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Polarization of recoil electron in elastic electron-deuteron scattering

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In the one-photon-exchange approximation for elastic electron-deuteron scattering, with space-reflection and time-reversal invariance, three form factors are needed to specify the deuteron current. In this paper the longitudinal polarization of the outgoing electron is calculated in terms of these form factors for arbitrary polarization of the incident electron and arbitrary density matrix for the initial deuteron spin state.

I. INTRODUCTION

The *ed* elastic scattering cross section, for either helicity of the incident electron and for arbitrary initial deuteron spin state, was given by Gourdin[1] in terms of the three form factors. The purpose of this paper is to carry the theory on to get the longitudinal polarization of the recoil electron for arbitrary mixed states of the incident particles.

Other calculations of polarization effects that have been made are as follows. Gourdin and Piketty[2] found the cross section for scattering by polarized deuterons and for production of polarized deuterons from unoriented input particles. Especially, they compared the cross section for production of longitudinally polarized deuterons with that for transversely polarized deuterons. Maravcsik and Ghosh[3] discussed the scattering cross section when the initial deuteron has spin component 0 or ± 1 in the direction of the momentum transfer. Several investigators studied the effect of the time-reversal noninvariance on the vector polarization of the recoil deuteron.[4]

The most recent experimental work has been done by Arnold *et al.*[5] and by Martin *et al.*[6] on the total cross sections up to a (momentum transfer)^2 of $q^2=155$ fm^{-2}. The only polarization measurement which has been made was by Prepost, Simonds, and Wiik.[7] They looked for the vector polarization of the recoil deuteron as a test of time-reversal noninvariance with negative results.

One motivation for doing some kind of spin-dependent measurement, and for calculation of these effects, is to get complete information about the scattering process. It is specified by three form factors G_0, G_1, and G_2. However, the cross section for scattering of unaligned particles, with output spins unobserved, serves to determine only ${G_1}^2$ and $({G_0}^2+{G_2}^2)$. Some other measurement must be made if G_0 and G_2 are to be determined separately.

In this paper we explore what information can be obtained by observing the longitudinal polarization of the recoil electron. For high-energy electron scattering in one-photon exchange, the longitudinal polarization of the electron plays a special role.[8] The cross section for scattering of a polarized electron from any object (except another electron) with output spin unobserved depends on the initial electron transverse polarization in a higher order in (mass/energy) than on the initial longitudinal polarization. Also the Møller scattering is more sensitive to the longitudinal polarization. Consequently it is the longitudinal polarization that is usually observed by a high-energy one-photon-exchange scattering and it is this component that we calculate below.

In general the density matrix for the spin of the initial deuteron depends on eight parameters. It is conceptually simpler to consider axially symmetric mixed states. They depend on only four parameters, two of them being the direction of the axis. Experiments in which the direction of the symmetry axis is varied can give decisive information about the form factors. Consequently calculations are reported below for the axially symmetric special case as well as for the general density matrix.

The end result of the work reported here is a general formula for final electron longitudinal polarization, as it depends on the initial electron longitudinal polarization and the initial deuteron spin state, in terms of the three form factors. As one specialization of the general formula, for initial pure states of definite helicity the final states are also pure with the same helicity. As another, if the initial deuteron has no polarization in the scattering plane, but otherwise has arbitrary orientation, the final longitudinal electron polarization is the same as the initial.

II. CALCULATION OF RECOIL-ELECTRON POLARIZATION

A spin-$\frac{1}{2}$ particle is described by its momentum $e^\mu=(e^0,\vec{e})$ and the polarization four-vector s^μ defined to be $(0,\vec{1})$ in its rest frame, where $\vec{1}$ specifies the spin state. (We use the notation of Bjorken and Drell.[9]) The explicit connection between s^μ and $\vec{1}$ is

$$\vec{s}=\vec{1}+\frac{\vec{e}(\vec{e}\cdot\vec{1})}{m_e(e^0+m_e)}\ , \tag{1a}$$

$$s^0=\vec{e}\cdot\vec{1}/m_e\ . \tag{1b}$$

Here $s\cdot e=0$ and $s\cdot s=-\vec{1}\cdot\vec{1}$ ranges from zero for an unpolarized state to -1 for a pure state. At high energy

$$\vec{s}=(\vec{1}\cdot\hat{e})(\vec{e}/m_e)+\vec{s}_T\ , \tag{2a}$$

$$s^0=(\vec{1}\cdot\hat{e})(e^0/m_e)\ , \tag{2b}$$

where $\vec{s}_T$ is $\vec{1}-(\vec{1}\cdot\hat{e})\hat{e}$, the transverse component of $\vec{1}$ and of $\vec{s}$, and $\hat{e}$ is a unit vector in the $\vec{e}$ direction. The first approximation for s^μ is $(\vec{1}\cdot\hat{e})e^\mu/m$, $\vec{s}_T$ comes in in the next order, and the neglected terms are of order m_e/e^0.

Following the established discussion,[10–12] we write the matrix element for *ed* elastic scattering in the one-photon-exchange approximation as

$$M_{fi}=\left[\frac{e^2}{q\cdot q}\right]\bar{u}_f\gamma_\mu u_i\xi_f^{*\alpha}\Omega^\mu_{\alpha\beta}\xi_i^\beta\ , \tag{3}$$

where in the deuteron rest frame ξ^μ is $(0,\vec{\xi})$, and the three components ξ_i describe the spin state in the representation in which the spin matrices are

$$(J_i)_{jk}=-i\epsilon_{ijk}\ . \tag{4}$$

The deuteron current consistent with Hermiticity, time reversal, space reversal, and gauge invariance is

$$\Omega^\mu_{\alpha\beta}=G_A g_{\alpha\beta}D^\mu+G_B(q_\alpha g^\mu{}_\beta-q_\beta g^\mu{}_\alpha) -G_C q_\alpha q_\beta D^\mu/2M^2\ . \tag{5}$$

Here q is d_f-d_i and D is d_f+d_i. These form factors are related to the standard physical type, in the notation reviewed in our previous paper,[12] by

$$\begin{aligned} G_0&=2M[(1+\tfrac{2}{3}\eta)G_A-\tfrac{2}{3}\eta G_B +\tfrac{2}{3}\eta(1+\eta)G_C]\ ,\\ G_1&=2MG_B\ ,\\ G_2&=(\tfrac{4}{3})2^{1/2}M\eta[G_A-G_B+(1+\eta)G_C]\ , \end{aligned} \tag{6}$$

where η is $q^2/4M^2$, q^2 denoting $|q\cdot q|$.

The density matrix for final lepton polarization is then

$$\rho_f=\frac{1}{16}\frac{e^4}{(q\cdot q)^2}\left[1+\frac{\not{e}_f}{m_e}\right]\gamma_\mu\left[1+\frac{\not{e}_i}{m_e}\right](1+\gamma_5\not{s}_i)\gamma_\nu\left[1+\frac{\not{e}_f}{m_e}\right][g^{\alpha\epsilon}-(d_f^\alpha d_f^\epsilon/M^2)]\Omega^\mu_{\alpha\beta}\rho_i^{\beta\zeta}\Omega^\nu_{\epsilon\zeta}\ , \tag{7}$$

where we have used the completeness relation

$$\sum_{m=0,\pm1}\xi^\alpha(m)\xi^{*\beta}(m)=d^\alpha d^\beta/M^2-g^{\alpha\beta}\ . \tag{8}$$

The initial spin-one density matrix as discussed by Mullin *et al.*[13] is

$$\rho=\tfrac{1}{3}+\tfrac{1}{2}\vec{P}\cdot\vec{J}+\tfrac{1}{4}Q_{ij}(J_iJ_j+J_jJ_i-\tfrac{4}{3}\delta_{ij}) \tag{9}$$

which in the representation of Eq. (4) becomes

$$\rho_{ij}=\tfrac{1}{3}\delta_{ij}-\frac{i}{2}P_l\epsilon_{lij}-\tfrac{1}{2}Q_{ij}\ . \tag{10}$$

The covariant deuteron density matrix $\rho^{\mu\nu}$ has components $(\rho^{ij},\rho^{0j},\rho^{i0},\rho^{00})$ defined to be $(\rho_{ij},0,0,0)$ in its rest frame. Explicitly

$$\rho_i^{\mu\nu}=\tfrac{1}{3}(d_i^\mu d_i^\nu/M^2-g^{\mu\nu})-\frac{i}{2M}d_{i\alpha}P_\beta\epsilon^{\alpha\beta\mu\nu}-\tfrac{1}{2}Q^{\mu\nu}\ . \quad (11)$$

Finally, the electron polarization four-vector is found from

$$s_f^\rho=\mathrm{Tr}(\gamma^\rho\gamma_5\rho_f)/\mathrm{Tr}\rho_f\ . \quad (12)$$

At high energy

$$\frac{1}{2}\left[1+\frac{\not{e}_i}{m_e}\right]\tfrac{1}{2}(1+\gamma_5\not{s}_i)=\frac{e_i}{4m_e}(1-\vec{1}_i\cdot\hat{e}_i\gamma_5+\vec{s}_{iT}\cdot\vec{\gamma}\gamma_5)\ . \quad (13a)$$

Also

$$\left[1+\frac{e_f}{m_e}\right]\gamma^\rho\gamma_5\left[1+\frac{\not{e}_f}{m_e}\right]=2\frac{e_f^\rho}{m_e}\gamma_5\left[1+\frac{\not{e}_f}{m_e}\right]+2\gamma^\rho\gamma_5\left[1+\frac{e_f}{m_e}\right], \quad (13b)$$

where the second term is negligible compared to the first. It is straightforward to evaluate the traces. The cross section is proportional to

$$\mathrm{Tr}\rho_f=\frac{1}{2m_e^2}\frac{e^4}{q^4}(\vec{1}_i\cdot\hat{e}_iC+D) \quad (14)$$

and the longitudinal component of electron polarization is

$$\vec{1}_f\cdot\hat{e}_f=\frac{C+\vec{1}_i\cdot\hat{e}_iD}{\vec{1}_i\cdot\hat{e}_iC+D}\ , \quad (15)$$

where C and D are defined by

$$C=i\epsilon_{\mu\rho\nu\sigma}e_i^\rho e_f^\sigma[g^{\alpha\epsilon}-(d_f^\alpha d_f^\epsilon/M^2)]\Omega^\mu_{\alpha\beta}\rho_i^{\beta\zeta}\Omega^\nu_{\epsilon\zeta}\ , \quad (16)$$

$$D=-(e_{i\mu}e_{f\nu}-g_{\mu\nu}e_i\cdot e_f+e_{f\mu}e_{i\nu})\times[g^{\alpha\epsilon}-(d_f^\alpha d_f^\epsilon/M^2)]\Omega^\mu_{\alpha\beta}\rho_i^{\beta\zeta}\Omega^\nu_{\epsilon\zeta}\ . \quad (17)$$

In the laboratory frame C and D are given by

$$C=M^2\eta(\eta^2+\eta)^{1/2}\cot\tfrac{1}{2}\theta\{(4G_0G_1+2^{1/2}G_1G_2)\vec{P}\cdot\hat{f}-2\eta^{1/2}[1+(1+\eta)\tan^2\tfrac{1}{2}\theta]^{1/2}G_1{}^2\vec{P}\cdot\hat{q}\}\ , \quad (18)$$

$$\begin{aligned}D=M^2\eta\cot^2\tfrac{1}{2}\theta(&\tfrac{8}{3}\eta(\eta+1)\tan^2\tfrac{1}{2}\theta\,G_1{}^2+2(G_0{}^2+\tfrac{2}{3}\eta G_1{}^2+G_2{}^2)\\&-Q_{kl}\{\eta G_1{}^2\hat{f}_k\hat{f}_l+3(2)^{1/2}\eta^{1/2}[1+(1+\eta)\tan^2\tfrac{1}{2}\theta]^{1/2}G_1G_2\hat{f}_k\hat{q}_l\\&+[\eta G_1{}^2+\eta(1+\eta)\tan^2\tfrac{1}{2}\theta\,G_1{}^2+\tfrac{3}{2}G_2{}^2+3(2)^{1/2}G_0G_2]\hat{q}_k\hat{q}_l\})\ ,\end{aligned} \quad (19)$$

where

$$\vec{f}=-2\vec{e}_i+\frac{e_i^0/M+1}{\eta+1}\vec{q}\ .$$

A useful specialization to consider is when the mixed state has axial symmetry about some direction, say $\hat{t}$. In this case ρ and $\vec{J}\cdot\hat{t}$ must commute so ρ can be written as

$$\rho_{ij}=\tfrac{1}{3}\delta_{ij}+\tfrac{1}{2}P(\vec{J}\cdot\hat{t})_{ij}+c(\tfrac{1}{3}\delta_{ij}-\hat{t}_i\hat{t}_j)\ , \quad (20)$$

where c is $\pm(\tfrac{3}{8}Q_{ij}Q_{ij})^{1/2}$. From the requirement that the eigenvalues be positive one finds that the ranges of the parameters are

$$-1\le c\le\tfrac{1}{2},\quad |P|\le\tfrac{2}{3}(1+c)\ .$$

The extreme value $c=-1$ occurs only with $P=0$. For it the density matrix is simply

$$\rho_{ij}=\hat{t}_i\hat{t}_j$$

corresponding to a pure $m=0$ state of $\vec{J}\cdot\hat{t}$ aligned about the t axis. The other extreme value $c=\tfrac{1}{2}$ may occur with a range of P values. One possibility is $c=\tfrac{1}{2}$, $P=\pm1$,

$$\rho_{ij}=\tfrac{1}{2}\delta_{ij}\pm\tfrac{1}{2}(\vec{J}\cdot\hat{t})_{ij}-\tfrac{1}{2}\hat{t}_i\hat{t}_j\ ,$$

which describes the pure $m=\pm 1$ state of $\vec{J}\cdot\hat{t}$ polarized in the $\pm\hat{t}$ direction. Although eight parameters are needed to determine P_i and Q_{ij}, only four are needed to determine the axially symmetric states. They are two for the direction of the axis $\pm\hat{t}$, one for the strength of the antisymmetric polarization term P, and one for the strength of the alignment term c.

Consider the case of the axis $\hat{t}$ in the scattering plane. Let $\vec{P}=P\hat{t}$ with P positive. Suppose $\hat{t}$ makes an angle β with $\vec{f}$ on the side toward $\hat{q}$ so that $\hat{t}\cdot\hat{q}=\sin\beta$ and $\hat{t}\cdot\hat{f}=\cos\beta$. Then the formulas for C and D simplify to

$$C=M^2\eta(\eta^2+\eta)^{1/2}\cot\tfrac{1}{2}\theta\, P\{(4G_0G_1+2^{1/2}G_1G_2)\cos\beta-2\eta^{1/2}[1+(1+\eta)\tan^2\tfrac{1}{2}\theta]^{1/2}G_1{}^2\sin\beta\}\ , \tag{21}$$

$$\begin{aligned}D=M^2\eta\cot^2\tfrac{1}{2}\theta(\ &\tfrac{8}{3}\eta(\eta+1)\tan^2\tfrac{1}{2}\theta\, G_1{}^2+2(G_0{}^2+\tfrac{2}{3}\eta G_1{}^2+G_2{}^2)\\ &+[\eta(1+\eta)\tan^2\tfrac{1}{2}\theta\, G_1{}^2+\tfrac{3}{2}G_2{}^2+3(2)^{1/2}G_0G_2]\cos2\beta\\ &-3(2)^{1/2}\eta^{1/2}[1+(1+\eta)\tan^2\tfrac{1}{2}\theta]^{1/2}G_1G_2\sin2\beta\})\ .\end{aligned} \tag{22}$$

III. DISCUSSION OF RECOIL-ELECTRON POLARIZATION

The final result for the longitudinal polarization of the scattered electron is Eq. (15) with the quantities C and D given in terms of the initial deuteron density matrix by Eqs. (18) and (19) and given for an initially axially symmetric deuteron spin state by Eqs. (21) and (22).

It is seen immediately from Eq. (15) that, if the initial electron has definite helicity, say $\vec{1}_i\cdot\hat{e}_i=\pm 1$, then the final electron state has the same helicity, $\vec{1}_f\cdot\hat{e}_f=\pm 1$, regardless of the initial deuteron spin state. Thus nothing can be learned about the form factors by watching the longitudinal polarization of beams with definite helicity. Another uninformative case is when the initial deuteron vector polarization $\vec{P}$ is zero or perpendicular to the scattering plane. Then Eq. (18) implies that $C=0$ and Eq. (15) that the longitudinal polarization is conserved, $\vec{1}_f\cdot\hat{e}_f=\vec{1}_i\cdot\hat{e}_i$.

Excluding the above special arrangements, a measurement of $\vec{1}_f\cdot\hat{e}_f$ for known $\vec{1}_i\cdot\vec{e}_i$ gives C/D from Eq. (15) and thus gives information about the form factors. There are too many possibilities for setting up an experiment to make a full discussion. As an example of what kind of information could in principle be obtained, suppose the initial deuteron state has axial symmetry, $c=0$, and has nonzero $\vec{P}$ in the scattering plane at various angles β. Suppose also that $\vec{1}_f\cdot\hat{e}_f$ is measured as a function of β for a given fixed $\vec{1}_i\cdot\hat{e}_i$. The only dependence on β is in Eq. (21). It is seen that C/D as a function of β oscillates about zero with period 2π and thus that $\vec{1}_f\cdot\hat{e}_f$ as a function of β oscillates about $\vec{1}_i\cdot\hat{e}_i$ with period 2π. At the points where $C/D=0$ and $\vec{1}_f\cdot\hat{e}_f=\vec{1}_i\cdot\hat{e}_i$, β satisfies

$$\tan\beta=\frac{G_1(4G_0+2^{1/2}G_2)}{2\eta^{1/2}[1+(1+\eta)\tan^2\frac{1}{2}\theta]^{1/2}G_1{}^2}\ ,$$

so by observing this value of β one gets this information about the form factors. The measurements of cross section with unpolarized beams give $G_1{}^2$ and $(G_0{}^2+G_2{}^2)$ so this gives new information, about the quantity $(4G_0+2^{1/2}G_2)$.

IV. DISCUSSION OF CROSS SECTION

The scattering cross section, for arbitrary initial spin states and with the final spins unobserved, is

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{\rm NS}\frac{\vec{1}_i\cdot\hat{e}_iC+D}{M^2 2\eta\cot^2\frac{1}{2}\theta}\ , \tag{23a}$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm NS}=\left(\frac{e^2}{8\pi e_i^0}\right)^2\frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta}\times\frac{1}{1+2(e_i^0/M)\sin^2\frac{1}{2}\theta} \tag{23b}$$

(NS denotes "no structure.") Evidently the cross section could also be used to determine the C/D ratio, as long as a polarized incident beam is available. When specialized to definite helicity, $\vec{1}_i \cdot \hat{e}_i = \pm 1$, this result agrees with Gourdin[1] except for some misprints. On his page 71, it should read $G_1{}^2$ instead of G_1 in the equation for S, there should be an additional factor of $\frac{1}{2}$ in the equation for A_3, and the factor multiplying $8\eta(G_0 + \frac{1}{3}\eta G_2)$ should be G_2 instead of G_1 in the equation for $B_{11} + B_{22} - 2B_{33}$.

Scattering cross sections in case the initial deuteron is an eigenstate of $\vec{J} \cdot \hat{t}$, as first discussed by Gourdin and Piketty,[2] are included in the above results. For example for an initial pure $m = 0$ state aligned in the scattering plane one uses Eqs. (23) for the cross section, C and D given by Eqs. (21) and (22) with $P = 0$ and $c = -1$. For an initial pure $m = 1$ state polarized in the scattering plane one uses $P = +1$, $c = \frac{1}{2}$.

Moravcsik and Ghosh[3] considered the scattering cross section for unpolarized electrons incident on deuterons in $m = 1$ and $m = 0$ eigenstates of $\vec{J} \cdot \hat{q}$. They introduced the parameter

$$P_{\mathrm{MG}} = \frac{2^{1/2}}{3} \frac{(d\sigma/d\Omega)_0 - (d\sigma/d\Omega)_1}{\frac{1}{3}(d\sigma/d\Omega)_0 + \frac{2}{3}(d\sigma/d\Omega)_1} . \quad (24)$$

It is straightforward to express this parameter in general in terms of the form factors by using Eq. (22) for D. In this case β is $\pi/2$ and $c = \frac{1}{2}$ for $m = 1$, $c = -1$ for $m = 0$. The result in general is

$$P_{\mathrm{MG}} = \frac{2G_0 G_2 + 2^{-1/2} G_2{}^2 + \frac{1}{3} 2^{-1/2} \eta [1 + 2(1+\eta)\tan^2\frac{1}{2}\theta] G_1{}^2}{G_0{}^2 + G_2{}^2 + \frac{2}{3}\eta [1 + 2(1+\eta)\tan^2\frac{1}{2}\theta] G_1{}^2} \quad (25)$$

which coincides with their result when the terms with the factors of η are negligible.

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