Radiative decays of massive neutrinos

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General formulas are given for the decay rate $\nu_2 \rightarrow \nu_1 + \gamma$ in the SU(2)×U(1) model for neutrinos with a small mass. The emphasis is on distinguishing between the cases of Dirac and Majorana neutrinos. Possible enhancements of the rate due to methods of eluding the Glashow-Iliopoulos-Maiani suppression and due to charged Higgs bosons are considered.

If neutrinos are massive and if the mass eigenstates are not degenerate, then it is possible to have a radiative decay of the form $\nu_2 \rightarrow \nu_1 + \gamma$. The possibility that massive relic neutrinos from the big bang might be detected as a result of this radiation has been discussed recently.[1,2] In addition, such decays have been discussed in a variety of astrophysical contexts.[3] Formulas for the rate of these decays have been given explicitly by Petcov[4] and by Goldman and Stephenson[5] and can be derived from the general results of Marciano and Sanda[6] and of Lee and Shrock.[7] All these results are given for the case of Dirac neutrinos whereas most present theoretical ideas[8] about neutrino mass yield Majorana neutrinos. In this paper we discuss the general case involving either Majorana or Dirac neutrinos. Since the predicted rates within the standard model are small, we consider some possibilities of enhancing the rate.

In order to understand the differences between the Majorana and Dirac cases, it is necessary first to review the calculation for the Dirac case, which we carry out in the Feynman—'t Hooft gauge. We assume the standard SU(2) × U(1) model with the leptons in left-handed doublets and right-handed singlets plus a single Higgs doublet. The relevant diagrams are shown in Fig. 1. Because of the Glashow-Iliopoulos-Maiani (GIM) cancellation the transition moment vanishes in the limit that all charged lepton masses are taken equal to zero. As a result, the diagrams involving the unphysical ϕ^+ cannot be ignored even though the coupling of ϕ^+ is proportional to a lepton mass. This coupling may be written

$$-2(G_F/\sqrt{2})^{1/2} \sum_{\alpha,a} \overline{\nu}_\alpha U_{\alpha a}(m_{la} R - m_\alpha L) l_a \phi^+ + \text{H.c.} \; , \tag{1}$$

where m_{la} $(=m_e, m_\mu$, etc.) is the charged-lepton mass, m_α is the neutrino mass, G_F is the Fermi constant, and $U_{\alpha a}$ is the unitary matrix relating the neutrino mass eigenstates $\nu_{\alpha L}$ $(\alpha = 1,2 \cdots)$ to the weak eigenstates ν_{aL} $(a = e, \mu \cdots)$

$$\nu_{\alpha L} = \sum_a U_{\alpha a} \nu_{aL} \; . \tag{2}$$

For simplicity, we assume CP invariance and choose $U_{\alpha a}$ to be real. The helicity projection

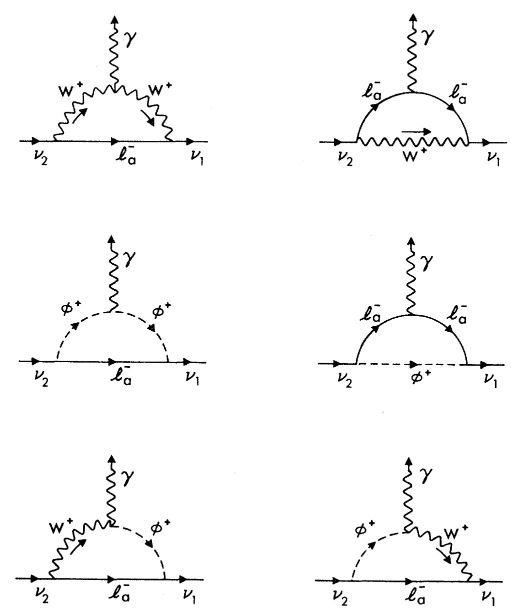

FIG. 1. Diagrams in the 't Hooft—Feynman gauge contributing to the process $\nu_2 \rightarrow \nu_1 + \gamma$ for Dirac neutrinos ν_2 and ν_1.

operators are $R=\frac{1}{2}(1+\gamma_5)$ and $L=\frac{1}{2}(1-\gamma_5)$.

With the approximation that $m_\alpha << M_W$ and $m_\alpha << m_{la}$ for all α and a, the transition amplitude is given by

$$T=-\frac{eG_F}{4\sqrt{2}\pi^2}\sum_a U_{1a}U_{2a}F(r_a)$$

$$\times\bar{\nu}_1(p')(m_2R+m_1L)\sigma_{\lambda\rho}q^\rho\epsilon^\lambda\nu_2(p)\ , \quad (3)$$

where $r_a=(m_{la}/M_W)^2$, $q=p-p'$, ϵ^λ is the photon polarization, and

$$F(r)=(1-r)^{-2}\left[-\frac{3}{4}(2-5r+r^2)+\frac{3}{2}\frac{r^2\ln r}{1-r}\right]\ . \quad (4)$$

This gives for the decay rate

$$\Gamma=\frac{\alpha G_F^2}{128\pi^4}\left[\frac{m_2^2-m_1^2}{m_2}\right]^3(m_2^2+m_1^2)$$

$$\times\left[\sum_a U_{1a}U_{2a}F(r_a)\right]^2\ . \quad (5)$$

For the case of three generations all the r_a are very small and we may approximate

$$F(r)\approx-\frac{3}{2}+\frac{3}{4}r\ . \quad (6)$$

The first term in Eq. (6) does not contribute because of the GIM cancellation; as a result, the rate is given by

$$\Gamma\approx\frac{\alpha}{2}\left[\frac{3G_F}{32\pi^2}\right]^2\left[\frac{m_2^2-m_1^2}{m_2}\right]^3(m_2^2+m_1^2)$$

$$\times\left[\sum_a U_{1a}U_{2a}r_a\right]^2$$

$$\approx(10^{29}\ \text{yr})^{-1}\left[\frac{m_2}{30\ \text{eV}}\right]^5(1-x^2)^3$$

$$\times(1+x^2)(U_{1\tau}U_{2\tau})^2\ , \quad (7)$$

where $x=m_1/m_2$ and we have assumed in the last line that only the term with $r_a=r_\tau=m_\tau^2/M_W^2$ is significant. For our later discussion we note that if the term in Eq. (1) proportional to the neutrino mass m_α is omitted, the leading term in T is proportional to $r_a\ln r_a$ rather than just r_a.[9]

A considerably larger decay rate becomes possible if we imagine[1] there are four generations with a much heavier charged lepton σ in the fourth generation. In this case to a good approximation we set $m_e=m_\mu=m_\tau=0$ so that

$$\sum_a U_{1a}U_{2a}F(r_a)=U_{1\sigma}U_{2\sigma}[F(r_\sigma)-F(0)]\ . \quad (8)$$

The function $[F(r_\sigma)-F(0)]^2$, which now is proportional to the rate, is plotted in Fig. 2. Contrary to the suggestion of Ref. 1 this function does not peak near $r_\sigma\approx 1$. For values of $r_\sigma >> 1$,

$$F(r_\sigma)-F(0)\approx\frac{3}{4}-\frac{3\ln r_\sigma}{2r_\sigma}\ . \quad (9)$$

In the limit that r_σ approaches infinity,

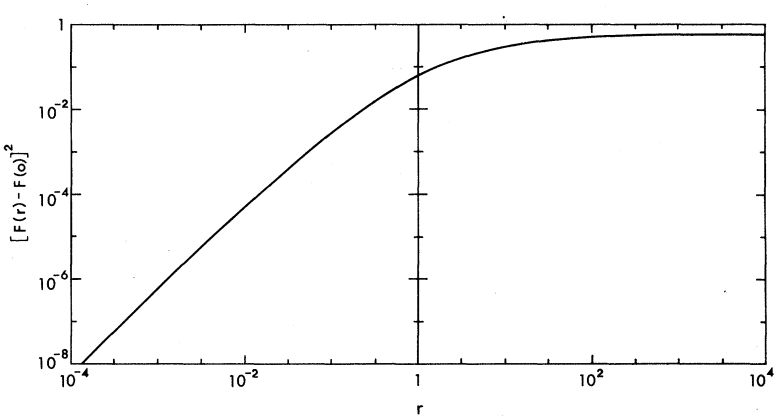

FIG. 2. Plot of $[F(r)-F(0)]^2$ vs r (note logarithmic scale on both axes).

$$\Gamma \to \frac{\alpha}{2}\left[\frac{3G_F}{32\pi^2}\right]^2\left[\frac{m_2{}^2-m_1{}^2}{m_2}\right]^3(m_2{}^2+m_1{}^2)(U_{1\sigma}U_{2\sigma})^2$$
$$\approx(2\times10^{22}\ \text{yr})^{-1}\left[\frac{m_2}{30\ \text{eV}}\right]^5(1-x^2)^3$$
$$\times(1+x^2)(U_{1\sigma}U_{2\sigma})^2 \ . \qquad (10)$$

This limit is approached relatively slowly so that for $m_\sigma \approx M_W$ the rate is still a factor of 9 below the asymptotic value. This corresponds to one method, but not a unique one, for eliminating the GIM suppression. An alternative method[1] is to add a fourth generation of neutrinos for which both $\nu_{\sigma L}$ and $\nu_{\sigma R}$ are singlets with no corresponding charged leptons. The Lagrangian then contains a bare mass term

$$-\mathscr{L}=\sum_{a=1}^{4} m_{\sigma a}\bar{\nu}_{\sigma L}\nu_{aR}+\text{H.c.}$$

Combining this with the usual mass term obtained from the Higgs vacuum expectation values one obtains a set of four neutrino eigenstates given by Eq. (2). The decay rate is given by Eq. (5) with the sum over a running only from 1 to 3. In this case, the first term in Eq. (6) dominates the sum so that

$$\sum_{a=1}^{3} U_{1a}U_{2a}F(r_a)\approx-\tfrac{3}{2}\sum_{a=1}^{3}U_{1a}U_{2a}=\tfrac{3}{2}U_{1\sigma}U_{2\sigma} \ . \qquad (11)$$

The rate is then four times as large as given in Eq. (10).

We turn now to the case of Majorana neutrinos. There are two differences to be considered. In the first place, we must recalculate the diagrams of the type shown in Fig. 1. In addition, however, we must consider the question of how the neutrinos acquire their Majorana mass. In the case of Dirac Dirac neutrinos the extension of the standard model to include a mass is completely straightforward. However, a Majorana mass for the usual doublet neutrinos corresponds to a violation of weak isospin by one unit. Thus, starting with our SU(2)×U(1)-invariant Lagrangian we must introduce this violation of weak isospin in a consistent manner. The simplest model is to add a Higgs triplet to the theory in addition to the Higgs doublet. The details of this model are developed in the Appendix. As a result of adding the Higgs triplet, new diagrams involving physical charged Higgs bosons occur (shown in Fig. 3) and we calculate

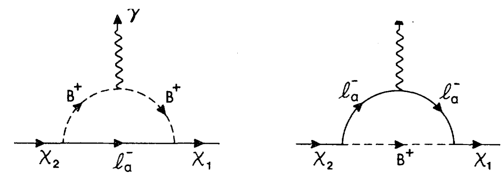

FIG. 3. Diagrams involving physical charged Higgs boson contributing to the radiative decay of a Majorana neutrino. There are additional diagrams where B^+ and l_a^- are replaced by B^- and l_a^+.

these additional contributions.

We first limit ourselves to the diagrams of Fig. 1. An interesting question that arises is the coupling of the unphysical Higgs boson proportional to the neutrino mass, the second term in Eq. (1). This term was originally derived for the Dirac case; since the Majorana mass arises from a different mechanism, it is not immediately obvious whether this term is present. As noted above, this term plays an essential role in the calculation in the gauge we use. However, since it is possible to avoid this term altogether by going to the unitary gauge, it would seem that this term must be required by gauge invariance. In the Appendix we show explicitly how this term arises in the model with a Higgs triplet.

We first consider the model with three fermion generations. The Majorana mass matrix links ν_{eL}, $\nu_{\mu l}$, $\nu_{\tau L}$ to the right-handed antiparticles $(\nu_e^c)_R$, $(\nu_\mu^c)_R$, $(\nu_\tau^c)_R$. Again, assuming *CP* invariance, we can diagonalize the mass matrix by an orthogonal matrix U. At this stage, however, some of the eigenvalues may be negative. We call the eigenvalues $\eta_\alpha m_\alpha$, where m_α is real and positive, and η_α is $+1$ or -1. In order that the fields describe particles with positive masses m_α, we define the mass eigenstates by the Majorana fields[10]

$$\chi_\alpha=\sum_a U_{\alpha a}(\nu_{aL}+\eta_\alpha\nu_{aR}^c) \ . \qquad (12)$$

(We use χ instead of ν just to remind ourselves that the neutrino is a Majorana one now.) In calculating $\chi_2\to\chi_1+\gamma$, we will have to consider separately the cases (a) $\eta=\eta_2/\eta_1=+1$ and (b) $\eta=\eta_2/\eta_1=-1$.

The next point to note is that for each diagram of Fig. 1, there exists a second diagram in which the W^+, ϕ^+, and l^- lines are replaced by W^-, ϕ^-, and l^+. [It is understood in these diagrams that $\phi^\pm$ stands for the unphysical Higgs boson; thus for the case discussed in the Appendix the ϕ^+ lines should be relabeled S^+ as defined by Eq. (A11)]. Such a contribution is absent in the Dirac case because the right-handed neutrinos have no weak interaction, but occurs in the Majorana case since $\nu_{\alpha R} \equiv \nu_{\alpha R}^c$. The contribution from this set of diagrams is proportional to $(m_2 L + m_1 R)$, but its sign with respect to the contribution from diagrams in Fig. 1 will depend on the ratio η. (See Appendix for details.) As a result, the factor $(m_2 R + m_1 L)$ in Eq. (3) should be replaced in case (a) by

$$(m_2 R + m_1 L) - (m_2 L + m_1 R) = \gamma_5 (m_2 - m_1) . \tag{13a}$$

Then in place of Eqs. (3) and (5) we have

$$T = -\frac{eG_F}{4\sqrt{2}\pi^2} \sum_a U_{1a} U_{2a} F(r_a) \times (m_2 - m_1) \bar{\chi}_1(p') \gamma_5 \sigma_{\lambda\rho} q^\rho \epsilon^\lambda \chi_2(p) , \tag{14a}$$

$$\Gamma = \frac{\alpha G_F^2}{64\pi^4} \left[\frac{m_2^2 - m_1^2}{m_2} \right]^3 (m_2 - m_1)^2 \times \left[\sum_a U_{1a} U_{2a} F(r_a) \right]^2 . \tag{15a}$$

The minus sign in Eq. (13a) corresponds to the requirement that if we were calculating the diagonal moment for a single Majorana neutrino we would get the answer zero.

In case (b), Eq. (13a) should be replaced by

$$(m_2 R + m_1 L) + (m_2 L + m_1 R) = m_2 + m_1 . \tag{13b}$$

Then, in place of Eqs. (14a) and (15a), we have

$$T = -\frac{eG_F}{4\sqrt{2}\pi^2} \sum_a U_{1a} U_{2a} F(r_a) (m_2 + m_1) \times \bar{\chi}_1(p') \sigma_{\lambda\rho} q^\rho \epsilon^\lambda \chi_2(p) , \tag{14b}$$

$$\Gamma = \frac{\alpha G_F^2}{64\pi^4} \left[\frac{m_2^2 - m_1^2}{m_2} \right]^3 (m_2 + m_1)^2 \times \left[\sum_a U_{1a} U_{2a} F(r_a) \right]^2 . \tag{15b}$$

For this case the limit $m_1 = m_2$ corresponds to the merger of two Majorana particles into a Dirac particle and the transition moment can be reinterpreted as a diagonal moment for this Dirac particle. However, this is not the normal type of Dirac neutrino employed in the first part of this paper but rather one of the unusual varieties discussed by one of us recently.[11]

The physical distinction between the two cases is that in case (a) χ_2 and χ_1 have the same *CP* property, whereas in case (b) they have opposite *CP* properties. It follows from *CP* invariance that the matrix element must be either of the magnetic dipole type $\sigma_{\lambda\rho}$ or electric dipole $\sigma_{\lambda\rho}\gamma_5$, but not a mixture of the two.[12] If $m_1 << m_2$, then for both cases the rate is twice the rate for Dirac neutrinos. However, as m_1 approaches m_2, the rate for case (a) approaches zero, whereas for case (b) it approaches four times the Dirac rate.

For the case in which one of the neutrinos is Majorana and the other Dirac, the formulas for the Dirac case hold. However, if the heavier neutrino is the Majorana there exists a second decay mode $\chi_2 \to \nu_1^c + \gamma$ with the same rate as $\chi_2 \to \nu_1 + \gamma$ and the total decay rate is thus a factor of 2 larger than the Dirac case. Note that in the case $m_1 = 0$ it is irrelevant whether ν_1 is called a Dirac or Majorana particle.

It is now necessary to consider the diagrams of Fig. 3 due to the exchange of the charged Higgs boson B^+, the coupling of which is given by Eq. (A23). The result is that in Eqs. (14) and (15) we must replace $F(r_a)$ by

$$F(r_a) \to F(r_a) + f(\rho_a), \quad \rho_a = (m_{la}/m_B)^2 , \tag{16}$$

$$f(\rho) = (1-\rho)^{-2} \{ (\rho^2 - \rho \ln\rho - \rho) + \tan^2\alpha [\tfrac{1}{4}\rho^2 + \tfrac{1}{4}\rho + \tfrac{1}{2}(1-\rho)^{-1} \rho^2 \ln\rho] \} , \tag{17}$$

where m_B is the mass of B^+. With the limit $\tan^2\alpha < 0.065$, which follows from Eq. (A27), the second term can generally be neglected. For m_B infinite the decay rate is given by Eq. (15a) or (15b), and we may refer to this rate as Γ_∞. We then have, with $\rho_e = r_e = \rho_\mu = r_\mu \approx 0$,

$$\frac{\Gamma}{\Gamma_\infty} = \left[\frac{F(r_\tau) + f(\rho_\tau) - F(0)}{F(r_\tau) - F(0)} \right]^2 \tag{18}$$

and, with the approximation $r_\tau << 1$, $\rho_\tau << 1$,

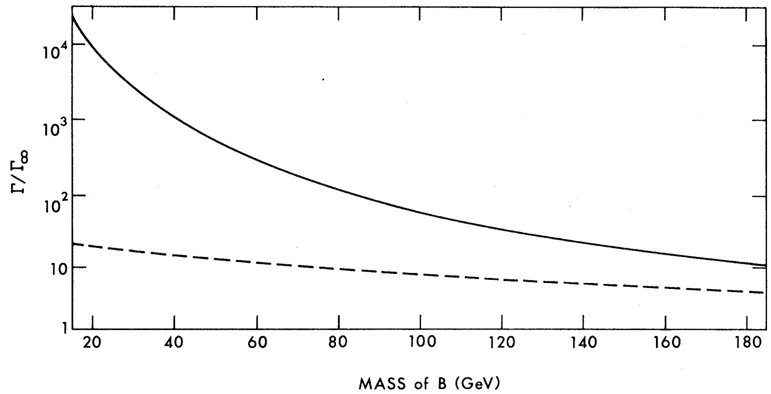

FIG. 4. Effect on the rate due to the diagrams in Fig. 3. Γ_∞ is the rate if the diagrams in Fig. 3 are negligible, viz., if $m_B \to \infty$. We have taken $\tan^2\alpha = 0.05$. The solid curve is for the three-generation case, the dashed curve is for the four generation case with $m_\sigma = M_W$.

$$\frac{\Gamma}{\Gamma_\infty} = \frac{16}{9}\left[\frac{\rho_\tau}{r_\tau}\right](1+\ln\rho_\tau)^2 . \tag{19}$$

Figure 4 shows Γ/Γ_∞ as a function of m_B for $M_W = 85$ GeV. For values of m_B between 20 GeV and M_W we find that the contribution of the B^+ diagram increases the rate by a factor of 10^4 to 10^2.

If we add a fourth generation, Eqs. (14)–(16) are unchanged with the sum going over a including the new generation σ. If we neglect the contribution from the diagrams in Fig. 3, we get a large increase in the rate compared to the rate in the three-generation case. This enhancement factor is just the same as that in the Dirac case. On top of that, the diagrams in Fig. 3 will enhance the rate by a factor obtained by replacing the subscript τ by σ in Eq. (18). For example, for $m_\sigma = M_W$, this increase is by a factor between 5 and 20, as shown in Fig. 4. Considering both the factors, with a physical charged Higgs boson of mass 20 GeV, the rate for $m_\sigma = M_W$ is given by

$$\Gamma = (5\times 10^{21}\ \text{yr})^{-1}\left[\frac{m_2}{30\ \text{eV}}\right]^5 (1-x^2)^3 \times (1-\eta x)^2 (U_{1\sigma}U_{2\sigma})^2 . \tag{20}$$

If we add a fourth generation of neutrino with no charged lepton to accompany it, we can again use Eqs. (14)–(16) with a running over the three charged leptons. In the contributions from diagrams in Fig. 1, the leading term—which is independent of lepton masses—does not cancel now by the orthogonality of U. The contribution from the diagrams in Fig. 3 always involves some factor of ρ_a. So, for values of m_B for which $\rho_\tau << 1$, diagrams of Fig. 3 can be neglected so that we get the same result as Eq. (11):

$$\sum_{a=1}^{3} U_{1a}U_{2a}[F(r_a)+f(\rho_a)] \simeq -\tfrac{3}{2}\sum_{a=1}^{3} U_{1a}U_{2a} = \tfrac{3}{2} U_{1\sigma}U_{2\sigma} . \tag{21}$$

This will give a rate

$$\Gamma = (2.5\times 10^{21}\ \text{yr})^{-1}\left[\frac{m_2}{30\ \text{eV}}\right]^5 \times (1-x^2)^3 (1-\eta x)^2 (U_{1\sigma}U_{2\sigma})^2 . \tag{22}$$

In conclusion, we have discussed in this paper how the results for the decay rate $\nu_2 \to \nu_1 + \gamma$ must be modified from previous calculations using Dirac neutrinos for the case of Majorana neutrinos. When both neutrinos are massive, it is necessary to distinguish two cases: (a) both neutrinos have the same CP property; (b) they have opposite CP properties. If we need consider only the diagrams of the type of Fig. 1, the results are given in Eqs. (14) and (15); if $m_1 = 0$, the cases (a) and (b) give the same result, which is twice the rate for a Dirac neutrino. For the case of the usual three generations, the rates are very low; much larger rates are possible if we extend the lepton content of the $SU(2)\times U(1)$ model either by adding a fourth generation or by mixing singlet neutrinos with doublets. There clearly are other possibilities of modifying the standard model to get larger rates.

An important point we have emphasized is that even if we give the mixing matrix and CP properties we cannot get an unambiguous answer for the rate for the case of Majorana neutrinos. The basic reason is that we need a well-defined consistent theory from which the Majorana mass arises. To illustrate this point we have used the theory in which the Majorana mass arises from the vacuum expectation value of a Higgs triplet. In this case it is necessary to consider the contribution of the diagrams of Fig. 3 in addition to those of Fig. 1. The final result then depends sensitively on the mass m_B of the physical charged Higgs boson B^+; for $m_B \lesssim M_W$, these diagrams are dominant and the rate is increased over that given by eqs. (14) and (15) by the factor shown in Fig. 4. It should be emphasized that we are using the Higgs triplet as an illustrative example; more attractive theories of the Majorana mass, such as grand unified theories, do not employ this mechanism. It may well be in those theories that Fig. 1 does dominate and that Eqs. (14b) and (15b) do provide a reasonable estimate of the rate. This question is now under study.

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APPENDIX

A simple model[13] that gives a Majorana mass to neutrinos is the standard model with a Higgs triplet $\vec{\mathrm{H}}$ in addition to the usual doublet:

$$\phi=\begin{bmatrix}\phi^+\\ \phi^0\end{bmatrix}\ , \quad \vec{\mathrm{H}}=\begin{bmatrix}H^{++}\\ H^+\\ H^0\end{bmatrix}\ . \qquad \text{(A1)}$$

With the convention $Q=T_3+Y$, ϕ has $Y=\frac{1}{2}$ and H has $Y=1$.

In the unbroken theory, the derivative terms involving these scalars are

$$(D^\lambda\phi)^\dagger(D_\lambda\phi)+(D^\lambda\vec{\mathrm{H}})^\dagger\cdot(D_\lambda\vec{\mathrm{H}})\ , \qquad \text{(A2)}$$

where

$$D_\lambda\phi=(\partial_\lambda-ig\frac{\vec{\tau}}{2}\cdot\vec{\mathrm{A}}'_\lambda-\frac{ig'}{2}B_\lambda)\phi\ , \qquad \text{(A3)}$$

$$D_\lambda\vec{\mathrm{H}}=(\partial_\lambda-ig\frac{\vec{\mathrm{t}}}{2}\cdot\vec{\mathrm{A}}'_\lambda-ig'B_\lambda)\vec{\mathrm{H}}\ . \qquad \text{(A4)}$$

Here, $\vec{\mathrm{A}}'_\lambda$ and B_λ represent SU(2) and U(1) gauge fields. $\vec{\tau}/2$ and $\vec{\mathrm{t}}/2$ are the 2×2 and 3×3 representations of SU(2) generators, i.e.,

$$\left[\frac{\tau_i}{2},\frac{\tau_j}{2}\right]=i\epsilon_{ijk}\frac{\tau_k}{2}\ , \qquad \text{(A5)}$$

and an exactly similar relation replacing τ by t. Thus, τ_1, τ_2, τ_3 are the Pauli matrices and

$$t_1=\sqrt{2}\begin{bmatrix}0&1&0\\1&0&1\\0&1&0\end{bmatrix}\ , \quad t_2=\sqrt{2}\begin{bmatrix}0&-i&0\\i&0&-i\\0&i&0\end{bmatrix}\ ,$$

$$t_3=2\begin{bmatrix}1&&\\&0&\\&&-1\end{bmatrix}\ . \qquad \text{(A6)}$$

The symmetry breaking is determined from

$$\langle\phi\rangle=\begin{bmatrix}0\\ v_2/\sqrt{2}\end{bmatrix}\ , \quad \langle\vec{\mathrm{H}}\rangle=\begin{bmatrix}0\\0\\ v_3/\sqrt{2}\end{bmatrix}\ . \qquad \text{(A7)}$$

Now we will have to replace ϕ by $\phi'+\langle\phi\rangle$, and H by $H'+\langle H\rangle$. We will omit the primes.

From Eq. (A2), we can now obtain the gauge-boson masses

$$M_W{}^2=\tfrac{1}{4}g^2(v_2{}^2+2v_3{}^2)\ ,$$
$$M_Z{}^2=\tfrac{1}{4}(g^2+g'^2)(v_2{}^2+4v_3{}^2)\ . \qquad \text{(A8)}$$

As usual, the Fermi constant is given by

$$G_F=\sqrt{2}g^2/8M_W{}^2\ . \qquad \text{(A9)}$$

Equation (A2) will also give couplings of the form

$$M_W W_\lambda^-\partial^\lambda S^+ + \text{H.c.}\ , \qquad \text{(A10)}$$

where

$$S^+=\cos\alpha\,\phi^+ + \sin\alpha\,H^+ \qquad \text{(A11)}$$

with

$$\tan\alpha=\sqrt{2}v_3/v_2\ . \qquad \text{(A12)}$$

Equation (A10) serves to identify S^+ as the unphysical charged Higgs boson. The orthogonal combination is the physical charged Higgs boson

B^+ of undetermined mass:

$$B^+ = -\sin\alpha\phi^+ + \cos\alpha H^+ . \quad \text{(A13)}$$

The fermion content is

$$\psi_{aL} = \begin{bmatrix} \nu_a \\ l_a \end{bmatrix}_L , \quad l_{aR}$$

and the CP-conjugate fields required by CPT invariance,

$$\psi^c_{aR} , \quad l^c_{aL} .$$

In the unbroken theory, the Yukawa couplings of the fermions are

$$\mathscr{L}_Y = -\sum_a f_a \bar{\psi}_{aL} \phi l_{aR} - \sum_{a,b} f_{ab} \bar{\psi}^c_{aR} (i\tau_2 \vec{\tau} \cdot \vec{\mathrm{H}}) \psi_{bL} + \text{H.c.} \quad \text{(A14)}$$

Here, for convenience, we have taken a fermion basis in which the charged leptons are diagonal. From our assumption of CP invariance, f_a, f_{ab} are real. By construction, f_{ab} is symmetric. The charged leptons get their masses from $\langle \phi \rangle$,

$$m_{la} = f_a v_2 / \sqrt{2} . \quad \text{(A15)}$$

Majorana mass terms for neutrinos come from $\langle H \rangle$,

$$-\mathscr{L}_m = \sum_{a,b} m_{ab} \bar{\nu}_{aL} \nu^c_{bR} + \text{H.c.} , \quad \text{(A16)}$$

where

$$m_{ab} = v_3 f_{ab} .$$

Using the orthogonal matrix U, we can diagonalize m_{ab},

$$\sum_{a,b} U_{\alpha a} m_{ab} (U^T)_{b\beta} = \delta_{\alpha\beta} \eta_\alpha m_\alpha , \quad \text{(A17)}$$

where $\eta_\alpha = \pm 1$ and m_α is a positive real number. We then define the fields $\nu_{\alpha L}$ (Greek index),

$$\nu_{\alpha L} = \sum_a U_{\alpha a} \nu_{aL} , \qquad \nu^c_{\alpha R} = \sum_a U_{\alpha a} \nu^c_{aR} , \quad \text{(A18)}$$

and the Majorana fields

$$\chi_\alpha = \nu_{\alpha L} + \eta_\alpha \nu^c_{\alpha R} . \quad \text{(A19)}$$

In terms of χ_α, $\mathscr{L}_m$ is diagonal,

$$-\mathscr{L}_m = \sum_\alpha m_\alpha \bar{\chi}_\alpha \chi_\alpha . \quad \text{(A20)}$$

Note that χ_α is an eigenstate of CP with eigenvalue η_α. The factor η_α must be included in (A19) in order that the χ_α satisfy field equations for positive mass.[10]

In terms of χ_α the Yukawa couplings of the neutrinos with charged Higgs bosons can be written from Eq. (A14),

$$\mathscr{L}_y^{\text{charged}} = -\sum_{\alpha,a} \frac{g m_{la}}{\sqrt{2} M_W \cos\alpha} \bar{\chi}_\alpha U_{\alpha a} l_{aR} \phi^+ + \sum_{\alpha,a} \frac{g m_\alpha}{\sqrt{2} M_W \sin\alpha} \bar{\chi}_\alpha U_{\alpha a} l_{aL} H^+ + \text{H.c.} . \quad \text{(A21)}$$

The coupling with the unphysical S^+ of Eq. (A11) is thus

$$-\frac{g}{\sqrt{2} M_W} \sum_{\alpha,a} \bar{\chi}_\alpha U_{\alpha a} (m_{la} R - m_\alpha L) l_a S^+ + \text{H.c.}, \quad \text{(A22)}$$

which is just the form given in the text in Eq. (1). The coupling with the physical Higgs boson B^+ is

$$\frac{g}{\sqrt{2} M_W} \sum_{\alpha,a} \bar{\chi}_\alpha U_{\alpha a} (m_{\alpha a} \tan\alpha\, R + m_\alpha \cot\alpha\, L) l_a B^+ + \text{H.c.} \quad \text{(A23)}$$

The couplings with the W boson may be expressed in an explicitly CP-invariant form as

$$\mathscr{L}_W = \frac{g}{\sqrt{2}} \sum_{\alpha,a} U_{\alpha a} (\bar{\nu}_{\alpha L} \gamma^\lambda l_{aL} W^+_\lambda + \bar{\nu}^c_{\alpha R} \gamma^\lambda l^c_{aR} W^-_\lambda) \quad \text{(A24)}$$

$$= \frac{g}{\sqrt{2}} \sum_{\alpha,a} U_{\alpha a} \bar{\chi}_\alpha (\gamma^\lambda l_{aL} W^+_\lambda + \eta_\alpha \gamma^\lambda l^c_{aR} W^-_\lambda) . \quad \text{(A25)}$$

The couplings with S and B can be written in a form analogous to that of Eq. (A25) in which case the S^- and B^- terms [terms mentioned as H.c. in Eqs. (A22) and (A23)] also contain the factor η_α.

It is now straightforward to see that the contribution from the diagrams in Fig. 1 involves $\eta_1(m_2 R + m_1 L)$. The diagrams of the type in Fig. 1 with l_a^-, W^+, S^+, B^+ lines replaced by l_a^+, W^-, S^-, B^- lines will contribute $-\eta_2(m_2 L + m_1 R)$. Thus the factor $(m_2 R + m_1 L)$ for the Dirac calculation should be replaced by (omitting an overall phase factor η_1)

$$(m_2 R + m_1 L) - \eta(m_2 L + m_1 R) = (m_2 - \eta m_1)(R - \eta L) , \quad \text{(A26)}$$

where $\eta = \eta_2/\eta_1$. The consequences are discussed in the text.

In calculating the graphs with the exchange of B^+, the major contribution comes from using each of the terms in Eq. (A23) once so that the result is independent of $\tan\alpha$. There is a second contribution as given in Eq. (17) proportional to $\tan^2\alpha$; from the data on neutral currents we can determine

$$\left[\frac{M_W}{M_Z\cos\theta_W}\right]^2 = 0.981 \pm 0.037 \; . \qquad \text{(A27)}$$

Comparing with Eqs. (A8) and (A12), we get the bound on $\tan\alpha$ mentioned in the text. The other contribution proportional to $\cot^2\alpha$ must involve (neutrino mass)^3 and so is assumed to be negligible.

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[8]For a review see L. Wolfenstein, in *Proceedings of the Neutrino Mass Miniconference, Telemark, Wisconsin, 1980* (Ref. 3).

[9]Our Eq. (7) agrees with the rate given in Ref. 4. Reference 1 quotes an erroneous result for Γ proportional to

$$\left[\frac{m_\tau{}^2}{M_W{}^2}\ln\frac{m_\tau{}^2}{M_W{}^2}\right]^2 .$$

Such a logarithmic term is given in the text of Ref. 4, but corrected in the erratum. It is also given in the text of Ref. 5, but a note at the end says that the logarithmic term should be deleted. More recently, E. Ma and A. Pramudita [Phys. Rev. D 24, 1410 (1981)] have given results in which m_α/M_W need not be small. In the limit used by us, their amplitude agrees with ours.

[10]Methods of doing this have been discussed in various papers. See, for example, T. P. Cheng and L. F. Li, Phys. Rev. D 22, 2860 (1980); V. Barger *et al.*, Phys. Rev. Lett. 45, 692 (1980).

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[12]While we were completing this paper, we received a paper by J. Schechter and J. W. F. Valle [Phys. Rev. D 24, 1883 (1981); 25, 283(E) (1982)]. Assuming *CP* invariance, they give the result of our Eq. (14a) for Majorana neutrinos. They also present the form in our Eq. (14b) for the case of *CP* violation. However, it is our contention that it is only the simultaneous presence of these two amplitudes that implies *CP* violation.

[13]See, for example, Cheng and Li (Ref. 10).