Two-photon decay of the pseudoscalar glueball

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We derive two sum rules for the amplitudes for two-photon decays of the pseudoscalar mesons including a glueball. The two-photon decay width of the pseudoscalar glueball can be estimated with inputs of $\Gamma(\pi\to 2\gamma)$, $\Gamma(\eta\to 2\gamma)$, and $\Gamma(\eta'\to 2\gamma)$, assuming Ward identities and two other conditions to be useful. If the $\iota(1440)$ seen in the radiative decay of J/ψ is the pseudoscalar glueball, then we obtain $\Gamma(\iota\to 2\gamma)=10\pm 26$ keV, where the large error comes mainly from the experimental ambiguity of $\Gamma(\eta'\to 2\gamma)$.

The discovery of glueballs seems to be one of the interesting problems with respect to hadron physics in the framework of QCD. But detailed information on glueballs has not yet been obtained from QCD alone, in contrast with the ordinary hadrons which are composed of quarks. Therefore, many phenomenological studies are necessary to clarify the dynamics of glueballs.[1] Then, in this note, we will study the two-photon decay of a pseudoscalar glueball based on the current-algebra approach.[2] Since quarks and gluons are strongly coupled, there must exist some $q\bar{q}$ component in the glueballs. Thus, the glueballs will be identified through the interactions with ordinary hadrons and photons. Especially, a pseudoscalar glueball (G) mixes with η', so that it is important to investigate the two-photon decay of the pseudoscalar glueball.

We consider the equations expressing the presence of the electromagnetic anomaly for the axial-vector currents[3]

$$\partial_\mu A_\mu^a = D^a + S_a \mathfrak{A}_{\rm em}, \quad a=3,8,0\,,$$
$$D^a = i\bar{q}\gamma_5\left\{\frac{\lambda^a}{2},m\right\}q+\delta^{a0}\left[\frac{2}{3}\right]^{1/2}\frac{3\alpha_s}{4\pi}G^b_{\mu\nu}\tilde{G}^b_{\mu\nu}\,, \tag{1}$$

where $\mathfrak{A}_{\rm em}=(\alpha_{\rm em}/4\pi)F^{\rm em}_{\mu\nu}F^{\rm em}_{\mu\nu}$, $(S_3, S_8, S_0)=(\sqrt{3}/18)(\sqrt{3},1,2\sqrt{2})$, and $G^b_{\mu\nu}$ is the gluon field tensor. Here A_μ^a is defined as the usual axial-vector current with three quark flavors,

$$A_\mu^a \equiv \bar{q}\gamma_\mu\gamma_5\left[\frac{\lambda^a}{2}\right]q, \quad a=3,8,0\,. \tag{2}$$

We assume that D^a can be expressed in terms of the interpolating fields of ϕ_π, ϕ_η, $\phi_{\eta'}$ and ϕ_G (the glueball field) and decay constants,[3] i.e.,

$$D^3 = m_\pi{}^2F_\pi\phi_\pi\,,$$
$$D^8 = m_\eta{}^2F_\eta\phi_\eta + m_{\eta'}{}^2F_{\eta'8}\phi_{\eta'}\,, \tag{3}$$
$$D^0 = m_\eta{}^2F_{\eta0}\phi_\eta + m_{\eta'}{}^2F_{\eta'}\phi_{\eta'} + m_G{}^2F_G\phi_G\,,$$

in which η-G and π-G mixings are neglected. So we obtain

$$\partial_\mu A_\mu^3 = m_\pi{}^2F_\pi\phi_\pi + S_3\mathfrak{A}_{\rm em}\,,$$
$$\partial_\mu A_\mu^8 = m_\eta{}^2F_\eta\phi_\eta + m_{\eta'}{}^2F_{\eta'8}\phi_{\eta'} + S_8\mathfrak{A}_{\rm em}\,, \tag{4}$$
$$\partial_\mu A_\mu^0 = m_\eta{}^2F_{\eta0}\phi_\eta + m_{\eta'}{}^2F_{\eta'}\phi_{\eta'} + m_G{}^2F_G\phi_G + S_0\mathfrak{A}_{\rm em}\,.$$

Here, only the presence of the $m_G{}^2F_G\phi_G$ term differs from the previous analyses.[2] By taking the matrix elements of the above equations between the vacuum and the 2γ state and proceeding in the standard way,[2] we have

$$m_\pi{}^2F_\pi\langle 2\gamma|\phi_\pi|0\rangle + S_3\langle 2\gamma|\mathfrak{A}_{\rm em}|0\rangle = 0\,,$$
$$m_\eta{}^2F_\eta\langle 2\gamma|\phi_\eta|0\rangle + m_{\eta'}{}^2F_{\eta'8}\langle 2\gamma|\phi_{\eta'}|0\rangle + S_8\langle 2\gamma|\mathfrak{A}_{\rm em}|0\rangle = 0\,, \tag{5}$$
$$m_\eta{}^2F_{\eta0}\langle 2\gamma|\phi_\eta|0\rangle + m_{\eta'}{}^2F_{\eta'}\langle 2\gamma|\phi_{\eta'}|0\rangle + m_G{}^2F_G\langle 2\gamma|\phi_G|0\rangle + S_0\langle 2\gamma|\mathfrak{A}_{\rm em}|0\rangle = 0\,.$$

These equations lead to the following sum rules among the amplitudes including $G\to 2\gamma$:

$$F_\eta\langle 2\gamma\,|\,\eta\rangle=-F_{\eta'8}\langle 2\gamma\,|\,\eta'\rangle+\frac{1}{\sqrt{3}}F_\pi\langle 2\gamma\,|\,\pi^0\rangle\ , \tag{6}$$

$$F_G\langle 2\gamma\,|\,G\rangle=-F_{\eta'}\langle 2\gamma\,|\,\eta'\rangle+\frac{2\sqrt{2}}{\sqrt{3}}F_\pi\langle 2\gamma\,|\,\pi^0\rangle-F_{\eta 0}\langle 2\gamma\,|\,\eta\rangle\ . \tag{7}$$

Although it is difficult to estimate the amplitudes of $G\to 2\gamma$ from these sum rules under the present experimental and theoretical situation, it is important to study how to evaluate $\langle 2\gamma\,|\,G\rangle$. Thus, assuming the $\iota(1440)$ seen in the radiative decay of J/ψ to be the pseudoscalar glueball,[4] we will try to investigate how far the $\langle 2\gamma\,|\,G\rangle$ can be determined with the presently available information.

In the sum rules (6) and (7) the decay constants $F_{\eta'8}$, $F_{\eta'}$, F_η, and $F_{\eta 0}$ are unknown parameters except for $F_G\langle 2\gamma\,|\,G\rangle$. The amplitudes $\langle 2\gamma\,|\,\pi\rangle$, $\langle 2\gamma\,|\,\eta\rangle$, and $\langle 2\gamma\,|\,\eta'\rangle$ are fixed from the experimental values as follows[5]:

$$\langle 2\gamma\,|\,\pi\rangle:\langle 2\gamma\,|\,\eta\rangle:\langle 2\gamma\,|\,\eta'\rangle=1:(0.78\pm 0.06):(1.37\pm 0.26)\ . \tag{8}$$

In order to estimate $F_G\langle 2\gamma\,|\,G\rangle$, four unknown parameters mentioned above must be determined. For this purpose, we perform an analysis of the broken SU(3)×SU(3) chiral Ward identities (WI's).[3,6] Then we show that these parameters are determined uniquely. The WI's relevant for us are expressed as[6]

$$m_\eta{}^2F_\eta{}^2+m_{\eta'}{}^2F_{\eta'8}{}^2=\tfrac{1}{3}(4m_K{}^2F_K{}^2-m_\pi{}^2F_\pi{}^2)\ , \tag{9}$$

$$m_\eta{}^2F_\eta(F_\eta+\sqrt{2}\widetilde{F}_{\eta 0})+m_{\eta'}{}^2F_{\eta'8}(F_{\eta'8}+\sqrt{2}\widetilde{F}_{\eta'}=m_\pi{}^2F_\pi{}^2\ , \tag{10}$$

$$m_\eta{}^2(F_\eta+\sqrt{2}\widetilde{F}_{\eta 0})^2+m_{\eta'}{}^2(F_{\eta'8}+\sqrt{2}\widetilde{F}_{\eta'})^2-2\bar{S}=3m_\pi{}^2F_\pi{}^2\ , \tag{11}$$

where $\bar{S}$ is the surface term contributed by η, η', and G,

$$\bar{S}=m_\eta{}^2(F_{\eta 0}-\widetilde{F}_{\eta 0})^2+m_{\eta'}{}^2(F_{\eta'}-\widetilde{F}_{\eta'})^2+m_G{}^2F_G{}^2\ , \tag{12}$$

and $F_\pi=F_K$ are taken in the following. The decay constants $\widetilde{F}_\eta$ and $\widetilde{F}_{\eta'}$ are defined by[6]

$$\left\langle 0\,\left|\,\partial_\mu A^8_\mu-\left[\frac{2}{3}\right]^{1/2}\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}\,\right|\,\eta\right\rangle\equiv m_\eta{}^2\widetilde{F}_{\eta 0}\ ,$$

$$\left\langle 0\,\left|\,\partial_\mu A^0_\mu-\left[\frac{2}{3}\right]^{1/2}\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}\,\right|\,\eta\right\rangle\equiv m_\eta{}^2\widetilde{F}_{\eta'}\ . \tag{13}$$

At this point, we cannot yet obtain a solvable set of equations. Therefore, (6) should be added to this set together with inputs of $\langle 2\gamma\,|\,\pi\rangle$, $\langle 2\gamma\,|\,\eta\rangle$, and $\langle 2\gamma\,|\,\eta'\rangle$. Moreover, two other conditions are necessary to arrive at the solved set. One of these is the ratio of $\Gamma(\psi\to\eta\gamma)$ to $\Gamma(\psi\to\eta'\gamma)$ written in the form

$$\frac{\Gamma(\psi\to\eta\gamma)}{\Gamma(\psi\to\eta'\gamma)}=\frac{K_\eta{}^3}{K_{\eta'}{}^3}\frac{m_\eta{}^4}{m_{\eta'}{}^4}\frac{(F_{\eta 0}-\widetilde{F}_{\eta 0})^2}{(F_{\eta'}-\widetilde{F}_{\eta'})^2}\ , \tag{14}$$

which was derived by Novikov *et al.*[7] The other is given by Milton *et al.*[6]:

$$\frac{A(\eta\to\pi^+\pi^-\pi^0)}{A(\eta'\to\eta\pi^+\pi^-)}=\left[\frac{2}{3}\right]^{1/2}\frac{m_d-m_u}{m_d+m_u}\frac{F_{\eta'}-\widetilde{F}_{\eta'}}{F_\pi}\ , \tag{15}$$

where the π and η' are at zero momentum. We use these equations as known input, although the left-hand side of (15) was predicted in the analysis of Milton *et al.*[6] The experimental values of (14) and (15) are, respectively, 0.26 ± 0.03 in MARK II,[4] and 0.11 ± 0.02.[5] The latter value is corrected at zero momentum of π. The ratio of quark masses is given by Gross *et al.*[8] from the electromagnetic mass differences of pions and kaons, i.e.,

$$\frac{m_d-m_u}{m_d+m_u}=0.29\pm 0.01\ . \tag{16}$$

Solving (9)−(12), and (6), (14), and (15), six unknown decay constants F_η, $F_{\eta 0}$, $\widetilde{F}_{\eta 0}$, $F_{\eta'}$, $F_{\eta'8}$, and $\widetilde{F}_{\eta'}$ are completely fixed. That is, F_η and $F_{\eta'8}$ are derived by solving (6) and (9) to be

$$F_\eta=(1.00\pm 0.15)F_\pi\ ,$$
$$F_{\eta'8}=(-0.15\pm 0.06)F_\pi\ . \tag{17}$$

Here the errors reflect the experimental uncertainty of two-photon decay amplitudes of π, η, and η'. In order to evaluate $\bar{S}$ in (12), we must in addition give a numerical value of $m_G{}^2F_G{}^2$. Then, we assume the $\iota(1440)$ seen in radiative decay of J/ψ to be the pseudoscalar glueball,[4] and so $|F_G|=0.3F_\pi$

is taken as a reasonable value[9] from the analysis of the ratio $\Gamma(\psi\to\iota\gamma)/\Gamma(\psi\to\eta'\gamma)$. Finally, by solving (10) and (11) subject to the conditions (12), (14), and (15), we obtain

$$\begin{aligned} F_{\eta'}&=(1.22\pm0.11)F_\pi\ ,\\ F_{\eta 0}&=(0.34\pm0.23)F_\pi\ ,\\ \widetilde{F}_{\eta'}&=(0.75\pm0.14)F_\pi\ ,\\ \widetilde{F}_{\eta 0}&=(-0.31\pm0.30)F_\pi\ . \end{aligned} \tag{18}$$

Putting these solutions into the left-hand side of (7), $F_G\langle 2\gamma\,|\,G\rangle$ is estimated as follows:

$$F_G\langle 2\gamma\,|\,G\rangle=(-0.31\pm0.39)F_\pi\ . \tag{19}$$

From the value of $|F_G|=0.3F_\pi$ given above, the decay width of $G\to2\gamma$, as the final results of our analysis, becomes

$$\Gamma(G\to2\gamma)=10\pm26\ \text{keV}\ . \tag{20}$$

The large error in (20) comes mainly from the experimental ambiguity of $\Gamma(\eta'\to2\gamma)$. This result asserts that our sum rules given by (6) and (7) are very delicate as to deriving $\Gamma(G\to2\gamma)$ under the presently experimental situation.

We have derived two sum rules for the amplitudes of two-photon decays of the pseudoscalar mesons including the glueball. Then, the magnitude of $\Gamma(G\to2\gamma)$ can be predicted with inputs of $\Gamma(\pi\to2\gamma)$, $\Gamma(\eta\to2\gamma)$, and $\Gamma(\eta'\to2\gamma)$, assuming WI's and two other conditions (14) and (15). The latter two conditions lead to the absolute value of anomaly couplings of η and η'. In that case, the $\iota(1440)$ seen in the radiative decay of J/ψ is assumed to be the pseudoscalar glueball. Unfortunately, our result (20) involves a large error. If more definite relations are available to derive the anomaly coupling of η and η', and if the precise experimental value of $\Gamma(\eta'\to2\gamma)$ is given, we shall obtain a more definitive prediction for $\Gamma(G\to2\gamma)$.

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