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Slope parameter and zero trajectories in $\pi^- p$ scattering

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This paper is a comment on several recent papers by Parida and collaborators on diffraction scattering. It is pointed out that these authors use assumptions on πN zero trajectories which differ strongly from results derived from phase shifts, and that a more critical attitude towards published data for the slopes at $t = 0$ is needed. The proposed applications of the conformal-mapping method are criticized.

In a series of papers[1,2] Parida, and Parida and Mahapatra, have recently presented in detail a method for a description of πN diffraction scattering by means of a "convergent polynomial expansion", emphasizing in particular a relation between the slope parameters at $t=0$ and zero trajectories. The most recent paper[2] is devoted to a study of the $\pi^- p$ diffraction peak. The authors take the slope parameters from a compilation[3] which gives values derived by empirical fits to differential cross sections outside the Coulomb-interference range and is based on the data available in 1971. Considerable parts of their input for the zero trajectories are simple extrapolations of results determined in a limited energy interval by a Barrelet-type analysis.[4]

It is the purpose of this paper to point out that much better information on the slope parameter is available from recent phase-shift analyses,[5-7] which lead to a more reliable extrapolation to $t=0$ and take into account the new experiments performed in 1971–1978, including some in the Coulomb-interference region.[8] Furthermore, the amplitudes of the new phase-shift analyses have also been used for a detailed study of zero trajectories.[9-11] It turns out that the results are at variance with the extrapolations of Parida and Mahapatra[2] and also with their prediction for the low-energy region. Consequently, their conclusions have to be reconsidered.

Finally we add a remark, explaining why we do not think that the proposed expansion is useful for an "understanding" of the energy dependence of the slope or of other features of diffraction scattering, even if the input were replaced by a more accurate one.

I. ZERO TRAJECTORIES

Transversity amplitudes are related to experimental data by

$$|F(\pm)| = \frac{d\sigma}{d\Omega}(1 \pm P)\,. \tag{1}$$

Therefore, the zeros of $F(\pm; s,t)$ at real s and complex t, $t=\tau(s)$, can be determined from data alone up to a sign ambiguity of $\mathrm{Im}\tau$ (Refs. 4 and 12). We prefer to use zero trajectories calculated from amplitudes which have been reconstructed from phase shifts. One reason is that one obtains a unique value for $\mathrm{Im}\tau$. Furthermore, gaps in the data are filled and the result is smoothed by the application of isospin and analyticity constraints in phase-shift analysis.[5,6]

In Figs. 1 and 2 we show zero trajectories derived from the Carnegie-Mellon–LBL analysis,[6] because this solution is smoother due to the use of new preliminary $\pi^- p$ data[8] and to the amalgamation procedure. Below its energy range ($s < 1.7$ GeV^2) the curves follow from our solution.[5] The agreement with Barrelet *et al.*[4] in their momentum range and with the result shown in Fig. 6 of Barrelet's earlier paper[12] is reasonable.

In a certain range ($2.4 < s < 3.1$ GeV^2) trajectory F has a very small imaginary part and runs almost along $\theta = 180°$, i.e., it belongs to both $F(\pm)$. Outside this range it lies in $F(+)$.

A comparison with Figs. 1 and 2 of Parida and Mahapatra[2] shows the following differences.

(i) The linear extrapolation of the trajectories in the s,t plane to the low-energy side in Fig. 1 of Ref. 2 is wrong. Odorico's straight-line pat-

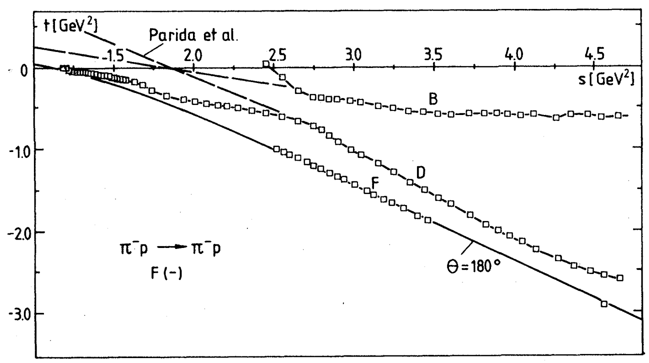

FIG. 1. Projection of zero trajectories belonging to the $\pi^- p \to \pi^- p$ transversity amplitude $F(-)$ onto the real s,t plane. The points have been calculated from recent phase-shift analyses (Refs. 5 and 6). The straight lines are the extrapolations assumed by Parida *et al.* (Ref. 2).

tern (Ref. 13) is only a crude approximation. The results of our study of the zero trajectories (Refs. 9–11) show that there are considerable distortions in regions where two trajectories come close to each other ("intersections"). In an earlier paper[9] we have shown that the pattern at intersections can be understood as a consequence of Weierstrass's "preparation theorem" for holomorphic

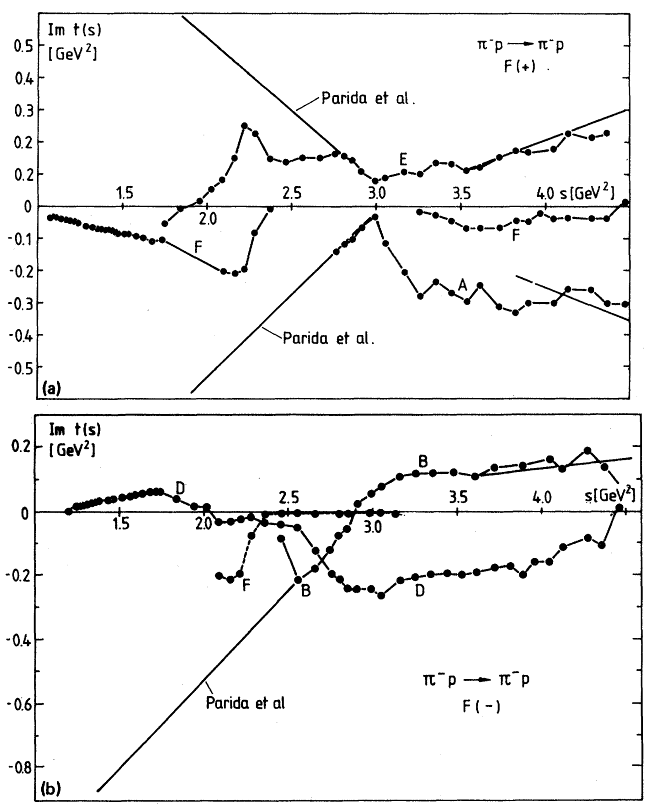

FIG. 2. (a) and (b) Imaginary parts of the zero trajectories of the $\pi^- p \to \pi^- p$ transversity amplitudes $F(+)$ and $F(-)$ as calculated from phase-shift analyses. In the range $2.4 < s < 3.2$ GeV^2 trajectory F runs along the backward direction and belongs, therefore, to both $F(+)$ and $F(-)$. In order to show the continuation, we have plotted in (b) between 2.2 and 2.4 GeV^2 again part of trajectory F which belongs to $F(+)$. Curve B cannot be continued to lower energies, because its real part leaves the physical region (Fig. 1). The straight lines are the extrapolation assumed by Parida *et al.* (Ref. 2).

functions of two complex variables.

(ii) The linear extrapolation of the imaginary parts of the zero trajectories in Fig. 2 of Ref. 2 is also at variance with our result. It is unclear why Parida and Mahapatra[2] assume that the absolute values of the imaginary parts of trajectories A and E and trajectories B and D are the same (see Ref. 4). This is not so and the linear extrapolation assumed there to lower energies is equally unjustified (see Fig. 2). Figure 2 and similar figures for other amplitudes (Refs. 10 and 11) show that, in general, there is a tendency of the trajectories to remain near the physical s,t plane. This agrees with the theoretical expectation, because the zero trajectories can pass the s-channel resonance surfaces $s=M_i^2$ (M_i = complex resonance mass) only at "Legendre zeros" and these have small imaginary parts (see Refs. 12, 13, 9 and 11).

The discrepancy is far outside the uncertainty of our calculation as can be seen from the fact that it occurs with the results derived from four different phase-shift analyses (see Refs. 4–6 and 10) which are in reasonable agreement with each other.

II. THE SLOPE

Parida and Mahapatra[2] are interested in the "oscillatory pattern" of the logarithmic slope $b(k)$, where k = pion laboratory momentum,

$$b(k)=\frac{(d/dt)d\sigma/dt}{d\sigma/dt}\bigg|_{t=0}, \qquad (2)$$

which, in their opinion, "has eluded predictions from many models." It is surprising that they do not mention the usual interpretation, according to which the structure belongs to the contributions of the *nucleon resonances* which, at higher energies, are superimposed on a diffractive background.

This mechanism produces structures in $b(k)$ at least up to 5 GeV/c, because resonantlike "circles" in Argand diagrams of partial waves have been seen in this range.[14] It is true that their magnitude is rapidly decreasing with increasing k, but the contribution of a partial wave to $b(k)$ has a factor $\sim l^3$ and the angular momentum l is growing with k for peripheral resonances. Furthermore, one observes the interference with the large background. In order to search for these structures of $b(k)$, one has to perform cross-section measurements at very small $|t|$ including the Coulomb-interference region, which do not yet exist in this momentum range.

Parida and Mahapatra[2] mention a structure of $b(k)$ at even higher momenta (10–30 GeV/c) which follows from the slopes derived by Foley *et al.*[15] from their data. We have reanalyzed these data[16] admitting a possible normalization error of a few percent and using all data points, whereas the authors have fitted only the small t range $t>-0.05$ GeV^2. Our conclusion is that the $b(k)$ values are systematically smaller, leaving no evidence for a structure in this range.

In discussing the relation between slopes and zeros one should also consider the possibility that a zero trajectory passes $t=0$, entering the physical region from the outside. This leads frequently to a dip in the energy dependence of the forward cross section which is usually accompanied by a structure in $b(k)$. For instance, there is a dip in the forward π^-p cross section at $s=2.5$ GeV^2, where trajectory B enters the physical region (Fig. 1).

III. THE ANALYTIC STRUCTURE

As mentioned by Parida and Mahapatra[2], their method can be applied only to amplitudes which have a *real* zero in the physical $\cos\theta$ region. Figure 2 shows that it happens that an almost real zero exists in a limited energy range, but the existing data and phase-shift analysis show that there are other energy ranges in which this does not occur.

Another problem is that the transversity amplitudes have a kinematical cut in addition to the singularities of the invariant amplitudes. One could suspect that this invalidates the treatment by the authors, but they just mention this point without being very worried. Zero trajectories of invariant amplitudes are shown in Ref. 11. There is a remarkable similarity between the zero patterns of all four invariant amplitudes $A^{\pm}$, $B^{\pm}$ ($\pm$ denotes the isospin evan and odd combination), which is quite unexpected from Odorico's extension of the Veneziano model to πN scattering.[13]

The authors say that their "spurious cut" is not serious, because a similar cut has not affected the convergence of Barrelet's expansion.[12] However, the two cases are quite different: Barrelet uses a conformal mapping in order to get rid of the kinematical cut and to be able to describe *both* transversity amplitudes $F(\pm)$ by a single analytic function. There is no cut along the physical region of his $w=\exp(i\theta)$ plane.[17] On the other hand, Parida and Mahapatra[2] consider expansions for each of the transversity amplitudes separately, for instance [$x=\cos\theta$, see their Eqs. (7), (8), (14), and (15)],

$$F(x)/[(x-x_1)(x-x_2)]=e^{-\alpha z/2}\sum_{n=0}^{\infty}a_nL_n(\alpha z)\,. \qquad (3)$$

For all z values in the physical range, their mapping gives *two* x values. Their expansion cannot be valid in any complex neighborhood of the physical region because, for z values in the physical range, the left-hand side of Eq. (3) has two values and the right-hand side has only one.

Even if the above-mentioned shortcomings would not exist and the slopes were fitted by an expansion which also takes into account the zeros, one has only one of many possible *descriptions* of a few properties of the πN amplitudes; but, in our opinion, it is difficult to qualify this as a progress in the "understanding" of diffraction scattering. The "prediction" of zeros can certainly not compete with other methods which take into account the whole of the experimental information, and it is hard to see why there should be a general and accurate relation between zeros and slopes, even if one also uses the boundaries of the spectral functions. If the authors would take into account a larger part of the experimental information in their fits, they would need more terms in their expansion and finally they would arrive at a first step towards phase-shift analysis.

NOTE ADDED

In several new papers of this series (Refs. 18–20), which came to our attention more recently, Parida, and Parida and Giri, realized that the spurious cuts "may be an objectionable feature." They chose another conformal mapping such that a spurious cut does not occur. As a consequence, the zero trajectories which played an important role in the earlier method are not of interest any more. It is true that the new method does not have some of the shortcomings of the earlier version, but we still cannot see what one can learn from its results for diffraction scattering because of the following reasons (our arguments are given for πN scattering[18]).

The expansion is a very flexible one, being restricted only by the location of the branch points in the Mandelstam plane, i.e., the theoretical input is very weak. After having discussed at length optimized polynomial expansions and convergence properties, the reader is surprised to see that the authors use *only the first term* of the expansion and claim that this is a suitable approximation for diffraction scattering. Another expansion for the energy dependence is truncated after the second term. In order to demonstrate that the truncations lead to a successful description of diffraction scattering the authors plot

$$f(s,t)=\frac{d\sigma(s,t)/dt}{d\sigma(s,0)/dt}$$

versus a scaling variable χ, whose parameters have been determined by a fit to the published values of the forward slope b.

We agree that it is of some interest if an empirical ansatz describes accurately a large number of data points but, without a physical argument, we do not see why it should be relevant that the ansatz happens to coincide with the first two terms of one of the possible expansions. Furthermore, the demonstration of the success of the proposed scaling law is not impressive, if one remembers that simple plots of $f(s,t)$ vs t show a "scaling" of a comparable quality, in particular, if one subtracts the contribution of the real part (see our earlier papers Refs. 14, 16, and 21).

Finally, the authors have used the published data for the logarithmic slope b at $t=0$ in an uncritical way. Many of the published b values are based on the popular extrapolation $f(s,t)=\exp(bt+ct^2)$. The data of Burq *et al.*[22] (which have been ignored by Parida and collaborators) in the Coulomb-interference region have shown that this ansatz is misleading (see our papers Refs. 14, 16, and 23). This conclusion has been confirmed by the recent Fermilab experiment at 200 GeV/c (Ref. 24).

Note added in proof. Our comments on Parida's reply are given in the Karlsruhe Report No. TKP 81-4 (unpublished). Further results on the behavior of zero trajectories and slopes can be found in *Landolt-Börnstein I/9b*, edited by H. Schopper (Springer, Berlin, to be published).

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