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Spin-zero quarks in e^+e^- annihilation

Chiara R. Nappi
The Institute for Advanced Study, Princeton, New Jersey 08540
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We study the phenomenology of scalar-quark resonances in e^+e^- annihilation.

I. INTRODUCTION

Both supersymmetry and hypercolor predict the existence of scalar quarks, i.e., particles that carry color, but no spin. In supersymmetry they are the supersymmetric partners of fermionic quarks and in hypercolor theories they arise as pseudo-Goldstone bosons. In this paper, without reference to any particular model of supersymmetry or hypercolor, we study the phenomenology of such particles.[1] More precisely, we analyze the production of scalar-quark—antiquark resonances in e^+e^- annihilation and their subsequent decays, using a potential model of ordinary heavy quarks.[2]

Let us first remark that one could hope to detect the process $e^+e^-\to\phi\bar{\phi}$ (ϕ hereafter indicates scalar quarks) as a new threshold in the cross section

$$R=\frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)}\ .$$

It is known, however, that the thresholds of scalar particles are hard to observe. This is because scalar particles are produced in p waves and their threshold, therefore, rises very slowly, $\Delta R\propto|\vec{p}\,|^3$. In contrast, fermions have $\Delta R\propto|\vec{p}\,|$. Moreover, even asymptotically, i.e., for $|\vec{p}\,|>>m$, the change in R due to scalar quarks is only $\frac{1}{4}$ what it would be for a fermion of the same charge.

Another possibility is that scalar quarks might be discovered by detecting quark-antiquark bound states as resonances in e^+e^- annihilation. (This is of course one of the main ways of studying charm and bottom quarks.) As we will see in detail, bound states of scalar quarks are narrow resonances (unless the scalar itself has a very large width, as discussed later). The detectability of a narrow resonance depends primarily on its leptonic width. Therefore our first step (Sec. II) will be to compute Γ_e.

The experimental search for narrow resonances in e^+e^- annihilation has set very stringent upper bounds on the magnitude of the leptonic width of possible resonances, in some range of energies. The best upper bounds are in the region up to 8 GeV (Refs. 3 and 4); less strong upper bounds have been set at PETRA at higher energies, up to 35 GeV.[5] We will compute Γ_e for scalar-quarkonium ($\phi\bar{\phi}$ bound state) and compare it with experimental bounds, wherever they exist. The purpose is to see whether scalar quarks can be detected as resonances in e^+e^- annihilation and whether they have already been ruled out experimentally in some range of energies. It turns out that Γ_e is very small, the reason being once more that scalar-quark bound states must be produced in p waves. Above quark masses of about 3 GeV, Γ_e is much below the experimental upper limits, even for a scalar quark of charge $\frac{2}{3}$. Moreover Γ_e decreases rapidly with the quark mass.

In Sec. III we will compute the hadronic decay width of the $2P$ scalar-quarkonium, and in Sec. IV the radiative decay width to the $1S$ bound state It turns out that the radiative decay dominates for very heavy quarks. The $1S$ state, on the other side, decays almost exclusively into hadrons (Sec. III), since leptonic decay is forbidden. We will conclude this paper with a few comments on some model-dependent decay modes.

II. LEPTONIC WIDTH

A bound state $\phi\bar{\phi}$ can be produced from e^+e^- according to Fig. 1. It must have therefore the quantum numbers of the intermediate photon, i.e., $J^{PC}=1^{--}$. Because scalar quarks carry no spin, a scalar resonance must be created in a state of angular momentum $J=1$. The current matrix element between the vacuum and a bound state $\phi\bar{\phi}$ of $J=1$ and given polarization j is, to the lowest order in the quark momentum,

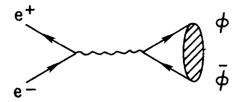

FIG. 1. One-photon production of scalar-quarkonium from e^+e^-.

$$\langle 0 | J_i | P,j \rangle = \frac{\sqrt{3}}{m} \partial_i \phi_P^j(0) , \quad (1)$$

where ϕ_P^j is the $2P$ wave function with polarization j, and the factor $\sqrt{3}$ is associated with color SU(3). From (1) it follows that the analog of the Van Royen-Weisskopf formula for the leptonic width of a scalar-quark resonance is

$$\Gamma_e(2P \to e^+e^-) = 24\alpha^2 Q^2 \frac{|R_P'(0)|^2}{M^4} , \quad (2)$$

where R_P is the radial wave function, M the mass of the bound state, α the fine-structure constant, and Q the charge of the quark in units of the electron charge.

In order to evaluate the wave function appearing in (2) we have numerically solved the Schrödinger equation for the standard potential of heavy-quark spectroscopy,

$$V(r) = -\frac{\kappa}{r} + \frac{1}{a^2} r , \quad (3)$$

where κ is normalized to $\simeq 0.3$ at the charmonium mass, and $1/a^2$ is inferred from charmonium fits to be about 0.18 GeV^2.

In Table I we have listed the values of Γ_e for a large range of quark masses. In the computation we have taken $Q=\frac{2}{3}$. For scalar quarks of charge $Q=\frac{1}{3}$ these numbers must be divided by four. For comparison (see Table V), let us remark that for a purely linear potential Γ_e is

$$\Gamma_e = 6.4\alpha^2 \left[\frac{1}{a^2} \right]^{5/3} Q^2 \frac{m^{5/3}}{M^4} ,$$

where m is the quark mass and M (equal to $2m$ plus the energy eigenvalue) is the bound-state mass. For a purely coulombic potential, instead,

$$\Gamma_e = \frac{1}{512} \alpha^2 \kappa^3 Q^2 m .$$

As stated in the Introduction, scalar-quark resonances can be considered narrow resonances. This will become clear when we compute hadronic and radiative widths. Also, as we will see, the leptonic width is a small fraction of the total width. Experimentally, for narrow resonances (whose width is much smaller than the beam-energy spread) one measures the integrated area of the resonance cross section. For a Breit-Wigner-type resonance this is connected to the leptonic width by the formula

$$\int \sigma_{\text{res}} dE = \frac{2\pi^2(2J+1)}{M^2} \frac{\Gamma_e \Gamma_h}{\Gamma} ,$$

and $\Gamma_h/\Gamma \simeq 1$ if the hadronic width predominates.

Results of the search at SPEAR for narrow resonances in the region between 5.7 and 6.4 GeV are published in Ref. 3. To summarize them roughly we say that they set an upper bound of $\cong 100$ eV on the leptonic width. Measurements in the region around 7 GeV give $\Gamma_e < 60$ eV for $7.0 < E_{\text{cm}} < 7.4$ GeV.[4] Scans intended to search for $t\bar{t}$ bound states have been performed at PETRA.[5] In the energy ranges from 29.9 to 31.5 GeV and from 35.0 to 35.6 GeV, these scans have set upper limits on Γ_e of 0.7 and 0.4 KeV, respectively.

If we compare the above upper bounds to the values in Table I, we conclude that scalar-quark resonances are ruled out experimentally only for quark masses below 3 GeV. For a quarkonium mass above 6 GeV, Γ_e is below the experimental bounds, so that scalar-quark resonances might well have been missed. In fact the conclusion of this analysis seems to be that scalar-quark bound states are very difficult to detect as resonances in e^+e^- annihilation. Moreover, the difficulty increases with increasing energy.

TABLE I. Leptonic widths of the $2P$ scalar-quark resonances for different values of the quark mass and $Q=\frac{2}{3}$. $M \equiv 2m+\epsilon$, where ϵ is the energy eigenvalue and m is the resonance mass.

m (GeV)	M (GeV)	Γ_e (eV)	m (GeV)	M (GeV)	Γ_e (eV)
1	3.0	180	10	20	7.3
2	4.8	100	15	30	3.7
3	6.7	57	20	40	2.3
4	8.6	35	30	60	1.2
5	10.5	24	50	100	0.6

III. HADRONIC WIDTHS

Before computing the width of any scalar-quarkonium decay mode, it is important to understand whether the $2P$ scalar-quark bound state produced in e^+e^- annihilation is above or below the continuum threshold. If it were above threshold, it would be a broad resonance which decays mainly into "scalar-containing" hadron, i.e., fermions made of a scalar quark and an ordinary spin-$\frac{1}{2}$ quark. Our $2P$ bound state turns out, however, to be located below threshold. In fact, the $2P$ state is below threshold in the case of charm, and according to potential models it will be further below for heavier quarks.[6]

In order to compute the decay width of the $2P$ scalar resonance into hadrons, we make the usual assumption that the annihilation into hadrons proceeds through gluons, the dominant contribution coming from the minimum number of intermediate gluons. One-gluon decay is excluded by color conservation, two-gluon decay is excluded because the two-gluon state is even under C (while the $2P$ scalar-quarkonium has $J^{PC}=1^{--}$). So the first possible decay is into three gluons which must come in a C-odd color-singlet state. Therefore the three final gluons must be in a d-coupling state. This state is color symmetric, so the three-gluon diagram of Fig. 2(c) is forbidden because of the antisymmetric coupling at the vertex. The only three-gluon diagrams are the ones in Figs. 2(a) and 2(b). In the radiation gauge, however, the amplitude for Fig. 2(b) is quadratic in the quark momentum. Therefore it has an extra suppression factor of the order α_s ($|\vec{p}| \propto 1/a_0 \propto \alpha_s m$, where a_0 is the Bohr radius of scalar-quarkonium) with respect to the amplitude of Fig. 2(a), that is only linear in the quark momentum. We will restrict ourselves to the diagram of Fig. 2(a).

The calculation of the decay rate for this diagram presents the same features as the corresponding annihilation rate for charmonium.[7] The graph

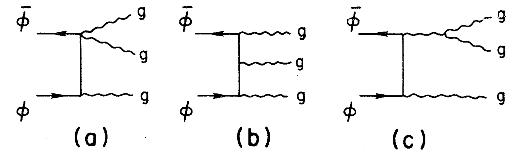

FIG. 2. Three-gluon decay of the $2P$ scalar-quarkonium state.

becomes infrared singular if the momentum of the gluon emitted from an external quark tends to zero. The singularity arises from using in the calculation the free propagator for the internal quark line and neglecting the fact that the quark comes in a bound state. An exact calculation would have no divergence, but would have a logarithmically enhanced term. The exact calculation would be quite complicated,[8] so we will derive here only a rough estimate of this rate, by calculating the coefficient of the logarithmically enhanced term. Actually one may calculate the diagram 2(a) as if the gluons were photons and then multiply the answer by the factor $\frac{5}{54}\,\alpha_s^{\,3}/\alpha^3 Q^6$ due to color and to the difference of the coupling constants. So in the logarithmic approximation we get

$$\Gamma_h(2P \rightarrow 3g) = \tfrac{640}{9}\alpha_s^{\,3}\frac{|R_P'(0)|^2}{M^4}\ln\frac{m}{\Delta}\ .$$

Δ is a cutoff that we can choose of the order a_0^{-1}, where a_0 is the Bohr radius of scalar-quarkonium, $a_0 = 2/m\kappa$.

We report in Table II the hadronic width of the $2P$ bound state for various values of the quark mass. In the computation we have taken $\alpha_s \cong 0.2$ at the charmonium mass and scaled it logarithmically thereafter.

For completeness, we also compute the hadronic decay width of the $1S$ resonance via the minimum possible number of gluons, i.e., at the lowest possible order in α_s. The allowed diagrams for decay of the $1S$ state into two gluons are shown in Figs.

TABLE II. Hadronic decay widths of the $2P$ scalar-quark resonance for $Q=\frac{2}{3}$.

M (GeV)	$\Gamma_h(2P\rightarrow 3g)$ (keV)	M (GeV)	$\Gamma_h(2P\rightarrow 3g)$ (keV)
3.0	500	30	1.4
6.7	200	40	0.8
10	18	60	0.3
20	3.6	100	0.04

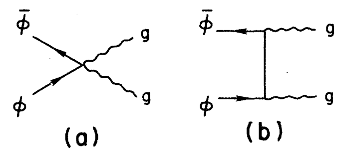

FIG. 3. Two-gluon decay of the $1S$ scalar-quarkonium state.

3(a) and 3(b). As above, we choose to work in the radiative gauge and ignore the diagram 3(b) which has an extra suppression factor of the order α_s with respect to the amplitude of 3(a). The hadronic decay of the $1S$ state therefore turns out to be, to lowest order in quark velocity,

$$\Gamma_h(1S \to 2g) = \frac{4}{3}\alpha_s^2 \frac{|R_S(0)|^2}{M^2} ,$$

where $R_S(0)$ is the $1S$ radial wave function at the origin.

In Table III we list the values of this decay width for various quark masses, using the wave function at the origin computed from the potential (3). The hadronic decay is the predominant decay of the $1S$ bound state, which of course has no leptonic decay. The photonic decay into 2γ is much weaker. In fact the ratio between the two is

$$\frac{\Gamma(1S \to 2g)}{\Gamma(1S \to 2\gamma)} = \frac{2}{9}\frac{\alpha_s^2}{Q^2\alpha^2} \cong 850$$

for $Q = \frac{2}{3}$.

IV. TRANSITIONS BETWEEN BOUND STATES

A $2P$ scalar-quark bound state can decay into a $1S$ state by emitting a photon with width given by

$$\Gamma_\gamma(2P \to 1S + \gamma) = \frac{4}{9}\alpha Q^2 |E_{12}|^2 \omega^3 ,$$

where $\omega = E_{2P} - E_{1S}$ and $E_{12} = \int_0^\infty R_{1S}(r) R_{2P}(r) dr$. Again calculating the matrix element from the charmonium model, we list in Table IV the radiative width for some values of the quark mass. Notice that the electromagnetic width of the $2P$ state starts out bigger than the corresponding hadronic one, and decreases much more slowly with the quark mass. This is compatible with the fact that for a purely linear potential $\Gamma_\gamma(2P \to 1S + \gamma)$ is proportional to $m^{-5/3}$, unlike $\Gamma_h(2P \to 3g)$ that goes like $m^{-7/3}$ (see Table V). So the radiative decay predominates for heavy enough quarks. A possible way to discover scalar-quarkonium might be by detecting the monochromatic photons from the radiative transition from the $2P$ to the $1S$ state.

The $2P$ bound state could also decay to the $1S$ state by emitting gluons. By color conservation, two is the minumum number of gluons involved in the process. If we could neglect the recoil of the $1S$ bound state, and assume that it is produced with zero momentum, then the decay $2P \to 1S + 2g$ would be analogous to the decay of a spin-1 particle decaying into two massless spin-1 particles, which is known to be forbidden.[9] As a result, the amplitude for this process is proportional to the recoil momentum of the bound state. This is of the order of the energy of the emitted gluons, therefore of the order $\alpha_s^2 m$. It follows that the cross section for this process is suppressed by a factor of α_s^4 with respect to the corresponding decay for fermion quarks. We therefore neglect this decay.

V. CONCLUSIONS

To conclude, let us remark that the scalar-quark resonance could have some model-dependent decay modes which might be much larger than the model-independent ones we have calculated. For instance, the scalar quark could decay into an ordinary quark and an antilepton according to the Yukawa coupling $\bar{l}(\lambda + \lambda'\gamma_5)q\phi$. This decay would contribute a width $(\lambda^2 + \lambda'^2/8\pi)m$ to the scalar and twice that, $\lambda^2 + \lambda'^2/4\pi$, to the bound state (of

TABLE III. Hadronic decay rates of the $1S$ scalar-quark resonance for various values of the quark mass and $Q = \frac{2}{3}$.

m (GeV)	M (GeV)	$\Gamma_h(1S \to 2g)$ (keV)	m (GeV)	M (GeV)	$\Gamma_h(1S \to 2g)$ (keV)
1	2.6	3900	15	30	270
3	6.3	1200	30	60	180
5	10.5	650	50	100	150
10	20.4	490			

TABLE IV. Photonic transition decay rates for various values of the quark mass, and $Q=\frac{2}{3}$.

M (GeV)	$\Gamma_\gamma(2P \to 1S+\gamma)$ (keV)	M (GeV)	$\Gamma_\gamma(2P \to 1S+\gamma)$ (keV)
2.6	680	30	15
6.3	130	60	6.9
10.5	65	100	3.8
20.4	25		

TABLE V. Scaling behavior of the various decay widths for power-law potentials.

	$V(r) \propto r^\alpha$	$V(r) \propto r$	$V(r) \propto r^{-1}$
$\Gamma_e(2P \to e^+e^-)$	$m^{-3/4\alpha/\alpha+2}$	$m^{-7/3}$	m
$\Gamma_h(2P \to 3g)$	$m^{-3+4\alpha/\alpha+2}$	$m^{-7/3}$	m
$\Gamma_\gamma(2P \to 1S+\gamma)$	$m^{-2+3\alpha/\alpha+2}$	$m^{-5/3}$	m
$\Gamma_h(1S \to 2g)$	$m^{-1+2\alpha/\alpha+2}$	m^{-1}	m

course for sufficiently low masses lepton conservation imposes upper bounds on the above coupling constants); or the scalar could decay into two antiquarks with a similar formula for the width. However, these decays could not both be allowed or baryon-number conservation would be violated.

Some models also allow decay into an octet quark and an ordinary quark. According to supersymmetry the coupling constant for this decay is $\sqrt{2}g$, where g is the strong coupling constant. The quarkonium decay rate for this decay would then be $\frac{8}{3}\alpha_s m$, almost of the same order as the quark mass. The decays computed in this paper, instead, have been model-independent ones, with the bound state assumed to be a narrow resonance. If the model-dependent decays give the scalar quark a width bigger than the energy splitting between bound states, the description in terms of scalar-quark bound states would break down.

Note added. Some of the issues discussed in this paper have been analyzed by G. Barbiellini, G. Bonneaud, G. Coignet, J. Ellis, M. K. Gaillard, C. Matteuzzi, and B. H. Wiik, DESY Report No. 79/67 (unpublished). I would like to thank M. Peskin for drawing this reference to my attention. Note, however, a factor-of-three discrepancy between Eq. (2) of this paper and the corresponding equation on page 23 of Barbiellini *et al.*

ACKNOWLEDGMENT

I thank M. Cavalli-Sforza for many useful conversations.

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