons and quarks

In order to illustrate the importance of heavy flavors for our considerations let us assume for the sake of discussion that we can identify the D and B mesons (generical name for any meson including c or b and one of the nonstrange quarks) on an event-by-event basis. Due to negligible values of ξ_c and ξ_b, the assumption of "leading" is unnecessary in this case. In Table III we present the comparison of heavy-flavor asymmetries in the case of 1LP nonleading heavy flavors and alternating jets. Although the results are certainly unrealistic, they clearly show the potential of heavy flavors if only we could find an effective method of their identification.

Actually, such a method does exist for bottom

TABLE III. Examples of heavy-meson jet asymmetries at $\sqrt{s}=30$ GeV, assuming full identification of D and B mesons. For 1LP jets the same values of parameters as in Table II were taken. Note that the quark-mass effects are not negligible in $\tilde{A}_b$ and $\tilde{A}_c$. It follows from the calculations in Ref. 1 that at $\sqrt{s}=30$ GeV we have $\tilde{A}_u=0.102$, $\tilde{A}_d=\tilde{A}_s=0.200$, $\tilde{A}_c=0.101$, and $\tilde{A}_b=0.178$.

Particles identified	Jet type	$\tilde{A}_k^{\rm obs}$ (%)	η_k (%)	P_k
D^+	1LP	-10.06	3.38	3.68
B^+		$+17.71$	0.85	4.73
D^+	NLH	-10.07	15.96	0.78
B^+		$+17.77$	4.00	1.00
$B^+/\bar{B}^0/D^-/\bar{D}^0$	NLH ALT	$+11.61$	39.93	0.23

mesons. In Ref. 6 we described the idea of identifying B-mesons production through electrons or muons with large momentum component perpendicular to the jet axis, $k_\perp$. For the sake of completeness let us briefly repeat some of the results described there. It follows from the discussion in Ref. 6 that, in an idealized case when one neglects the smearing due to the experimental jet axis, for $k_\perp > 1.2$ GeV/c the contamination of bottom sample by charm is negligible and only $\omega_b^{k_\perp}$ and $\bar{\omega}_b^{k_\perp}$ are different from zero. Note that $\bar{\omega}_b^{k_\perp}$ is derived from the number of electrons emitted backward with respect to the parent quark. This number is quite sensitive to the fragmentation function of B mesons. In view of theoretical arguments[10] that the heavy flavors should have a much harder spectrum than the light ones, we consider two different forms[6]

$$D_{\rm BS}(z)=2z \tag{5.7}$$

(where BS refers to Bjorken and Suzuki) and

$$D_{\rm FF}(z)=0.45+1.65(1-z)^2\ , \tag{5.8}$$

although we believe the first choice to be closer to reality. The results based on a $k_\perp$ cut of 1.2 GeV/c are given in Table IV. We also consider an obvious improvement of statistics there due to the inclusion of positrons (with an appropriate change of definition of forward direction) and muons.

A comparison of Tables II and IV shows that this should be by far the best available method for measuring jet asymmetries at the present time. For a good electron and muon detector it requires only about three times the luminosity necessary for $e^+e^-\to\mu^+\mu^-$ asymmetries.

To make these numbers more credible, we applied our method of estimating the jet-axis smearing effect on the charm contamination of the bottom sample described in detail in Ref. 6 and the new results are presented in the second part of Table IV. Obviously, this is still only a theoretical estimate and the final comparison should be done by experimental groups independently, using individual features of their detectors.

TABLE IV. B-meson jet asymmetries at $\sqrt{s}=30$ GeV, calculated based on large-$k_\perp > 1.2$ GeV/c prompt-lepton B identification. Separately considered are the two cases: (i) theoretically idealized situation neglecting the experimental-jet-axis misidentification effect and (ii) more realistic situation including this effect by means described in Ref. 6. The charm fragmentation function was assumed constant for this calculation. The values of quark asymmetries are as in Table III.

Particles identified	Jet type	B fragmentation function	Jet-axis smearing	$\tilde{A}_k^{\rm obs}$ (%)	η_k (%)	P_k
B^-/B^0	$k_\perp^{e^-}$	FF	Neglected	-11.55	0.54	17.54
B^-/B^0	$k_\perp^{e^-}$	BS	Neglected	-13.21	0.53	13.53
$B^-/B^0/B^+/\bar{B}^0$	$k_\perp^{e^-/e^+}$	BS	Neglected	-13.21	1.06	6.77
$B^-/B^0/B^+/\bar{B}^0$	$k_\perp^{l}$	BS	Neglected	-13.21	2.12	3.38
B^-/B^0	$k_\perp^{e^-}$	FF	Included	-8.30	0.58	31.27
B^-/B^0	$k_\perp^{e^-}$	BS	Included	-10.26	0.68	17.66
$B^-/B^0/B^+/\bar{B}^0$	$k_\perp^{e^-/e^+}$	BS	Included	-10.26	1.36	8.83
$B^-/B^0/B^+/\bar{B}^0$	$k_\perp^{l}$	BS	Included	-10.26	2.72	4.41

Finally, let us briefly discuss the situation at higher energies. As is well known, the asymmetries will grow when the energy gets closer to the mass of the Z^0 resonance[1] and the measurability of the effects considered here should increase substantially. Even if the numbers quoted so far have cast some doubt on the feasibility of this type of measurement at PETRA or PEP in view of their luminosity problems, the same effects will be of major interest for the next generation of accelerators. In Table V we give the comparison of our best results (most suitable channels for electroweak jet asymmetry measurements) at three energies, $\sqrt{s} = 30$, 50, and 70 GeV. All numbers have been calculated neglecting any possible influence of a top quark; if it exists and has a threshold in this energy region, the quoted asymmetry values should be modified.

One obvious conclusion from Table V is that if there is any future for quark asymmetry measurements, it is most probably connnected with heavy-flavor identification. Our method from Ref. 6 already gives some hope that this effect might eventually become observable and we strongly believe that in the near future other more sophisticated and more efficient methods will be found that will lead us ultimately to indirect measurements of quarks couplings to the neutral heavy boson Z^0.

TABLE V. The energy dependence of asymmetries and statistics for the best channels from Tables II and IV. In the case of bottom mesons only BS-fragmentation-function results are given. Possible top-quark effects and jet-axis smearing are neglected.

Particles identified	Jet type	$\sqrt{s}$ (GeV)	$\tilde{A}_k^{\text{obs}}$ (%)	η_k (%)	P_k
μ^--μ^+		30	−6.8	27.2	1.00
		50	−24.1	25.1	1.00
		70	−65.1	13.5	1.00
π^+	1LP	30	−1.41	12.3	51.7
		50	−4.67	12.3	54.0
		70	−7.27	12.0	90.1
K^+	1LP	30	+1.94	3.99	84.1
		50	+5.60	4.06	113.8
		70	+5.11	4.45	491.8
π^+/K^-	1LP	30	−1.54	16.3	32.7
	ALT	50	−4.90	16.3	37.0
		70	−6.68	16.4	78.1
K^+-K^-	1LP	30	+9.16	0.17	90.2
	COR	50	+26.0	0.18	119.1
		70	+22.4	0.24	474.5
π^+/K^+-π^-/K^-	1LP	30	−1.65	1.82	127.6
	ALT	50	−5.70	3.63	122.9
	COR	70	−9.72	3.60	168.0
$B^-/B^0/B^+/\bar{B}^0$	$k_\perp^{e^-/e^+} > 1.2$	30	−13.2	1.06	6.77
	GeV/c	50	−45.3	1.24	5.70
		70	−34.2	2.02	24.19
$B^-/B^0/B^+/\bar{B}^0$	$k_\perp^l > 1.2$ GeV/c	30	−13.2	2.12	3.38
		50	−45.3	2.48	2.85
		70	−34.2	4.04	12.09

VI. SUMMARY AND CONCLUSIONS

Using the standard model of the electroweak interaction in the lowest order and the Field-Feynman fragmentation model, we have investigated hadronic-jet angular asymmetries in e^+e^- annihilation, in the energy range 30 to 70 GeV. We emphasized the practical limitations in calculating the asymmetry when subject to experimental restrictions, such as efficiency and statistics. These, combined with theoretical estimates, give the realistic expectations of the various possible channels considered by us and by others. The numerical results should give good indications about the observable size of the effects and suggest rules for reasonable jet selection. An objective criterion in the comparison of various channels allowed us to eliminate the misleading conceptions in selecting channels based on previous theoretical considerations. This overindulgence in theoretical considerations often seriously limits the statistics of a data sample to such an extent that the effect can never be observed within a reasonable length of time of data acquisition. Furthermore, this can also lead to a bias against some channels having smaller observed asymmetries but turning out more suitable with all realistic restrictions taken into account.

In this paper we have concentrated on the analysis of quark fragmentation, leaving the simplest version of quark production cross section from Ref. 1 unchanged. In Ref. 1 only the lowest-order diagrams in quark production are considered. We feel that we owe the readers some explanation why we neglect the higher-order QED and QCD corrections and what are their expected contributions.

1. Higher-order QED corrections (α^3 and α^4) have been analyzed in the literature[12] for the process $e^+e^-\to l^+l^-$ and $e^+e^-\to l^+l^-\gamma$, where $l=e$ or μ and are known to give substantial contributions to asymmetries. However, a recent estimate of these effects in quark production[5] shows that if one cuts out the forward and backward cones where their contribution is largest, the remaining QED asymmetries should be much smaller than the electroweak ones. Cutting out the curves of $\theta_{\min}=20°$, as suggested in Ref. 5, has negligible effect on our asymmetries, increases the required luminosity by only 17% [see Fig. 4(b)], and reduces QED asymmetries to less than 50% of the electroweak asymmetries. A detailed discussion of QED including possible quark mass effects will be deferred to a future publication.

2. Higher-order QCD corrections to quark asymmetries have been recently calculated in Ref. 15. The main result is that apart from the jet-axis-misidentification effect, hard gluons give rise to only a small modification to the asymmetries which is factorizable and no more than 6% of the asymmetries calculated in the previous sections. Hence the QCD modifications of the asymmetry are certainly smaller than the uncertainties existing in the first-generation experiments and may be safely neglected at present.

Our present limited knowledge of fragmentation processes introduces sizable uncertainties in theoretical estimates of the observable effects. However, from the discussions of Secs. IV and V, several general important conclusions may be drawn.

1. The effects of electroweak jet asymmetries should be observable. A judicious selection of jets may give us a sample of events with much larger asymmetry than that of muons. However, as follows from the numbers listed in Table V, it may take us quite a long time with the present e^+e^- luminosities before sufficiently good statistics are achieved.

2. In no case considered here can quark jets compete with muons in the feasibility of the asymmetry experiment. However, some of them come close to muons (a factor of 3 in our best case) and it is quite possible that with the discovery of some new method of heavy-flavor identification they will become comparable. Even if this test of standard model should be carried out first in muonic channel, there is a good deal of interesting physics contained in the hadronic jet asymmetries and they should be measured.

3. The heavy flavors, especially bottom, have a much better chance to be isolated due to their large mass.

4. Before we try to measure the asymmetries of jets involving light-flavored particles only, it is crucial that we know to good precision the value of the parameter ξ_s, e.g., from one-particle inclusive data. The asymmetries of jets defined by strange particles are extremely sensitive to the change of this parameter.

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APPENDIX: RANK-VIOLATION PROBABILITIES

In principle, the rank-violation probabilities P_α and $P_{\alpha\beta}$, defined in Sec. II A, depend only on parameter a in Eq. (3.4). However, introducing the cuts in variable z to avoid problems connected with leading secondaries (see Sec. III), we have to replace P_α and $P_{\alpha\beta}$ by conditional probabilities defined as follows: $P_\alpha(z_1^0)=$ probability that the particle of rank α is leading and has a $z\geq z_1^0$; $P_{\alpha\beta}(z_1^0,\, z_2^0)=$ probability that the particle of rank α is leading with $z\geq z_1^0$ and the particle of rank β is next to leading with $z\geq z_2^0$. Obviously now the sum of probabilities is not equal to 1,

$$\sum_\alpha P_\alpha(z)=\widetilde{P}(z)<1\ . \tag{A1}$$

To give the reader some feeling as to how strongly these probabilities depend on parameter a and indices α, β, we present in Tables VI and VII the numerical values of matrix $\hat{P}_{\alpha\beta}(z_1^0=\frac{1}{3},\, z_2^0=0)$ for $a=0.88$ (Ref. 7) and $a=0.55$ (Ref. 9).

Note that in general

$$P_\alpha(z_1^0)\geq \sum_\beta P_{\alpha\beta}(z_1^0,z_2^0) \tag{A2}$$

and the equality holds only for $z_2^0=0$.

From quantities $P_\alpha(z)$ we can now derive the required probabilities ω_i^k. Consider first 1LP jets. Let particle a consist of quarks $q_j\bar{q}_l$. Then

$$\omega_i^a(z_1^0)\equiv\omega_i^{j\bar{l}} =\frac{1}{2}\sum_\alpha C_\alpha^{ijl}P_\alpha(z_1^0)\ , \tag{A3}$$

where

$$C_\alpha^{ijl}=\begin{cases}0 & \text{for } \alpha=1,\ i\neq j\\ \xi_l & \text{for } \alpha=1,\ i=j\\ \xi_j\xi_l & \text{for } \alpha\neq 1\end{cases} \tag{A4}$$

or, equivalently,

$$C_\alpha^{ijl}=\delta_{\alpha 1}\delta_{ij}\xi_l+(1-\delta_{\alpha 1})\xi_j\xi_l\ . \tag{A4$'$}$$

As discussed in Sec. III, the factor $\frac{1}{2}$ in (A3) is responsible for missing the vector-meson production. Equations (A4) and (A4′) follow in an obvious way from Feynman-Field cascade fragmentation picture.[7] Substituting (A4′) into (A3), we get

TABLE VI. Rank-violation probabilities.

$\alpha \backslash \beta$	1	2	3	4	5	6
	$\hat{P}_{\alpha\beta}(z_1^0=\frac{1}{3},z_2^0=0;a=0.88)$					
1	0	0.125	0.082	0.052	0.030	0.017
2	0.086	0	0.041	0.025	0.016	0.008
3	0.037	0.033	0	0.012	0.008	0.004
4	0.015	0.013	0.011	0	0.004	0.002
5	0.006	0.004	0.004	0.003	0	0.001
6	0.002	0.002	0.002	0.001	0.001	0
	$\hat{P}_\alpha(z_1^0=\frac{1}{3},a=0.88)$					
α	1	2	3	4	5	6
	0.323	0.182	0.098	0.047	0.019	0.008
	$\widetilde{P}(z_1^0=\frac{1}{3},\ a=0.88)=0.680$					

TABLE VII. Rank-violation probabilities.

α \ β	1	2	3	4	5	6
	$\hat{P}_{\alpha\beta}(z_1^0=\frac{1}{3},z_2^0=0,a=0.55)$					
1	0	0.190	0.111	0.063	0.032	0.016
2	0.125	0	0.051	0.027	0.014	0.006
3	0.046	0.040	0	0.014	0.008	0.003
4	0.017	0.014	0.011	0	0.004	0.002
5	0.006	0.004	0.004	0.003	0	0.001
6	0.002	0.001	0.001	0.001	0.001	0
	$\hat{P}_{\alpha}(z_1^0=\frac{1}{3},a=0.55)$					
α	1	2	3	4	5	6
	0.422	0.227	0.114	0.049	0.018	0.006
	$\tilde{P}(z_1^0=\frac{1}{3},\ a=0.55)=0.836$					

$$\omega_i^{j\bar{l}}=\tfrac{1}{2}\delta_{ij}\xi_l P_1(z_1^0)+\tfrac{1}{2}\xi_j\xi_l[\tilde{P}(z_1^0)-P_1(z_1^0)]\ . \tag{A5}$$

It follows that for practical purposes we only need to know two parameters, $\tilde{P}$ and P_1 instead of the whole $\hat{P}_\alpha$ matrix.

The generalization of Eqs. (A3)–(A5) into the case of 2LP jets is straightforward, although the notation becomes complicated due to the number of indices involved. Let particle a consist of quarks $q_j\bar{q}_l$ as before, and particle b, $q_p\bar{q}_r$. Then the following relations hold:

$$\omega_i^{a,b}(z_1^0,z_2^0)\equiv\omega_i^{j\bar{l},p\bar{r}}(z_1^0,z_2^0)=\tfrac{1}{2}\sum_{\alpha,\beta}C_{\alpha\beta}^{ijlpr}P_{\alpha\beta}(z_1^0,z_2^0)\ , \tag{A6}$$

where

$$C_{\alpha\beta}^{ijlpr}=\begin{cases}0 & \text{for } \alpha=1,\ i\neq j\ ,\\ 0 & \text{for } \beta=1,\ i\neq p\ ,\\ 0 & \text{for } \beta=\alpha+1,\ l\neq p\ ,\\ 0 & \text{for } \alpha=\beta+1,\ j\neq r\ ,\\ \xi_l\xi_r & \text{for } \alpha=1,\ \beta=2,\ i=j,\ l=p\ ,\\ \xi_j\xi_l & \text{for } \alpha=2,\ \beta=1,\ i=p,\ j=r\ ,\\ \xi_l\xi_p\xi_r & \text{for } \alpha=1,\ \beta>2,\ i=j\ ,\\ \xi_j\xi_l\xi_r & \text{for } \beta=1,\ \alpha>2,\ i=p\ ,\\ \xi_j\xi_l\xi_r & \text{for } \beta=\alpha+1\geq 2,\ l=p\ ,\\ \xi_j\xi_l\xi_p & \text{for } \alpha=\beta+1\geq 2,\ j=r\ ,\\ \xi_j\xi_l\xi_p\xi_r & \text{in all other cases}\ ,\end{cases} \tag{A7}$$

or, equivalently,

$$\begin{aligned}C_{\alpha\beta}^{ijlpr}=(1-\delta_{\alpha\beta})\{&\delta_{\alpha1}\delta_{ij}[\delta_{\beta2}\delta_{lp}\xi_l\xi_r+(1-\delta_{\beta2})\xi_l\xi_p\xi_r]\\&+\delta_{\beta1}\delta_{ip}[\delta_{\alpha2}\delta_{jr}\xi_j\xi_l+(1-\delta_{\alpha2})\xi_j\xi_l\xi_r]\\&+(1-\delta_{\alpha1})(1-\delta_{\beta1})[\delta_{\alpha,\beta+1}\delta_{jr}\xi_j\xi_l\xi_p+\delta_{\alpha,\beta-1}\delta_{lp}\xi_j\xi_l\xi_r\\&+(1-\delta_{\alpha,\beta+1})(1-\delta_{\alpha,\beta-1})\xi_j\xi_l\xi_p\xi_r]\}\ .\end{aligned} \tag{A7$'$}$$

Substituting (A7′) into (A6), one could derive an equation analogous to (A5) but it is too complicated to be of any use for discussion purposes. However, it seems worthwhile to mention that in this case the entire information required is given by seven parameters. These are

$$\begin{aligned}
\widetilde{P}_1(z_1^0,z_2^0) &= \sum_{\beta} P_{1\beta}(z_1^0,z_2^0) \, , \\
\widetilde{P}_2(z_1^0,z_2^0) &= \sum_{\alpha} P_{\alpha 1}(z_1^0,z_2^0) \, , \\
\widetilde{P}_3(z_1^0,z_2^0) &= \sum_{\alpha=\beta+1} P_{\alpha\beta}(z_1^0,z_2^0) \, , \\
\widetilde{P}_4(z_1^0,z_2^0) &= \sum_{\alpha=\beta-1} P_{\alpha\beta}(z_1^0,z_2^0) \, , \\
\widetilde{P}(z_1^0,z_2^0) &= \sum_{\alpha,\beta} P_{\alpha\beta}(z_1^0,z_2^0) \, ,
\end{aligned} \tag{A8}$$

and two original matrix elements, $P_{12}(z_1^0,z_2^0)$ and $P_{21}(z_1^0,z_2^0)$.

In closing, let us point out that the probability for a quark q to fragment into a 1LP jet with h_1 leading may be written in compact form in terms of the fragmentation functions for $z_1^0 > \frac{1}{3}$. If $D_q^{h_1}(z)$ and $D_q^{h_1 h_2}(z_1,z_2)$ are the single- and two-particle fragmentation functions,[7] we may write the probability as

$$\begin{aligned}
\omega_i^{h_1} = &\int_{z_1^0}^{1} D_q^{h_1}(z_1) dz_1 \\
&- \sum_{h_2} \int_{z_1^0}^{1} dz_1 \int_{z_1}^{1} dz_2 D_q^{h_1 h_2}(z_1,z_2) \, ,
\end{aligned} \tag{A9}$$

where the sum in the second term on the right-hand side runs over all observed hadrons.

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[1]M. J. Puhala, Z. J. Rek, and B. L. Young, Phys. Rev. D 23, 89 (1981).

[2]L. Camillieri *et al.*, CERN Yellow Report No. 76-18, 1976 (unpublished).

[3]J. Ellis and M. K. Gaillard, Ref. 2.

[4]D. H. Schiller, Z. Phys. C 3, 21 (1979); G. Schierholz and D. H. Schiller, DESY Reports Nos. DESY 79/29, 1979 and DESY 80/88, 1980 (unpublished); F. W. Bopp and D. H. Schiller, Z. Phys. C 6 161 (1980).

[5]C. Iso and Y. Ishihara Munehisa, Prog. Theor. Phys. 65, 1959 (1981).

[6]M. J. Puhala, Z. J. Rek, B. L. Young, and X. T. Zhu, Phys. Lett. (to be published); Phys. Rev. D (to be published).

[7]R. D. Field and R. P. Feynman, Nucl. Phys. B136, 1 (1978).

[8]R. Orava (private communication); V. V. Amosov *et al.*, Phys. Lett. 93B, 210 (1980).

[9]S. L. Wu, in *High-Energy Physics—1980,* Proceedings of the XXth International Conference, Madison, Wisconsin, edited by L. Durand and L. G. Pondrom (AIP, New York, 1981), p. 604; S. Yamada, *ibid.*, p. 606.

[10]M. Suzuki, Phys. Lett. 71B, 139 (1977); J. D. Bjorken, Phys. Rev. D 17, 171 (1978).

[11]We should stress here that our calculations and numbers do not apply directly to Ref. 5 since a different cut in momentum was applied there (a cut for the sum of z variables simultaneously for both leading particles in opposite jets) and a different fragmentation model is used. It is our belief, however, that qualitatively that method leads to a serious worsening of statistics.

[12]R. A. Berends, K. J. F. Gaemers, and R. Gastmans, Nucl. Phys. B63, 381 (1973); F. A. Berends and G. J. Komen, Phys. Lett. 63B, 432 (1976); G. Pessarino and M. Veltman, Nucl. Phys. B160, 151 (1979); F. A. Berends and R. Kleiss, *ibid.* 177, 237 (1981).

[13]Even if the amount of charm turns out to be surprisingly large in fragmentation functions as it is now believed to be in proton structure functions [see e.g., S. Brodsky, P. Hoyer, C. Peterson, and N. Sakai, Phys. Lett. 93B, 451 (1980) or P. Hoyer, in *High Energy Physics—1980* (Ref. 9), it will not change our results qualitatively.

[14]G. G. Hanson, in *Gauge Theories and Leptons,* proceedings of the XIIIth Rencontre de Moriond, Les Arcs, France, edited by J. Trân Thanh Vân (Editions Frontieres, Dreux, France, 1978), p. 15; C. Bebek *et al.*, Phys. Lett. 46, 84 (1981); G. Wolf, in *The Weak Interactions,* proceedings of the Summer Institute on Particle Physics, SLAC, 1980, edited by Anne Mosher (SLAC, Stanford, 1981).

[15]J. Jersak, E. Laermann, and P. M. Zerwas, Phys. Lett. 98B, 363 (1981).