Comment on effective-Lagrangian formulations of the U(1) axial anomaly

R. Arnowitt and Pran Nath
Department of Physics, Northeastern University, Boston, Massachusetts 02115
(Received 19 March 1981)

Recent effective-Lagrangian formulations of the U(1) axial anomaly are examined to determine their consistency and the extent to which the fundamental quantum-chromodynamics theory is replicated. One formulation is shown to possess the desired properties, and some difficulties with two other formulations are pointed out.

I. INTRODUCTION

Effective Lagrangians which obey the principles of current algebra and the PCAC (partial conservation of axial-vector current) condition[1,2] have had a long history of successful applications for low-energy hadronic phenomena. Recent work of Witten[3] has made it possible to relate these previous effective-Lagrangian approaches to the fundamental quantum-chromodynamic (QCD) theory under the assumptions of color confinement and the $1/N$ expansion. However, the earlier effective Lagrangians[1,2] did not include effects of the U(1) axial anomaly, and hence the topological aspects of QCD were not treated in these effective-Lagrangian methods.

As one attempts to extend the effective-Lagrangian formulations so as to encompass the topological aspects of QCD one would like to preserve as much of the structure of the original Lagrangian as possible. In particular, the original theory allows for U(1) symmetry transformation and possesses the charge $\tilde{Q}_9^5$ which accomplishes this, i.e.,

$$\tilde{Q}_9^5 \equiv \int d^3x \tilde{A}_9^0(x) . \tag{1.1}$$

Here $\tilde{A}_9^0$ is the time component of the symmetry current $\tilde{A}_9^\mu$ which is obtained from the gauge-invariant current A_9^μ by subtracting the QCD topological current density K_{QCD}^μ. Thus one has

$$\tilde{A}_9^\mu(x) = A_9^\mu(x) - (\tfrac{2}{3})^{1/2} N_l K_{\mathrm{QCD}}^\mu . \tag{1.2}$$

$$K_{\mathrm{QCD}}^\mu(c) = \frac{g^2}{32\pi^2} \epsilon^{\mu\alpha\beta\gamma} A_\alpha^A (F_{\beta\gamma}^A - \tfrac{1}{3} C^{ABC} A_\beta^B A_\gamma^C) . \tag{1.3}$$

In Eq. (1.2), $N_l (=3)$ is the number of light quarks, $A_\mu^A(x)$, $A=1,\ldots,8$ is the octet of color gluons, and $F_{\mu\nu}^A$ are its field strengths.

In order to reproduce correctly this aspect of the QCD theory in the effective-Lagrangian formulation, one is led in a natural way to introduce a phenomenological four-vector field $K^\mu(x)$ which plays the role of the topological current density analogous to the role that K_{QCD}^μ plays in QCD. Further, as in QCD the topological charge density $Q(x)$ must be given by the divergence of K^μ, i.e., $Q(x) = \partial_\mu K^\mu$.

Recently, three detailed models[4-6] of the U(1) axial anomaly have been proposed and applications of these formulations to a number of phenomena have also been made.[7,8] The formulation of Ref. 5 introduces a four-vector field K^μ and a corresponding kinetic energy $(\partial_\mu K_\nu)^2/2C$ where C is an *a priori* undetermined constant. It can be characterized by the Lagrangian[9]

$$L_1 = \frac{1}{2C} (\partial_\mu K_\nu)^2 + G\partial_\mu K^\mu + L_{\mathrm{CA}} - \theta\partial_\mu K^\mu . \tag{1.4}$$

Here G and L_{CA} are functions of all the other $J^P = 0^\pm$ and $1^\pm$ meson fields and θ is the CP-violating coupling constant of QCD. (The determination of G and L_{CA} by current algebra is discussed in Sec. II below.) The formulation of Ref. 4 also introduces a four-vector field K^μ but with a kinetic energy term which depends only on its divergence:

$$L_2 = \frac{1}{2C} (\partial_\mu K^\mu)^2 + G\partial_\mu K^\mu + L_{\mathrm{CA}} - \theta\partial_\mu K^\mu . \tag{1.5}$$

Both L_1 and L_2 allow the construction of a $\tilde{Q}_9^5$ generator of the U(1) symmetry transformations. However, because Eq. (1.5) of Ref. 4 depends only on $\partial_\mu K^\mu$, the K^μ canonical commutation relations are actually inconsistent with the field equations. Thus the dynamics allows one to eliminate K^μ in terms of the other meson fields, but as we will see, in some calculations one gets the right answer by carrying out calculations *prior* to the elimination while in other situations one may eliminate $\partial_\mu K^\mu$ and perform the calculations. One of the purposes of this paper is to explain how one is to use L_2 so that one does not get incorrect results.

In the third effective-Lagrangian formulation of the U(1) anomaly of Ref. 6, the axial four-vector field K^μ does not appear at all. Rather only a pseudoscalar constraint field $Q(x)$ representing the topological charge density is introduced:

$$L_3 = \frac{1}{2C}[Q(x)]^2 + GQ(x) + L_{CA} - \theta Q(x). \quad (1.6)$$

Because of the absence of the K^μ field, this formulation does not possess the generator of U(1) symmetry transformations, and hence, does not replicate QCD in this regard.

Thus, only the formulation of Ref. 5 can represent fully the topological properties of the original QCD theory, i.e., it both possesses the generator of U(1) symmetry transformations, and is free of inconsistencies.

II. MATRIX ELEMENTS OF TOPOLOGICAL CHARGE

We begin by briefly reviewing the determination of L_{CA} and G of Eq. (1.4) from the current-algebra constraints. L_{CA} turns out to be governed by the nonanomalous sectors of the current-algebra conditions and hence is unchanged from its previous value[1,2] while G is subject to the anomalous PCAC condition.[4-6] The most general determination of L_{CA} is given in Ref. 2 where it was found that

$$\begin{aligned} L_{CA} = & -\tfrac{1}{2} V_a^\mu W_{ab}^{-1} V_{\mu b} - \tfrac{1}{4} H_a^{\mu\nu} H_{\mu\nu a} \\ & - \tfrac{1}{2}[\delta_{ab} + M_c^2 (g^{-1}F)_{ca} (g^{-1}F)_{cb}] \gamma_a^\mu \gamma_{\mu b} \\ & - \tfrac{1}{2}\mu_{ab}\chi_a\chi_b + L_I'(\chi_a, \gamma_a^\mu, H_{\mu\nu a}) \end{aligned} \quad (2.1)$$

and V_a^μ are the 18-plet of U(3) × U(3) currents

$$V_a^\mu(x) = g_{ab} v_b^\mu + F_{ab}\partial^\mu \chi_b. \quad (2.2)$$

In Eqs. (2.1) and (2.2) χ_a and v_a^μ, $a = 1, \ldots, 18$ are the 18-plet of fields for the physical $J^P = 0^\pm$ and $1^\pm$ mesons,[10] M_c are the vector masses and $W_{ab} \equiv g_{ac}g_{bc}/M_c^2 + F_{ac}F_{bc}$. Further, γ_a^μ and $H_{\mu\nu a}$ appearing in Eq. (2.3) have the definitions

$$\gamma_a^\mu = \partial^\mu \chi_a - F_{ca} W^{-1}{}_{cb} V_b^\mu - Z_{1dab} W^{-1}{}_{dc} \chi_b V_c^\mu \quad (2.3)$$

and

$$H_{\mu\nu a} \equiv \partial_\mu v_{\nu a} - \partial_\nu v_{\mu a} + g^{-1}{}_{ad} W^{-1}{}_{ed} C_{ebc} V_{\mu b} V_{\nu c}, \quad (2.4)$$

where C_{abc} are the U(N_l)×U(N_l) structure constants. The coupling constant Z_{1abc} in Eq. (2.3) is determined through the current-algebra commutation relations obeyed by currents of Eq. (2.2) which give

$$Z_{1abc} = (S^{-1}A_a S)_{bc}, \quad (2.5)$$

where $A_{abc} = f_{abc}$ when a = natural parity and $A_{abc} = d_{abc}$ when a, b = unnatural parity and S_{ab} is the wave-function renormalization matrix of Glashow and Weinberg.[11]

The form of L_{CA} stated above [Eqs. (2.1)–(2.5)] is required by the current-algebra commutation relations only[7] and is thus unchanged from the earlier current-algebra evaluation.[2] On the other hand, G of Eq. (1.4) and L_I of Eq. (2.1) are left undetermined by commutation relations and one needs the PCAC condition for their determination:

$$\partial_\mu V_a^\mu = F_{ac}\mu_{cb}\chi_b + (\tfrac{2}{3})^{1/2} N_l \delta_{a9} \partial_\mu K^\mu \quad (2.6)$$

[In Eq. (2.6) μ_{ab} is the chiral $0^\pm$ mass matrix and δ_{a9} is unity when a is in the ninth axial channel and zero otherwise.] Thus it is only here that the anomaly enters the discussion. The function G is assumed to have the general functional dependence $G = G(\chi_a, \gamma_a^\mu, H_{\mu\nu a})$. Then Eq. (2.6) implies that it obeys the equation[5,7]

$$\begin{aligned} F_{ab}\frac{\partial G}{\partial \chi_b} = & -Z_{1abc}\left(\frac{\partial G}{\partial \chi_b}\chi_c + \frac{\partial G}{\partial \gamma_b^\mu}\gamma_c^\mu\right) \\ & - C_{abc} g_{bd} (g^{-1})_{ec} \frac{\partial G}{\partial H_d^{\mu\nu}} H_e^{\mu\nu} - (\tfrac{2}{3})^{1/2} N_l \delta_{a9}. \end{aligned} \quad (2.7)$$

The complexity of Eq. (2.7) arises due to two causes: the inclusion of SU(3) and chiral breaking in Z_{1abc} of Eq. (2.6) and the presence of nonminimal couplings due to the dependence of G on γ_a^μ and $H_{\mu\nu a}$ in Eq. (2.7). The nonminimal couplings are consistent with the $1/N$ expansion and a complete description of the low-energy domain should of course include them. However, if one ignores these couplings (and assumes that G is a function of χ_a only) one may integrate Eq. (2.7) in closed form. Thus defining $\omega_a = (u_a, v_a)$ by

$$\omega_a \equiv (S\chi)_a + G_a, \quad (2.8)$$

where $G_a = \langle 0|\omega_a|0\rangle$ arises from the chiral-breaking condensate,[12] one sees that the u_a and v_a transform as the $0^\pm$ components of a (3, 3*) multiplet and $G(\chi_a)$ obeys

$$A_{abc}\frac{\partial G}{\partial \omega_b}\omega_c = -\delta_{a9}(\tfrac{2}{3})^{1/2} N_l. \quad (2.9)$$

Integration of Eq. (2.9) gives

$$G = \tfrac{1}{2}(\ln\det\xi - \ln\det\xi^\dagger), \quad (2.10)$$

where $\xi \equiv (u_a + iv_a)\lambda_a$ and λ_a are the Gell-Mann matrices.

Equation (2.10), arising when nonminimal couplings in G are ignored, is similar to, but more general than, the σ-model analyses used by the authors of Refs. 4 and 6 since it includes the chiral and SU(3) symmetry-breaking effects involved in relating the (3, 3*) densities to the physical 18-plet of meson fields χ_a. [See Eqs. (2.5) and (2.8).] For example, if one expands Eq. (2.10) in a power series, one finds

$$G = -\left(\tfrac{2}{3}\right)^{1/2} N_I \left[(F^{-1})_{a9}\chi_a - (F^{-1})_{ar} Z_{1rbc}(F^{-1})_{b9}\chi_a \chi_c + \cdots \right]. \tag{2.11}$$

As discussed in Ref. 7, these symmetry-breaking effects are numerically considerable. Thus the σ model may be viewed as a very interesting but unrealistic model since it does not represent the reality of the low-energy domain. Of course, one may extend the analyses of Refs. 4 and 6 to include these symmetry-breaking and nonminimal coupling effects. We will assume in the following that this has been done [i.e., that the functions G and L_{CA} of Eqs. (1.5) and (1.6) are the same as the ones determined above for Eq. (1.4)] so that the three models, Eqs. (1.4), (1.5), and (1.6), can be directly compared.

We first compare the matrix elements of the topological charge between physical meson states, and show under the above assumption it is the *same* for the three models. First, in the formalism of Eq. (1.4), K^μ obeys the equation

$$-\frac{\Box^2}{C} K_\mu(x) = \partial_\mu G(x) \tag{2.12}$$

and so the matrix elements of K_μ between physical meson states give

$$\langle a| K_\mu | b\rangle = i\frac{q_\mu}{q^2} C\langle a| G(\chi_a)| b\rangle , \tag{2.13}$$

where $q_\mu \equiv (q_a - q_b)_\mu$. Equation (2.13) exhibits explicitly the ghost pole of Kogut and Susskind.[13] The divergence of Eq. (2.13) gives

$$\langle a| \partial_\mu K^\mu | b\rangle = - C\langle a| G(\chi_a)| b\rangle . \tag{2.14}$$

Thus C represents the strength of the coupling of the topological charge density to the mesic fields of $G(\chi_a)$ in all mesic matrix elements.

In the formulation of Eq. (1.5) one may similarly calculate the matrix element of $\partial_\mu K^\mu$. Here K^μ satisfies the equation of motion

$$\partial_\nu \left[\frac{1}{C}\partial_\mu K^\mu + G(\chi_a)\right] = 0 \tag{2.15}$$

and integration gives

$$\frac{1}{C}\partial_\mu K^\mu + G(\chi_a) = C_1 , \tag{2.16}$$

where C_1 is a constant.[14] If one chooses $C_1 = 0$ (as in the analysis of Ref. 4), one has precisely Eq. (2.14). Finally, in the formulation of Eq. (1.6) one has on varying the equation

$$Q(x) = - C\, G(\chi_a) \tag{2.17}$$

which again yields Eq. (2.14) with $\partial_\mu K^\mu$ replaced by Q.

Thus all three effective-Lagrangian formulations give the same matrix elements for the topological charge density $Q(x) = \partial_\mu K^\mu$ between physical states. Next we discuss questions of quantization and internal consistency of the three formulations.

III. QUANTIZATION OF TOPOLOGICAL VARIABLES

For the Lagrangian of Eq. (1.4), the quantization of the axial four-vector field K^μ is quite straightforward. The canonical momenta Π_μ corresponding to the four-vector K^μ is

$$\Pi_\mu = \frac{1}{C}\partial^0 K_\mu + G(\chi)\delta^0_\mu , \tag{3.1}$$

which implies the equal-time relations

$$[K^\mu, \partial_0 K_\nu] = iC\,\delta^\mu_\nu \delta^3 . \tag{3.2}$$

Equation (3.2) is consistent with the field equations and thus the theory of Eq. (1.4) is internally consistent.

Next in the theory of Eq. (1.5), the canonical momenta conjugate to K^μ are

$$\Pi_\mu = \frac{1}{C}\delta^0_\mu \partial_\nu K^\nu + G(\chi)\delta^0_\mu . \tag{3.3}$$

Here there is only one canonical variable Π_0 which is conjugate to K^0 while the remaining components Π_i all vanish, i.e., $\Pi_i \equiv 0$. Thus the Lagrangian of Eq. (1.5) contains no variables that determine the dynamics of K^i, and one has only the canonical commutation relation for Π_0 and K^0:

$$[K^0, \Pi_0] = i\,\delta^3 . \tag{3.4}$$

Then the essential problem for the Lagrangian of Eq. (1.5) arises when one combines Eq. (3.4) with the equations of motion Eq. (2.16). Choosing $C_1 = 0$ (as in the analysis of Ref. 4), one may write Eq. (2.16) as

$$\Pi_0 = 0 \quad (C_1 = 0) \tag{3.5}$$

and so the equations of motion are inconsistent with the canonical commutation relation of Eq. (3.4).

Finally, in the effective-Lagrangian formulation Eq. (1.6), Q is a constraint variable, and its commutation relations with the other fields are determined by solving for Q according to Eq. (2.17). Thus, here, since the field K^μ is dispensed with altogether, the quantization of the theory is safe, and consistent with equations of motion.

IV. THE U(1) GENERATORS AND *CP*-VIOLATING INTERACTION

In the fundamental QCD Lagrangian it is found useful to perform a U(1) rotation to transform the CP-violating interaction from the gluon sector to the quark sector.[15] The generator which allows one to effect this transformation is the generator of U(1) symmetry transformation defined in Eq. (1.1). This generator requires in its definition the existence of an explicit K^0 which is defined in the Lagrangian of Eq. (1.4) as well as of Eq. (1.5). For the Lagrangian of Eq. (1.4) one can show[7] using the canonical commutation relations that $[\tilde{Q}_9^5, \partial_\mu K_\nu]=0$.

The generator of the U(1) symmetry transformation in the formulation of Eq. (1.5) also yields

$$[\tilde{Q}_9^5, \partial_\mu K^\mu]=0 \tag{4.1}$$

upon using the canonical commutation relation Eq. (3.4). However, there is once again a consistency problem here. Thus if one uses the field equations of Eq. (2.16) one has[16]

$$[\tilde{Q}_9^5, \partial_\mu K^\mu]=-i2N_f C \neq 0 . \tag{4.2}$$

Thus two ways of computation lead to conflicting results. This pitfall may, of course, be avoided by simply arranging the calculations based on Eq. (1.5) so that the results identical to that of Eq. (1.4) are reproduced. In the third formulation, i.e., of Eq. (1.6), only the topological charge density enters the Lagrangian and an explicit representation of the symmetry charge $\tilde{Q}_9^5$ does not exist there. Thus one cannot carry out the U(1) symmetry transformations in this theory. However, the analog of the Baluni transformations consists here of actually eliminating the constraint variable from the effective Lagrangian.

Finally as an illustration of the essential correctness of ideas of representing the topological aspects of QCD by effective-field methods, we shall apply the effective-field method to the fundamental Lagrangian itself to show that well-known results may consistently be derived. We shall represent the topological aspects of QCD by effective-field methods of Eq. (1.4) and thus write the modified fundamental Lagrangian in the form

$$L=\frac{1}{2C}(\partial_\mu K_\nu)^2+G(q,\bar{q})\partial_\mu K^\mu-\theta\partial_\mu K^\mu \\ -\bar{q}\frac{1}{i}\gamma^\mu D_\mu q-\frac{1}{4}F_{\mu\nu a}F^{\mu\nu a}-\bar{q}Mq , \tag{4.3}$$

where M is the quark mass matrix with diag $M=(m_u,m_d,m_s)$. Equation (4.3) obeys the PCAC condition

$$\partial_\mu A_a^{\mu}=\bar{q}i\gamma^5\left\{\frac{\lambda_a}{2},M\right\}q+\delta_{a9}(\tfrac{2}{3})^{1/2}N_f\partial_\mu K^\mu . \tag{4.4}$$

In an analysis parallel to the one that gives Eq. (2.10) one has

$$G(q,\bar{q})=\frac{i}{2}(\mathrm{Tr}\ln\bar{q}_L q_R-\mathrm{Tr}\ln\bar{q}_R q_L) , \tag{4.5}$$

where $q_L=(1-\gamma_5)q/2$ and $q_R=(1+\gamma_5)q/2$. Next we wish to make a U(1) transformation that eliminates the θ dependence from the K^μ sector and introduces it in the quark sector. Following the method of Coleman and Crewther,[17] it is found convenient to introduce a set of CP-violating phases in the vacuum expectation value of $\bar{q}_{Li}q_{Ri}$ $(i,j=1,2,3)$ so that $\langle 0|\bar{q}_{Li}q_{Rj}|0\rangle=\delta_{ij}\exp(-i\beta_i)$. The vacuum expectation value of the interaction Hamiltonian in the presence of the phases is then

$$\langle H\rangle_{\beta_i}=2\sum_i m_i\cos\beta_i+\Big(\theta-\sum_i\beta_i\Big)\partial_\mu K^\mu . \tag{4.6}$$

One now minimizes the vacuum energy with respect to β_i which gives $m_i\sin\beta_i=m_j\sin\beta_j=m_K\sin\beta_K\equiv\lambda_\theta$ and chooses $\sum\beta_i=\theta$ to eliminate the CP-violating interaction out of the K^μ sector. For β_i small one determines

$$\lambda_\theta=\left(\sum_{i=1}^{3}m_i^{-1}\right)\theta . \tag{4.7}$$

Next, one makes a transformation $q_{L(R)i}\to q_{L(R)i}\times\exp[+(-)i\beta_i/2]$ to eliminate the CP-violating phases from the vacuum expectation value of the quark fields and finds a CP-violating interaction in the quark sector:

$$\delta L_{\mathrm{CP}}=i\lambda_\theta\sum_i\bar{q}_i\gamma_5 q_i . \tag{4.8}$$

Equation (4.8) is identical to the result of Baluni[15] and so one finds that the effective-field treatment of the topological aspects of QCD parallel correctly the treatment of the original theory.

V. CONCLUSION

We have compared three different effective-Lagrangian treatments of the U(1) axial anomaly. In many respects the three approaches give identical results. Thus the matrix elements of the topological charge density are the same in all three formulations if the same functions of the meson fields are used for G and L_{CA} of Eqs. (1.4), (1.5) and (1.6). The apparent complexity of Ref. 5 is due to the fact that there G and L_{CA} include effects due to chiral and SU(3) breaking as well as couplings to higher spin mesons. Since these effects can be shown to be large,[7] only Ref. 5 represents a realistic model in the low-energy domain.

In addition, one encounters some formal difficulties in the formulation of Ref. 4. In this formulation, the canonical commutation relations involving K^0 and its conjugate momentum Π_0 are seen to be inconsistent with the field equation which determines $\partial_\mu K^\mu$. In order to get the correct results from this formulation, one must sometimes do the calculation first and eliminate $\partial_\mu K^\mu$ only at the end. At other times (e.g., in computation of meson matrix elements of $\partial_\mu K^\mu$ in Sec. II) initial elimination of $\partial_\mu K^\mu$ leads to correct results.

While the formalism of Ref. 6 is internally consistent, it does not produce a full realization of the symmetry structure of the original QCD theory. Specifically, one does not have a field realization of the generator of the U(1) symmetry transformation here. In a sense the formalism of Ref. 6 is complementary to that of Ref. 4 in that it overcomes the inconsistency of canonical commutation relations with field equations by never introducing the K^μ field, but because of that it also cannot define the U(1) generator. It is only in the formulation of Ref. 5 that the inadequancies of both formulations, i.e., of Refs. 4 and 6, are overcome.

Note added in proof. In the following paper, J. Schechter has reformulated the model of Rosenzweig *et al.*[4] by making use of Dirac's theory of constraints.[18] We agree that this formulation eliminates the inconsistencies of Ref. 4, and is indeed equivalent to the prescriptions proposed in our Comment above. We believe, however, that the statement made in the following paper that an inconsistency can arise when the Kogut-Susskind ghost field K_μ is included in the U(1) effective Lagrangian[5,19] is incorrect. This is because the anomalous PCAC condition implies that only $\partial_\mu K^\mu$ couples to the physical mesic fields $\chi_a(x)$. Thus if one makes the "gauge" transformation

$$K_\mu = \tilde{K}_\mu + \partial_\mu \Lambda\,, \tag{N1}$$

where $\Box^2\Lambda = -C\,G(\chi_a)$, one finds that Eq. (1.4) above reduces to (neglecting θ effects)

$$L_1 = \frac{1}{2C}\,(\partial_\mu \tilde{K}_\nu)^2 - \tfrac{1}{2}C\,[G(\chi_a)]^2 \tag{N2}$$

and the Kogut-Susskind ghost field can be decoupled completely from the mesic interactions. Hence no inconsistency is possible due to the ghost nature of the K_μ field (as happens also in the fundamental QCD theory). Note that the mesic interactions of Eq. (N2) are identical to those of Ref. 4.

The advantage of treating the K_μ field in the fashion of Ref. 5 is that it shows quite directly how the Kogut-Susskind mechanism[13] operates to eliminate the $\eta\to 3\pi$ and other U(1) paradoxes. [This is the content of Eqs. (2.13) and (2.14) above.] Thus it adds additional insight into the possible workings of the fundamental QCD.

ACKNOWLEDGMENT

Research for this paper was supported in part by the National Science Foundation under Grant No. PHY80-08333.

---

[1]J. Wess and B. Zumino, Phys. Rev. 163, 1727 (1967); R. Arnowitt, M. H. Friedman, and P. Nath, Phys. Rev. Lett. 19, 1085 (1967); C. G. Callan, S. Coleman, and B. Zumino, Phys. Rev. 177, 2239 (1968); 177, 2247 (1968).

[2]R. Arnowitt, M. H. Friedman, P. Nath, and R. Suitor, Phys. Rev. D 3, 594 (1971).

[3]E. Witten, Nucl. Phys. B160, 57 (1979).

[4]C. Rosenzweig, J. Schechter, and G. Trahern, Phys. Rev. D 21, 3388 (1980).

[5]P. Nath and R. Arnowitt, Phys. Rev. D 23, 473 (1981).

[6]P. DiVecchia and G. Veneziano, Nucl. Phys. B171, 253 (1980).

[7]R. Arnowitt and P. Nath, Report No. NUB 2468, 1980 (unpublished); P. Nath and R. Arnowitt, Report No. NUB 2469, 1980 (unpublished).

[8]P. DiVecchia, F. Nicodemi, R. Pettorino, and G. Veneziano, Nucl. Phys. B181, 318 (1981); E. Witten, Ann. Phys. (N. Y.) 128, 363 (1980); K. Kawarbayashi and N. Ohta, Nucl. Phys. B175, 477 (1980).

[9]One is free to add to the L_1 of (1.4) the term $(1/2D)(\partial_\mu K^\mu)^2$ (provided $D/C \neq -1$ so that the K^μ kinetic energy matrix has an inverse) without changing any of its physical consequences. This freedom corresponds to the freedom of gauge fixing of the K^μ field.

[10]We decompose the 18-plet of spin-zero fields χ_a as $\chi_a \equiv (\phi_a, \sigma_a)$ where ϕ_a, σ_a, $a = 1, \ldots, 9$ are the nonet of pseudoscalar and scalar fields. We shall also denote the set of axial-vector currents by $A^\mu_a(x)$, $a = 1, \ldots, 9$.

[11]S. L. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968). We use the notation $S_{ab} = (\sqrt{Z})_{ab}$.

[12]One may show from the current-algebra constraints that $A_{abc}G_c = F_{ab}S_{bc}$ (Ref. 2). The only nonvanshing elements of G_a are the natural-parity channel component G_8 and G_9. These may be parametrized in terms of a mixing angle ψ according to $G_8 = \sqrt{\frac{2}{3}}\cos\psi + \sqrt{\frac{1}{3}}\sin\psi$, $G_9 = -\sqrt{\frac{1}{3}}\cos\psi + \sqrt{\frac{2}{3}}\sin\psi$, and the angle ψ can be explicitly evaluated by current algebra (Ref. 7).

[13]J. Kogut and L. Susskind, Phys. Rev. D 11, 3594 (1975).

[14]C_1 is an example of a hidden constant that is absent in the action but arises as a constant of integration in the equations of motion. An analysis of theories that possess hidden constants is given in A. Aurilia,

H. Nicolai, and P. K. Townsend, Nucl. Phys. B176, 509 (1980).

[15] V. Baluni, Phys. Rev. D 19, 1227 (1979).

[16] Equation (4.2) follows on using the relation $[\tilde{Q}_9^5, G(\chi_a)] = -i(\frac{2}{3})^{1/2} N_l$.

[17] See, e.g., R. J. Crewther, in *Field Theoretical Methods in Particle Physics*, proceedings of the NATO Advanced Study Institute on Field Theoretical Methods in Particle Physics, Kaiserslautern, 1979, edited by W. Ruhl (Plenum, New York, 1980), p. 529.

[18] P. Dirac, *Lectures on Quantum Mechanics* (Academic, New York, 1965).

[19] See also Eq. (1.4).