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Neutrino decay and spontaneous violation of lepton number

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The orders of magnitude of decay rates for relatively light neutrinos are studied in the framework of the $SU(2)\times U(1)$ gauge group. The assumption is made that a hierarchy parameter ϵ ($\approx$ (muon mass)$\div$[some new mass scale (possibly much smaller than the grand unification scale)]) plays a meaningful role in the full theory. For orientation it is first noted that the traditional $\nu\gamma$ decay channel as well as the 3ν decay channel give neutrino lifetimes which for "typical" parameters are substantially longer than the age of the universe. Then we examine in detail some recent proposals which are claimed to result in greatly speeded-up decays into ν+Majoron, where the Majoron is a true Goldstone boson associated with spontaneous breakdown of lepton number. In a theory in which the usual Higgs doublet is augmented by a complex singlet (1-2 model) it is noted that the decay width into ν+Majoron actually *vanishes* to order ϵ^5. In a theory where the doublet is augmented by a complex triplet (2-3 model) this decay is shown to vanish exactly, neglecting radiative corrections. A more general Majoron theory (1-2-3 model) is constructed and shown to also yield a vanishing tree-level decay rate for ν+Majoron decay to order ϵ^5. Finally, the tree amplitudes in the 1-2 and 1-2-3 models are shown to give decay widths for ν+Majoron of order ϵ^9 which correspond to lifetimes *much greater* than the age of the universe.

I. INTRODUCTION

If neutrinos have mass[1] the heavier ones should decay into lighter ones. These decays are quite interesting from the standpoint of cosmology as well as elementary-particle physics. Let us discuss the general situation in the framework of the $SU(2)\times U(1)$ gauge theory of weak-electromagnetic interactions. We shall assume that the theory contains n two-component neutrino spinors belonging to SU(2) doublets and n two-component neutrino spinors which are singlets under $SU(2)\times U(1)$. The mass term in the Lagrangian has the following general form[2] in terms of the column vector of two-component *bare* spinors ρ:

$$\mathscr{L}_{\text{mass}}=-\tfrac{1}{2}\rho^T\sigma_2 M\rho+\text{H.c.}\ , \tag{1.1}$$

$$M=\begin{bmatrix} M_1 & D\\ D^T & M_2\end{bmatrix}. \tag{1.2}$$

Here σ_2 is the Pauli matrix. M is in general a complex symmetric matrix wherein the first n indices refer to SU(2)-doublet fields. The sub-blocks D and D^T are the ordinary Dirac terms while M_1 is a Majorana piece which arises from an invariant term when the neutral member of a complex Higgs triplet acquires a vacuum expectation value. M_2 is another Majorana piece which is already $SU(2)\times U(1)$ invariant without the need for any extra Higgs fields. Physical neutrino fields ν are defined by

$$\rho=U\nu\ , \tag{1.3}$$

where U is a unitary matrix satisfying

$$U^TMU=\text{real}\ ,\ \text{diagonal}\ . \tag{1.4}$$

The description above is quite general and can accommodate many models of interest. In a large class of models the theory is arranged so that $M_1=0$, giving

$$M=\begin{bmatrix} 0 & D\\ D^T & M_2\end{bmatrix}. \tag{1.5}$$

First consider models of type (1.5). The order of magnitude of the entries of D is about 100 MeV (muon mass) while the order of magnitude of M_2 represents new physics. One could work only in the group $SU(2)\times U(1)$ and choose this magnitude arbitrarily, but it seems more reasonable (or in any event quite interesting) to imagine a model in which $SU(2)\times U(1)$ is embedded in a larger group and where the magnitude of M_2 is associated with an appropriate larger scale. The latter scale need not, of course, be the scale of grand unification. In fact, our discussion is more interesting when the scale M_2 is considerably lower than 10^{14} GeV. Let us define the hierarchy parameter

$$\epsilon=O\left[\frac{D}{M_2}\right], \tag{1.6}$$

in an evident symbolic notation. We shall assume that ϵ is small. Then, when (1.5) is brought to diagonal form there will be n "superheavy" Majorana neutrinos whose mass scale is M_2 and n "light" Majorana neutrinos whose mass scale is of order

$$m\,(\text{light neutrinos})=O\left[\frac{D^2}{M_2}\right]$$
$$=\epsilon^2 O(M_2)\ . \tag{1.7}$$

We will concentrate our discussion on these n light neutrinos. For orientation, if the light-neutrino mass scale is 100 eV we find $M_2 \sim 10^8$ MeV and $\epsilon=10^{-6}$. It is algebraically trivial but very useful to notice that (1.7) enables us to rewrite (1.6) as

$$\epsilon=O\left[\frac{\text{light-neutrino mass}}{D}\right]. \tag{1.8}$$

Since the order of magnitude of D is known the entire discussion can be given in terms of the scale ratio ϵ.

Now consider the decays of ν_H, one of the heavier light neutrinos into ν_L, one of the lighter of the light neutrinos. The standard mode for this decay is

$$\nu_H \to \nu_L + \gamma\ , \tag{1.9}$$

which arises due to radiative corrections involving charged intermediate boson virtual emission and reabsorption. The order of magnitude of the decay width for (1.9) is estimated[3] as

$$\Gamma(\nu_H \to \nu_L+\gamma) \sim \alpha(\text{mixing factor})\left[\frac{\text{charged}-\text{lepton mass}}{W-\text{meson mass}}\right]^4 \left|\frac{m\,(\nu_H)}{m\,(\mu)}\right|^5 \Gamma(\mu\to e\bar{\nu}_e\nu_\mu)\ . \tag{1.10}$$

In (1.10), $\alpha \simeq \frac{1}{137}$, $m\,(\nu_H)$ is the ν_H mass, $m\,(\mu)$ is the muon mass, while the mixing factor involves elements of the leptonic Kobayashi-Maskawa-Cabibbo matrix. With "reasonable" numbers and using (1.8) one has roughly

$$\Gamma(\nu_H \to \nu_L+\gamma) \sim 10^{-12}\epsilon^5\Gamma(\mu \to e+\bar{\nu}_e+\nu_\mu)$$
$$\sim 10^{-28}\epsilon^5\text{MeV}\ . \tag{1.11}$$

It has been noted[4] that in a model of the present type the leptonic Glashow-Iliopoulos-Maiani mechanism does not hold due to the fact that only n independent linear combinations of the $2n$ neutrino fields interact with the W and Z intermediate vector bosons. This leads to Z-boson-mediated flavor-changing neutral-current decays of which the dominant type is

$$\nu_H \to \nu_L + \nu_L' + \nu_L'\ , \tag{1.12}$$

where ν_L' is another (or even ν_L itself) of the light neutrinos. The width for this mode is estimated[5] as

$$\Gamma(\nu_H \to 3\nu_L) \sim \epsilon^4 \left|\frac{m\,(\nu_H)}{m\,(\mu)}\right|^5 \Gamma(\mu \to e+\bar{\nu}_e+\mu)$$
$$\sim 10^{-16}\epsilon^9\text{MeV}\ . \tag{1.13}$$

Comparing (1.13) with (1.11) shows that the $\nu_L\gamma$ mode will dominate the $3\nu_L$ mode for sufficiently small hierarchy ratio ϵ (i.e., sufficiently high new energy scale). However for roughly $\epsilon > 10^{-3}$, the $3\nu_L$ mode will dominate.[6] Of course both of these decays are extremely slow for small ϵ. As an example, take $\epsilon=10^{-6}$ in (1.11) to predict a lifetime $\Gamma^{-1} \sim 10^{58}$ MeV^{-1}. This is much longer than the age of the universe which is usually estimated as 10^{39} MeV^{-1}.

Thus, it is interesting to ask if there exist possible extensions of the standard weak-interaction theory which can speed up neutrino decay. Recently Chikashige, Mohapatra, and Peccei[7,8] have proposed adding to the theory a complex SU(2)$\times$U(1) Higgs singlet Φ which carries lepton number minus 2. We will refer to this as the 1-2 model. The Lagrangian is arranged to be invariant with respect to lepton-number transformations. When Φ acquires a vacuum expectation value the lepton number is broken spontaneously, the submatrix M_2 in (1.5) is generated, and a massless Goldstone boson, the "Majoron," appears. It is expected that the interactions of the Majoron with ordinary matter are sufficiently weak that it ought to have escaped detection. The authors claim however that the decay ν_H goes to ν_L plus Majoron is rather rapid. Their estimate[8] is

$$\Gamma(\nu_H \to \nu_L + J) \sim 10^{-2} h^2(\text{mixing factor}) \times \epsilon^4 m(\nu_H) , \tag{1.14}$$

where we have denoted the Majoron by J. h is a dimensionless coupling constant whose value is not very severely restricted. Taking h^2(mixing factor) $\approx 10^{-6}$ and using (1.8) would give

$$\Gamma(\nu_H \to \nu_L + J) \sim 10^{-6} \epsilon^5 \text{MeV} . \tag{1.15}$$

This has the same ϵ dependence as the $\nu_L \gamma$ mode (1.11) but may be very much stronger, implying lifetimes less than the age of the universe. However a more detailed analysis to be presented here shows that $\Gamma(\nu_H \to \nu_L + J)$ actually *vanishes* to order ϵ^5 in the 1-2 model. This means that the lifetime for a neutrino to decay into a lighter one plus Majoron will *not* be less than the age of the universe in the present scheme (for typical parameters).

A natural question to ask then is whether one can find a different Majoron scheme in which a formula like (1.14) or (1.15) holds true. One Majoron scheme (2-3 model) involving a complex Higgs triplet was proposed by Gelmini and Roncadelli.[9] Their model is essentially the same as a usual Higgs-triplet model[10] with the restriction that the lepton-number-violating term in the Higgs potential is deleted. In this model, however, it is easy to see that the decay $\nu_H \to \nu_L + J$ is completely forbidden at tree level. We shall also construct a generalized Majoron scheme by including all three Higgs multiplets—singlet, doublet, and triplet—in a lepton-number-conserving Lagrangian. This theory (1-2-3 model) is characterized by (1.2) rather than (1.5) and also yields the result that the decay $\nu_H \to \nu_L +$ Majoron vanishes to order ϵ^5.

In Sec. II the mass terms and Majoron interaction terms of the general 1-2-3 model are discussed. It is necessary to know which linear combination of the Higgs fields corresponds to the Majoron. This is worked out in the Appendix.

Section III contains the specialization of the general discussion to the 1-2 model. The transformation matrix in (1.3) between bare and physical neutrino fields is calculated to an accuracy of order ϵ^2. This is used to demonstrate that the amplitude for $\nu_H \to \nu_L + J$ vanishes to order ϵ^2, which implies the stated result. The transformation matrix is also used to demonstrate the validity of (1.13), which is independent of the Majoron scheme. In Sec. IV we demonstrate the vanishing of the amplitude for $\nu_H \to \nu_L + J$ in the 2-3 model. The vanishing of the amplitude for $\nu_H \to \nu_L + J$ to order ϵ^2 is demonstrated in Sec. V for the general 1-2-3 model in the nontrivial case when M_1 is assumed to be of order $\epsilon^2 M_2$. It is also shown that the coupling of the Majoron to ordinary (charged) fermions is drastically suppressed in this model. In a recent paper Georgi, Glashow, and Nussinov[11] examine the 2-3 model in great detail. They claim that the vanishing of the Majoron coupling to two different fermions in tree order is guaranteed by the fact that it is a Goldstone boson. While this holds trivially in the 2-3 model it *does not* hold in the more complicated 1-2 and 1-2-3 models. We shall demonstrate in the discussion of Sec. VI how the usual argument fails for these two cases and why a more detailed discussion of the type given here is required. We shall also explicitly calculate the off-diagonal Majoron couplings to order ϵ^4.

II. THE NEUTRINO YUKAWA TERMS IN THE 1-2-3 MODEL

Both the neutrino mass terms and neutrino-Majoron interaction terms are assumed to arise from the most general $SU(2) \times U(1) \times$ lepton-number invariant Yukawa Lagrangian in the usual way. We shall first write these terms for the general case when the Higgs singlets, doublet, and triplet are all present and then we shall specialize to the 1-2 and 2-3 cases. Our notation and lepton-number (l) assignments for the Higgs multiplets are as follows[12]:

$$\begin{aligned} &\Phi \quad (l=-2) , \\ &\phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} \quad (l=0) , \\ &h = \begin{bmatrix} h^+/\sqrt{2} & h^{++} \\ h^0 & -h^+/\sqrt{2} \end{bmatrix} \quad (l=-2) . \end{aligned} \tag{2.1}$$

We assume that the theory breaks down in such a way that the vacuum values of the neutral fields are real (no spontaneous CP violation):

$$\begin{aligned} \langle \Phi \rangle &= \langle \overline{\Phi} \rangle \equiv x \ , \\ \langle \phi^0 \rangle &= \langle \overline{\phi}^0 \rangle \equiv \lambda \approx 2^{-1/4} G_F{}^{-1/2} \ , \\ \langle h^0 \rangle &= \langle \overline{h}^0 \rangle \equiv y \ . \end{aligned} \tag{2.2}$$

Here G_F is the Fermi constant and we have assumed that y is so small that it does not contribute much to $G_F{}^{-1/2}$. It is convenient to separate the neutral fields into their real and imaginary parts:

$$\begin{aligned} \Phi &= \Phi_r + i\Phi_i \ , \\ \phi^0 &= \phi_r^0 + i\phi_i^0 \ , \\ h^0 &= h_r^0 + ih_i^0 \ . \end{aligned} \tag{2.3}$$

In the Appendix we show that the zero-mass particle corresponding to the Majoron is given by the linear combination:

$$\begin{aligned} J = {} & [4\lambda^2 y^4 + x^2(\lambda^2+4y^2)^2 + y^2\lambda^4]^{-1/2} \\ & \times[-2\lambda y^2 \phi_i^0 + x(\lambda^2+4y^2)\Phi_i + y\lambda^2 h_i^0] \ . \end{aligned} \tag{2.4}$$

This holds in the 1-2-3 model. We can specialize to the 1-2 model[7] by setting the triplet vacuum value y to zero:

$$J = \Phi_i \quad (\text{1-2 model}) \ . \tag{2.5}$$

We similarly arrive at the 2-3 model[9] by setting x to zero:

$$J = (4y^2+\lambda^2)^{-1/2}(-2y\phi_i^0 + \lambda h_i^0) \quad (\text{2-3model}) \ . \tag{2.6}$$

Now let us write the part of the invariant lepton-Yukawa interaction involving the neutral Higgs fields. Absorbing the arbitrary coupling constants into equivalently arbitrary mass parameters in a manner consistent with Eqs. (1.1) and (1.2), we have

$$\mathscr{L} = -\tfrac{1}{2}\rho^T \sigma_2 \begin{bmatrix} M_1 \dfrac{h^0}{\langle h^0 \rangle} & D\dfrac{\phi^0}{\langle \phi^0 \rangle} \\ D^T \dfrac{\phi^0}{\langle \phi^0 \rangle} & M_2 \dfrac{\Phi^*}{\langle \Phi^* \rangle} \end{bmatrix} \rho + \text{H.c} \ . \tag{2.7}$$

A more detailed discussion of this notation and related topics is given in Ref. 2. Note that the quantities M_1 and M_2 may be arbitrary complex symmetric $n \times n$ matrices, while D is just an arbitrary $n \times n$ matrix. The fields ρ are the bare ones, related to the physical ones [see (1.3)] by

$$\rho = U\nu \equiv \begin{bmatrix} U_a & U_b \\ U_c & U_d \end{bmatrix} \begin{bmatrix} \widetilde{\nu} \\ N \end{bmatrix} \ . \tag{2.8}$$

The $\widetilde{\nu}$ are the n light neutrinos we are interested in here while the N are the n superheavy ones. The unitary matrix U is determined by requiring

$$\begin{aligned} U^T M U &= \text{real positive, diagonal} \\ &\equiv \begin{bmatrix} m & 0 \\ 0 & m_s \end{bmatrix} \ . \end{aligned} \tag{2.9}$$

We previously showed[2] that such a U can always be found for a general M.

III. SUPPRESSION OF $\nu_H \to \nu_L J$ IN THE 1-2 MODEL

In this case the triplet h is absent and the upper left submatrix in (2.7) should be deleted. It is possible to diagonalize the resulting mass matrix M of the type (1.5), neglecting terms of order ϵ^3 [where $\epsilon = O(D/M_2)$], by making the ansatz

$$\begin{aligned} & U = (\exp iH)V \ , \\ & H = \begin{bmatrix} 0 & S \\ S^+ & 0 \end{bmatrix} , \quad V = \begin{bmatrix} V_1 & 0 \\ 0 & V_2 \end{bmatrix} \ . \end{aligned} \tag{3.1}$$

Here S is taken to be $O(\epsilon)$, while V_1 and V_2 are unitary and $O(1)$. Substituting (3.1) as well as (1.5) into (2.9) then determines S (from the requirement that the off-diagonal sub-blocks vanish) to be

$$S = -iD^*(M_2^*)^{-1} \ . \tag{3.2}$$

V_1 and V_2 are the diagonalizing matrices which satisfy

$$-V_1^T D M_2{}^{-1} D^T V_1 = m = \text{real positive, diagonal} \ , \tag{3.3}$$

$$V_2^T[M_2 + \tfrac{1}{2}(M_2^*)^{-1}D^+D + \tfrac{1}{2}D^T D^*(M_2^*)^{-1}]V_2 = m_s = \text{real positive, diagonal} \ . \tag{3.4}$$

The transformation matrix U is then[13]

$$U=\begin{bmatrix} U_a & U_b \\ U_c & U_d \end{bmatrix}=\begin{bmatrix} [1-\frac{1}{2}D^*(M_2^*)^{-1}M_2{}^{-1}D^T]V_1 & D^*(M_2^*)^{-1}V_2 \\ -M_2{}^{-1}D^TV_1 & [1-\frac{1}{2}M_2{}^{-1}D^TD^*(M_2^*)^{-1}]V_2 \end{bmatrix}+O(\epsilon^3)\ . \tag{3.5}$$

We are interested in the Yukawa coupling to the Majoron J which is given by (2.5) in the 1-2 model. Using this as well as (2.8) in (2.7) gives the Majoron Yukawa interaction

$$\mathscr{L}=\frac{iJ}{2x}(\tilde{\nu}^TN^T)\sigma_2\begin{bmatrix} U_c^TM_2U_c & U_c^TM_2U_d \\ U_d^TM_2U_c & U_d^TM_2U_d \end{bmatrix}\begin{bmatrix}\tilde{\nu} \\ N\end{bmatrix}+\text{H.c.}\ , \tag{3.6}$$

where x is the Φ vacuum value [of $O(M_2)$]. For the decay $\nu_H\to\nu_L+J$ to take place one requires nondiagonal elements in the upper left submatrix in (3.6). However with U_c from (3.5) and (3.3) we obtain

$$U_c^TM_2U_c=-m=\text{diagonal}\ . \tag{3.7}$$

Thus there are no transitions from one generation of light neutrinos to another correct to $O(m/x)=\epsilon^2$ in amplitude. Note that the result in (3.7) does *not* follow to all orders in ϵ just by diagonalizing the mass matrix. The latter would require

$$U_a^TDU_c+U_c^TD^TU_a+U_c^TM_2U_c=\text{diagonal}\ , \tag{3.8}$$

which does not in general coincide with (3.7).

The estimate for $\Gamma(\nu_H\to\nu_L+J)$ in (1.14) and (1.15) was based on an amplitude of $O(\epsilon^2)$. We shall see in Sec. VI that Γ will start at $O(\epsilon^9)$. For a typical ν_H of mass about 100 eV we had $\epsilon\simeq10^{-6}$. The extra suppression of a factor 10^{-24} would then give $\Gamma^{-1}\approx10^{60}\ \text{MeV}^{-1}$, which is substantially longer than the age of the universe. Thus the Majoron decay channel in the 1-2 model seems much less spectacular than before from the standpoint of cosmology.

The matrix U in (3.5) is also useful for discussing the $\nu_H\to3\nu_L$ decay which takes place by Z-boson exchange. We showed[14] that the neutrino neutral-current$-Z$ interaction term is

$$\mathscr{L}=\frac{ig'}{2\sin\theta_W}Z_\mu\bar{\nu}\gamma_\mu\left[\frac{1+\gamma_5}{2}\right]P\nu\ , \tag{3.9}$$

where we have switched to the standard four-component notation for the neutrino fields, and $g'=-e/\cos\theta_W$ is the weak-hypercharge U(1) coupling constant. The matrix P is given by

$$P=\begin{bmatrix}U_a^\dagger \\ U_b^\dagger\end{bmatrix}(U_a\ \ U_b)=\begin{bmatrix} 1-V_1^\dagger D^*(M_2^*)^{-1}M_2{}^{-1}D^TV_1 & V_1^\dagger D^*M_2{}^{*-1}V_2 \\ V_2^\dagger M_2{}^{-1}D^TV_1 & V_2^\dagger M_2{}^{-1}D^TD^*(M_2^*)^{-1}V_2 \end{bmatrix}+O(\epsilon^3)\ . \tag{3.10}$$

The last term in the upper left sub-block of (3.10) differs from (3.3) so we now expect $\nu_H\to\nu_L+\text{virtual } Z$ with amplitude of $O(\epsilon^2)$. Since the first term in the upper left sub-block gives virtual $Z\to2\nu_L'$ with $O(1)$, we find the amplitude for $\nu_H\to\nu_L+2\nu_L'$ to be of $O(\epsilon^2)$. This explains the ϵ^4 factor in (1.13). Note that the validity of (3.9) and (3.10) is completely independent of the Majoron scheme.

IV. ABSENCE OF $\nu_H\to\nu_LJ$ IN THE 2-3 MODEL

In this case the two-component neutrino singlet fields are simply absent from the theory. Equation (2.7) for the Yukawa term becomes

$$\mathscr{L}=-\frac{1}{2}\frac{h^0}{\langle h^0\rangle}\rho^T\sigma_2M_1\rho+\text{H.c.}\ , \tag{4.1}$$

where M_1 is an arbitrary symmetric matrix. From (2.6) we find that the field h^0 contains a piece of the Majoron J:

$$h^0=h_r^0+ih_i^0=\frac{i\lambda}{(4y^2+\lambda^2)^{1/2}}J+\cdots\ . \tag{4.2}$$

Substituting (4.2) into (4.1) gives the interaction of the Majoron with the "bare" neutrinos:

$$\mathscr{L}=-\frac{i}{2}\frac{\lambda}{y}(4y^2+\lambda^2)^{-1/2}J\rho^T\sigma_2M_1\rho+\text{H.c.} \tag{4.3}$$

Finally we must rewrite [see (1.3)] ρ in terms of the physical fields ν as $\rho = U\nu$. U, which now is an $n \times n$ (rather than $2n \times 2n$) unitary matrix, is to be chosen to make $U^T M_1 U =$ real positive, diagonal. But this transformation also diagonalizes the Majoron Yukawa interaction in (4.3). Thus the amplitude for $\nu_H \to \nu_L + J$ vanishes in this model, neglecting radiative corrections.

V. $\nu_H \to \nu_L J$ IN THE 1-2-3 MODEL

Now we must deal with the full mass matrix in (1.2). Clearly if M_1 is of order less than $\epsilon^2 O(M_2)$ the results will be the same as for the 1-2 model and the ν_H decay will be similarly suppressed. Thus let us make the assumption

$$O(M_1) = \epsilon^2 O(M_2) \ . \tag{5.1}$$

In this case it is easy to see that the transformation matrix U in (3.5) still diagonalizes the full M with neglect of terms of (ϵ^3), provided that (3.3) is replaced by

$$V_1^T(-DM_2{}^{-1}D^T + M_1)V_1 = m = \text{real positive, diagonal} \ . \tag{5.2}$$

In other words, V_1 is determined from a different formula.

The Majoron Yukawa interaction is to be extracted from the following specialization of (2.7) to the light-neutrino sector,

$$\mathcal{L} = -\tfrac{1}{2}\tilde{\nu}^T \sigma_2 \left[\frac{h^0}{\langle h^0 \rangle} U_a^T M_1 U_a + \frac{\phi^0}{\langle \phi^0 \rangle}(U_a^T D U_c + U_c^T D^T U_a) + \frac{\Phi^*}{\langle \Phi \rangle} U_c^T M_2 U_c \right] \tilde{\nu} + \text{H.c.} \ , \tag{5.3}$$

where (2.8) was also used. We write the normalization factor in (2.4) as

$$A = [4\lambda^2 y^4 + x^2(\lambda^2 + 4y^2)^2 + y^2\lambda^4]^{-1/2} \tag{5.4}$$

[roughly one has $A \approx (x\lambda^2)^{-1}$] and notice that each of the fields in (5.3) contains a piece of the Majoron,

$$\Phi^* = -iAx(\lambda^2 + 4y^2)J + \cdots \ , \quad \phi^0 = -2iA\lambda y^2 J + \cdots \ , \quad h^0 = iAy\lambda^2 J + \cdots \ . \tag{5.5}$$

Substituting (5.5) into (5.3) and using U_a and U_c from (3.5) gives finally the Majoron Yukawa interaction[15]

$$\mathcal{L} = -\frac{i}{2} AJ\lambda^2 \tilde{\nu}^T \sigma_2 V_1^T(M_1 - DM_2{}^{-1}D^T)V_1\tilde{\nu} + \text{H.c.} + O(\epsilon^3) \ . \tag{5.6}$$

Comparing this with (5.2) shows that the Yukawa interaction in the 1-2-3 model is again proportional to the mass matrix m correct to order ϵ^2.

We conclude this section with a remark about the Majoron coupling to *charged* fermions. In either the 2-3 or 1-2-3 model the neutral member of the ordinary Higgs doublet is seen to contain a little piece of the Majoron. Thus the ordinary Yukawa interaction of the standard theory gives a Majoron Yukawa interaction with charged fermions. Using (2.6) ones finds this interaction in the 2-3 model to be[9]

$$\mathcal{L} \simeq \mp \frac{2iym_\psi}{\lambda^2} J\bar{\psi}\gamma_5\psi \ , \tag{5.7}$$

where m_ψ is the mass of fermion ψ and the upper (lower) sign corresponds to a positively (negatively) charged fermion. The magnitude of (5.7) has already been noted to be rather small. Using (5.5) we find that the result (5.7) should be multiplied by approximately y/x (which is of order ϵ^2) in the 1-2-3 model. Thus in the 1-2-3 model the magnitude of (5.7) is further suppressed, making the Majoron even harder to detect.

VI. DISCUSSION

We have shown that Majoron schems do not, for reasonable parameters, yield lifetimes for $\nu_H \to \nu_L + J$ shorter than the age of the universe. More precisely we have found that in the 1-2 and 1-2-3 models the Yukawa coupling matrix in the light-neutrino sector is essentially given by $\langle \Phi \rangle^{-1} m$ (m being the diagonal light-neutrino mass matrix) correct to order ϵ^2. It is given exactly by $\langle h^0 \rangle^{-1} m$ in the 2-3 model. Since $\langle \Phi \rangle >> \langle \phi^0 \rangle >> \langle h^0 \rangle$ the latter coupling (although exactly diagonal) can be rather large and this may have important phenomenological effects.

One might, based on experience with the properties of U(1) Goldstone bosons in other contexts, expect that their tree couplings to fermions are exactly diagonal. Hence it might be considered unnecessary for us to have discussed the properties of the transformation matrix U of (3.5) in detail. However in models involving Majorana neutrinos such as the 1-2 and 1-2-3 theories the usual arguments *fail* for an interesting reason and the diagonality of the Yukawa matrix only holds to a certain order in the scale hierarchy parameter. We shall demonstrate this now for the 1-2 model and also use this technique to calculate the Yukawa matrix to order ϵ^4 where it is explicitly seen to be nondiagonal in general.

In the 1-2 theory let ρ_l stand for the n bare light neutrinos and ρ_h stand for the n *bare* heavy neutrinos. The lepton-number current computed from Noether's theorem is

$$L_\mu=\sum_{l=1}^{n}\rho_l^\dagger\sigma_\mu\rho_l-\sum_{h=1}^{n}\rho_h^\dagger\sigma_\mu\rho_h +2\Phi_r\overleftrightarrow{\partial}_\mu\Phi_i+\cdots , \tag{6.1}$$

where the ellipsis represents the charged fields which will not concern us. It is crucial that the first and second terms differ in sign. This is because ρ_l and ρ_h have *opposite* lepton numbers as one notices from (2.7) and that the usual Higgs field ϕ^0 carries zero lepton number while Φ and h^0 each carry -2 units. Because the total Lagrangian is lepton-number invariant we have $\partial_\mu L_\mu=0$ even in the presence of spontaneous breakdown where $\Phi_r\approx\langle\Phi_r\rangle\equiv x$. Then (6.1) yields

$$\partial_\mu\left[\sum_{l=1}^{n}\rho_l^\dagger\sigma_\mu\rho_l-\sum_{h=1}^{n}\rho_h^\dagger\sigma_\mu\rho_h\right]+2x\,\Box J=0 , \tag{6.2}$$

where we have identified $J=\Phi_i$ [see (2.5)]. Now suppose that the relative sign between the first two terms of (6.2) were positive (or that the second term was absent as in the 2-3 theory). Then we could use the transformation (2.8) to replace the first two terms by

$$\partial_\mu\sum_{\text{all } i}\nu_i^\dagger\sigma_\mu\nu_i ,$$

where the ν_i are the physical (mass diagonal) fields. Sandwiching (6.2) between two *different* neutrino states would then result in a contribution only from the field $\Box J$ and we would conclude that the off-diagonal Majoron-Yukawa couplings vanish. However, the minus sign in (6.2) invalidates this conclusion. To carry this approach further substitute (2.8) into (6.2) and specialize to the light-neutrino sector. In matrix notation ($\tilde{\nu}$ stands for the light-*physical*-neutrino fields)

$$\partial_\mu(\tilde{\nu}^\dagger\sigma_\mu R\tilde{\nu})+2x\,\Box J+\cdots=0 , \tag{6.3}$$

where the matrix R is given by

$$\begin{aligned}R&=U_a^\dagger U_a-U_c^\dagger U_c\\ &=1-2V_1^\dagger D^* M_2^{*\,-1}M_2^{-1}D^T V_1+\cdots .\end{aligned} \tag{6.4}$$

Equation (3.5) was used in the last step. We have in mind to take matrix elements of (6.3) between any two neutrino states. This can be simplified by using the free equation of motion[2] for the physical field

$$\partial_\mu\tilde{\nu}_j^\dagger\sigma_\mu=-\mathrm{I}m_j\nu_j^T\sigma_2 , \tag{6.5}$$

and by writing the Majoron Yukawa interaction with light *physical* neutrinos as

$$\mathscr{L}_{\text{Yukawa}}=\frac{iJ}{2}\sum_{k,n}\tilde{\nu}_k^T\sigma_2\tilde{\nu}_n g_{kn}+\text{H.c.} , \tag{6.6}$$

thereby defining the coupling matrix g_{kn}. The quantity $\Box J$ is to be replaced by

$$-\frac{\partial}{\partial J}\mathscr{L}_{\text{Yukawa}}+\cdots ,$$

using Lagrange's equation. Then (6.3) simply becomes

$$-i\sum_{k,n}\tilde{\nu}_k^T\sigma_2(m_kR_{kn}+xg_{kn})\tilde{\nu}_n+\text{H.c.}=0 , \tag{6.7}$$

which evidently results in the final formula

$$g_{kn}=-\frac{1}{x}m_k\delta_{kn}+\frac{2}{x}m_k(V_1^\dagger D^* M_2^{*\,-1}M_2^{-1}D^T V_1)_{kn}+\cdots . \tag{6.8}$$

The first term of (6.8) has a leading contribution of order ϵ^2 [see (3.3)] and agrees with our previous result in (3.6) and (3.7). The second term is of order ϵ^4 and represents the leading amplitude for the tree contribution to $\nu_H\to\nu_L+J$. It contains a factor which is identical to the matrix P in (3.10) restricted to the light-neutrino sector. Notice that the present technique has enabled us to "escalate" our knowledge of U [given in (3.5)] to order ϵ^2 to a result for the nondiagonal Yukawa matrix g_{kn} accurate to order ϵ^4. A similar argument can be carried out in the 1-2-3 model with essentially the same result.

As remarked before the above amplitude leads to $\Gamma(\nu_H \to \nu_L + J)$ of order ϵ^9 and for reasonable parameters a numerical value of roughly 10^{-60} MeV. Such an extremely low rate makes it comparable to the width for $\Gamma(\nu_H \to \nu_L + \gamma)$. In fact at this level the radiative correction (by W exchange) to $\Gamma(\nu_H \to \nu_L + J)$ [which is expected to be similar to $\Gamma(\nu_H \to \nu_L + \gamma)$ in magnitude] is comparable to the tree contribution. In the 2-3 model, as remarked by Georgi *et al.*,[11] the radiative contribution is the only one. These authors have also investigated the cosmological implications in the 2-3 model of the processes $\nu_H + \nu_H \to J \to \nu_L + \nu_L$ and $\nu_L + \nu_L \to J + J$. The amplitudes for these in the 1-2 model involve the "square" of (6.8).

Throughout this work we have assumed that it is sensible to organize things by the hierarchy of scales given in (1.6), (1.8), and (5.1). This of course may be questioned since, for example, the spread of masses in the known three-generation structure is a factor of 10^3. Nevertheless the assumption of the existence of a new scale is perfectly in tune with the idea that SU(2)$\times$U(1) is embedded in a larger group which does not show up at usual "resolutions." We do not, however, want to rule out the conservative possibility that the SU(2)$\times$U(1) structure is the only correct feature of the present fashion in gauge theories. Then one might want to allow, e.g., M_2 to be of the same order as D and to have special mixing angles, etc. Such a possibility is more difficult to analyze in general but Hosotani[5] has recently shown that some relaxation of the bounds on neutrino masses may be obtained due to the reaction (1.12). A conceptually straightforward experimental test for such a situation would be the observation of neutrino oscillations in neutral-current hadronic interactions.[4]

Note added. In our discussion we have assumed that only a single Higgs multiplet of each isospin was present. One might wonder whether decays into Majoron + lighter neutrino are also suppressed in the $1^p2^q3^r$ model which contains p isosinglets, q doublets, and r triplets. In this multiple-Higgs-field case the general discussion of Sec. VI should still be applicable. In particular, Eq. (6.2) will continue to hold if J is the particular linear combination of fields corresponding to the Majoron and x is its "decay constant." The discussion which follows (6.2) only requires us to assume that a single hierarchy parameter exists. Thus (6.8) would predict a tree level decay amplitude for $\nu_H \to \nu_L \gamma$ of order $\epsilon^4 O(M_2/x)$. Generally x should be of the same order as M_2 if only a single hierarchy exists so we expect the same suppression to hold. For example, in the case where two $l=-2$ singlets, Φ and Φ' (with vacuum values x and x') are present, x is simply to be replaced by $(x^2+x'^2)^{1/2}$.

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APPENDIX

We will identify the correct combination of Higgs fields to give the Majoron by using the invariance properties[16] of the Higgs potential V. In order that V be invariant under lepton number [taking SU(2)$\times$U(1) for granted] it is necessary that terms such as

$$c_1\phi^\dagger\phi\Phi + c_2\phi^\dagger h\phi_c + \text{H.c.}$$

(where $\phi_c = \tau_2\phi^*$) be excluded. In fact, to convert the general SU(2)$\times$U(1)-invariant 2-3 model into a lepton-number-conserving one it is only necessary to remove the c_2 term from the Higgs potential. An interesting lepton-number-conserving term in the 1-2-3 model is, however,

$$c_3\phi^\dagger h\phi_c\Phi^* + \text{H.c.}$$

The possibility of such a term illustrates the need for mixing among neutral fields belonging to all three Higgs multiplets.

The invariance of V under an infinitesimal global lepton-number transformation leads immediately [see (2.1) and (2.3)] to the equation

$$\Phi_i\frac{\partial V}{\partial \Phi_r} - \Phi_r\frac{\partial V}{\partial \Phi_i} + h_i^0\frac{\partial V}{\partial h_r^0} - h_r^0\frac{\partial V}{\partial h_i^0} + \cdots = 0 \,. \tag{A1}$$

The ellipsis in (A1) represents the contributions from fields which are electrically charged. Similarly the invariance of V under weak-hypercharge gauge transformations leads to (Φ has $Y=1$, h has $Y=2$, and Φ has $Y=0$)

$$\phi_i^0\frac{\partial V}{\partial\phi_r^0}-\phi_r^0\frac{\partial V}{\partial\phi_i^0}+2h_i^0\frac{\partial V}{\partial h_r^0}-2h_r^0\frac{\partial V}{\partial h_i^0}+\cdots=0\,. \tag{A2}$$

Differentiating (A1) with respect to Φ_i and evaluating the resulting equation in the vacuum state gives

$$\left\langle\frac{\partial V}{\partial\Phi_r}\right\rangle-\langle\Phi_r\rangle\left\langle\frac{\partial^2 V}{\partial{\Phi_i}^2}\right\rangle-\langle h_r^0\rangle\left\langle\frac{\partial^2 V}{\partial\Phi_i\partial h_i^0}\right\rangle=0\,.$$

Here we have assumed no *spontaneous* breakdown of *CP* conservation as discussed in Sec. II. Using (2.2) and setting $\langle\partial V/\partial\Phi_r\rangle=0$ gives

$$x\left\langle\frac{\partial^2 V}{\partial{\Phi_i}^2}\right\rangle+y\left\langle\frac{\partial^2 V}{\partial\Phi_i\partial h_i^0}\right\rangle=0\,. \tag{A3}$$

By differentiating (A1) and (A2) we easily find four more relations:

$$\begin{aligned}
&x\left\langle\frac{\partial^2 V}{\partial h_i^0\partial\Phi_i}\right\rangle+y\left\langle\frac{\partial^2 V}{\partial h_i^0\partial h_i^0}\right\rangle=0\,,\\
&\lambda\left\langle\frac{\partial^2 V}{\partial\phi_i^0\partial\phi_i^0}\right\rangle+2y\left\langle\frac{\partial^2 V}{\partial\phi_i^0\partial h_i^0}\right\rangle=0\,,\\
&\lambda\left\langle\frac{\partial^2 V}{\partial\phi_i^0\partial h_i^0}\right\rangle+2y\left\langle\frac{\partial^2 V}{\partial h_i^0\partial h_i^0}\right\rangle=0\,,\\
&x\left\langle\frac{\partial^2 V}{\partial\Phi_i\partial\phi_i^0}\right\rangle+y\left\langle\frac{\partial^2 V}{\partial h_i^0\partial\phi_i^0}\right\rangle=0\,.
\end{aligned} \tag{A4}$$

Equations (A3) and (A4) yield enough information so that the mass squared matrix of neutral (imaginary part) Higgs fields is completely determined in terms of a single parameter, e.g.,

$$a\equiv\left\langle\frac{\partial^2 V}{\partial\phi_i^0\partial\phi_i^0}\right\rangle\,.$$

In the basis ϕ_i^0,Φ_i,h_i^0 this matrix is

$$\begin{bmatrix} 1 & \dfrac{\lambda}{2x} & -\dfrac{\lambda}{2y}\\ \dfrac{\lambda}{2x} & \dfrac{\lambda^2}{4x^2} & -\dfrac{\lambda^2}{4xy}\\ -\dfrac{\lambda}{2y} & -\dfrac{\lambda^2}{4xy} & \dfrac{\lambda^2}{4y^2}\end{bmatrix}a\,. \tag{A5}$$

The determinant of (A5) vanishes so there is at least one Goldstone boson. Trying to solve for it uncovers a two-dimensional null subspace so there are in fact two. The linear combination which is absorbed by the Z^0 vector boson is defined from the feature that it cannot contain any component of Φ_i (which does not couple to the Z). The absorbed Goldstone boson is proportional to

$$\lambda\phi_i^0+2yh_i^0\,, \tag{A6}$$

while the Majoron given in (2.4) is orthogonal to this. Finally the massive linear combination is proportional to

$$\phi_i^0+\frac{\lambda}{2x}\Phi_i-\frac{\lambda}{2y}h_i^0\,. \tag{A7}$$

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[1]An up-to-date survey of the situation is given in *Proceedings of the Neutrino Mass Miniconference, Telemark, Wisconsin, 1980,* edited by V. Barger and D. Cline (University of Wisconsin, Madison, 1981).

[2]Our notation follows that given in several earlier papers, J. Schechter and J. W. F. Valle, Phys. Rev. D 22, 2227 (1980); 23, 1666 (1981); 24, 1883 (1981). In these papers we have attempted to give fair references to the already extensive recent literature on the theory of massive neutrinos.

[3]Calculations of this decay mode done almost 20 years ago are discussed in the talk of M. Nakagawa in Ref. 1. The most recent discussion is given by P. B. Pal and L. Wolfenstein, preceding paper, Phys. Rev. D 25, 766 (1982).

[4]In addition to the first of Ref. 2, see V. Barger, P. Langacker, J. P. Leveille, and S. Pakvasa, Phys. Rev. Lett. 45, 692 (1980); M. Gell-Mann, G. Stephenson, and R. Slansky (unpublished); D. D. Wu, Phys. Lett. 96B, 311 (1980).

[5]See the first of Ref. 2 and Y. Hosotani, Enrico Fermi Institute Report No. EFI 81/12, 1981 (unpublished). Also I. Y. Kobzarev *et al.*, ITEP Report No. ITEP-90, 1980 (unpublished). Earlier related references are given by S. Pakvasa, in *High Energy Physics—1980,* proceedings of the XXth International Conference, Madison, Wisconsin, edited by L. Durand and L. G. Pondrom (AIP, New York, 1981).

[6]Of course if ϵ becomes this large the concept of a scale hierarchy becomes blurred. Also the important decay channel $\nu_H\to e^+e^-\nu_L$ opens up when $\epsilon\approx10^{-2}$.

[7]Y. Chikashige, R. N. Mohapatra, and R. D. Peccei, Phys. Rev. Lett. 98B, 265 (1981).

[8]Y. Chikashige, R. N. Mohapatra, R. D. Peccei, Phys. Rev. Lett. 45, 1926 (1980).

[9]G. B. Gelmini and M. Roncadelli, Phys. Lett. 99B, 411 (1981).

[10]See, for example, Ref. 2 above; T. P. Cheng and L. F.

Li, Phys. Rev. D 22, 2860 (1980).

[11]H. Georgi, S. Glashow, and S. Nussinov, Harvard Report No. HUTP-8/A026, 1981 (unpublished).

[12]Equation (1.2) of Ref. 2 should be replaced by the expression for h given here.

[13]The same result for the special case when M is real has been obtained in a slightly different way by Hosotani in Ref. 5.

[14]See Eq. (3.15) of Ref. 2. Notice that Eq. (3.9) is written in terms of the Majorana field operators ν. We may take matrix elements of this equation by using (A4) of Ref. 2 to arrive at the more complicated structure proportional to (4.10) of Ref. 2.

[15]Equation (5.6) is written in terms of Majorana field operators. We may take matrix elements and express the result in terms of auxiliary four-component Dirac spinors by using

$$\left[\frac{E_p E_p'}{mm'}\right]^{1/2} \langle r'\vec{\mathrm{p}}\,'\,|\,(i\tilde{\nu}^T\sigma_2 F\tilde{\nu}+\mathrm{H.c.})\,|\,\mathrm{r},\vec{\mathrm{p}}\rangle = -2\bar{u}^{(r')}(\vec{\mathrm{p}}\,')\mathrm{Im}F u^{(r)}(\vec{\mathrm{p}}) + 2i\bar{u}^{(r')}(\vec{\mathrm{p}}\,')\mathrm{Re}F\gamma_5 u^{(r)}(\vec{\mathrm{p}})\ .$$

Here F is a symmetric matrix in neutrino generation space and $u^{(r)}(\vec{\mathrm{p}})$ is an ordinary Dirac spinor.

[16]This method was recently used in a similar problem by J. Kandaswamy, Per Salomonson, and J. Schechter, Phys. Rev. D 19, 2757 (1979).