Pion-double-charge-exchange contribution to neutrinoless double-β decay

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(Received 13 October 1981)

It is shown that the double charge exchange of pions in flight between two nucleons makes a substantial contribution to neutrinoless nuclear double-β decay if it is mediated by heavy Majorana neutrinos.

Neutrinoless double-β decay has recently become[1–6] a very interesting process due to recent developments in gauge theories[7–12] which predict that lepton charge (number) must be broken at some level. In this Communication we explore a new mechanism for neutrinoless double-β decay which involves the double charge exchange of pions in flight between two nucleons.

In a previous paper,[13] which will be referred to as I, we have examined the neutrinoless double-β decay

$$(A,Z)\rightarrow(A,Z+2)+e^-+e^- \ , \tag{1}$$

concentrating solely on a mechanism which involved only nucleons. We showed that even if the mediating Majorana neutrino is very massive the two-body transition operator behaves as e^{-mr}/r with $m\approx m_p$ provided that the nucleon form factor is taken into account. Thus, this operator does not suffer from severe short-range pathologies and the two-nucleon mechanism will substantially contribute to neutrinoless double-β decay even in the presence of the nuclear hard core.[13] It is tempting, however, to consider the contribution of other particles which can undergo a change of charge by two units and have a reasonable chance to be found in the nuclear medium. Such candidates are the nucleon isobars[14] and in particular

$$n\rightarrow\Delta^{++}+e^-+e^- \ . \tag{2}$$

Such a process, however, does not contribute to the $0^+\rightarrow0^+$ nuclear decay due to angular momentum selection rules.[15] Other processes, such as

$$\Delta^0\rightarrow\Delta^{++}+e^-+e^-, \quad \Delta^-\rightarrow\Delta^++e^-+e^- \ , \tag{3}$$

may contribute to second order but their contribution will be small since they are proportional to $P_\Delta{}^2\approx10^{-4}$ (P_Δ is the probability of finding such an isobar inside the nucleus). More recently[16] Halprin has investigated the contribution of processes like $n\rightarrow\Delta(1910)+e^-+e^-$ to reaction (1). Such a process is indeed allowed from the point of view of angular momentum. Kotani, however, has argued[16] that even this mechanism will not substantially contribute to $0^+\rightarrow0^+$ nuclear transitions since the $\Delta(1910)$ must predominantly have $l=2$ (around its center of mass). Furthermore, the probability of finding such a particle inside the nucleus is not known. Kotani argues again that, due to the heavy mass of $\Delta(1910)$, this probability must be very small.[16] Thus the situation at this point is not completely clear.

In the present paper we will investigate another mechanism which does not suffer from the above ambiguities, namely, the double charge exchange of pions in flight between the two nucleons:

$$\pi^-\rightarrow\pi^++e^-+e^- \ . \tag{4}$$

The relevant Feynman diagrams which give rise to (4) are shown in Fig. 1. The amplitude associated with Fig. 1(a) takes the form

$$\mathcal{M}=[\bar N(p_3)g_r\gamma_5\tau_+N][\bar N(p_4)g_r\tau_+\gamma_5N(p_2)]\frac{1}{(p_1-p_3)^2-m_\pi{}^2}\frac{1}{(p_4-p_2)^2-m_\pi{}^2}\mathcal{M}_1 \ . \tag{5}$$

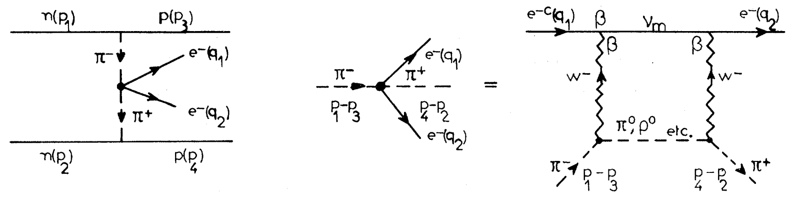
All the essential new physics is contained in the amplitude $\mathcal{M}_1$ associated with Fig. 1(b). In evaluating this diagram we will follow a procedure analogous to that described previously[13] in I. The charged current in the leptonic vertex is taken to be of the form

$$\begin{pmatrix}\nu_e+\beta N_0\\ e^-\end{pmatrix}_L \ , \tag{6}$$

where ν_e and N_0 are the light and heavy components of the intermediate Majorana neutrino ν_m given by

$$\nu_e=\sum_j X_{ej}^{(1)}\nu_j, \quad N_0=\sum_j X_{ej}^{(2)}N_j \ , \tag{7}$$

where ν_j (N_j) are the (Majorana) mass eigenstates in the light (heavy) sector and β is a convenient parametrization of the mixing between the two groups.



(a) (b)

FIG. 1. (a) Lepton-number-violating diagrams associated with the double charge exchange of a pion in flight between the two nucleons. (b) The elementary process $\pi^- \to e^- + e^- + \pi^+$ is exhibited.

The amplitude associated with the heavy sector takes the form

$$\mathcal{M}_1 = \frac{G_F}{m_p}\eta_\sigma I_{\lambda\mu}(1-P_{12}) \times [\bar{e}^-(q_2)\gamma^\lambda\gamma^\mu(1+\gamma_5)e^{-c}(q_2)] \ , \quad (8a)$$

where G_F is the Fermi coupling constant,

$$\eta_\sigma = \beta^2\frac{m_p}{m_\sigma}, \quad \frac{1}{m_\sigma} = \sum_j (X_{ej}^{(2)})^2\frac{1}{M_j} \ , \quad (8b)$$

P_{12} is the permutation operator exchanging the spins and momenta of the two electrons, and M_j are the heavy-mass eigenvalues.

The quantity $I_{\lambda\mu}$ will be calculated in two ways.

Case (i). The pions will be treated as "elementary." Only the π^0 will be included in the intermediate mesonic states. The vertex functions are taken[17] to be $\sqrt{2}(p_1-p_2+q)_\lambda$ and $\sqrt{2}(p_4-p_2+q)_\mu$, each multiplied by a dipole shape form factor[13]

$$F(k^2) = \frac{1}{(1-k^2/m_A{}^2)^2}, \quad m_A = 0.85 \text{ GeV}/c^2 \ , \quad (9)$$

where k is the momentum transfer at the vertex. Ignoring the external momenta in front of m_A and employing standard loop-integral techniques[18] we obtain

$$I_{\lambda\mu} \approx 2\int \frac{d^4q}{(2\pi)^4}\frac{(p_1-p_3+q)_\lambda}{q^2-m_\pi{}^2}\frac{(p_4-p_2+2)_\mu}{(1-q^2/m_A{}^2)^4} \cong g_{\lambda\mu}\frac{M_A{}^4}{192\pi^2} \ , \quad (10)$$

$$\mathcal{M}_1 = \tfrac{1}{2}(G_F m_p)^2\eta_\sigma 4 m_\pi \xi_\pi (1-P_{12}) \times [\bar{e}^-(q_2)(1+\gamma_5)e^{-c}(q_1)] \ , \quad (11)$$

with

$$\xi_\pi = \frac{1}{96\pi^2}\frac{m_A}{m_\pi}\left(\frac{m_A}{m_p}\right)^3 \ . \quad (12)$$

It is now pretty straightforward to make a nonrelativistic reduction of the amplitude of Eq. (5). Furthermore, since the momenta of the electrons are negligible in front of m_π we can easily transform this amplitude to coordinate space to obtain

$$\mathcal{M} = \tfrac{1}{2}(G_F m_p)^2\eta_\sigma\frac{m_e}{m_p}f_A{}^2\frac{\hbar c}{m_p c^2 R_0}\langle f|\Omega_\pi|i\rangle \times (1-P_{12})[\bar{e}^-(q_2)(1+\gamma_5)e^{-c}(q_1)] \ , \quad (13)$$

where $R_0 = r_0 A^{1/3}$ = nuclear radius, $f_A = 1.24$, and

$$\Omega_\pi = \alpha_\pi\frac{m_p}{m_e}\sum_{i\neq j}\tau_+(i)\tau_+(j)\frac{R_0}{r_{ij}} \times [F_1(x_\pi)\vec{\sigma}_i\cdot\vec{\sigma}_j + F_2(x_\pi)(3\vec{\sigma}_i\cdot\hat{r}\,\vec{\sigma}_j\cdot\hat{r} - \vec{\sigma}_i\cdot\vec{\sigma}_j)] \ , \quad (14a)$$

$$\hat{r} = \frac{\vec{r}_{ij}}{r_{ij}}, \quad \vec{r}_{ij} = \vec{r}_i - \vec{r}_j, \quad x_\pi = r_{ij}\frac{m_\pi c^2}{\hbar c} \ , \quad (14b)$$

$$F_1(x) = (x-2)e^{-x}, \quad F_2(x) = (x+1)e^{-x} \ , \quad (14c)$$

$$\alpha_\pi = \frac{1}{72\pi}\left(\frac{m_A}{m_p}\right)^2\left(\frac{m_A}{m_\pi}\right)^2\frac{(f_{\pi NN})^2}{f_A{}^2} = 0.0072 \ , \quad (14d)$$

$$(f_{\pi NN})^2 = 0.08 \ .$$

Case (ii). The pions are considered as composite particles described in terms of a bound quark-antiquark system. The bound wave function can be treated in the fashion of Adler *et al.*[19] or in the quark-bag-model approach.[20] For our purposes it is adequate to consider the quarks as nonrelativistic with a quark density corresponding to the phenomenological form factor of Eq. (9). The operator at the quark level can be obtained as in I. Employing closure for the intermediate meson states we finally obtain

$$I_{\lambda\mu} \approx \frac{m_A{}^4}{96\pi^2}\langle\pi^+|\omega_{\lambda\mu}|\pi^-\rangle \ , \quad (15a)$$

where

$$\omega_{\lambda\mu} = \sum_{i\neq j}\tau_+(i)\tau_+(j)L_\lambda(i)L_\mu(j) \ ,$$

$$L_\lambda(i) = \begin{cases} 1, & \lambda = 0 \\ -\vec{\sigma}(i), & \lambda \neq 0 \ . \end{cases} \quad (15b)$$

In the above expression unlike Eq. (14a), the indices i and j label the quarks and $|\pi^+\rangle$, $|\pi^-\rangle$ describe the spin-isospin part of the quark-antiquark wave function. Combining Eqs. (15a), (15b), and (8) we once again obtain (3) with

$$\alpha_\pi = 0.06 \quad . \tag{16}$$

The larger value is attributed to the fact that, by invoking closure, we included here intermediate states other than π^0. Equation (13) is the same with that obtained in I except for the nuclear matrix element which in I was found to be[13] $\langle f|\Omega_\sigma|i\rangle$ with

$$\Omega_\sigma = \sum_{i\neq j} \tau_+(i)\tau_+(j)\left(\frac{f_V^2}{f_A^2} - \vec{\sigma}_i\cdot\vec{\sigma}_j\right)\frac{R_0}{r_{ij}}F(x_A) \quad ,$$

$$x_A = \frac{m_A c^2 r_{ij}}{\hbar c}, \quad F(x) = \tfrac{1}{48}x(x^2+3x+3)e^{-x} \quad .$$

Thus one can apply the analysis of double-β decay given in I with the only modification that

$$\langle f|\Omega_\sigma|i\rangle \rightarrow \langle f|\Omega_\sigma + \Omega_\pi|i\rangle \quad .$$

In order to estimate the pionic contribution to double-β decay we will examine the ^{48}Ca $\rightarrow$ ^{48}Ti transition.[5,13,19] We will employ a nuclear model which has been described in some detail elsewhere.[6,13] For the reader's convenience we summarize its basic features here. The $N=0,1,2$ harmonic-oscillator shells are assumed completely filled (inert core) and the active nucleons are distributed in the $1f_{7/2}$ shell. Thus the initial ^{48}Ca state is a closed $1f_{7/2}$ neutron shell, while the final state contains two protons and six neutrons in the $1f_{7/2}$ shell. The precise final nuclear wave function depends on the effective two-nucleon interaction[13] used, but the final results reported here are not too sensitive to such variations.

Our numerical results were obtained using

$$^{48}\mathrm{Ti(g.s.)} = 0.8451|1\rangle - 0.5233|2\rangle + 0.1094|3\rangle - 0.0009|4\rangle \quad ,$$

with

$$|i\rangle = |f_{7/6}^6(n)J_i; f_{7/2}^2(p)J_i; 0^+\rangle, \quad J_i = 0,2,4,6 \quad .$$

With the above nuclear wave function we obtained

$$\langle f|\Omega_\pi|i\rangle = 14 \quad [\text{(i)}]$$

and

$$\langle f|\Omega_\pi|i\rangle = 94 \quad [\text{(ii)}] \quad .$$

This must be compared with the matrix element of the two-nucleon mode

$$\langle f|\Omega_\sigma|i\rangle = 72 \quad .$$

With the above pionic contribution included, the limits obtained in I change by approximately a factor of 2. Thus using the experimental lower limit[21] on the lifetime of ^{48}Ca $\rightarrow$ ^{48}Ti we obtain

$$\beta^2 \frac{m_p}{m_\sigma} \leq 5\times10^{-8} \quad ,$$

$$m_\sigma \gtrsim (2\times10^4) - (2\times10^5)\ \mathrm{GeV}/c^2 \quad .$$

Analogous calculation for the light-neutrino sector shows that the pionic matrix element is less than 1% of that involving only nucleons. So in this case it can safely be neglected.

In conclusion, we have shown that if η_σ is not negligible, the heavy component of the Majorana neutrino will make an important contribution to neutrinoless double-β decay. A substantial part of this contribution may come from the pionic mode discussed in this work.

[1]See, e.g., H. Primakoff and S. P. Rosen, Phys. Rev. 184, 1925 (1969), and references therein.

[2]For a review, particularly of the experimental situation, and a complete list of references, see D. Bryman and C. Picciotto, Rev. Mod. Phys. 50, 11 (1978); Yu. G. Zdesenko, Fiz. Elem. Chastits At. Vatra 11, 1369 (1980) [Sov. J. Part. Nucl. 11, 542 (1980)]; see also, H. Primakoff and S. P. Rosen, Purdue University report 1981 (unpublished).

[3]W. C. Haxton, G. J. Stephenson, and S. Strottman, Phys. Rev. Lett. 47, 153 (1981).

[4]M. Doi, T. Kotani, H. Nishiura, K. Otuda, and E. Takusugi, Phys. Lett. 30B, 219 (1981).

[5]E. Fiorini *et al.*, Nuovo Cimento A 13, 747 (1973).

[6]J. D. Vergados, Phys. Rev. C 13, 865 (1976).

[7]In the standard Weinberg-Salam model [S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory, Relativistic Groups and Analyticity (Nobel Symposium No. 8),* edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367; S. L. Glashow, Nucl. Phys. 22, 579 (1961)], lepton number is conserved.

[8]Grand unification schemes, with the one exception of the "minimal" SU(5), contain $\Delta L=2$ terms. For SU(5) see, e.g., H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974); for SO(10) see, e.g., H. Fritzsch and P. Minkowski, Ann. Phys. 93, 193 (1975); H. Georgi and D. V. Nanopoulos, Nucl. Phys. B155, 52 (1979).

[9]M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity,* edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979), p. 315.

[10]R. Barbieri, D. V. Nanopoulos, G. Morchio, and F. Strocchi, Phys. Lett. 90B, 91 (1980).

[11]E. Witten, Phys. Rev. Lett. 91B, 81 (1980). For a clear review on the Majorana mass see, e.g., E. Witten, Harvard University Report No. HUTP-80/A051 (unpublished).

[12]R. N. Mohapatra and G. Senjanovic, Phys. Rev. D 23, 165 (1981).
[13]J. D. Vergados, Phys. Lett. (to be published).
[14]A. Halprin, P. Minkowski, H. Primakoff, and S. P. Rosen, Phys. Rev. D 13, 2567 (1976); J. D. Vergados and M. Ericson, Nucl. Phys. (to be published).
[15]M. Doi, T. Kotani, H. Nishiura, K. Okuda, and E. Takasugi, Osaka University Report No. OS-GE 81-28, 1981 (unpublished).
[16]A. Halprin, Phys. Rev. D 24, 2988 (1981); T. Kotani (private communication).
[17]E. D. Commins, *Introduction to Weak Interactions* (McGraw-Hill, New York, 1973), p. 92.
[18]G. 't Hooft and M. Veltman, Report No. CERN 73-9, 1973 (unpublished).
[19]S. L. Adler, E. W. Colglazier, Jr., J. B. Healy, T. Karliner, J. Lieberman, Y. J. Ng, and H. S. Tsao, Phys. Rev. D 11, 3309 (1975).
[20]A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D 10, 2599 (1974); A. Chodos *et al., ibid.* 9, 3471 (1974); W. A. Bardeen *et al.,* 11, *ibid.* 1094 (1975).
[21]B. J. Cleveland, W. R. Leo, C. S. Wu, P. J. Callon, and J. D. Ullman, Phys. Rev. Lett. 35, 757 (1975); R. K. Burdin, P. J. Callon, J. D. Ullman, and C. S. Wu, Nucl. Phys. A158, 1337 (1970); E. Mateosian and M. Goldhaber, Phys. Rev. 146, 810 (1966).