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Majoron emission by neutrinos

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The presence of a massless boson coupled to neutrinos in the Gelmini-Roncadelli gauge model can be tested in π, K leptonic decays and in wrong-sign single-lepton production by neutrinos. We place bounds on Majoron-neutrino couplings from experimental limits on these processes.

A Nambu-Goldstone boson, called the Majoron, arises in gauge models that have a spontaneous breaking of a $B-L$ global symmetry.[1,2] There are two possible models of this kind, with and without right-handed neutrinos. In the latter case, proposed by Gelmini and Roncadelli (GR),[1] the Majoron may have interesting observable consequences, some of which have been recently considered.[3] In this paper we examine possible effects of the Majoron in leptonic weak decays of kaons and pions, and in deep-inelastic scattering of neutrinos; the processes of interest are illustrated in Fig. 1. We derive experimental upper limits on the Yukawa coupling of the Majoron to neutrinos.

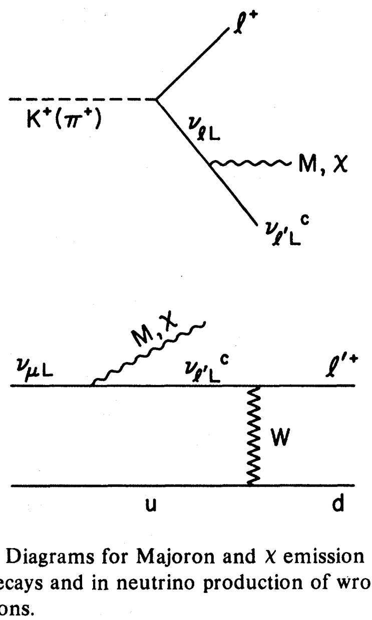

FIG. 1. Diagrams for Majoron and χ emission in K or π leptonic decays and in neutrino production of wrong-sign single leptons.

In the GR model the standard $SU(2)\times U(1)$ model is modified by a triplet of Higgs bosons Ψ $(\psi^{++}, \psi^{+}, \psi^{0})$ in addition to the usual doublet Φ. Ψ is assigned a nonzero $B-L$ number and $B-L$ as a global symmetry is preserved in the Lagrangian, but is broken spontaneously by a vacuum expectation value v_T of ψ^0. There is a genuine zero-mass Nambu-Goldstone boson (Majoron) M and a light neutral Higgs boson χ, whose couplings to neutrinos are given by

$$\mathcal{L}_{\nu M+\nu\chi}=\frac{1}{2}\sum_{l,l'} g_{ll'}\bar{\nu}_l(v_T+i\gamma_5 M+\chi)\nu_{l'} \ , \qquad (1)$$

where l,l' go over e, μ, τ and $\nu=\nu_L+\nu_L^c$. M and χ also couple to other fermions, but much more weakly; those couplings are of order $m_f v_T/v_D^2$, where $v_D \sim 250$ GeV is the vacuum expectation value of the usual $I=\frac{1}{2}$ Higgs field and $v_T<\frac{1}{10}v_D$ from measurements of the neutral-current strength.[4] We will be only concerned with the couplings of Eq. (1) in this paper.

Neutrino masses are obtained by diagonalizing the mass matrix whose elements are $g_{ll'}v_T$. If the resulting flavor mixings are small, then approximately

$$m_{\nu_e} \simeq g_{ee} v_T, \quad \text{etc.}$$

However, if the $g_{ll'}$ are all comparable, then all mixing angles are large and two of the three neutrino masses are nearly zero. In Feynman amplitudes for Majoron-emission processes, the neutrino eigenmasses appear in the virtual-neutrino propagators and the diagonal couplings are related to the $g_{ll'}$ by a unitary transformation. For simplicity we shall frequently use a generic label g to denote the effective overall coupling, with the understanding that g is process dependent.

Some bounds on the parameters in the GR model have been derived by Georgi, Glashow, and Nussinov.[3] These are $v_T < 100$ keV (lest cores of red supergiant stars lose energy too soon), $m_{\nu_e} < 15$ eV for a Majorana ν_e from analysis[5] of neutrinoless double-β decay of ^{76}Ge and ^{82}Se, and $g \leqslant 10^{-3}$ from analysis[3,6] of the ratio of double-β decays ^{128}Te and ^{130}Te. By using $v_T < 100$ keV, m_X is expected to be of order 100 keV or less. In this paper we deduce limits on Majoron couplings to ν_e and ν_μ from analyses of leptonic decays.

$K(\pi) \to l\nu M$ decay. A Majoron (or χ) can be emitted from the neutrino of $K(\pi) \to l\nu$ decays. The differential decay rate for Majoron emission is

$$d\Gamma = \frac{g^2 G_F{}^2 f_K{}^2 \sin^2\theta_C}{128\pi^5 m_K} \frac{(2K\cdot M)(K\cdot l) - m_K{}^2 M\cdot l}{m_x{}^2 - m_\nu{}^2} \times \delta^4(K - l - \nu - M)\frac{d^3l}{E_l}\frac{d^3\nu}{E_\nu}\frac{d^3M}{E_M} \quad , \tag{2}$$

where f_K is the kaon decay constant and θ_C is the Cabibbo angle. The four-momenta of the kaon, charged lepton, Majoron, and virtual neutrino are denoted by K, l, M, and N, respectively, and $m_x{}^2 = N^2$. We introduce the dimensionless quantities $x \equiv m_x{}^2/m_K{}^2 = 1 + \alpha - 2E_l/m_l$, $\alpha \equiv m_l{}^2/m_K{}^2$, and $\beta \equiv m_\nu{}^2/m_K{}^2$. The invariant decay-lepton distribution in x can be expressed in the factorized form

$$d\Gamma(K \to l\nu M) = dR\;\Gamma(K \to lN) \quad , \tag{3}$$

where

$$dR = dx(x - \beta)g^2/32\pi^2x^2 \tag{4}$$

and $\Gamma(K \to lN)$ is the leptonic-decay rate to a neutrino N of mass m_x

$$\Gamma(K \to lN) = G_F{}^2 f_K{}^2 \sin^2\theta_C m_K{}^3[x + \alpha - (x - \alpha)^2]\lambda^{1/2}(1,x,\alpha)/8\pi \quad . \tag{5}$$

Note that $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. The kinematic limits on x are $\beta \leqslant x \leqslant (1 - \sqrt{\alpha})^2$. Equations (3)–(5) also apply to $\pi^+ \to l^+\nu M$ decay, replacing m_K, f_K, $\sin\theta_C$ by m_π, f_π, $\cos\theta_C$. For χ emission dR is given by

$$dR = dx(x^2 + \beta^2 + 6x\beta - \gamma x - \gamma\beta)(x - \beta)^{-2}\lambda^{1/2}(x,\beta,\gamma)g^2/32\pi^2x^2 \quad , \tag{6}$$

where $\gamma \equiv m_\chi{}^2/m_K{}^2$. The lower limit on x in this case is $(\sqrt{\beta} + \sqrt{\gamma})^2 \leqslant x$. For x values much greater than the lower limits, the contributions of χ and M emission are essentially equal.

Upper bounds on g^2 can be placed from the experimental limits from searches for heavy neutrinos in $K(\pi) \to l\nu$ decays and from searches for $K(\pi) \to l +$ neutrals over particular ranges of m_x. Figure 2 compares the predicted branching fractions for Majoron plus χ emission based on $g^2 = 2\times10^{-4}$ with experimental bounds.[7–9] We deduce the following 90%-C.L. upper limits

$$(g^2)_{ee} < 1.8\times10^{-4} \quad (K \to e,\ \text{CERN, Ref. 7}) \quad ,$$
$$(g^2)_{ee} < 2.7\times10^{-4} \quad (\pi \to e,\ \text{TRIUMF, Ref. 8}) \quad , \tag{7}$$
$$(g^2)_{\mu\mu} < 2.4\times10^{-4} \quad (K \to \mu,\ \text{LBL, Ref. 9}) \quad .$$

Here g^2 is the square of the matrix whose elements are $g_{ll'}$.

Electron-muon universality tests for total leptonic widths can also be used to bound g^2. The Majoron and χ contributions constitute a much larger fraction of $K \to e$ than of $K \to \mu$, since $\Gamma(K \to e\nu) \simeq 2.5\times10^{-5}\Gamma(K \to \mu\nu)$ and $\Gamma(K \to eM\nu) \approx 0.1\Gamma(K \to \mu M\nu)$ for comparable $(g^2)_{ee}$ and $(g^2)_{\mu\mu}$. The predicted deviation from e, μ universality for m_ν, $m_X > 1$ eV is

$$R_K \equiv \frac{\Gamma(K \to eL^0)/\Gamma(K \to \mu L^0)}{\Gamma(K \to e\nu)/\Gamma(K \to \mu\nu)} = 1 + 1970(g^2)_{ee} \quad , \tag{8}$$

where L^0 includes ν, νM, and $\nu\chi$ final states. For π decay the corresponding prediction is

$$R_\pi = 1 + 157.5(g^2)_{ee} \quad . \tag{9}$$

The experimental results,[10,11] including radiative corrections[12] to $l\nu$ theoretical rates in the denominator of Eq. (8), are $R_K = 1.016 \pm 0.06$ and R_π

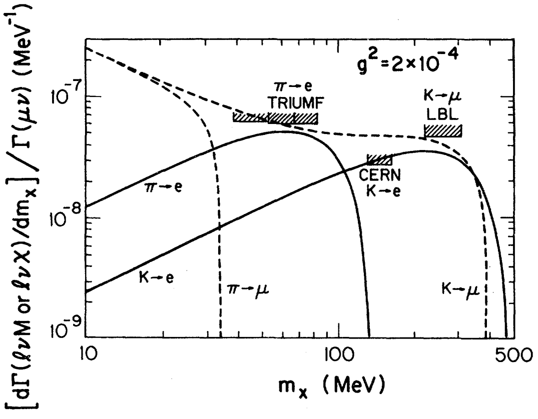

FIG. 2. Predictions for the differential leptonic-decay rates of K or π mesons into final states with $l\nu$ and Majoron or χ. The variable m_x is the square root of the virtual-neutrino four-momentum squared. Solid curves represent electron decays and dashed curves represent muon decays. Data are from Refs. 7–9. All $K(\pi)$ differential rates are normalized to the $K(\pi) \to \mu\nu$ rate.

$=1.033 \pm 0.019$. The corresponding universality bounds on g^2 at the 90% C.L. are

$$
\begin{aligned}
(g^2)_{ee} &< 4.5\times10^{-5} \quad (K\to l\nu,\ \text{Ref. } 10)\ ,\\
(g^2)_{ee} &< 3.1\times10^{-4} \quad (\pi\to l\nu,\ \text{Ref. } 11)\ .
\end{aligned}
\tag{10}
$$

$\nu N\to l^+M+$ hadrons. Majoron or χ emission changes incident neutrinos to charge-conjugate neutrinos which produce charge-conjugate leptons in scattering processes. For deep-inelastic scattering of neutrinos on a u-parton target, the differential cross section for Majoron emission is

$$
d\hat\sigma = \frac{G_F{}^2 g^2}{8\pi^5(\hat s - m_u{}^2)}\,\frac{l\cdot u M\cdot d}{2M\cdot\nu}
\times\delta^4(u+\nu-l-M-d)\frac{d^3\vec{l}}{E_l}\,\frac{d^3\vec{M}}{E_M}\,\frac{d^3\vec{d}}{E_d}\ , \tag{11}
$$

where the momenta are labeled by ν (incident neutrino), l (charged lepton), M (Majoron), u (initial up parton), and d (final down parton); $\hat s$ is the energy squared of the consituent subprocess. Equation (11) can be recast in the form

$$
d\hat\sigma = \frac{G_F{}^2 g^2 (W^2 - m_d{}^2)}{32\pi^3(\hat s - m_u{}^2)^2}\left(\frac{\hat s - Q^2 - W^2 - m_\nu{}^2}{\hat s - Q^2 - m_u{}^2 - m_l{}^2}\right)
\times\ln\left(\frac{1+\beta}{1-\beta}\right)\frac{dW^2\,dQ^2}{\beta}\ , \tag{12}
$$

where

$$
\beta = [1 - 4W^2 m_\nu{}^2/(\hat s - Q^2 - m_l{}^2 - m_u{}^2)^2]^{1/2}\ . \tag{13}
$$

In Eq. (12), $W^2=(M+d)^2$ is the invariant mass squared of the M and d, and $Q^2=-(l-\nu)^2$ is the four-momentum transfer squared from ν to l. The integration limits are $m_d \leq W \leq (\hat s)^{1/2} - m_l$ and $0 \lesssim Q^2 \lesssim (\hat s - m_u{}^2)(\hat s - W^2)/\hat s$. To a good approximation Eq. (11) applies to χ emission as well, with the denominator factor $2M\cdot\nu$ replaced by $2M\cdot\nu - m_\chi{}^2$. The modification to Eq. (12) in the χ case is to replace β with γ, where

$$
\gamma \equiv \frac{\beta[1 - 4W^2 m_\chi{}^2/(W^2 - m_d{}^2)^2]^{1/2}}{1 - 2W^2 m_\chi{}^2/(\hat s - Q^2 - m_l{}^2 - m_u{}^2)(W^2 - m_d{}^2)}\ . \tag{14}
$$

Folding in the parton distributions of an average nucleon target from Ref. 13, and taking a single ν mass with coupling g, we find an integrated cross section for Majoron emission of

$$
\sigma(\nu N\to l^+Mh) = 0.21 g^2 E_\nu[\ln(M_N E_\nu/m_\nu{}^2) - 0.9]
\times 10^{-41}\text{cm}^2\ , \tag{15}
$$

where M_N is the nucleon mass and E_ν is the neutrino beam energy in GeV. For χ emission with $m_\chi >> m_\nu$, the factor $m_\nu{}^2$ in Eq. (15) is replaced by $2.1 m_\chi{}^2$.

Experimental upper limits on "wrong-sign" single leptons produced by neutrinos can be used to bound g^2. The limits are $\sigma(\nu_\mu\to\mu^+)/\sigma(\nu_\mu\to\mu^-) < 1.6\times10^{-4}$ for $E_{\text{vis}} > 100$ GeV (Ref. 14) and $\sigma(\nu_\mu\to e^+)/\sigma(\nu_\mu\to\mu^-) < 3\times10^{-4}$ for $\langle E\rangle \simeq 3$ GeV.[15] From appropriate spectrum averages of the $M+\chi^0$ cross section with energy-acceptance cuts, we find that these limits imply

$$
\begin{aligned}
(g_{\mu\mu})^2 &< 2.5\times10^{-2} \quad (\nu_\mu\to\mu^+)\ ,\\
(g_{\mu e})^2 &< 1.8\times10^{-2} \quad (\nu_\mu\to e^+)\ ,
\end{aligned}
\tag{16}
$$

taking a common neutrino mass $m_\nu = 10$ eV and $m_\chi = 100$ keV.

Figure 3 shows the energy ratios E_l/E_ν and E_M/E_ν for the $\nu N\to l^+Mh$ or $l^+\chi h$ processes at $E_\nu = 100$ GeV. Appreciable missing energy E_M, carried by the M or χ, results in a charged lepton at relatively low energy.

Quasielastic production of wrong-sign leptons near threshold where form-factor effects can be neglected can be estimated from Eq. (12) in the approximation $g_A = g_V$. From the LAMPF limit[16] on $\sigma(\nu_e p\to e^+ n)$ we deduce only that $(g_{ee})^2 < 1$.

The limits on the Majoron coupling to neutrinos obtained in the preceding analysis can be summarized

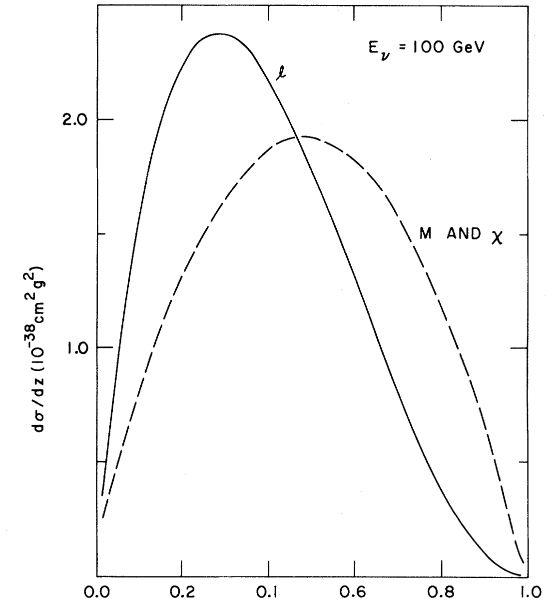

FIG. 3. Distributions in E_l/E_ν and E_M/E_ν in neutrino production of wrong-sign single leptons via $\nu N\to lMh$ or $l\chi h$. E_M is the missing energy carried by M or χ.

as follows:

$$\begin{aligned}(g^2)_{ee} &< 4.5\times10^{-5}\ ,\\ (g^2)_{\mu\mu} &< 2.4\times10^{-4}\ ,\\ (g_{\mu\mu})^2 &< 2.5\times10^{-2}\ ,\\ (g_{\mu e})^2 &< 1.8\times10^{-2}\ ,\end{aligned}\qquad(17)$$

where the latter two limits are based on an effective neutrino eigenmass of 10 eV and a χ mass of 100 keV. Leptonic-decay experiments in progress can improve on these limits or find evidence for Majoron emission.

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