Baryon magnetic moments in the broken-SU(6)-symmetry model

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A simple factor representing the symmetry-breaking effects due to quark masses is introduced in the computation of the magnetic moments of baryons in the context of broken SU(6) symmetry.

Recent precision measurements[1] of the magnetic moments of hyperons have revived lively discussions of broken SU(6) symmetry and the quark-model predictions. After the pioneering works on SU(3) and SU(6) symmetry,[2,3] symmetry-breaking effects due to the mass difference of quarks were introduced. The broken symmetry may be exhibited in the following two ways: (I) the magnetic-moment operator is expressed as a sum of Dirac magnetic moments of quarks which are inversely proportional to their masses, and (II) the matrix elements reflect the symmetry breaking of the strong interactions in the wave functions.

The magnetic-moment operator $\vec{\mu}$ is expressed as[4,5]

$$\vec{\mu}=\frac{e\overline{Q}}{2m_u c}\vec{\sigma}\ , \tag{1}$$

where

$$Q=(2/3,-\lambda/3,-\xi/3)\ , \tag{2}$$

with

$$\lambda=\frac{m_u}{m_d} \quad \text{and} \quad \xi=\frac{m_u}{m_s}\ , \tag{3}$$

and m_u, m_d, and m_s being the masses of the constituent up, down, and strange quarks, respectively.

Symmetry breaking of type II has been introduced in the form of a mass-scale factor,[6-8] SU(3)-symmetry-breaking terms,[9] or the configuration mixing of the SU(6) wave functions.[10] When Ref. 6 took into consideration the mass-scale factor $1/m_B$ in the context of broken SU(6) symmetry (m_B being the physical baryon mass), the experimental data of the hyperon magnetic moments except for Λ were not accurate enough to make a sensible comparison. The remarkable progress of experimental measurements since then has demonstrated that the correction by the scale factor $1/m_B$ is in a right direction.

The experimental data on the Ξ^0 and Σ^+ magnetic moments seem to be halfway between the theoretical predictions with and without the mass-scale factor. (Teese and Settles[8] pointed out that the mass scale $1/\sqrt{m_B}$ would give better agreement.) There is a way to understand this situation. The physical mass reflects symmetry-breaking effects and therefore such a scale factor correctly represents a symmetry (breaking) property. But the correction may not be the right one since symmetry-breaking effects of types I and II presumably originated from the same source (quark masses) and therefore one should use the same parameters (quark mass ratios) for both effects. Although Ref. 5 determines the ξ parameter from an analysis of the baryon masses, it is more natural to determine it from the magnetic moments of hyperons now that precise measurements are available.

In this paper, we propose a symmetry-breaking mechanism of type II which keeps ground-state SU(6) wave functions and introduces a mass-scale factor which satisfies the conditions: (a) it is a symmetric function of quark masses and (b) it has the dimension of inverse mass. The simplest form of such a quantity is given by

$$\text{(A)}\quad \frac{1}{m_1+m_2+m_3} \tag{4}$$

or

$$\text{(B)}\quad \frac{1}{m_1}+\frac{1}{m_2}+\frac{1}{m_3}\ , \tag{5}$$

where m_1, m_2, m_3 are the masses of the quarks which constitute the baryons.

Then the mass-scale factor entering in the expression for the magnetic moments of the Λ relative to that of the proton is

$$\frac{2m_u+m_d}{m_u+m_d+m_s}=\frac{2+\lambda^{-1}}{1+\lambda^{-1}+\xi^{-1}} \quad \text{for case (A)}$$

and (6)

$$\frac{2/m_u+1/m_d}{1/m_u+1/m_d+1/m_s}=\frac{2+\lambda}{1+\lambda+\xi} \quad \text{for case (B)}\,.$$

The magnetic-moment ratio for Λ and p is given by

$$\frac{\mu(\Lambda)}{\mu(p)}=\begin{cases}\dfrac{-\frac{1}{3}\xi}{\frac{1}{9}(8+\lambda)}\dfrac{2+\lambda^{-1}}{1+\lambda^{-1}+\xi^{-1}} & \text{for case (A)}\\[2ex] \dfrac{-\frac{1}{3}\xi}{\frac{1}{9}(8+\lambda)}\dfrac{2+\lambda}{1+\lambda+\xi} & \text{for case (B)}\,.\end{cases} \tag{7}$$

For $\lambda=1$, we determine the parameter ξ in Eq. (7), using the experimental data of $\mu(p)$ and $\mu(\Lambda)$, to be $\xi=0.738$. For $\lambda\neq1$, we further use the magnetic-moment ratio of n and p as input,

$$\frac{\mu(n)}{\mu(p)}=\frac{-\frac{2}{9}(1+2\lambda)}{\frac{1}{9}(8+\lambda)}\frac{1+2\lambda}{2+\lambda} \quad \text{for both cases (A) and (B)}\,. \tag{8}$$

This leads to $\lambda=1.031$ and $\xi=0.726$.

In Tables I and II we list the prediction for the hyperon magnetic moments based on the mass-scale factors (4) and (5) for $\lambda=1$ and $\lambda\neq1$, along with the current experimental data. As is seen there both cases give essentially the same result. This is because the mass-scale factors (4) and (5) represent the same first-order SU(3)-breaking effects and differ only in the higher-order corrections. In particular we notice that the agreement for the Ξ^0 magnetic moment with experiment is excellent.

We have tried other forms of the mass-scale factor, $1/(m_1{}^2+m_2{}^2+m_3{}^2)^{1/2}$ and $(1/m_1{}^2+1/m_2{}^2+m_3{}^2)^{1/2}$, and obtained results for the magnetic moments which are very close to those for cases (A) and (B). [The parameter values for these cases are given by $\xi=0.743$ and $\xi=0.720$ for $\lambda=1$, respectively, and we have the prediction that $\mu(\Xi^0)$ is -1.24 and -1.25, respectively.] In Table III we made a comparison of our results with some of the other works which use a different mass scale, configuration mixing, or SU(3)-symmetry-breaking terms.

TABLE I. Baryon magnetic moments in nuclear magneton. (The underlined values are inputs.)

	SU(6) matrix element of Eq. (1)	Magnetic moments (A) ($\xi=0.738$) Mass factor		(B) ($\xi=0.726$) Mass factor		Experiment
p	1	1	$\underline{2.793}$	1	$\underline{2.793}$	2.793
n	$-\frac{2}{3}$	1	-1.86	1	-1.86	-1.913
Λ	$-\frac{1}{3}\xi$	$\frac{3}{2+\xi^{-1}}$	$\underline{-0.614}$	$\frac{2+\xi}{3}$	$\underline{-0.614}$	-0.614 ± 0.005
Σ^+	$\frac{1}{9}(8+\xi)$	$\frac{3}{2+\xi^{-1}}$	2.42	$\frac{2+\xi}{3}$	2.46	2.33 ± 0.13
Σ^0	$\frac{1}{9}(2+\xi)$	$\frac{3}{2+\xi^{-1}}$	0.76	$\frac{2+\xi}{3}$	0.77	
Σ^-	$-\frac{1}{9}(4-\xi)$	$\frac{3}{2+\xi^{-1}}$	-0.91	$\frac{2+\xi}{3}$	-0.92	-1.41 ± 0.25
Ξ^0	$-\frac{2}{9}(1+2\xi)$	$\frac{3}{1+2\xi^{-1}}$	-1.24	$\frac{1+2\xi}{3}$	-1.24	-1.25 ± 0.014
Ξ^-	$-\frac{1}{9}(4\xi-1)$	$\frac{3}{1+2\xi^{-1}}$	-0.49	$\frac{1+2\xi}{3}$	-0.48	-0.75 ± 0.06
$\Lambda\Sigma^0$	$\frac{1}{3}\sqrt{3}$	$\frac{3}{2+\xi^{-}1}$	1.44	$\frac{2+\xi}{3}$	1.47	$1.82^{+0.25}_{-0.18}$
Ω^-	$-\xi$	ξ	-1.52	ξ	-1.47	

TABLE II. Baryon magnetic moments (in nuclear magneton units) for $\lambda=(m_u/m_d)=1.0308$. (The underlined values are inputs.)

	SU(6) matrix element of Eq. (1)	Mass correction (A) ($\xi=0.740$)		Mass correction (B) ($\xi=0.727$)	
p	$\frac{1}{9}(1+\lambda)$	1	$\underline{2.793}$	1	$\underline{2.793}$
n	$-\frac{2}{9}(1+2\lambda)$	$\frac{2+\lambda^{-1}}{1+2\lambda^{-1}}$	$\underline{1.913}$	$\frac{1+2\lambda}{2+\lambda}$	$\underline{1.913}$
Λ	$-\frac{1}{3}\xi$	$\frac{2+\lambda^{-1}}{1+\lambda^{-1}+\xi^{-1}}$	$\underline{0.614}$	$\frac{1+\lambda+\xi}{2+\lambda}$	$\underline{0.614}$
Σ^+	$\frac{1}{9}(8+\xi)$	$\frac{2+\lambda^{-1}}{2+\xi^{-1}}$	2.40	$\frac{2+\xi}{2+\lambda}$	2.43
Σ^0	$\frac{1}{9}(4-2\lambda+\xi)$	$\frac{2+\lambda^{-1}}{1+\lambda^{-1}+\xi^{-1}}$	0.74	$\frac{1+\lambda+\xi}{2+\lambda}$	0.75
Σ^-	$-\frac{1}{9}(4\lambda-\xi)$	$\frac{2+\lambda^{-1}}{2\lambda^{-1}+\xi^{-1}}$	-0.94	$\frac{2\lambda+\xi}{2+\lambda}$	-0.97
Ξ^0	$-\frac{2}{9}(1+2\xi)$	$\frac{2+\lambda^{-1}}{1+2\xi^{-1}}$	-1.23	$\frac{1+2\xi}{2+\lambda}$	-1.23
Ξ^-	$-\frac{1}{9}(4\xi-\lambda)$	$\frac{2+\lambda^{-1}}{\lambda^{-1}+2\xi^{-1}}$	-0.48	$\frac{\lambda+2\xi}{2+\lambda}$	-0.48
$\Lambda\Sigma^0$	$\frac{1}{9}\sqrt{3}(1+2\lambda)$	$\frac{2+\lambda^{-1}}{1+\lambda^{-1}+\xi^{-1}}$	1.45	$\frac{1+\lambda+\xi}{2+\lambda}$	1.48
Ω^-	$-\xi$	$\frac{2+\lambda^{-1}}{3\xi^{-1}}$	-1.51	$\frac{3\xi}{2+\lambda}$	-1.46

TABLE III. Comparison of our results on baryon magnetic moments (in nuclear magnetons) with other works. [The underlined values are inputs (Ref. 11).]

	This article (mass scale in terms of quark masses) (A)	(B)	Tesse-Settles (Ref. 8) (mass scale by $1/\sqrt{m_B}$)	Isgur-Karl (Ref. 10) (configuration mixing)	Teese (Ref. 9) [SU(3) breaking-decouplet term]	Verma (Ref. 9) [SU(3) breaking]
p	$\underline{2.793}$	$\underline{2.793}$	$\underline{2.79}$	2.85	$\underline{2.793}$	$\underline{2.79}$
n	-1.86	-1.86	$\underline{-1.91}$	-1.91	$\underline{-1.913}$	-1.86
Λ	$\underline{-0.614}$	$\underline{-0.614}$	$\underline{-0.612}$	-1.61	$\underline{-0.614}$	$\underline{-0.61}$
Σ^+	2.42	2.46	2.39	2.54	2.40	2.46
Σ^0	0.76	0.77		0.77	0.79	0.76
Σ^-	-0.91	-0.92	-0.95	-1.00	-0.82	-0.92
Ξ^0	-1.24	-1.24	-1.27	-1.20	-1.25	$\underline{-1.20}$
Ξ^-	-0.49	-0.48	-0.48	-0.43	-0.68	-0.45
$\Lambda\Sigma^0$	1.44	1.47	1.45	-1.51	1.52	1.46
Ω^-	-1.52	-1.48				
r_Σ	-0.37	-0.38	-0.40	-0.39	-0.34	-0.37
r_Ξ	0.39	0.39	0.38	0.36	0.54	0.38

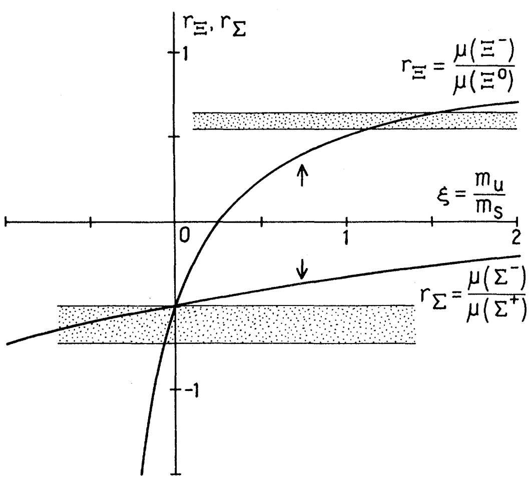

FIG. 1. The magnetic-moment ratios,

$$r_\Sigma = \frac{\mu(\Sigma^-)}{\mu(\Sigma^+)} \text{ and } r_\Xi = \frac{\mu(\Xi^-)}{\mu(\Xi^0)} ,$$

as functions of $\xi = m_u/m_s$. The horizontal lines represent the experimental values and the arrows indicate a theoretical prediction.

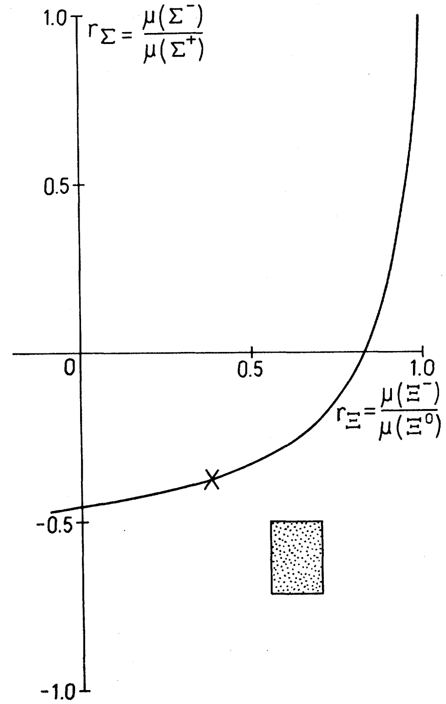

FIG. 2. The relationship between r_Σ and r_Ξ. The box represents the region of the current experimental data and the cross represents a theoretical prediction.

Finally, we will discuss the predictions for SU(6)-symmetry breaking of type I which are independent of the corrections due to symmetry breaking in terms of mass-scale factors discussed in this paper. For that purpose, define the ratios of the magnetic moments,

$$r_\Sigma = \frac{\mu(\Sigma^-)}{\mu(\Sigma^+)} = -\frac{4-\xi}{8+\xi} \tag{9}$$

and

$$r_\Xi = \frac{\mu(\Xi^-)}{\mu(\Xi^0)} = \frac{4\xi-1}{4\xi+2} . \tag{10}$$

It is natural to assume that the corrections due to symmetry breaking of the wave functions or the mass-scale factors would cancel in the above ratios.[12] Further eliminating the parameter $\xi = m_u/m_s$, in Eqs. (9) and (10), we obtain

$$r_\Sigma = -\frac{5-6r_\Xi}{11-10r_\Xi} . \tag{11}$$

The experimental values for r_Σ and r_Ξ are

$$(r_\Sigma)_{\text{exp}} = \frac{-1.41 \pm 0.25}{2.33 \pm 0.13} = -0.61 \pm 0.11 , \tag{12}$$

$$(r_\Xi)_{\text{exp}} = \frac{0.75 \pm 0.06}{1.25 \pm 0.014} = 0.60 \pm 0.05 , \tag{13}$$

to be compared with the theoretical predictions, -0.37 and 0.39, respectively. In Table III, the predictions for r_Σ and r_Ξ given by various authors are listed. As is seen there, there is little difference in their results. Figure 1 depicts r_Σ and r_Ξ as functions of ξ. It shows that the current experimental data does not comply with the theoretical prediction; the discrepancy is of the order of 3 standard deviations. In Fig. 2, we draw a graph of the function $r_\Sigma = f(r_\Xi)$ [Eq. (11)], and indicate the range of the experimental data (the box area) and a theoretical expectation (cross). It seems a curious coincidence that both experimental and theoretical values satisfy an approximate equation,

$$r_\Sigma \approx -r_\Xi . \tag{14}$$

In fact, Eqs. (9), (10), and (14) lead to the value of the ξ to be 0.71 which is sufficiently close to our adopted values (A) $\xi = 0.738$ and (B) $\xi = 0.726$. The ongoing experiment to measure hyperon magnetic moments is expected to clarify this situation in the near future.

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[11]See Ref. 6, the Erratum.

[12]Lipkin considered magnetic-moment ratios which involve particles in different isomultiplets as a test of broken SU(6) symmetry. We may notice that his ratios are more likely subject to further corrections of type II than those of Eqs. (9)—(11), as long as the effect of the orbital angular momentum (Ref. 13) is neglected.

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