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Reply to "Slope parameter and zero trajectories in $\pi^- p$ scattering"

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Shortcomings of the new zero trajectories of Höhler and Sabba Stefanescu are pointed out. The errors in the imaginary part could be large enough to make their results consistent with at least one real zero for every energy. Whether or not a real zero exists, it is possible to have convergent polynomial expansion (CPE) without any spurious cut with (very nearly) the same ansatz for the forward slope. We disagree with most of the criticisms on the applications of CPE to scaling without any spurious cuts. Even if one uses slope values obtained from parametrizations in the Coulomb-nuclear interference region, scaling is described in an excellent fashion.

I. INTRODUCTION

Höhler and Sabba Stefanescu[1] have criticized some of our papers on recent applications of convergent polynomial expansion (CPE)[2,3] dealing with two different aspects of hadronic collision processes: (i) Possible connection of zeros with forward slope parameters[4,5] and also scaling of differential cross sections in diffraction scattering processes[6]; the conformal mapping used for these processes possesses a spurious cut, as already pointed out.[4-9] (ii) Representation of scaling of differential cross sections in two-body elastic diffractive[7-9] and inelastic nondiffractive[10] processes; the conformal mapping used here does not possess any spurious cut. In Secs. II and III we furnish replies to the comments dealing with (i) and (ii), respectively.

II. APPLICATIONS DEALING WITH ZERO TRAJECTORIES

At the outset it should be mentioned that these[4,5] are phenomenological "models" of diffraction scattering assuming that the diffractive part of the amplitude possesses the same Barrelet-type[11] zeros.[5] Correlation with the forward slope parameter $b(s)$ was made from considerations of the available data[11,12] and their linear extrapolations. It may turn out that linear extrapolations are wrong,[1] but this does not prevent us from establishing our model with actual trajectories, the possibility for which is demonstrated in this section.

A. New trajectories of Hohler and Sabba Stefanescu

Some of the main criticisms are based upon their[1] new computation of zero trajectories, but here we point out certain shortcomings of their trajectories.[1]

Since the zeros of the transversity amplitudes are directly related to the experimental data on transversity cross sections, the trajectories computed in Refs. 11 and 12 are more reliable. Among these,[11,12] whereas only real parts of zero trajectories for $\pi^- p$ have been computed in Ref. 11 from relatively older data, both the real and the imaginary parts have been computed in Ref. 12 using the CERN-Holland[13] data first, then merging with their own data, and also using Carnegie-Mellon–LBL amalgamation of the data of other groups[14] in the energy range $3.5 \lesssim s \lesssim 4.5$ GeV^2, for which their own data were lacking. Also the authors[12] have used a local parametrization to determine the near backward zeros.

Höhler and Sabba Stefanescu have computed trajectories somewhat indirectly using amplitudes reconstructed from phase-shift analyses; they have also not exercised as much care as in Ref. 12 for the determination of zeros in the backward regions, which are likely to be ill determined in phase-shift analyses.[5] Further, the data of the Bristol group has not been used for analysis in some important energy ranges,[1] whereas the amalgamated data appear to have been used by Barrelet *et al.*[12,14] There are certain serious discrepancies from the results of Barrelet *et al.*[12] especially on the F trajectory.

1. Absence of the F trajectory

The trajectory F has been stated to belong to $F^{(-)}$ only for $2.4 < s < 3.1$ GeV^2 and to $F^{(+)}$ for all values of energy of interest.[1] This contradicts Ref. 12 according to which F belongs to both at least for $2.5 \lesssim s \lesssim 4.5$ GeV^2 and has the following implications. The number of zeros of an amplitude gives the order of the polynomial necessary for data fitting which increases with energy. It is generally expected[11] that, once a zero trajectory appears at lower energies, it would continue to exist for higher energies, besides, possibly, other new trajectories revealing their existence. This is generally true for Barrelet-type trajectories,[11,12]

but the F trajectory of Ref. 1 does not satisfy this general expectation, which, further, implies the improbable fact that the order of the polynomial and the number of partial waves necessary for higher energies with $s > 3.1$ GeV^2 is less than that necessary at lower energies for $s < 3.1$ GeV^2.

There are also disagreements between the results stated in the text[1] and those displayed in the figures, and also inconsistency among results displayed in Figs. 1 and 2(b). The trajectory F of $F^{(-)}$ has been shown in Fig. 1 to exist for $2.5 \lesssim s \lesssim 3.5$ GeV^2 and again at a single point at $s \simeq 4.6$ GeV^2 (open squares) as against the statement (upper limits $s = 3.1$ GeV^2). Similarly, in Fig. 2(b), the imaginary part of F has been shown even up to $s \simeq 2.1$ GeV^2, which falls clearly outside the range stated. We also note that the imaginary part of F in Fig. 2(b) continues till $s \simeq 2.1$ GeV^2, whereas the real part of the same trajectory ceases to exist below $s \simeq 2.5$ GeV^2 in Fig. 1.

2. *Errors in the imaginary parts and the question of the real zero*

For the construction of the proposed mapping,[4-6] it is not necessary that the same trajectory continues to be real for all energies, but it is sufficient that there is one real zero for every value of energy of interest. The computed results[1] do not rule out at least one real zero for every s value of interest, partly because of the shortcomings noted above, and partly for lack of actual computation of errors at least in the imaginary parts of the trajectories. Barrelet[11] has computed errors for $\pi^+ p$ scattering alone. However, the possibility of large errors of the imaginary parts has been discussed in detail by Barrelet *et al.*[12] according to which a realistic 8% uncertainty in the target polarization transforms into 30% uncertainty of the imaginary part of the B trajectory. From Table I of Ref. 12, in addition to the systematic and binning errors, the normalization error is 6% and the statistical error varies from 10–12%, in the polarization, near $\cos\theta$ values for the B trajectory; the statistical error in the polarization near $\cos\theta$ values for the D trajectory varies from 17–135%. Taking into account all possible errors in the polarization and differential cross section, one can argue that the uncertainty in the imaginary parts could be large enough to make the results[1] consistent with at lease one real zero for every energy value. Specifically, such errors in the B and/or D trajectories[1] in Fig. 2(b) of Ref. 1 for $s \gtrsim 3.1$ GeV^2, E and/or F trajectories in Fig. 2(a) of Ref. 1 for $s \lesssim 2.3$ GeV^2 would imply at least one real zero for every s. When possible errors are added to already small imaginary parts[1] of the D trajectory in Fig. 2(b) for $s \lesssim 2.3$ GeV^2 and the F trajectory in Fig. 2(a) for $s \gtrsim 3.2$ GeV^2, it is very difficult to rule out a real zero in these regions. Model-independent results provide strong evidence[15] in favor of zeros, although at high energies, which should appear as real ones in experiments.

B. Analytic structure

The clarifications provided below in terms of the mapping functions originally suggested in Ref. 4 (some of which have been already used in Refs. 9 and 10) was not thought to be necessary, as the spurious cut was not thought to be serious at that time.[4,5]

1. *With a real zero*

Independently of the comments of Höhler and Sabba Stefanescu, besides admitting that the spurious cut is present in the conformal mapping using a real zero,[4,5] it has been also pointed out that the cut might affect analyticity and convergence.[9] Even though the conformal mapping[4,5] may invalidate convergence of CPE in terms of Laguerre polynomials, a CPE exists in terms of

$$Z' = \sqrt{Z} = g(x) \cosh^{-1}\sqrt{w_0}, \tag{1a}$$

where

$$g(x) = \frac{C + x}{x + x_0}, \tag{1b}$$

$$w_0 = \frac{x_+ - x}{x_- + x} \frac{x_- + 1}{x_+ - 1}, \tag{1c}$$

as defined in the same papers.[4,5] The spurious cut that arose in the Z plane, because of the use of $g^2(x)$ which folds a part of the physical region on top of the other part, is absent in the Z' plane in which the image of the physical region is the entire real axis and that of the cuts in the boundary of a striplike figure moving with energy, provided $C \leq 1$ and $C \neq x_0$. This is shown in Fig. 1 for $C = 0.9$ and $x_0 = 1$. Even for low energies there exists a strip around the ReZ' axis containing a large portion of the image of the x plane in the interior of which the Hermite-polynomial expansion converges in the sense of CPE.[2-5] Since the corresponding weight function for the Hermite polynomial expansion can be taken as $\exp(-\alpha Z'^2/2) = \exp(-\alpha Z/2)$, the first terms of the analogous expansions[4,5] yields the same formulas for the forward slopes.[4,5]

2. *In the presence or absence of a real zero*

Whether or not a real zero exists, there is a CPE for every energy in terms of the variable Z_0 constructed earlier,[4,5] but the nature of the polynomials varies with energy.[9] Proceeding in

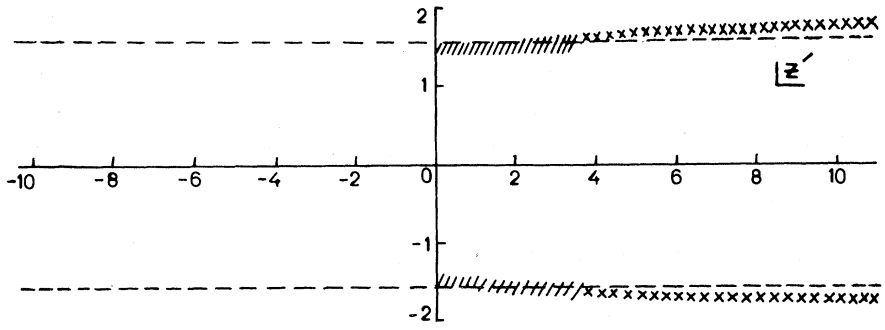

FIG. 1. Conformal mapping of the x plane onto the Z' plane. The dashed lines are the boundary of the strip inside which the Hermite-polynomial expansion converges.

the same way as in Ref. 5, one can obtain almost the same type of formula:

$$b(s)=\frac{\alpha}{t_R}\left(1+\frac{t_R}{4q^2+t_L-\Delta/s}\right)+b_0(s) \qquad (2)$$

with

$$b_0(s)=-\left[\prod_{i=1}^{m}\frac{\mathrm{Re}t_i(s)}{|t_i(s)|^2}+\prod_{i=1}^{n}\frac{\mathrm{Re}t_i'(s)}{|t_i'(s)|^2}\right], \qquad (3)$$

where F^+ (F^-) possesses m (n) zeros. The first term in (2) has been already derived and used at several places[4,9] and also in Ref. 5 with a different factor before the brackets, which is practically the same as α/t_R for all energies except near threshold values. Very nearly the same type of fit to the available data on $b(s)$ can be obtained using formula (2) and only the real trajectories of $F^{(+)}$ from Ref. 12, and all others from Ref. 1, in the energy range in which all of them have been supplied,[1] with $\alpha=0.236$ as shown in Fig. 2. This supports our conjecture[5] that the diffractive part has the same zeros as the nondiffractive part. We feel that it is also possible to fit the data on $b(s)$ in the whole of the energy range investigated[5] if all the trajectories[1] are provided.

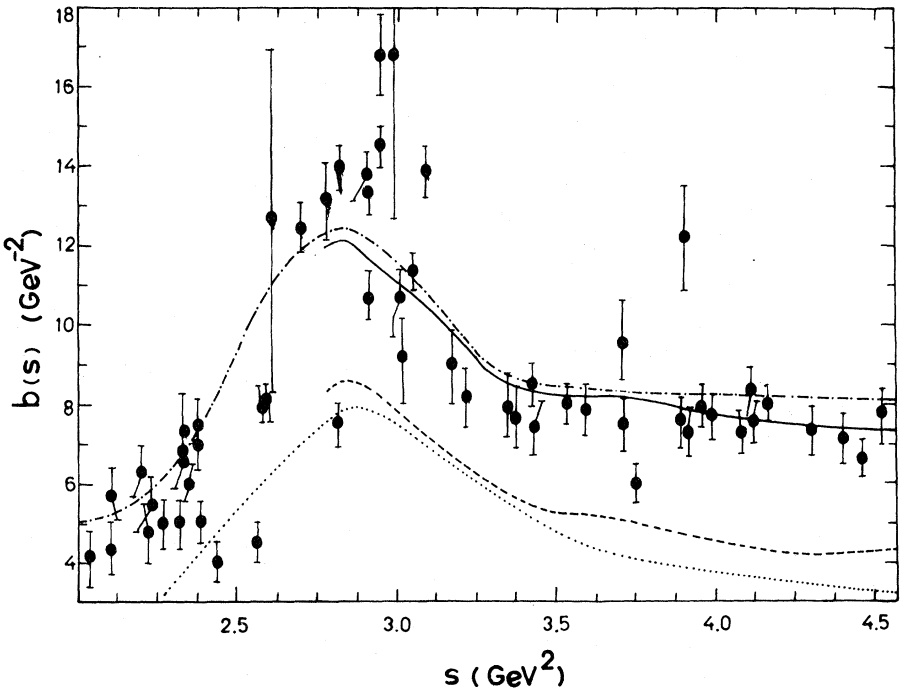

FIG. 2. Fit to the π^-p slope-parameter data using formula (2) and the new trajectories of Höhler *et al.*, as shown by the solid line and compared with the dot-dashed-line fit of Ref. 5 obtained using trajectories of Barrelet *et al.* and their linear extrapolations. The dashed (dotted)-line fit is the contribution calculated in this analysis (Ref. 5), without using any CPE, by means of formula (3) ((23)) with the zero trajectories of Höhler *et al.* (Barrelet *et al.*)

3. The effect of the kinematical cut

Although we have not been able to remove completely the suspicion of the authors on the possible effects of the kinematical cut, we suppose that this does not cause problems as we are considering the sum $\Sigma^+ + \Sigma^-$, but not the individual transversity cross sections. In the sum, the terms proportional to the polarization cancel out and thus, also the effect of the kinematical cut. The forward slope is related to this sum as a whole and thus is supposed to be free of the effect of the kinematical cut. Even if this term might be present in the individual amplitudes, being proportional to $\sin\theta$, this has negligible contribution as compared to that of the dynamical cut, according to our ansatz, near forward angles.

C. Reply to other questions

Instead of just relating the structure in $b(s)$ to the dips in forward cross section, as pointed out by Höhler and Sabba Stefanescu, in Sec. IV 3 of Ref. 5 we have observed the following criteria in a much better manner as a consequence of our analysis: A bump structure in $b(s)$ arises if, for one or more trajectories, $|\mathrm{Re}t_i|$ is small and $|\mathrm{Im}t_i| \ll |\mathrm{Re}t_i|$ for some value of s. Instead of occurring at $s = 2.5$ GeV^2, the bump occurs at $s \simeq 2.9$ GeV^2, where A and B trajectories[11,12] satisfy our criteria approximately, even in Ref. 1.

Reanalysis of the data of Foley *et al.*[16] has been reported[17] after our paper[5] was submitted. Naturally, if the structures in $b(s)$ are absent at higher momenta, the constraints[5] on trajectories are not to be satisfied at those energies. Since our representations[4,5] are models, obviously, other explanations are not ruled out. It is inappropriate to say that our ansatz describes only a few properties without testing it with the differential cross-section data, especially in the peak region. It has been already clarified that, using this method,[5] one can determine only one trajectory (either the real or the imaginary part) from the data on $b(s)$ provided others are known; therefore, the comment that this method of determination of zeros cannot compete with other methods is redundant.

III. APPLICATIONS WITHOUT USING ZERO TRAJECTORIES

The comment[1] that the conformal mapping used in Ref. 8 is the same as that used in Refs. 9 and 10 is incorrect. The mapping variable of the x plane in Ref. 8 is free of any spurious cut, possesses $t \leftrightarrow u$ symmetry, applies to pp scattering[7] only, and was developed as early as 1971.[3]

Scaling of differential cross sections, using analyticity and CPE, has been very successfully demonstrated for the first time for six diffractive and six nondiffractive processes.[9,10] The merit of the works should not be judged only by choosing $\pi^- p$ as an example and ignoring the total impression of scaling created along with others.[9,10] Even in the $\pi^- p$ case we disagree with most of the criticisms,[1] as they are based upon incorrect interpretations of our assumptions and results.[7-10]

A. Truncation of expansion and diffraction scattering

1. Truncation of the CPE in χ

The statement[1] that we have used *only the first term* in $f(s,t)$ and claimed that this is a suitable approximation for *diffraction scattering* is incorrect. It has been assumed,[3,6-10] for the determination of parameters in χ, that for moderate and high energies and only for $|t| \ll t_R \simeq 4m_\pi{}^2 = 0.078$ GeV^2, the first term represents the data on $f(s,t)$ reasonably well. This kinematical region in $|t|$ forms a very small part of the whole diffraction region for almost all diffraction scattering processes. As shown by the solid-line fit to the $\pi^- p$ data[18] at 200 GeV/c in Fig. 3(a) our assumption is justified.

2. Truncation of $\alpha(s)$

In reply to the comment[1] that the proposed expansion for $\alpha(s)$ has been truncated after the second term, we mention that no truncation has been forced *a priori* upon $\alpha(s)$ so far as data analysis is concerned, and the proposed expansion has the potentiality to satisfy general asymptotic behavior $(\ln s)^m$, $m = 0, 1, 2, 3, \ldots$. Since asymptotic behavior of η is $\ln s$ and $\xi = \eta^2$, and the maximum asymptotic rate of growth of $b(s)$ allowed by unitarity is $(\ln s)^2$, the expression, retaining only the first three (two) terms in $\eta(\xi)$, has been mentioned as the unitarity-restricted one, for the sake of bookkeeping[6-9]; but fitting the data on $b(s)$ did not need even the third term. They fit with the first two terms *which satisfy Regge asymptotic behavior.*

3. The scaling function

The reader may obtain from Ref. 1 the incorrect impression that we have already used only the first term in the proposed expansion for $f(s,t)$ as the scaling function in the whole diffraction region. The first term in $f(s,t)$ has been assumed to represent the data only for $|t| \ll 0.078$ GeV^2 in order to compute the parameters of $\alpha(s)$ occurring in the scaling variable,[7-10] but the entire CPE in χ, with $\{P_n(2\chi)\}$ replaced by the Laguerre polynomials $\{L_n(2\chi)\}$, has been proposed as the scaling func-

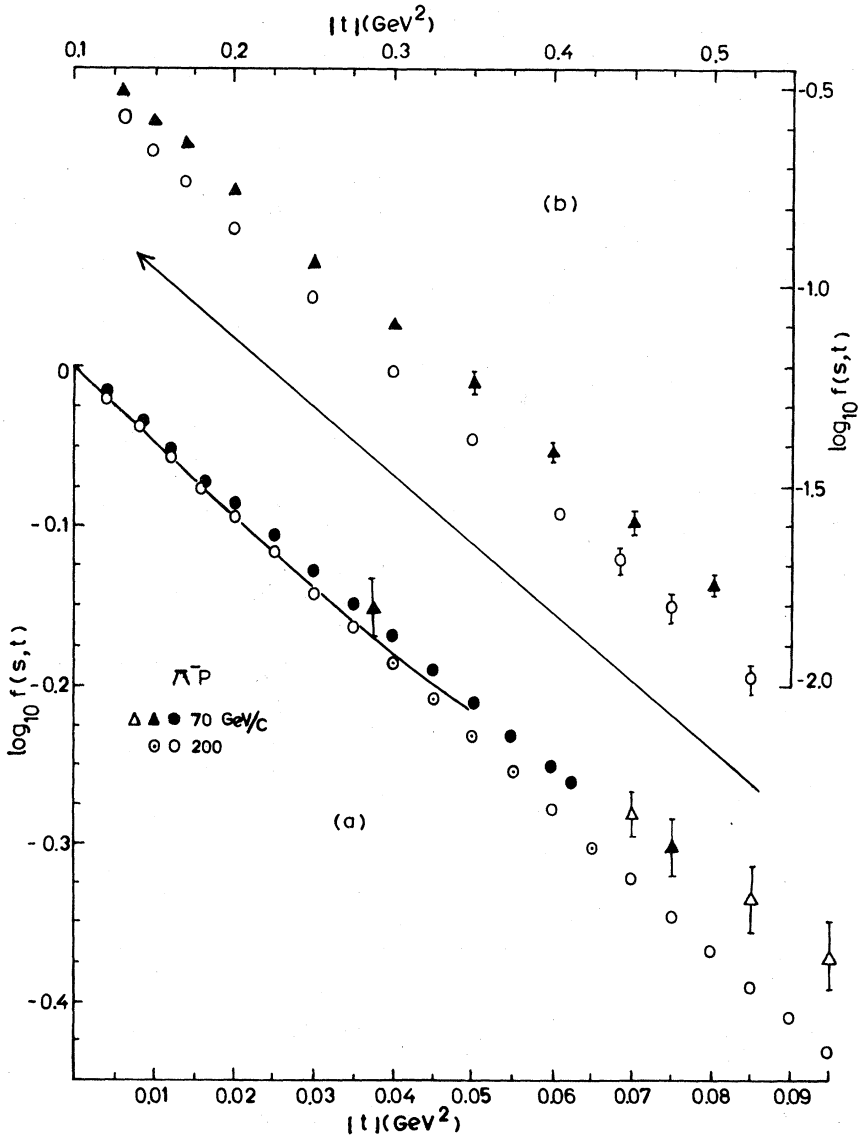

FIG. 3. Display of energy dependence in the $f(s,t)$ vs $|t|$ plot in the new (Ref. 18) and the old (Ref. 22) Fermilab data at 70 and 200 GeV/c for (a) smaller $|t|$ values and (b) larger $|t|$ values. The solid line in Fig. 3(a) shows the fit obtained using the first term in the proposed CPE for $f(s,t)$ to the data at $P_{\text{lab}}=200$ GeV/c, and justifies our assumption. The solid triangles and circles with dots represent the old Fermilab data. The open triangles represent locally extrapolated data of Ayres *et al.*, and the open and solid circles represent new Fermilab data. The arrow is the dividing line between (a) and (b).

tion. This function with the first 3–5 terms, fits the data on $f(s,t)$ for all the diffractive and nondiffractive processes in the peak region and somewhat outside it.[19]

B. Demonstration of scaling of the data

It is not clear why the authors of Ref. 1 examine scaling of the data after subtracting the real part, since in our ansatz, the real-part effects are included (pp. 2536, 2542, and 2544 in Ref. 9).[6-10] That such a potentiality to include real parts exists in the CPE makes it possible to propose the same scaling hypothesis and demonstrate its validity for the experimental data, for nondiffractive processes, where the imaginary part does not dominate.[10] Even the estimation of the real part[1,17] of the amplitude for nonforward $|t|$ values has not been verified experimentally and there also appears to be more or less deviation from recent experimental results,[18] even in the case of the real part in the forward direction. If one plots cross section ratio against $|t|$, without subtracting

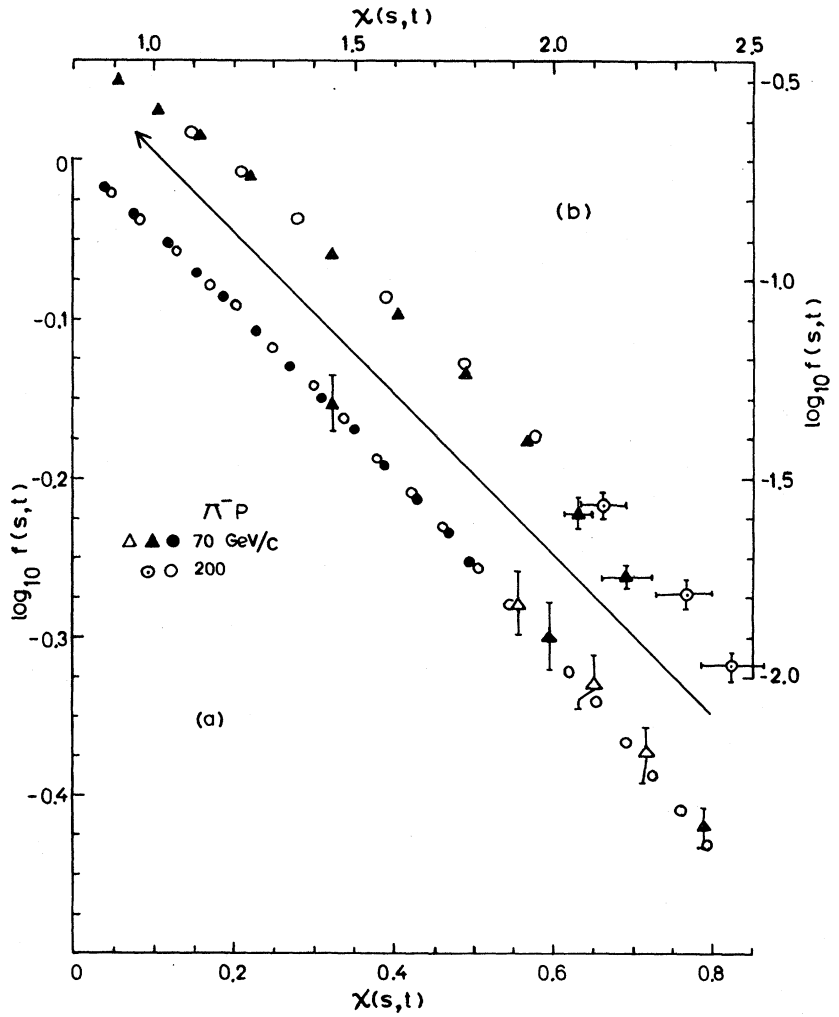

FIG. 4. Scaling of the cross-section-ratio data for $\pi^- p$ scattering for (a) smaller $|t|$ values displayed in Fig. 3(a), (b) larger $|t|$ values displayed in Fig. 3(b).

real parts, there exists sufficient energy dependence as shown in Figs. 3(a) and 3(b).

Since we were interested only in gross features of the fit, rather than in proposing the best acceptable fit, to the data on $b(s)$, a critical examination of the data was not necessary.[9] Although we missed the data of Burq *et al.*[20] inadvertently, no improvement or deterioration to the impression of scaling would have been made[9] had we included them along with others.[9] As our analyses[9] use the b values based on the popular extrapolation $f(s,t) = \exp(bt + ct^2)$, objections against which have been stronger[19,21] only after our papers[9,10] were submitted for publication, the related comments[1] imply that our ansatz may not work with the b values obtained from fits in the Coulomb-nuclear interference region.[17,18,21] As against such doubts we note that the first two terms in $\alpha(s)$ give an excellent fit to the data on $b(s)$ for $\pi^- p$ with

$$d_0 = 0.210,$$

$$d_1 = 0.166,$$

for $P_{\text{lab}} = 50$–200 GeV/c. Plotting the data[18,22] on $f(s,t)$ against χ removes energy dependence in excellent fashion both for larger and smaller $|t|$ values as can be noted comparing Figs. 3 and 4. Similar success in the demonstration of scaling of the data for other processes[18] has been noted.[19]

C. Reply to other questions

Our ansatz[7-10] is neither "empirical", nor devoid of a "physical argument"; rather several appropriate physical arguments have been provided at every stage of development of the "model"[7-10] using analyticity. The "flexibility" of the conformal-mapping method is well known; but this flexibility has been utilized to its advantage in these

papers[7-10] as in many other cases. In view of these responses to the comments of Höhler and Sabba Stefanescu, as noted in this section, the fact that our ansatz[7-10] realizes scaling, for the first time, for diffractive and nondiffractive processes using analyticity, whereas existing model-independent approaches fail,[23] and for many other physically appealing consequences stated in the relevant papers,[7-10] there is much to "learn" in contrast to the criticism of the authors.[1] This is more so if one considers $\pi^- p$ in the perspective of other processes.[7-10]

ACKNOWLEDGMENT

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