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Effective quark mass from $e^+e^- \to q\bar{q} \to$ hadrons

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A kinematic method is considered to estimate the quark mass from $e^+e^- \to q\bar{q} \to$ hadrons. It is based on a well-known linear relationship between the average meson multiplicity and the fireball mass, crucial tests being made with π production by pp collisions and $\bar{p}p$ *annihilations*. Applied to the multiplicity data of e^+e^- annihilation from ADONE, the method yields for the effective mass of the light quarks 0.33 ± 0.03 GeV.

I. INTRODUCTION

Results of well-known high-energy experiments of large-P_T, deep-lepton inelastic scattering, e^+e^- annihilations into hadrons, etc., indicate that hadrons are not elementary particles, but composites of quarks subject to confinement. This peculiar property of long-distance confinement raises the following question of primordial importance: What is meant by a quark mass? How does one measure the mass of a confined quark? In this regard, we mention that the masses of current quarks have been investigated within the framework of QCD,[1] as well as those of constituent quarks;[2] the latter include also the gluon cloud dragged along by the quarks in certain reactions.

In this paper we discuss a method to estimate the effective mass of constituent quarks using e^+e^- annihilation into hadrons, which proceeds via a virtual photon giving rise to a quark-antiquark pair,

$$e^+e^- \to q\bar{q} \to \text{hadrons} .$$

It is based on general grounds of kinematics, i.e., the average meson multiplicity is proportional to the fireball mass. We note that this property holds for various reactions of meson production covering a wide range of energy.[3(a)]

Consider, for example, the general case of meson production by the following reaction:

$$a + b \to \pi + \cdots .$$

We assume the observed average π multiplicity to be equal to the sum of those produced by the two fireballs in the forward and backward hemispheres of the c.m. system (c.m.s.). We recall that the mass of each fireball is related to its Lorentz factor with respect to the c.m.s.; for instance, for the mass of the forward fireball we have

$$M_a^* = E_a / \gamma_a^* \tag{1}$$

where E_a is the c.m. energy of the incident particle a, and a similar expression M_b^* for the backward fireball. As for γ_a^*, it is related to the velocity by $\beta_a^* = (1 - 1/\gamma_a{}^{*2})^{1/2}$, which is determined by means of the parameter λ_a as follows:

$$\beta_a^* = 1 - \lambda_a \tag{2}$$

with

$$\lambda_a = \frac{2\langle P_T \rangle_a}{\pi \langle P_L \rangle_a} , \tag{3}$$

where $\langle P_T \rangle_a$ and $\langle P_L \rangle_a$ are the averages of transverse and c.m. longitudinal momentum of mesons emitted by the fireball a; see Ref. 3(b). We note in passing that the parameter λ is related to Feynman-Yang scaling and that $\lambda = 1$ corresponds to the isotropic angular distribution.

We now write for the π multiplicity from the fireball a as

$$\langle n \rangle_a = A_a M_a^* + B_a , \tag{4}$$

where A_a and B_a are two parameters characteristic of the fireball under consideration, cf. Secs. II and III. A similar expression is for the other fireball b. Finally, we mention that in the scaling limit $M_a^* \sim E_a{}^{1/2}$; see Ref. 3(a). Thus, the linear relationship (4) leads to the well-known empirical law

$$\langle n \rangle \propto s^{1/4} , \tag{5}$$

where $s = (E_a + E_b)^2$. This law is found to be also adequate to describe the π multiplicity of e^+e^- annihilation.[4]

It is borne out that the relationship (4) contains implicitly the mass of the incident particle, as will be shown in Secs. II and III for pp collisions and $\bar{p}p$ annihilations. In Sec. IV we shall use this property to estimate the light-quark mass using e^+e^- annihilation data. A discussion will be given in Sec. V of the situation of high-energy data.

II. PROTON MASS FROM $pp \rightarrow \pi^- + \cdots$

We begin with a simple case of meson production by pp collisions. For definiteness, we consider the multiplicity of π^- and write, in view of the symmetry,

$$\langle n_- \rangle_{pp} = 2(A_p M_p^* + B_p) \ . \qquad (6)$$

We use the currently available accelerator data from $P_{\rm lab} = 10$ to 1500 GeV/c.[5] The plot of the negative average multiplicity vs the proton fireball mass computed according to Eqs. (1)–(3) is shown by open circles in Fig. 1.

By least-squares fit we find

$$2A_p = 0.56 \pm 0.02 \text{ GeV}^{-1} \ ,$$

$$2B_p = -0.46 \pm 0.10 \ .$$

This fit is shown by the dashed line, with χ^2/point = 1.79. Note that $B_p < 0$, indicating a threshold effect.

Indeed, regarding the produced pions as a photon gas, in other words, neglecting the pion mass, we expect no pion produced, if the fireball mass is equal to the proton mass. This condition leads to the following estimate for the proton mass:

$$m_p = -B_p / A_p = 0.81 \pm 0.19 \text{ GeV} \ , \qquad (7)$$

which differs from the expected value of 0.938 GeV by ~ 0.7 standard deviation.

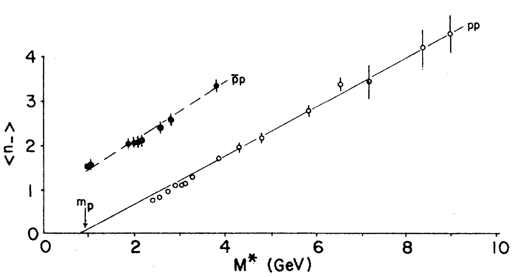

FIG. 1. Plots of the average negative-pion multiplicity against the fireball mass for pp collisions and $\bar{p}p$ annihilations. The straight lines represent least-square fits; see text.

Finally, it should be mentioned that the two parameters determined here are, within fitting errors, consistent with that of the previous one-parameter fit, account being taken of the threshold effect,

$$\langle n_- \rangle_{pp} = 2\alpha_p (M_p^* / m_p - 1) \qquad (8)$$

with $\alpha_p = 0.26 \pm 0.02$ as reported previously.[3(a)]

III. $\bar{p}p$ ANNIHILATION

We turn next to meson production by $\bar{p}p$ *annihilations,* excluding those by inelastic collisions which are similar to pp discussed in the previous section. We make use of available data for annihilations at rest as well as in flight for $P_{\rm lab}$ up to 100 GeV/c.[6] The plot of $\langle n_- \rangle_{\bar{p}p}$ against the fireball mass $M_{\bar{p}}^*$ is shown by full circles in Fig. 1. Note that at a given $P_{\rm lab}$, $M_{\bar{p}}^* \neq M_p^*$ of the pp collision. This is because, in general, $\langle n_- \rangle_{\bar{p}p} > \langle n_- \rangle_{pp}$, whereas the average c.m. momentum of pions from $\bar{p}p$ annihilation is, in general, smaller than that in the pp case. As $\langle P_T \rangle$ is different in the two cases of $\bar{p}p$ and pp, the scaling parameter as defined by Eq. (3) is larger for $\bar{p}p$ annihilation than for pp collision. Consequently, $M_{\bar{p}}^* < M_p^*$.

We analyze these data as in the pp case, assuming

$$\langle n_- \rangle_{\bar{p}p} = 2(A_{\bar{p}} M_{\bar{p}}^* + B_{\bar{p}}) \ , \qquad (9)$$

and find by least-squares fit

$$2A_{\bar{p}} = 0.62 \pm 0.04 \text{ GeV}^{-1} \ ,$$

$$2B_{\bar{p}} = 0.87 \pm 0.09 \ .$$

The fit is shown by the straight line, with χ^2/point = 1.33.

Note that here, the parameter $B_{\bar{p}}$ is positive, in contrast to the pp case, whereas the slope is the same as that of the pp case, within experimental errors. Furthermore, the difference between the two intercepts

$$2B_{\bar{p}} - 2B_p = 1.33 \pm 0.14$$

turns out to be consistent with the average negative multiplicity

$$\langle n_- \rangle_0 = 1.53 \pm 0.04$$

of $\bar{p}p$ annihilation at rest.[6]

This indicates that in terms of the fireball mass, the difference in multiplicity between $\bar{p}p$ annihilation, and pp collision can be accounted for by that of annihilation at rest. Thus, as in the pp case, we may estimate the proton (antiproton) mass by assuming

$$2B_{\bar{p}} - 2B_p = \langle n_- \rangle_0 , \tag{10}$$

and find

$$m_p = (\langle n_- \rangle_0 - 2B_{\bar{p}})/2A_{\bar{p}} = 1.07 \pm 0.13 \text{ Gev} . \tag{11}$$

Here again, the value thus estimated is consistent with the expected proton mass.

Suppose now the multiplicity of annihilation at rest $\langle n_- \rangle_0$ is not known, as in the case of $e^+e^- \to q\bar{q} \to$hadrons, which we are mainly concerned with. Then, we need other information to correlate the two parameters of Eq. (9). This can be achieved by using another parametrization, e.g., according to (5)

$$\langle n_- \rangle_{\bar{p}p} = a_{\bar{p}} E_{\text{c.m.}}{}^{1/2} + b_{\bar{p}} , \tag{12}$$

where $E_{\text{c.m.}}$ is the c.m. total energy of the $\bar{p}p$ system. A fit to the same $\bar{p}p$ data is shown by the dashed curve in Fig. 2. The parameters a and b of this alternative fit are

$$a_{\bar{p}} = 0.72 \pm 0.05 \text{ GeV}^{-1/2} ,$$
$$b_{\bar{p}} = 0.51 \pm 0.01 ,$$

the χ^2/point being 1.31 comparable to the previous fit Eq. (9).

Clearly, these two parametrizations, Eqs. (9) and (12), are not independent, since the kinematics of the fireball depend only on $E_{\text{c.m.}}$. One relation among these four parameters can be easily found by considering in particular the annihilation at rest, i.e., $\gamma^* = \gamma_{\text{c.m.}}$; thus

$$M_{\bar{p}}^* = m_p = E_{\text{c.m.}}/2 . \tag{13}$$

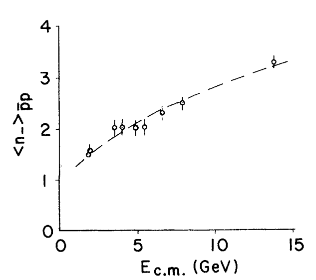

FIG. 2. Plot of the average negative-pion multiplicity against the c.m. energy for $\bar{p}p$ annihilations. The curve is a fit with the $E_{\text{c.m.}}{}^{1/2}$ law.

By means of Eqs. (9), (12), and (13), we find the relationship

$$2Am - a\sqrt{2m} + (2B - b) = 0 \tag{14}$$

between the two sets of parameters, A and B from Eq. (9) and a and b from Eq. (12), and the mass $2m$ of the annihilating particle-antiparticle, the subscript being omitted for simplicity.

Knowing the values of the two sets of parameters of fit, we can estimate m by solving the quadratic equation (14). We find

$$\sqrt{2m} = \begin{cases} 1.565 \\ 0.656 \end{cases} .$$

Consistency check of $\langle n_- \rangle_0 = 1.53$ with (9), (12) admits the larger root

$$m_p = 1.23 \pm 0.19 \text{ GeV} .$$

The error quoted here has been computed using the fitting errors on the two sets of parameters. It is interesting to note that the mass thus estimated agrees with the previous value using the multiplicity of annihilation at rest. Combining these two estimates, we find

$$m_p = 1.15 \pm 0.09 \text{ GeV} .$$

Finally, we note that if we assume a one-parameter fit as in the pp case mentioned above, Eq. (8), we write

$$\langle n_- \rangle_{\bar{p}p} = \langle n_- \rangle_0 + 2\alpha_{\bar{p}}(M_{\bar{p}}^*/m_p - 1) \tag{15}$$

with $\langle n_- \rangle_0 = 1.53$. We find $\alpha_{\bar{p}} = 0.25 \pm 0.08$, indicating that $\alpha_{\bar{p}} \simeq \alpha_p$, namely the same slope for both pp and $\bar{p}p$ fits; this similarity property has been noted in Ref. 3(a).

IV. QUARK MASS FROM $e^+e^- \to$HADRONS

We now apply the method to the meson production by e^+e^- annihilation. As is well known from the quark-parton model, this process proceeds via the formation of a quark-antiquark pair, which in turn fragments into hadrons,

$$e^+e^- \to q\bar{q} \to \text{hadrons} .$$

Thus, from the point of view of our fireball model, there exists certain kinematic analogy between these two processes, $q\bar{q} \to$hadrons on the one hand and $\bar{p}p \to \pi + \cdots$ on the other, notwithstanding the fact that the $q\bar{q}$ pair is in the virtual state. But as far as the kinematic properties are concerned, there is no difference between this case and the $\bar{p}p$ annihilation. Therefore we may proceed in a similar way.

Consider first the data below the J/ψ threshold. We shall use the experiment at ADONE for $E_{\text{c.m.}}=1.435-2.870$ GeV by the Rome-Bologna-Frascati collaboration.[7] They have measured both the charged and the neutral multiplicities. However, we have no experimental information on $\langle P_T\rangle$ and $\langle P_L\rangle$ to enable us to compute the fireball mass as outlined in Sec. I. In this regard, we note that at ADONE energy there is a simplification, namely the Lorentz factor of the fireball is practically equal to unity. Indeed, if we use the average π energy $\langle E_\pi\rangle$ from another experiment by the Frascati-Padova-Rome collaboration[8] and if we estimate the temperature T from our previous investigation of the SLAC and PETRA experiments,[4] we get for $E_{\text{c.m.}}=2.2$ GeV the values $\langle E_\pi\rangle=365\pm12$ MeV and $T\simeq 88$ MeV, which are related to the scaling parameter λ by

$$\langle E_\pi\rangle = T\frac{2(1+\lambda+\lambda^2)}{\lambda(1+\lambda)}\ , \tag{16}$$

according to the Bose-Einstein distribution modified for the scaling; see the Appendix of Ref. 4. Thus we deduce $\lambda=0.60$, indicating $\gamma^*\lesssim 1.09$ for the data we are dealing with.

Consequently, we assume $\gamma^*=1$ and set the fireball mass $M_q^*=E_{\text{c.m.}}/2$. With this simplification, we write Eq. (4) for the ADONE data of *charged and neutral* multiplicities as follows:

$$\langle n_{\pm 0}\rangle_{e\bar{e}} = 2(A_q E_{\text{c.m.}}/2+B_q)\ . \tag{17}$$

It is to be noted that this linear dependence of the average multiplicity on the c.m. energy has been predicted by the statistical model.[9] The plot of $\langle n_{\pm 0}\rangle$ vs $E_{\text{c.m.}}$ is shown in Fig. 3; the full line represents the fit with (17), χ^2/point being 0.37.

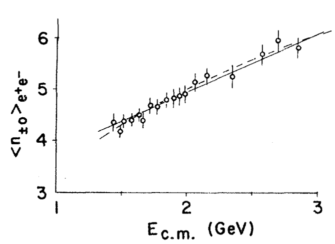

FIG. 3. Plot of the average meson multiplicities, charged and neutral, against the c.m. energy of e^+e^- annihilation. Data from ADONE experiment, Ref. 7. The full line is a linear fit, Eq. (17), and the dashed curve represents the fit with $\langle n\rangle\sim(E_{\text{c.m.}})^{1/2}$; see text.

The values of the parameters are

$$2A_q=1.34\pm0.67\ \text{GeV}^{-1}\ ,$$

$$2B_q=2.34\pm0.90\ .$$

Note that the intercept B_q is positive as in the case of $\bar{p}p$ annihilation.

Note that (5) holds also for e^+e^- annihilation as discussed previously,

$$\langle n_{\pm 0}\rangle_{e\bar{e}} = a_q E_{\text{c.m.}}{}^{1/2}+b_q\ . \tag{18}$$

The fit is shown by the dashed line in Fig. 3; the parameters are

$$a_q=3.74\pm0.17\ \text{GeV}^{-1/2}\ ,$$

$$b_q=0.25\pm0.08\ ,$$

the χ^2/point is 0.27 comparable to the previous fit.

With these two sets of parameters, we can estimate the effective quark mass by solving Eq. (14). We find

$$\sqrt{2m_q}=\begin{cases}4.75\\0.81\end{cases}.$$

Clearly, the larger root is ruled out by energy consideration; thus

$$m_q=0.33\pm0.03\ ,$$

and the quoted errors are computed from those on the fitted parameters.

A comparison of this estimate of quark mass with that of the proton mass $m_p=1.23\pm0.19$ GeV from $\bar{p}p$ annihilation, using the same method, indicates that the ratio

$$\frac{m_q}{m_p}=0.27\pm0.05$$

is in agreement with the predicted value $m_q/m_p=\frac{1}{3}$ of the naive quark-parton model, neglecting the binding energy of the three constituent quarks of the proton.

Finally, it should be mentioned that if instead of $\langle n_{\pm 0}\rangle$, we use only the *negative* multiplicity $\langle n_-\rangle=\frac{1}{2}\langle n_{\text{ch}}\rangle$, the parameters corresponding to this case are

$$2A_q'=0.22\pm0.06\ ,\quad 2B_q'=1.20\pm0.05\ ,$$

$$a_q'=0.86\pm0.11\ ,\quad b_q'=0.78\pm0.05\ .$$

We find for the quark mass

$$m_q'=0.50\pm0.18\ \text{GeV}\ ,$$

comparable to the previous estimate. However, we have to bear in mind that in the case of

$e^+e^- \to$hadrons, the use of charged multiplicity alone instead of charged and neutral multiplicities is bound to bias the mass estimation. We shall discuss this point in the concluding section.

V. REMARKS

We have estimated the effective mass of light quarks using low-energy data on $e^+e^- \to$hadrons from experiments at ADONE. It would be interesting to investigate heavy quarks using higher-energy e^+e^- data. In this regard, we note that the method requires the information on both charged and neutral multiplicities. This is because unlike the pp collisions and $\bar{p}p$ annihilations, here the ratio $\langle n_\pm \rangle / \langle n_0 \rangle$ decreases slowly with energy, as has been reported by the early experiments at SLAC.[10] This situation can also be seen from the fits carried out in the previous section using $\langle n_{\pm 0} \rangle = \langle n_{\rm ch} \rangle + \langle n_0 \rangle$ and $\langle n_- \rangle = \frac{1}{2} \langle n_{\rm ch} \rangle$ of the ADONE data. Comparing the fitted parameters, we note that they are not in the ratio of 3 to 1 as should be in the case of charge symmetry.

Bearing this in mind, we have studied the currently available data of PETRA experiments,[11] but we have limited ourselves to the results of the PLUTO collaboration for $E_{\rm c.m.} = 9-31$ GeV, for which we have at our disposal the values of $\langle P_T \rangle$ and $\langle P_L \rangle$.[12,13] Therefore we can compute the fireball mass using Eqs. (1)–(3). We have fitted the *negative* multiplicity as in the case of $\bar{p}p$ annihilation, namely using Eq. (6), instead of (17), which is a particular case for the low-energy data as discussed in Sec. IV. The parameters thus obtained are

$$2A_q = 0.34 \pm 0.02 \text{ GeV}^{-1} ,$$

$$2B_q = 2.19 \pm 0.10 ,$$

whereas the fit to Eq. (18) leads to

$$a_q = 0.94 \pm 0.08 \text{ GeV}^{-1/2} ,$$

$$b_q = 0.20 \pm 0.18 ,$$

in excellent agreement with a previous analysis using all $e^+e^- \to$hadron data from $E_{\rm c.m.} = 3-31$ GeV.[4]

With these two sets of parameters we arrive at a quadratic equation (14) for estimating the quark mass m_q. But here, we find the discriminant

$$\Delta = a^2 - 4A(2B - b) = -0.42^{+0.37}_{-0.54}$$

slightly negative, yielding an imaginary mass for the quark under consideration. This, in turn, implies a spacelike energy-momentum four-vector, which is contrary to what we are actually dealing with here, namely a virtual photon resulting from the e^+e^- annihilation, which is well known to be timelike.

Since Δ is consistent with zero within fitting errors, we may tentatively set $\Delta = 0$ and obtain from (14)

$$m_q = \tfrac{1}{2}(a_q / 2A_q)^2 = 3.83 \pm 0.46 \text{ GeV} ,$$

which is ~ 11 times heavier than the light-quark mass obtained in Sec. IV using low-energy data from ADONE.

What has caused this drastic increase of the estimated m_q cannot be investigated thoroughly at present because of the lack of necessary data on $\langle P_T \rangle$ and $\langle P_L \rangle$ in the energy region between the ADONE and PETRA experiments. It would be interesting to further investigate this problem of heavy-quark mass when additional information, in particular the neutral multiplicity, becomes available.

It should be mentioned that the fireball coefficient $2A_q$ estimated here is comparable to that of the π or K fireball, $(0.36 \pm 0.03)/m_p$ ($m_p = 0.938$ being the proton mass), in the case of $\pi^+ p$ and $K^+ p$ collisions.[14] This indicates that the meson production by π or K diffractive dissociation may proceed via quark-antiquark formation as discussed by Misra *et al.*[15]

In passing, we note that as far as the linear relationship between the average multiplicity and the fireball mass in concerned, Eq. (4), it has been extensively investigated in cases of π^- production by various collisions covering a wide range of energy.[3] In this regard, we note that for the pp case, $\langle n_- \rangle$ computed according to Eq. (8) agrees with experimental data within 3% for $P_{\rm lab} = 10-1500$ GeV/c and that this can be extended to 20 TeV/c of the cosmic-ray data as discussed in Ref. 3.

Finally, it is worth noting that the fireball coefficient A_p for π production by the pp collision is found to be the same as $A_{\bar{p}}$ of the $\bar{p}p$ annihilation, see Secs. II and III, and that a similar property has also been observed in a previous study of π^- production by $\pi^+ p$ and $K^+ p$ as well as ep, μp, γp and νp collisions.[14] Thus, we may expect, *mutatis mutandis,* the coefficient A_q of the quark-fireball mass in Eq. (17) remains the same, independent of the quark mass. If this property holds, then in case of quarks of different masses involved in e^+e^- annihilations, the superposition of light- and heavy-$q\bar{q}$ states leads to a parameter B_q of Eq. (17), depending on the average quark mass, weighted according to the percentages of various kinds of quarks contributing to $\langle n \rangle$, as is seen from Eq. (11) for $\bar{p}p$ annihilation. These remarks may be useful for further application of our method to investigate heavy-quark mass with high-energy e^+e^- annihilations above the charm- or b-quark threshold.

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